MULTIGRID CONVERGENCE ACCELERATION FOR TURBULENT FLOW WITH A NON-ORTHOGONAL FV ALGORITHM

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1. INTRODUCTION

The majority of practically relevant flows are three-dimensional, turbulent, confined within complex domains, highly convective and characterised by steep property variations. In such circumstances, adequate numerical accuracy hinges on the use of fine grids and high-resolution numerical approximations. Both tend to diminish iterative stability and solution economy, however. As regards the latter, the number of iterations required to achieve convergence with traditional relaxation methods, such as ADI, SOR, and SIP, is typically proportional to N^2-N^3, where N is the number of nodes. The more complex the turbulence model, the larger the exponent tends to be, and further deterioration arises with increasing geometric complexity, grid skewness, aspect ratio and accuracy of the convection scheme. There is, therefore, a pressing need to develop and apply solution strategies which diminish the above exponent. In ideal circumstances, the exponent would be close to 1.

The multigrid method [MGM, Brandt (1977)] is one solution practice which has proven itself to be a highly efficient for linear systems, returning convergence in computational efforts proportional to Nxlog(N). It is now widely applied in CFD, but rarely in conjunction with complex turbulence models incorporated into iterative pressure-based schemes for incompressible recirculating flows. In such a framework, inter-variate coupling, via the pressure field, plays an important role and must be carefully accounted for within the MGM implementation [Barcus et al (1988)]. When turbulence-transport equations are solved, additional measures must be introduced to secure realizability during prolongation and restriction which are key operations within the MGM. This summary deals with the latter aspect and illustrates the performance of the MGM implemented into a non-orthogonal, collocated FV method incorporating k-ε models and a second-moment closure.

2. APPROACH

A step-by-step description of the conversion of a single-grid algorithm to its MG counterpart, including the derivation of coarse-grid equations, restriction and prolongation operations, MG cycles, data structure and implementation of boundary conditions, can be found in Lien (1992). A more restricted account is given in Lien and Leschziner (1991). Two particular issues pertaining to the treatment of turbulence-transport equations are considered herein. Both are concerned with the treatment of sources.

With ~ denoting restricted (fine-to-coarse-grid-interpolated) values and ^ identifying coarse-grid quantities, evolving as the equations are solved on that grid, the following algorithmic practices are introduced:

1. Ignore changes in the *physical* sources on the coarse grid as coarse-grid solution progresses.

$$\hat{S}_\phi - \tilde{S}_\phi = 0 \qquad for \quad \phi = k, \epsilon, \overline{u_i u_j} \tag{1}$$

2. Arrange the *apparent* source in the coarse-grid equation as follows:

$$\hat{A}_P\hat{\phi}_P - \sum_m \hat{A}_m\hat{\phi}_m = \underbrace{\tilde{A}_P\tilde{\phi}_P - \sum_m \tilde{A}_m\tilde{\phi}_m - \tilde{R}_\phi}_{S_\phi^R} = S_\phi^{R+} + S_\phi^{R-} \tag{2}$$

where S_ϕ^{R+} and S_ϕ^{R-} are, respectively, unconditionally positive and negative fragments. The negative fragment is used to enhance diagonal dominance through:

$$\hat{A}_P \longleftarrow \hat{A}_P - S_\phi^{R-}/\phi_P \tag{3}$$

3. Having obtained the coarse grid correction $[\delta\phi]^c$, perform a positive-definite prolongation by means of:

$$[\,\delta\phi\,]^c = [\,\delta\phi^l\,]^c = [\,\delta\phi^l\,]^+ + [\,\delta\phi^l\,]^-$$

$$(\phi)^l_{new} = \frac{\phi^l_{old} + \delta\phi^{l+}}{\phi^l_{old} - \delta\phi^{l-}}\ \phi^l_{old} \qquad l = 1,2,3,4 \tag{4}$$

where $[\delta\phi]^-$ and $[\delta\phi]^+$ are unconditionally negative and positive fragments, respectively.

3. APPLICATION

Speed-up ratios, expressed in terms of work units (WU) and CPU times, for a 3D laminar flow in a 3D cavity whose side wall are tilted by 27° are given in Table 2. Calculations were performed with two convection schemes and three cycle types ('FMG' denoting "full multigrid scheme" and 'R' denoting a residual-controlled cycle). The main application is a turbulent flow through a sinusoidal diffuser at $Re_D=10^5$, computed with the QUICK scheme, the k-ε model and wall laws. The geometry and the ensuing streamfunction field are shown in Fig. 1, while Table 2 gives speed-up ratios obtained with a fixed V-cycle for five grids, all having the same lateral densities in order to maintain an invariant distribution of y^+-values along the near-wall grid line. Attention is drawn to the constancy of WU in Table 2, implying that the MGM performs in accordance with established behaviour in much simpler laminar conditions.

	V-cycle				FMG-V		FMG-R	
	HYBRID		QUICK		HYBRID		HYBRID	
GRID	WU sg/mg	CPU sg/mg	WU sg/mg	CPU sg/mg	WU sg/mg	CPU sg/mg	WU sg/mg	CPU sg/mg
24^3	3.3	3.7	3.0	3.7	3.3	4.4	4.5	6.0
32^3	5.7	6.3	5.0	5.9	5.7	7.5	7.6	9.9
40^3	9.1	9.1	6.7	7.5	10.3	11.7	12.6	15.0

Table 1: MG performance for laminar flow in 3D skewed cavity

GRID	WU	WU sg/mg	CPU sg/mg
240 × 40	125.69	25.53	14.29
200 × 40	107.64	20.68	11.98
160 × 40	91.23	17.63	9.80
120 × 40	101.80	9.65	5.41
80 × 40	122.40	3.71	2.09

Table 2: MG performance for turbulent flow through plane diffuser

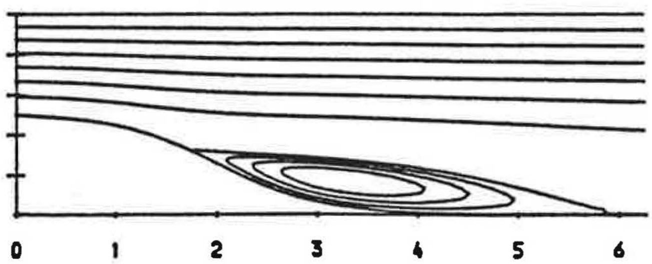

Fig. 1: Streamfunction pattern for turbulent flow through plane diffuser

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