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DIFFERENTIAL AND INTEGRAL METHOD FOR COMPUTING INTERZONAL AIRFLOWS USING MULTIPLE TRACER GASES

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ABSTRACT

The multiple tracer gas method is often used to predict interzonal airflows in buildings. The mass balances of tracer gases are expressed by a set of differential equations. These differential equations generally form the basis for calculating the airflows through the openings between zones. Two methods have been developed to solve the differential equations: the differential and integral methods.

In this paper, the two methods were evaluated by applying them to a laboratory case study with three inter-connected rooms where the interzonal airflows were controlled and measured. The tracer gases were CH_4 N_2O and SF_6. The results are discussed

KEYWORDS :Multiple tracer gas, Interzonal airflow. Decay mode

INTRODUCTION

As concern for indoor air quality has grown, so too has the need to measure interzonal airflows to assess the distribution of outdoor air in buildings. In many cases, these airflows can be evaluated by a multiple tracer gas method (e.g.. Aratani et al 1972: Sinden 1978; I'Anson et al 1982; Perera 1983). It involves the injection of a different tracer gas into each of several inter-connected spaces and the measurement of the tracer gas concentrations in each space or zone as a function of the elapsed time. Based on the simultaneously measured gas concentrations. the interzonal airflows can be calculated from the mass conservation equations for each tracer gas, and the mass flow balance equations for the air. There are two methods for solving the interzonal airflows. One is called the differential method (Enai and Shaw 1990). As the calculated airflows can vary with the set of tracer gas concentrations used. the accuracy of this method depends on selecting an appropriate set of concentrations from the measurements for calculating the airflow. The other is called the integral method (Sasaki and Aratani 1989). This method has the advantage that once the steady state condition is reached. the predicted airflows become essentially constant. even if the concentrations measured during the initial mixing period are included in the calculation. In this study, the two methods were tested in full-size rooms in a laboratory. The main objective was to compare the characteristics and feasibilities of the two methods for a case with three inter-connected rooms.

GOVERNING EQUATIONS AND PROCEDURES FOR COMPUTING

Figure 1 shows the case of three inter-connected zones. If three tracer gases denoted by A, B and C are injected into the zones, one for each zone, the rates of change in tracer gas concentrations in the three zones can be described by the following equations. assuming that the gas concentrations outside the zones are negligible:

$$V_1(dC_{A1}/dt)=-(F_{10}+F_{12}+F_{13})C_{A1}+F_{21}{}^{\bullet}C_{A2}+F_{31}{}^{\bullet}C_{A3}+Q_{A1} \quad (1)$$

$$V_1(dC_{B1}/dt)=-(F_{10}+F_{12}+F_{13})C_{B1}+F_{21}{*}C_{B1}+F_{31}{*}C_{B3} \quad (2)$$
$$V_1(dC_{C1}/dt)=-(F_{10}+F_{12}+F_{13})C_{C1}+F_{21}{*}C_{C1}+F_{31}{*}C_{C3} \quad (3)$$
$$V_2(dC_{A2}/dt)=F_{12}{*}C_{A1}-(F_{20}+F_{21}+F_{23})C_{A2}+F_{32}{*}C_{A3} \quad (4)$$
$$V_2(dC_{B2}/dt)=F_{12}{*}C_{B1}-(F_{20}+F_{21}+F_{23})C_{B2}+F_{32}{*}C_{B3}+Q_{B2} \quad (5)$$
$$V_2(dC_{C2}/dt)=F_{12}{*}C_{C1}-(F_{20}+F_{21}+F_{23})C_{C2}+F_{32}{*}C_{C3} \quad (6)$$
$$V_3(dC_{A3}/dt)=F_{13}{*}C_{A1}+F_{23}{*}C_{A2}-(F_{30}+F_{31}+F_{32})C_{A3} \quad (7)$$
$$V_3(dC_{B3}/dt)=F_{13}{*}C_{B1}+F_{23}{*}C_{B2}-(F_{30}+F_{31}+F_{32})C_{B3} \quad (8)$$
$$V_3(dC_{C3}/dt)=F_{13}{*}C_{C1}+F_{23}{*}C_{C2}-(F_{30}+F_{31}+F_{32})C_{C3}+Q_{C3} \quad (9)$$

The air flow balance equations for the three zones are:

$$-(F_{10}+F_{12}+F_{13})+F_{01}+F_{21}+F_{31}=0 \quad (10)$$
$$-(F_{20}+F_{21}+F_{23})+F_{02}+F_{12}+F_{32}=0 \quad (11)$$
$$-(F_{30}+F_{31}+F_{32})+F_{03}+F_{23}+F_{13}=0 \quad (12)$$

where

V = room volume m³ , F_{ij} = airflow rates from Zone i to Zone j m³/h
C = tracer gas concentration m³/m³ , Q = tracer gas release rates m³/h
t = elapsed time h

Using the differential method, the interzonal airflows can be calculated directly from the above equations using the tracer gas concentrations measured between 30 and 70 minutes after injection (Enai et al 1990). In the case of the integral method, Equations (1) to (9) have to be integrated for an adequate time interval starting from the time of injection. The concentration profiles of each tracer gas are defined by three equations, one for each zone. Of the three equations, only one has a source term. Only one general solution to the three equation set for a single tracer gas need be found, since the same solution will apply to the other two tracer gases. By dropping the subscript, Equations (1), (4) and (7) become the set of three general equations:

$$V_1(dtC_1/dt)=-(F_{10}+F_{12}+F_{13})C_1+F_{21}{*}C_2+F_{31}{*}C_3{*}Q_1 \quad (13)$$
$$V_2(dC_2/dt)=F_{12}{*}C_1-(F_{20}+F_{21}+F_{23})C_2+F_{32}{*}C_3 \quad (14)$$
$$V_3(dC_3/dt)=F_{13}{*}C_1+F_{23}{*}C_2-(F_{30}+F_{31}+F_{32})C_3 \quad (15)$$

Figure 2 shows the time interval for the calculations used by the two methods. dt is the time interval for the differential method and T is one for the integral method.

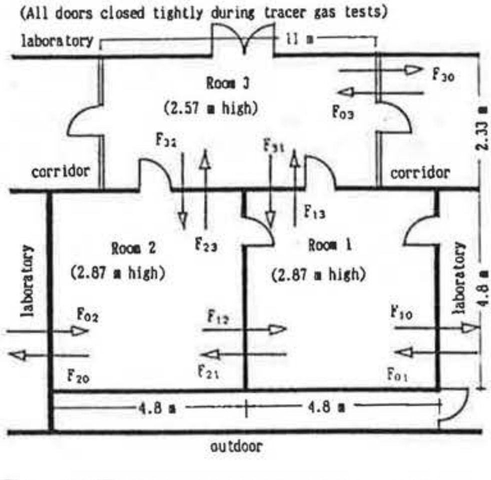

Figure 1 Test Rooms in the Building Performance Section of Institute for Research in Construction, NRC, Canada

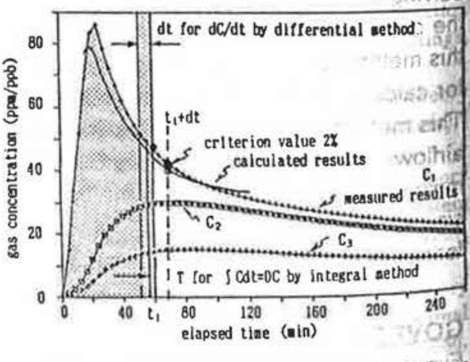

Figure 2 Time Interval for Calculations

$$\Sigma(C_1{}^{*}dt)_k=DC_1 \quad \Sigma[(dC_1/dt)^{*}dt]_k=C_1(T)-C_1(0) \quad \Sigma(Q_1{}^{*}dt)_k=DQ_1 \tag{16}$$

$$\Sigma(C_2{}^{*}dt)_k=DC_2 \quad \Sigma[(dC_2/dt)^{*}dt]_k=C_2(T)-C_2(0) \tag{17}$$

$$\Sigma(C_3{}^{*}dt)_k=DC_3 \quad \Sigma[(dC_3/dt)^{*}dt]_k=C_3(T)-C_3(0) \tag{18}$$

$$V_1\{C_1(T)-C_1(0)\}=-(F_{10}+F_{12}+F_{13})^{*}DC_1+F_{21}{}^{*}DC_2+F_{31}{}^{*}DC_3+DQ_1 \tag{19}$$

$$V_2\{C_2(T)-C_2(0)\}=F_{12}{}^{*}DC_1-(F_{20}+F_{21}+F_{23})^{*}DC_2+F_{32}{}^{*}DC_3 \tag{20}$$

$$V_3\{C_3(T)-C_3(0)\}=F_{13}{}^{*}DC_1+F_{23}{}^{*}DC_2-(F_{30}+F_{31}+F_{32})^{*}DC_3 \tag{21}$$

where T=ndt and k=1, 2, 3 .... n.

We can get three sets of the mass conservation equations like the above integrated equations. After changing the form of the equations like Equations (16), (17) and (18), the interzonal airflows are calculated simultaneously from the integrated gas concentrations.

From Equations (13), (14) and (15), the 3rd derivative of the three tracer gas concentration profiles $C_i(t)$ is,

$$(d^3Ci/dt^3)+K_a{}^{*}(d^2C_i/dt^2)+K_b{}^{*}(dC_i/dt)+K_c{}^{*}C_i+K_{di}=0 \tag{22}$$

The solution to $C_i(t)$ obtained by the Laplace Transformation (Enai et al 1990) is,

$$C_i(t)=X_i{}^{*}\exp(-a^{*}t)+Y_i{}^{*}\exp(-b^{*}t)+Z_i{}^{*}\exp(-c^{*}t)+W_i \tag{23}$$

where i=1, 2 and 3.

The tracer gas concentrations at (t_1+dt) can be obtained from Equation (23) on the voluntary initial condition of airflows calculated at (t_1) as shown in Figure 2. The initial conditions of airflows can be reset successively at the new sampling time of the gas concentrations. When the gas concentrations calculated at (t_1+dt) and the gas concentrations measured at (t_1+dt) agree with a criterion (e.g., 2% was used in this study), the interzonal airflows at (t_1) used for calculating the gas concentrations at the time (t_1+dt) become the appropriate airflow rates in the case of the differential method. Otherwise, we have to repeat the above procedure to compare the calculated gas concentrations with the measured gas concentrations at the next dt.

TEST METHODS

The test zones, as shown Figure 1, were three inter-connected rooms in a laboratory-office building. The walls, doors and ceilings of the rooms were sealed to minimize air leakage. Each connecting doorway was sealed with plywood panels through which two airflow systems were installed (one for each flow direction). These airflow systems consisted of a fan, an airflow measuring device, and an airflow controller. Each individual airflow was controlled at a constant rate. Except those through the airflow systems, the air leakage rates between the test rooms and their surroundings were not measured. The tracer gas injection tube was located at the center of each room. These rooms were each divided into eight volumetrically equal regions with a sampling tube installed at the center of each region. Based on the results, the sampling tubes in each room were connected to a manifold to produce an "average" sample. Each test began by adjusting the airflows in the six systems to rates between 0.2 and 1 air change per hour (ac/h) as shown in Table 1. Then, CH_4, SF_6 and N_2O were simultaneously introduced into Rooms 1, 2 and 3, respectively. Immediately after injection, the concentrations of the three gases at the manifolds in each room were measured at 4 minute intervals for a period of two to four hours.

TEST RESULTS

Fourteen multiple tracer gas tests were conducted (Enai et al 1990); In this paper, six tests which are listed in Table 1 are discussed. Figure 3 shows a typical set of gas concentration profiles measured in the test rooms. Each set consists of twelve profiles, three for the surrounding area and the others for the three rooms. In the case of the differential method, F_{12}, F_{21}, F_{13}, F_{31}, F_{23} and F_{32} were first calculated using the concentrations of CH_4, N_2O and SF_6 measured at 30 minutes after the gas injection. The calculation was repeated several times with a set of concentrations measured at 4 minutes after the previous set. Figure 4 shows a typical example of the calculated interzonal airflows. Similar calculations were carried out for all tests. They were decided by comparing the measured gas concentrations with the values obtained from Equation (23) as shown in Table 1. In the case of the integral method, it is not necessary to use such a trial and error approach to obtain the solutions.

Table 2 shows the integrated gas concentrations during the measurement period and the differences of gas concentrations between the start of measurement and the finish. Such results were used to calculate the interzonal airflows from the simultaneous equations of the integral method. Table 1 shows the calculated interzonal airflows and run times in this study. Also shown in Table 1, are the results calculated using the differential method. The measured

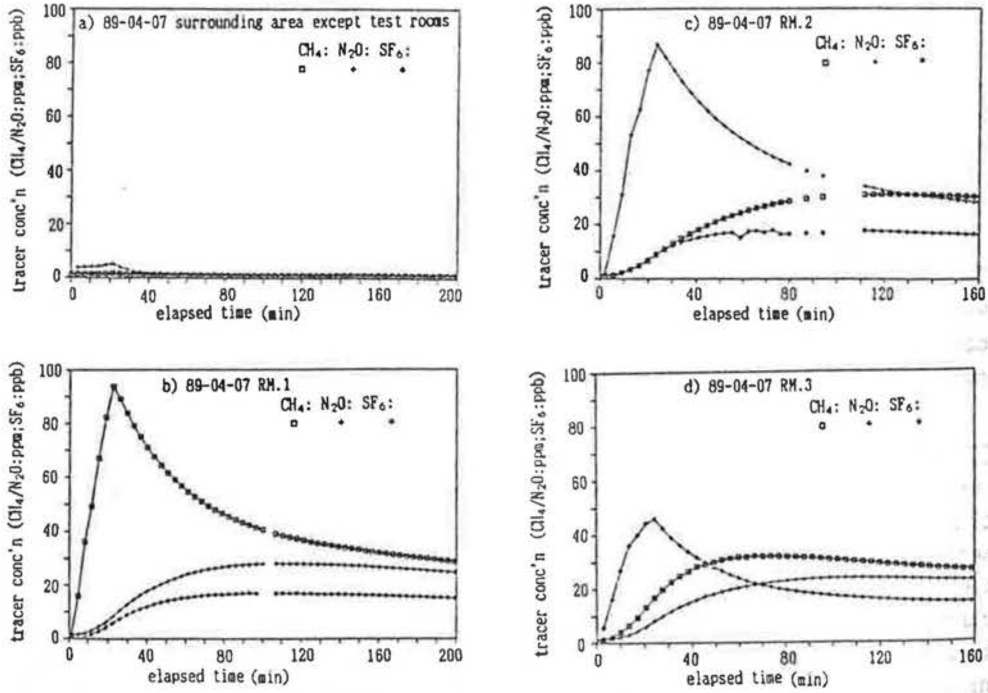

Figure 3 Change of Tracer Gas Concentration

Table 1 Test Conditions and Test Results

test No.	RM.1 ml	RM.2 ml	RM.3 ml		F_{12}	F_{13}	F_{21}	F_{23}	F_{31}	F_{32}	run time min.
101	SF_6	N_2O	CH_4	M	0.79	0.79	0.80	0.79	0.80	0.78	
	2.4	7548	7470	D	0.86	0.88	0.73	0.79	0.92	0.95	t_1 33.2
				I	0.83	1.03	0.53	0.65	0.63	0.86	T 268.7
104	SF_6	N_2O	CH_4	M	0.50	0.48	0.51	0.51	0.49	0.49	
	3.3	7516	7466	D	0.55	0.54	0.47	0.46	0.54	0.56	t_1 40.4
				I	0.83	0.83	0.45	0.52	0.60	0.78	T 182.5
108	CH_4	N_2O	SF_6	M	0.30	0.89	0.60	0.61	0.60	0.79	
	6760	6760	4.4	D	0.37	0.77	0.56	0.48	0.74	0.83	t_1 37.3
				I	0.36	0.80	0.50	0.43	0.64	0.83	T 272.6
109	CH_4	N_2O	SF_6	M	1.00	0.23	0.25	0.99	0.98	0.26	
	6760	6760	4.4	D	1.05	0.22	0.23	0.90	1.09	0.24	t_1 47.4
				I	1.07	0.26	0.20	0.90	1.07	0.28	T 201.5
111	CH_4	N_2O	SF_6	M	0.25	0.24	0.25	0.24	0.25	0.24	
	6000	6000	3.3	D	0.25	0.24	0.23	0.19	0.30	0.27	t_1 32.5
				I	0.25	0.21	0.21	0.19	0.25	0.28	T 211.3
113	CH_4	N_2O	SF_6	M	0.78	0.79	0.79	0.78	0.77	0.80	
	7520	7520	4.4	D	0.87	0.71	0.86	0.71	0.87	0.86	t_1 36.2
				I	0.81	0.76	0.77	0.67	0.84	0.84	T 158.8

(Header of the airflow columns: interzonal airflows ac/h)

M : measured airflows
D : calculated airflows with the differential method
I : calculated airflows with the integral method
t_1: appropriate time of the differential method
T : time interval of the integral method

and calculated values for F_{12}, F_{21}, F_{13}, F_{31}, F_{23} and F_{32} are given in Table 1 and in Figure 5. As shown in Table 1, the appropriate set of concentrations was found from the concentrations measured between 30 and 50 minutes after injection. The integral method does not have such a restriction. Figure 5-a shows that the airflow rates calculated from the gas concentrations agreed within 20 % (of the measured rates by the orifice) in the case of the differential method. Figure 5-b shows that the calculated flow rates by the gas concentration agreed within 30 % (on the above same conditions) in the case of the integral method.

Table 2 Integrated Concentration DC and Difference dC

test	RM.1			RM.2			RM.3			
No.	CH_4	N_2O	SF_6	CH_4	N_2O	SF_6	CH_4	N_2O	SF_6	
101	97.0	96.7	74.4	98.0	144.2	53.7	122.5	83.3	47.7	DC
	16.8	18.0	10.2	16.8	17.8	10.1	13.7	15.0	8.8	dC
104	47.6	38.9	66.5	45.0	83.9	35.0	79.6	31.3	30.5	DC
	16.8	15.7	14.6	16.9	16.7	14.8	12.8	12.0	11.3	dC
108	177.4	95.5	64.4	108.9	156.1	67.1	107.7	80.3	85.5	DC
	23.2	19.8	13.5	23.1	19.8	13.4	19.2	16.7	12.0	dC
109	133.9	59.1	52.5	98.4	119.3	43.8	65.1	66.8	68.0	DC
	24.9	20.1	14.2	26.6	21.4	14.7	20.9	17.2	12.5	dC
111	187.8	50.8	28.9	63.5	158.6	29.1	49.0	38.3	73.7	DC
	33.1	20.3	10.9	25.1	27.2	10.9	17.5	14.1	10.2	dC
113	119.9	61.7	38.7	73.4	107.2	39.9	64.5	53.8	59.9	DC
	29.8	26.6	16.7	29.2	28.1	16.6	24.6	22.6	14.9	dC

$DC = DC_{1n}$ ppm*h (ppb*h), $dC = C_n(T)-C_n(0)$ ppm (ppb)

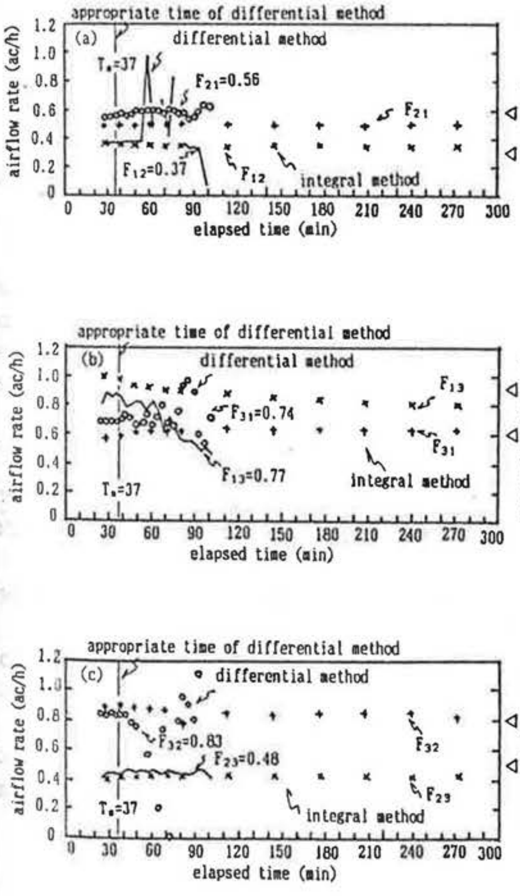

Figure 4 Calculated Interzonal Airflows by Differential or Integral Method

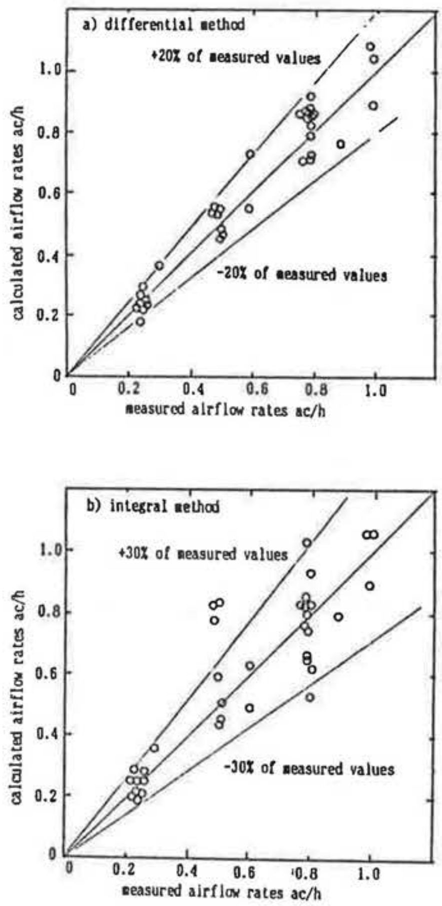

Figure 5 Comparison of Measured Airflow Rates and Calculated Ones in Test Rooms

The test conditions under which the measurements are conducted are rarely constant, for example, the ventilation rates can change rapidly during a test due to wind. When the test conditions are stable, the differential method is better than the integral method. Otherwise, the integral method is more accurate because it is less subject to the influence of unsteady test conditions. Thus, the integral method is more suitable than the differential method for field applications.

CONCLUSIONS

(1) Two different methods have been developed for calculating the interzonal airflows for three inter-connected zones. The differential method includes a procedure for checking the accuracy of the calculated airflows by comparing the measured gas concentration with the values obtained from the mathematical solutions.

(2) In the case of the differential method, the predicted results suggest that the gas concentration values measured between 30 to 50 minutes after the gas injection should be used for interzonal airflow calculations. The integral method , however, does not have such a restriction. In this paper, the predicted airflows by the integral method were stable even if the gas concentrations measured during the initial mixing period were used in the calculation.

(3) When using the proposed differential method, the predicted airflow rates agreed with the measured values within 20%. The integral method produced predicted airflow rates that agreed with the measured values within 30%. It must be noted that these conclusions were obtained using data measured under carefully controlled laboratory conditions. There is no clear evidence to suggest that this would also apply to data measured under field conditions. Because the integral method is easier to use and less subject to errors due to unsteady conditions (i.e., change every minute) than the differential method, it would be expected to be more suitable for practical applications.

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