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A Study on Differential Reynolds Stress Model for the Application to Air Flow in a Room

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ABSTRACT

This paper presents the numerical solutions of the room airflow by Differential Reynolds Stress Model(DSM) of turbulence and discusses some closure approximations in this model. A particular attention is given to the wall-reflection approximation in the pressure-strain term. Then the numerical solutions of the room airflow are compared with the experiment. The feasibility of DSM and K-ε model is also discussed.

The wall reflection term gives a great influence on the turbulent quantities and their distribution. This closure approximation shows the reverse energy re-distribution in the flow normal to the wall. The numerical solutions by DSM agree well with the experiment. Although the mean velocity predicted by K-ε model is also in good agreement with the experiment, K-ε model gives unrealistic turbulent quantities in this particular case.

KEYWORDS

Differential Reynolds-stress model, Room Airflow, Pressure-strain Term.

INTRODUCTION

Since many office buildings and residential buildings have been highly insulated and air-tightened recently, more sophisticated control of the air-conditioning is required than ever. Therefore it is very important to understand the characteristics of the room airflow.

The numerical methods to predict airflow have been developed recently. K-ε model is a typical one and used widely for many practical applications. However it could not be used for a certain flow such as strongly stratified flow. Several researchers(ex. Murakami et al. 1990) use a more precise model successfully employing Reynolds-stress equations, which consists of the algebraic expressions for Reynolds stresses.

Although this Algebraic Reynolds-stress model(ASM) has removed some shortcommings of K-ε model, ASM may have some limitations when applied to the flow where the convection or diffusion is dominant. In this paper, the characteristics of Differential Reynolds-stress model(DSM) are discussed with regard to the application to the room airflow.

In section 1, DSM is applied to a simple flow field and its basic characteristics are discussed. Attention is mainly given to pressure-strain term. The numerical solutions by DSM and K-ε model are compared with a simple experiment in section 2.
The applicability of DSM is then discussed.

1. EXAMINATION OF CLOSURE APPROXIMATIONS IN DSM

1-1. Basic equations of DSM

The turbulent model employed in this paper is mainly the one proposed by Launder, Rodi, Reece(Launder et al.1975). Two approximations proposed by Shir(Shir 1973) and Gibson,Launder (Gibson et al.1978) are used as the wall-reflection in the pressure-strain term. This is one of the most widely-used Reynolds-stress models. The DSM used in this paper, thus, are as follows.

Equation of continuity

$$\frac{\partial U_i}{\partial x_i}=0 \tag{1}$$

Equation of mean moment

$$\frac{\partial U_i}{\partial t}+\frac{\partial U_i U_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}-\frac{\partial \overline{u_i u_j}}{\partial x_j}-\frac{1}{Re}\frac{\partial}{\partial x_j}\left[\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right] \tag{2}$$

Transport equation of Reynolds-stress

$$\frac{\partial \overline{u_i u_j}}{\partial t}=-\frac{\partial U_k \overline{u_i u_j}}{\partial x_k}+P_{ij}+\phi_{ij}-\frac{2}{3}\varepsilon\delta_{ij}+D_{ij} \tag{3}$$

where,

$$P_{ij}=-\overline{u_i u_k}\frac{\partial U_j}{\partial x_k}-\overline{u_j u_k}\frac{\partial U_i}{\partial x_k} \quad (4) \qquad \phi_{ij}=\phi_{ij}^{(1)}+\phi_{ij}^{(2)}+\phi_{ij}^{(w1)}+\phi_{ij}^{(w2)} \quad (5)$$

$$\phi_{ij}^{(1)}=-C_1\frac{\varepsilon}{k}\left(\overline{u_i u_j}-\frac{2}{3}k\delta_{ij}\right) \quad (6) \qquad \phi_{ij}^{(2)}=-C_2'\left(P_{ij}-\frac{2}{3}P_k\delta_{ij}\right) \quad \text{(IPM)(7)}^{\dagger}$$

$$\phi_{ij}^{(2)}=-\frac{(C_2+8)}{11}\left(P_{ij}-\frac{2}{3}P_k\delta_{ij}\right)-\frac{(30C_2-2)}{55}k\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-\frac{(8C_2-2)}{11}\left(Q_{ij}-\frac{2}{3}P_k\delta_{ij}\right) \quad \text{(QIM)(8)}^{\dagger\dagger}$$

$$P_k=-\overline{u_i u_k}\frac{\partial U_i}{\partial x_k} \quad (9) \qquad Q_{ij}=-\overline{u_i u_k}\frac{\partial U_k}{\partial x_j}-\overline{u_j u_k}\frac{\partial U_k}{\partial x_i} \quad (10)$$

$$\phi_{ij}^{(w1)}=\Sigma C_{w1}\frac{\varepsilon}{k}\left\{\overline{u_k u_m}n_k^{(w)}n_m^{(w)}\delta_{ij}-\frac{3}{2}\overline{u_k u_i}n_k^{(w)}n_j^{(w)}-\frac{3}{2}\overline{u_k u_j}n_k^{(w)}n_i^{(w)}\right\}\frac{k^{3/2}}{C_l\varepsilon h_n^{(w)}} \tag{11}$$

$$\phi_{ij}^{(w2)}=\Sigma C_{w2}\left\{\phi_{km(2)}n_k^{(w)}n_m^{(w)}\delta_{ij}-\frac{3}{2}\phi_{ki(2)}n_k^{(w)}n_j^{(w)}-\frac{3}{2}\phi_{kj(2)}n_k^{(w)}n_i^{(w)}\right\}\frac{k^{3/2}}{C_l\varepsilon h_n^{(w)}} \tag{12}$$

$$D_{ij}=C_s\left\{\frac{\partial}{\partial x_k}\left(\frac{k}{\varepsilon}\overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l}\right)\right\} \quad (13)^{\ddagger}$$

$$D_{ij}=C_s'\left[\frac{\partial}{\partial x_k}\left\{\frac{k}{\varepsilon}\left(\overline{u_i u_l}\frac{\partial \overline{u_j u_k}}{\partial x_l}+\overline{u_j u_l}\frac{\partial \overline{u_k u_i}}{\partial x_l}+\overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l}\right)\right\}\right] \quad (14)^{\ddagger\ddagger}$$

Transport equation of turbulent energy dissipation

$$\frac{\partial \varepsilon}{\partial t}=-\frac{\partial U_k\varepsilon}{\partial x_k}-C_{\varepsilon1}\frac{\varepsilon}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}-C_{\varepsilon2}\frac{\varepsilon^2}{k}+C_\varepsilon\left\{\frac{\partial}{\partial x_k}\left(\frac{k}{\varepsilon}\overline{u_k u_l}\frac{\partial \varepsilon}{\partial x_l}\right)\right\} \tag{15}$$

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† proposed by Naot,Shavit,Wolfstain(Naot et al.1973)
†† proposed by Launder,Rodi,Reece(Launder et al.1975)
‡ proposed by Daly,Hallow(Daly et al.1970)
‡‡ proposed by Hanjalic,Launder(Hanjalic et al.1972)

where,

U_i:mean velocity
u_i:fluctuating velocity
P:mean pressure
ρ:air density
ν:viscocity
$\overline{u_i u_j}$:Reynolds stress
k:turbulent energy
ε:turbulent energy dissipation

n_i^w:i-direction component of unit vector normal to wall numbered w
δ_{ij}:kronecker's delta
h_n^w:normal distance from the wall numbered w
P_{ij}:production term of $\overline{u_i u_j}$
ϕ_{ij}:pressure-strain term of $\overline{u_i u_j}$
ε_{ij}:dissipation term of $\overline{u_i u_j}$
D_{ij}:diffusion term of $\overline{u_i u_j}$

Table 1 numerical constants in model

$C_1=1.8$	$C_2=0.4$ (Eq.(8))	$C_s=0.22$ (Eq.(13))
	$C_2'=0.6$(Eq.(7))	$C_s'=0.11$(Eq.(14))
$C_{w1}=0.5$	$C_{w2}=0.3$	$C_l=2.5$
$C_{\varepsilon1}=1.44$	$C_{\varepsilon2}=1.92$	$C_\varepsilon=0.16$

As is shown above, there are several different types of closure approximations with respect to the pressure-strain term(Equations (7) and (8)) or the turbulent diffusion term (Equations (13) and (14)). In this paper, the characteristics of these different approximations are examined when applied to the 2-dimensional isothermal airflow in a room.

1-2. Characteristics of pressure-strain term

This term contributes to the energy distribution between the normal stresses, but not to the turbulent energy. The pressure-strain term is modeled as sum of two terms caused by two different physical processes. One is the interaction between the turbulent components($\phi_{ij}^{(1)}$) and the other is the interaction between the turbulent component and the mean strain($\phi_{ij}^{(2)}$).

The similar terms which represent the wall effects ($\phi_{ij}^{(w1)}, \phi_{ij}^{(w2)}$) are modeled in a similar manner. The characteristics of the wall-reflection terms are described as following. They are modeled in such a way to reduce the stress normal to the wall and to increase another stresses in the near-wall turbulence. This is based on the experimental results near the wall. But it was pointed out that the term $\phi_{ij}^{(w2)}$ could show the reverse effect in the flow normal to the wall(Murakami et al.1990).

To examine the characteristics of these terms, some numerical calculations are tried to a simple flow. The calculated values of ϕ_{ij} are shown in Tables 2 and 3. They are typical values to the flow along a wall(Table 2) and the flow normal to the wall (Table 3). The model named DSM-1 includes $\phi_{ij}^{(w2)}$ and DSM-2 does not.

The magnitude of $\phi_{ij}^{(w2)}$ is small and $\phi_{ij}^{(w1)}$ and $\phi_{ij}^{(1)}$ are dominant in the area along the wall. Although $\phi_{ij}^{(w)}$ makes the stress normal to the wall($\overline{u^2}$) decrease and others($\overline{v^2}$, $\overline{w^2}$) increase, $\overline{w^2}$ is increased than $\overline{v^2}$. Though $\phi_{ij}^{(w)}$ reduces the stress normal to the wall correctly, the energy transfer to the other stresses is not enough compared the experimental results(Launder et al. 1975).

It is also apparent that $\phi_{ij}^{(w2)}$ shows the reverse energy transfer near the flow normal to the wall. The stress normal to a wall is $\overline{v^2}$ and it must be decreased by $\phi_{ij}^{(w)}$. But $\phi_{ij}^{(w2)}$ increases $\overline{v^2}$ and decreases $\overline{u^2}$ and $\overline{w^2}$ in the flow normalto the wall in the numerical results.

Since $\phi_{ij}^{(w2)}$ model was derived based on the flow parallel to the wall, it may give

Table 2 typical non-dimensional orders of ϕ_{ij} in flow along with a wall($\times 10^{-3}$)

	$\phi_{ij}^{(w2)}$	$\phi_{ij}^{(w1)}$	$\phi_{ij}^{(1)}$	$\phi_{ij}^{(2)}$
DSM-1				
i=1,j=1	-0.37	-2.67	5.12	0.56
i=2,j=2	0.25	0.75	-3.00	-1.31
i=3,j=3	0.12	1.92	-2.12	0.75
DSM-2				
i=1,j=1		-2.26	3.65	1.51
i=2,j=2		0.66	-2.71	-2.83
i=3,j=3		1.60	-0.95	1.32

Table 3 typical non-dimensional orders of ϕ_{ij} in flow attacked to a wall($\times 10^{-3}$)

	$\phi_{ij}^{(w2)}$	$\phi_{ij}^{(w1)}$	$\phi_{ij}^{(1)}$	$\phi_{ij}^{(2)}$
DSM-1				
i=1,j=1	-1.84	0.89	2.29	1.73
i=2,j=2	2.14	-2.56	-1.92	-2.75
i=3,j=3	-0.30	1.67	-0.37	1.02
DSM-2				
i=1,j=1		0.18	0.95	1.28
i=2,j=2		-1.14	-0.67	-1.75
i=3,j=3		0.96	-0.28	0.47

unrealistic results when applied to the flow normal to the wall. The same tendency appeared near the inlet.

It should be noted that the difference between Equations (7) and (8), or Equations (13) and (14) is small.

2. Comparison with experiment

In this section, 2-dimensional turbulent flow in the room is calculated by two different versions of DSM mentioned above and K-ε model. Simulations are compared with the experiment. The calculations are performed to the room shown in Figure 1 under the boundary conditions listed in Table 4. In the followings, DSM-1 implies inclusion of $\phi_{ij}^{(w2)}$ and DSM-2 does not include this term. Both models refer to IPM(Launder 1983) for $\phi_{ij}^{(2)}$ given by Equation (8). Equation (13) is also used as a turbulent diffusion term.

Figures 2, 3 and 4 show the mean velocity distributions calculated by each model. The mesured values are doneted by broken lines. The predictions and the experiment agree well. The results by two DSM are almost same. Disagreement with the mesured values near the corner may be due to not the shortcoming of models but lack of cell partition as shown later. The velocity at the edge of jet by k-ε is slightly smaller than that of DSM.

The turbulent stress distributions are shown in Figures 5, 6 and 7†. The level of each stress by DSM-1 is greater than that of DSM-2. As mentioned above, the predicted level of $\overline{u^2}$ by DSM-1 near the wall is much greater than the experimental results due to $\phi_{ij}^{(w2)}$. The same situation occurs near the inlet. The values of $\overline{u^2}$ by K-ε model are negative near the inlet††.

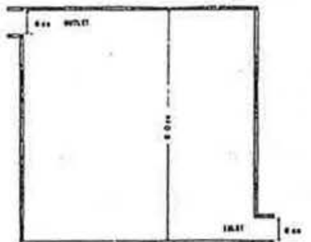

Figure 1 calculated space

Table 4 numerical conditions

outlet : U_n, $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$ measured values
$\overline{w^2}=(\overline{u^2}+\overline{v^2})/2$, $k=(\overline{u^2}+\overline{v^2}+\overline{w^2})/2$
l, ε referenced value

inlet : $\partial U_n/\partial x_n=0$, $\partial\overline{u_i^2}/\partial x_n=0.0$
$\partial\varepsilon/\partial x_n=0.0$, $\partial\overline{uv}/\partial x_n=0.0$

wall : $U_n=0.0$, $\partial U_p/\partial x_n=1/7$, $\partial\overline{u_i^2}/\partial x_n=0.0$
$\varepsilon=C_D^{3/4}k^{3/2}/(\kappa x_{n1})$, $-\overline{uv}=(\nu+\nu_t)\partial U/\partial x_n$

$\nu_t=C_D k^2/\varepsilon$, κ : Karman's constant(0.4), $C_D=0.09$
U_n : velocity component normal to wall
U_p : velocity component parallel to wall
x_{n1} : distance from wall to the first defined point of scalar

using upwind scheme for convection term
and Adams-Bashforth scheme for time differential term

† The negative values of measurement of $\overline{v^2}$ may be caused by the experimental uncertainty.

†† Boussinesq formula was used to estimate the values of $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$ in the computation by K-ε model.

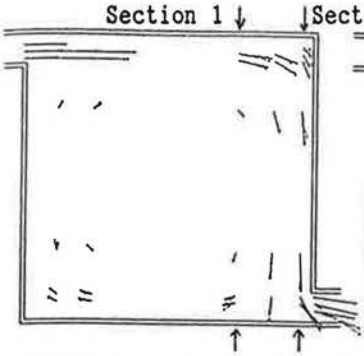

Figure 2 velocity (DSM-1)

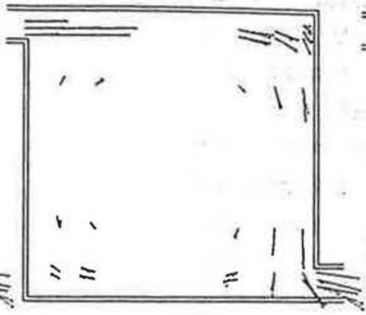

Figure 3 velocity (DSM-2)

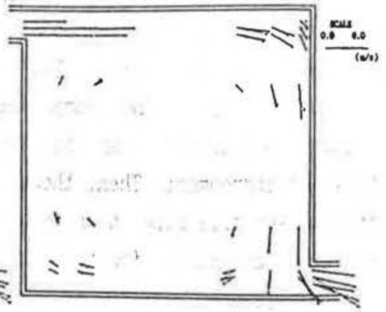

Figure 4 velocity (K-ε)

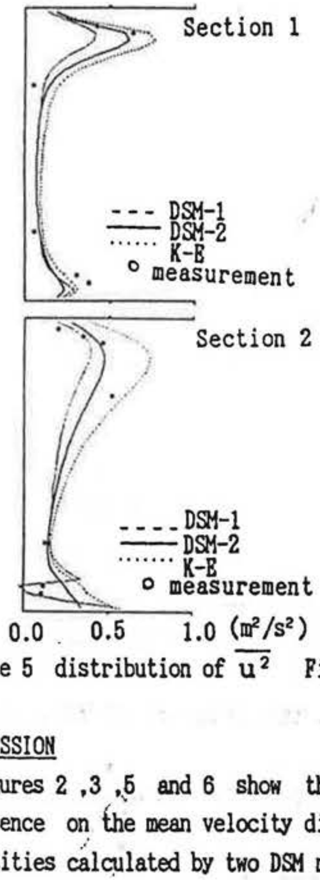

Figure 5 distribution of $\overline{u^2}$

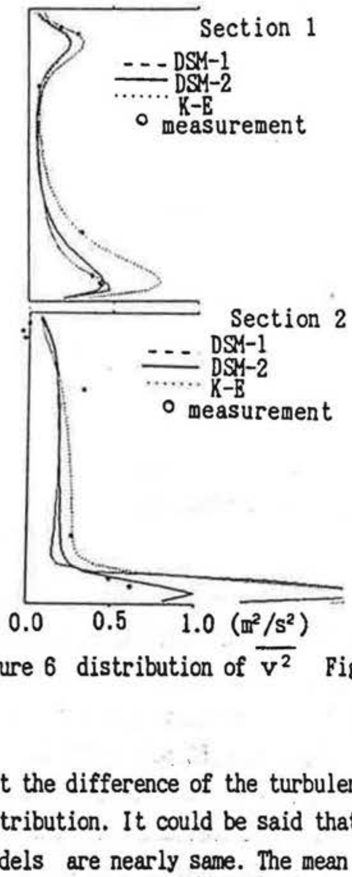

Figure 6 distribution of $\overline{v^2}$

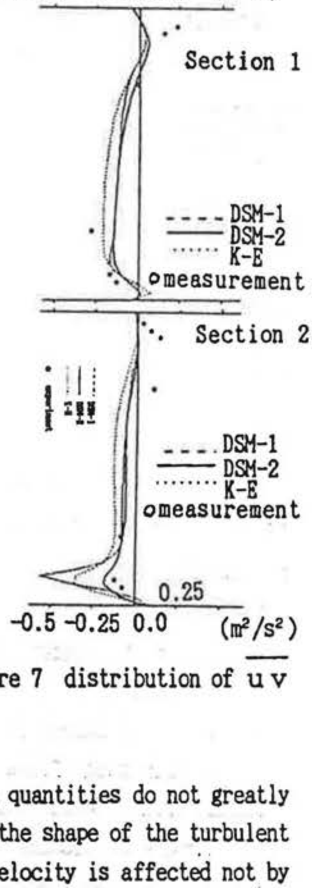

Figure 7 distribution of $\overline{uv}$

DISCUSSION

Figures 2 ,3 ,5 and 6 show that the difference of the turbulent quantities do not greatly influence on the mean velocity distribution. It could be said that the shape of the turbulent quantities calculated by two DSM models are nearly same. The mean velocity is affected not by the level of turbulent quantities but by the gradients of them. Extremely high values of $\overline{v^2}$ near the inlet predicted by DSM-1 may be unrealistic. The production rate of $\overline{v^2}$ is very large there and $\phi_{ij}{}^{(w2)}$ contribute to the energy gain of $\overline{v^2}$.

Altough K-ε model predicts the mean velocity reasonably well, some of the energy components are negative. Since this model does not contain $\overline{u^2}$, $\overline{v^2}$ or $\overline{uv}$ as variable and these values are estimated by using Boussinesq formula, it does not nucessarily mean that K-ε model can not be used in this case. It could be said, however, that Boussinesq formula is not valid when applied at least to the flow discussed here.

As shown above, the numerical results disagree in the flow normal to the wall and around the inlet. The computational conditions, i.e. the large cell size or boundary conditions may be the reasons for this disapgreement. Then, these are modified, the smaller cell size near wall regions and non-slip condition for velocity wall-B.C. are used. Calculated results is shown in figure 8. The improvement obtaind by these modifications is drastic and the numerical result is in good agreement with the experiment.

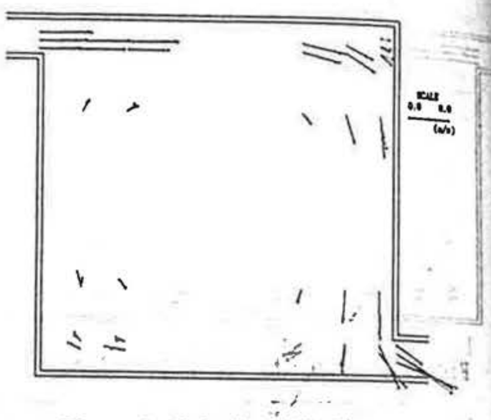

Figure 8 Velocity (DSM-2)

CONCLUSIONS

1. Differential Reynolds-stress Model has enough potential for application to room airflow. DSM can predict turbulent quantities even near wall and inlet with reasonable accuracy.
2. Wall-reflection approximation with regard to mean-strain in pressure-strain term may give the energy component normal to wall, which is different from the experimental results.
3. K-ε model may give the erroneous result when applied to room airflow. The discrepancy is large near the inlet.
4. The non-slip B.C. for the velocity may be sutable for the flow normal to the wall.

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