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Room Airflow Analysis by Means of Differential Stress Model

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ABSTRACT

The paper reports the way to prevent numerical instability in computing DSM by means of MAC method. There are three problems in calculating DSM numerically. The first is that mean flow equations of DSM have no diffusion term. The second is an instability due to turbulent diffusion term with cross diffusion. The other is a way to decide a stable time step. We proposed the solutions to the three problems and applied our algorithm to a calculation of two-dimensional room airflow field. The results showed that our method made it possible to compute DSM stably by the MAC method.

KEYWORDS Differential Stress Moment, Room Airflow, Realizability, Cross Diffusion, Pseudo-viscosity

INTRODUCTION

The purpose of this paper is to consider a way of computing room airflow by the Differential Stress Moment model (which will hereinafter be abbreviated to DSM).

Huang and Leschziner (1985) presented a stabilization for DSM. They succeeded in stably solving the mean flow equations, using pseudo-viscosities that are derived from Reynolds stress equations of DSM. We apply their stabilization to the MAC method and discuss a cross diffusion term, which was not considered by Huang and Leschziner. The cross diffusion term influences a numerical stability. We consider how to prevent a numerical instability, and show a way of determining a stable time step. We also apply our proposal to a two-dimensional room air flow field and investigate its effectiveness. We examine the effects of wall reflection term and difference scheme on the flow field.

We adopt IPM (Isotropization of Production Model, Launder 1983) among DSM models, because this method is most popularly used.

STABILIZATION IN CASE OF MAC METHOD

The stabilization of DSM was introduced by Huang and Leschziner (1985). They calculated DSM stably using the pseudo-viscosity representation. Their method is applicable to the MAC method, though in a case of the MAC method a time-differential component should be took into account. We replace this component with a one-step former residual of a finite difference equation of Reynolds stress.

If using the pseudo-viscosity representation of Huang and Leschziner, equations of mean flow have cross diffusion terms. We try to modify their pseudo-viscosity representation so as to have no cross diffusion term. In this modification, the term producing the cross diffusion term in turbulent shear production term is included in Sp term of Huang and Leschziner and the term not producing it is included in Su term. Please refer to the paper of Hiraoka and Nakamura (1992) for details.

MODIFICATION OF TURBULENT DIFFUSION TERM

When we use QUICK difference scheme for the equations of Reynolds stress and viscous dissipation rate, results often diverge. (Huang and Leschziner used the power-low-differencing scheme of Patanker 1980. In a region of high mesh Re number, this scheme becomes an upwind difference scheme of first order.) This is due to an effect of cross diffusion in the turbulent diffusion term. From stability analysis we can show that positive time step can not be obtained if the cross diffusion is relatively large.

The turbulent diffusion term (D) is expressed as follows,

$$D = c\left(\frac{k}{\varepsilon}\right)\tau_{ij}\frac{\partial^2\phi}{\partial x_i \partial x_j}+\cdots \qquad (1)$$

The next expression indicates a real symmetric matrix (τ_{ij}), which consists of Reynolds stress τ_{ij},

$$\left(\tau_{ij}\right) = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{12} & \tau_{22} & \tau_{23} \\ \tau_{13} & \tau_{23} & \tau_{33} \end{bmatrix} \qquad (2)$$

The turbulent diffusion term is diffusive if it is of an elliptic type. A condition that the turbulent diffusion term is of elliptic type is that matrix (τ_{ij}) is positive-definite. If this condition is not satisfied, the result is likely to diverge. There are several ways to keep the turbulent diffusion term elliptic. One of them is to modify the normal stresses so as to satisfy a realizability of Schumann (1977). If this matrix satisfies the realizabilty, the turbulent diffusion term is of diffusion type.

STABILITY ANALYSIS BY MEANS OF HIRT'S METHOD

In this section we make a stability analysis for an equation of convection-diffusion type with cross diffusion term. We determine a time step by applying Hirt's stability analysis to an equation with cross diffusion.

Consider the following equation of convection-diffusion type with cross diffusion term,

$$\frac{\partial \phi}{\partial t}+U_j\frac{\partial \phi}{\partial x_j}=\nu_{ij}\frac{\partial^2\phi}{\partial x_i\partial x_j} \tag{3}$$

where let U_i be constant and viscosity tensor ν_{ij} is symmetric. Φ is a reynolds stress or a viscous energy dissipation rate. If the pseudo-viscosity representation is used, Φ also indicates a mean velocity. Let the viscosity matrix (ν_{ij}) be positive-definite so that Equation 11 could be elliptic in (x_1,x_2,x_3) space.

When we make a forward difference with time step Δt and a central difference with mesh size h, we get an equation which corresponds to the equation really computed by the finite difference method. The equation becomes the same as Equation 3, in form, by replacing ν_{ij} with ν^e_{ij}. The effective viscosity ν^e_{ij} is expressed by the next equation.

$$\nu^e_{ij}=\nu_{ij}-\frac{\Delta t}{2}U_iU_j \tag{4}$$

If we use Hirt's stability analysis (1968), the condition for stability is that the matrix (ν^e_{ij}) is positive-definite.

In the case of two-dimensional space, a time step Δt is obtained from the above condition, as follows.

$$\Delta t<\frac{2\left[\nu_{11}\cdot\nu_{22}-(\nu_{12})^2\right]}{\nu_{22}\cdot U_1^2+\nu_{11}\cdot U_2^2-2\,\nu_{12}\cdot U_1U_2} \tag{5}$$

Condition 5 is verified by the result computed numerically with the Neumann's stability analysis, which is more theoretical, if mesh Re number is large enough (mesh Re > 5) (Hiraoka and Nakamura 1992). Hirt (1968) did not take a cross diffusion term of numerical viscosity into account. His result does not agree with that from Neumann in a case of two-dimensional equation.

NUMERICAL CALCULATION OF TWO DIMENSIONAL ROOM AIR FLOW

We try to calculate numerically a two-dimensional room air flow in order to verify our proposition for solving DSM. We adopt the a forward difference scheme for time step and the QUICK scheme for convection term of mean velocity equation. A time step is determined from Condition 5, considering cross diffusion term. We take up 12 cases shown in Table 1 in order to investigate the effects of wall reflection terms and finite difference schemes on a flow field.

RESULTS AND DISCUSSION

(1) [convergency]: Table 2 shows the results of convergence. CASEs 1 to 3 converged. In these cases we adopted the upwind scheme for convection term of the equations of Reynolds stress and energy dissipation rate. But CASEs 4 to 6 diverged. Convergence is possible if we treat cross diffusion term as we mentioned above (CASEs 7 to 12).

(2) [influence of finite difference scheme]: Figures 1 to 4 show the results of CASE 1 and CASE 7. A disparity on difference schemes of convection term in equations of Reynolds stress and dissipation rate does not affect the mean flow field in pattern. But the turbulent field is influenced.

(4) [influence of wall reflection term]: Figures 4 to 9 show the results of CASE 7 to 9. Differently from the result of ASM (Algebraic Stress Moment model) by Murakami et al. (1990) results converge even in incorporating Rapid term in wall reflection term when cross diffusion is properly compensated by the methods mentioned above. Wall reflection term affect turbulent quantities. In a case where only Rotta term is used as a wall reflection term, the result is largely different in both quantity and pattern from that in the case where both Rotta and Rapid terms are included. In the latter case τ_{11} becomes large in the neighborhood of the stagnation point above the exhaust.

(5) [difference in algebraic expression of Reynolds stress]: CASEs 10 to 12 show the results from the modified pseudo-viscosity representation. The results are almost the same as those of CASEs 7 to 9. If the pseudo-viscosity representation of Huang and Leschziner becomes hyperbolic, our modified representation seems better.

CONCLUSIONS

We proposed an algorism for calculating DSM using the MAC method and made it clear that the algorism was useful. Conclusions are :

(1) We applied the stabilization of Huang and Leschziner to a calculation by the MAC method.

(2) We made a stability analysis of a convection-diffusion type

equation with cross diffusion. We made it clear that the Hirt's stability analysis is usable for determining a stable time step for the equation of this type.

(4) The condition that the turbulent diffusion term is elliptic is equal to the realizability of Schumann.

(5) We made it clear from our calculation that the difference in approximation of a wall reflection term affects turbulent quantities of the flow field. It is necessary to approximate more accurately the wall reflection term in the neighborhood of corners, exhausts, intakes or stagnation points.

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TABLE 1 Combinations of Difference Scheme, Treatment of Diffusion term, Wall Reflection, and Pseudo-viscosity.

	difference scheme of eqs. of Reynolds stress and viscous dissipation	treatment of turbulent diffusion term	wall reflection term	pseudo-viscosity representation
CASE 1	upwind	non	1	A
CASE 2	upwind	non	2	A
CASE 3	upwind	non	0	A
CASE 4	QUICK	non	1	A
CASE 5	QUICK	non	2	A
CASE 6	QUICK	non	0	A
CASE 7	QUICK	realizability	1	A
CASE 8	QUICK	realizability	2	A
CASE 9	QUICK	realizability	0	A
CASE 10	QUICK	realizability	1	B
CASE 11	QUICK	realizability	2	B
CASE 12	QUICK	realizability	0	B

wall reflection term 0 : no wall refection
1 : Rotta term
2 : Rotta and Rapid terms
pseudo-viscosity representation : A : representation of Huang et al.
B : modified representation

TABLE 2 Results on Convergency.

	RESULT
CASE 1	converge
CASE 2	converge
CASE 3	converge
CASE 4	diverge
CASE 5	diverge
CASE 6	diverge
CASE 7	converge
CASE 8	converge
CASE 9	converge
CASE 10	converge
CASE 11	converge
CASE 12	converge

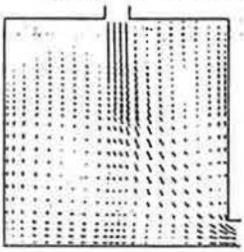

FLOW OF VELOCITY

FIGURE 1 Mean flow pattern (CASE 1).

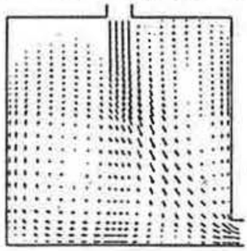
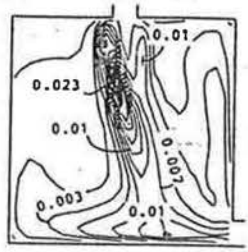
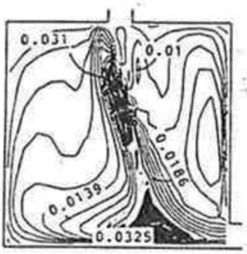

FLOW OF VELOCITY

FIGURE 2 Mean flow pattern (CASE 7).



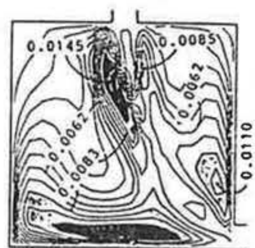
TURBULENT ENERGY

FIGURE 3 Turbulent energy (CASE 1).



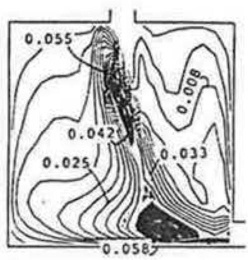
TURBULENT ENERGY

FIGURE 4 Turbulent energy (CASE 7).



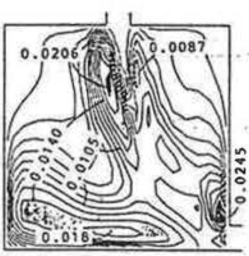
$\overline{u_1 u_1}$

FIGURE 5 Reynolds stress τ_{11} (CASE 7).



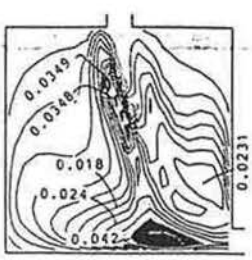
TURBULENT ENERGY

FIGURE 6 Turbulent energy (CASE 8).



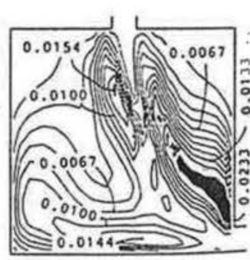
$\overline{u_1 u_1}$

FIGURE 7 Reynolds stress τ_{11} (CASE 8).



TURBULENT ENERGY

FIGURE 8 Turbulent energy (CASE 9).



$\overline{u_1 u_1}$

FIGURE 9 Reynolds stress τ_{11} (CASE 9).