6046

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NUMERICAL SIMULATION OF AN AIR FLOW WITH SMOKE PARTICLES CAUSED BY A FIRE

Yasuyoshi Horibata
Systems & Software Engineering Laboratory
Toshiba Corporation
70, Yanagi-cho, Saiwai-ku, Kawasaki 210, Japan

ABSTRACT

An air flow with smoke particles caused by a fire is simulated. The flow has a low Mach number and a large temperature variation; hence, compressibility is important. The compressible equations are modified by scale analysis. The modified equations are integrated by a finite difference scheme. The scheme is essentially the MAC method. Thermal convection caused by a fire is simulated in a tunnel.

INTRODUCTION

An air flow with smoke particles caused by a fire is simulated. The temperature variation in the flow is large. Hence, the assumption of incompressibility cannot be used. On the other hand, the Mach number of the flow is low.

As is well known, time-dependent compressible flow schemes become ineffective at low Mach numbers (Choi and Merkle 1985; Morinishi and Satofuka 1988). This ineffectiveness occurs because a wide disparity exists between the time scales associated with convection and the propagation of acoustic waves.

The present paper modifies the compressible equations by scale analysis (Horibata). A finite difference scheme similar to the MAC method integrates the modified equations. Finally, thermal convection caused by a fire is simulated in a tunnel.

THE COMPRESSIBLE EQUATIONS

Assume particles have negligible terminal velocity and hence always follow the fluid flow. The mixing ratio of particles is defined by $m = \rho_p / \rho_f$; ρ_f and ρ_p are the fluid and particle densities, respectively. Assume m is small relative to one in order of magnitude:

$$m \ll 1. \tag{1}$$

For simplicity, assume a laminar flow. Use Cartesian coordinates x, y, z, with z vertically upwards.

The Navier-Stokes equations are

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{1i}}{\partial x_i}, \tag{2}$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \sigma_{2i}}{\partial x_i}, \tag{3}$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial \sigma_{3i}}{\partial x_i} - \rho g, \tag{4}$$

where ρ is the total density; u, v, w are the x, y, z components of the velocity, respectively; p is the pressure; g is the gravitational acceleration; and σ_{ij} is the viscous stress tensor. The tensor σ_{ij} is given by

$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\,\delta_{ij}\frac{\partial u_l}{\partial x_l} \right),$$

where μ is the coefficient of viscosity.

The equation for internal energy is

$$\rho\frac{D\varepsilon}{Dt} = \frac{p}{\rho}\,\frac{D\rho}{Dt} - \frac{\partial q_i}{\partial x_i} + Q, \tag{5}$$

where ε is the internal energy per unit mass; Q is the rate of heat produced per unit volume by external agencies; and q_i is the heat flux density. The flux q_i is defined by $q_i = -\kappa\partial T/\partial x_i$, where T and κ are the temperature and thermal conductivity, respectively.

The compressible continuity equation is

$$\frac{D\rho}{Dt} = -\rho\frac{\partial u_i}{\partial x_i}. \tag{6}$$

The continuity equation for the particles is

$$\rho\frac{Dm}{Dt} = -\frac{\partial j_i}{\partial x_i} + S, \tag{7}$$

Here S is the rate of particles produced per unit volume by external agencies, and j_i is the diffusion flux density. The flux j_i is defined by $j_i = -\rho D\,(\partial m/\partial x_i)$, where D is the diffusion coefficient.

For a perfect gas, the equation of state is

$$p = (\gamma-1)(1-m)\rho\varepsilon, \tag{8}$$

where γ is the ratio of specific heats. Moreover, the following relationship exists: $\varepsilon = c_v T$, where c_v is the specific heat at constant volume.

<u>MODIFICATION OF THE COMPRESSIBLE EQUATIONS</u>

Let ρ_r, T_r, and p_r denote the representative density, temperature, and pressure of a flow, respectively. The equation of state relates these quantities: $p_r = \rho_r R T_r$; R is the gas constant. In general, ρ_r, T_r, and p_r are functions of time. Denoting by p_d the deviation from p_r, express the pressure as

$$p(t,x,y,z) = p_r(t) + p_d(t,x,y,z). \tag{9}$$

Using Eq.(9), if conditions

$$M^2,\ \frac{1}{\tau c} M,\ \frac{gh}{c^2} \ll \frac{\Theta}{T_r} \tag{10}$$

$$M^2,\ \frac{1}{\tau c} M,\ \frac{gh}{c^2} \ll m \tag{11}$$

are fulfilled, Eqs.(2), (3), (4), (5), (6), and (8) can be rewritten as

$$\rho\frac{Du}{Dt} = -\frac{\partial p_d}{\partial x} + \frac{\partial \sigma_{1i}}{\partial x_i}, \tag{12}$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p_d}{\partial y} + \frac{\partial \sigma_{2i}}{\partial x_i}, \tag{13}$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p_d}{\partial z} + \frac{\partial \sigma_{3i}}{\partial x_i} - \rho g. \tag{14}$$

$$\rho\frac{D\varepsilon}{Dt} = \frac{1}{\gamma}\left(-\frac{\partial q_i}{\partial x_i} + Q + \frac{dp_r}{dt} \right). \tag{15}$$

$$\frac{1}{\varepsilon}\frac{D\varepsilon}{Dt} - \frac{1}{p_r}\frac{dp_r}{dt} = \frac{\partial u_i}{\partial x_i}. \tag{16}$$

$$p_r = (\gamma-1)(1-m)\rho\varepsilon. \tag{17}$$

From Eqs.(15) and (16), the equation for the representative pressure is obtained:

$$V\frac{dp_r}{dt} + \left(\gamma \int u_i n_i \, df\right) p_r = (\gamma-1)\left(-\int q_i n_i \, df + \int Q \, dv \right), \tag{18}$$

where V is the volume of the integration space and n_i is the unit vector normal to the surface element df. When the flow is open to the atmosphere, take p_r as a constant.

NUMERICAL INTEGRATION

A finite difference scheme integrates the modified equations. The grid system is a standard spatially staggered mesh. The scheme is essentially the MAC method.

From Eqs.(12), (13), and (14), one obtains the equation for p_d^{n+1}; the superscript n+1 refers to the next time level. The successive over-relaxation (SOR) method solves the equation.

One obtains implicitly u^{n+1}, v^{n+1}, w^{n+1}, ε^{n+1}, and m^{n+1} from Eqs.(12), (13), (14), (15), and (7), respectively.

SIMULATION RESULT

Consider an air flow in a two-dimensional tunnel fire. Figure 1 shows the simulation region and used grid system of the tunnel. The tunnel has a slope of 4%.

Figure 2 and 3 show the flow pattern and mixing ratio of smoke particles, respectively.

The maximum values of the local Mach number and mixing ratio are about 3×10^{-3} and 10^{-2}, respectively. From $M \sim 3\times10^{-3}$, $m \sim 10^{-2}$, $\tau \sim 100s$, $l \sim 50m$, $h \sim 10m$, $\Theta \sim 400K$, $T_r \sim 300K$, one has

$$\frac{l}{\tau c} M \sim 4\times10^{-6}, \qquad \frac{gh}{c^2} \sim 8\times10^{-4}, \qquad \frac{\Theta}{T_r} \sim 1.3.$$

Hence conditions (1), (10), and (11) are fulfilled.

CONCLUSION

Numerical simulation of an air flow with smoke particles is described, and a simulation result is presented. The compressible equations are modified by scale analysis. The modified equations require less conditions than the Boussinesq equations. Thermal convection caused by a fire is simulated.

REFERENCES

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Figure 1. Simulation region and grid system of the tunnel.

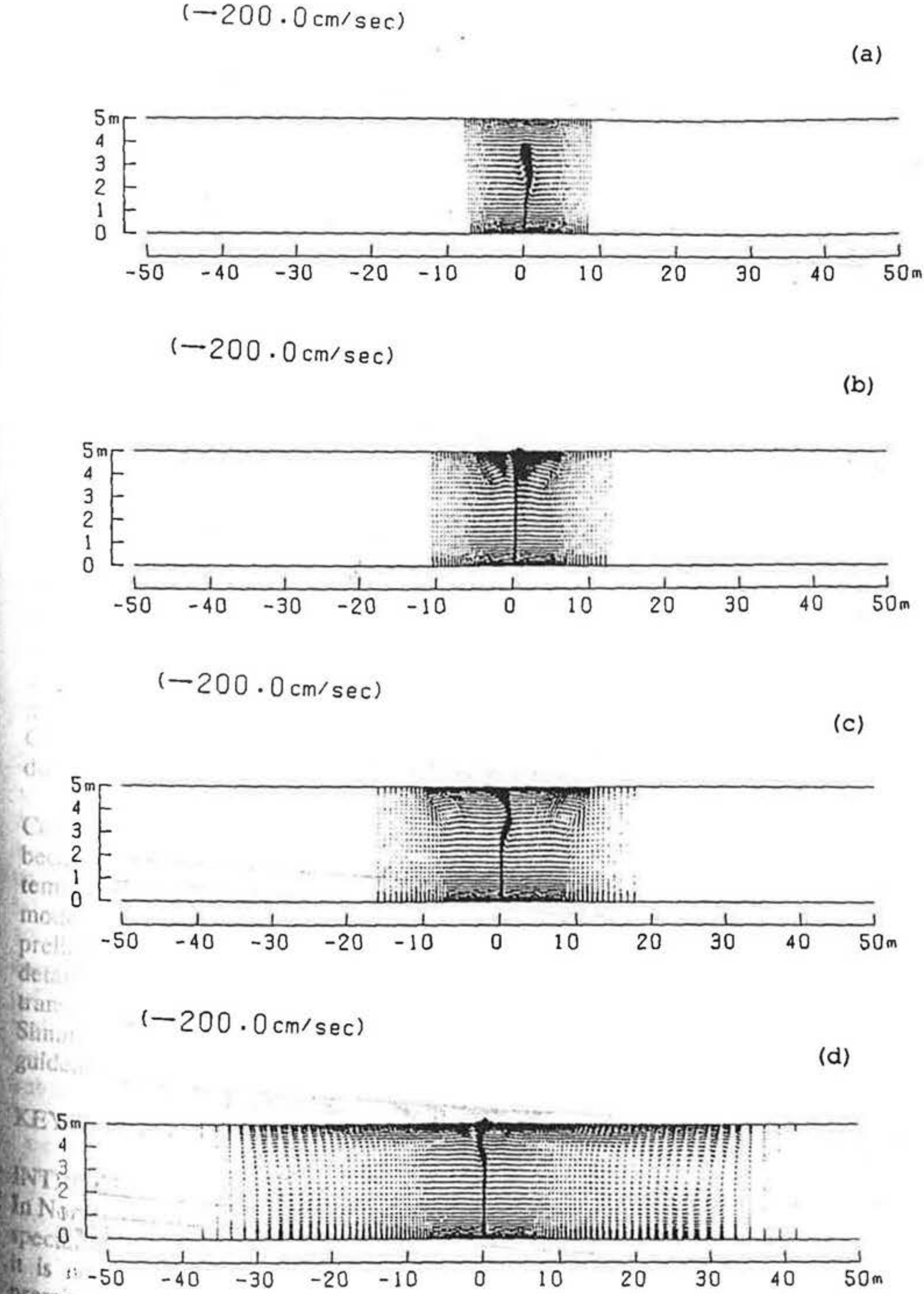

Figure 2. Evolution of the flow field. Figure 2a, 2b, 2c, and 2d correspond to 10, 20, 30, and 100s, respectively, after a fire breaks out.

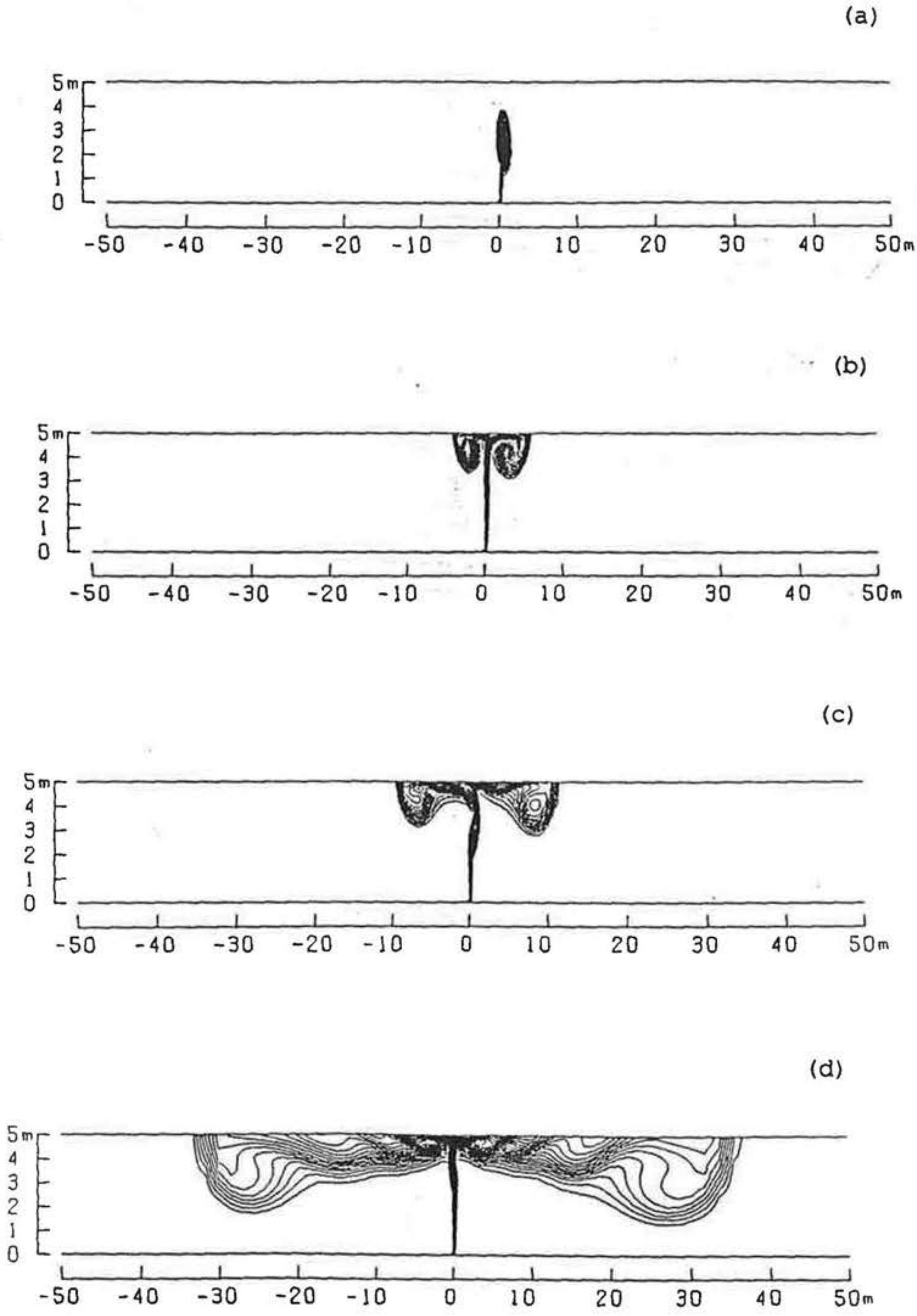

Figure 3. Evolution of the smoke particle field. Figure 3a, 3b, 3c, and 3d correspond to 10, 20, 30, and 100s, respectively, after a fire breaks out. The contours range from 0.01 to 10g/kg with a contour interval of 0.01g/kg.