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ERRORS IN THE CALCULATION OF VENTILATION FLOW RATES.

A Dissertation
Presented to
the Department of Civil Engineering and Building
Coventry Polytechnic

In Partial fulfilment
of the requirements for the Degree
Bachelor of Science

by
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May 1989

ACKNOWLEDGEMENTS

I would like to express my thanks to all those people who have helped to make this dissertation possible. In particular, I am much indebted to Dr. J.R. WATERS for his guidance, encouragement and enthusiasm throughout the past year, also to K. SINCLAIR for his technical support.

ABSTRACT

This project investigates the effect of rounding errors in the calculation of ventilation flow rates. The flow rates are determined from the concentrations of contaminant which are measured in a tracer decay experiment.

As shown in the present dissertation, the introduction of rounding errors leads to substantial changes in the final flows. Experimental errors are expected to be larger than rounding errors and therefore, they would produce greater errors in the computed flows.

The aim of this research is to find a way of measuring these errors, and as shown later, the condition number used in this report is not a satisfactory measure of these errors.

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INTRODUCTION

AND

OBJECTIVES

0. INTRODUCTION AND OBJECTIVES:

Why is ventilation and air movement important?

Because of:

(i) Energy,

(ii) Contaminants and air quality.

The measurement of ventilation is most easily accomplished by considering a building as a single cell, in which the air is fully mixed. In this case, measurement is easily achieved by using a tracer gas. This is the single zone theory.

The single zone theory is satisfactory if the ventilation rate only is to be studied.

However, in large buildings, it is advantageous, and often necessary to know how fresh and contaminated air is distributed internally. For this, the building needs to be considered as a set of interconnected zones, rather than as a single zone. This is the multizone theory, as described in Sinden's work.

If the flows between zones are known, the distribution of contaminant can be calculated from the theory.

However, interzone flows are often not known, in which case it is necessary to calculate or measure them.

This investigation is concerned with some of the problems of obtaining interzone flows from measurements.

One of the most commom method of measurement is the tracer decay technique, in which tracer gas is introduced into one (or more) of the zones, and then the changes in the tracer gas concentration are measured over a period of time. Sinden's theory can then be used to obtain values for the interzone flows. However, Waters and Simons discovered that the results were often inconsistent, and this inconsistency was thought to be due to the sensitivity of the derived flows to errors in the measured concentrations.

This problem was further studied by Waters and Simons who pointed out that there may be problems due to certain combinations of interzone flows creating redundancy in the solution.

The objective of this study is to examine the extent to which errors in the measured concentrations create errors in the flows which are computed from the concentrations.

The method which has been adopted is to take two hypothetical buildings, each of three zones, in which the flows are specified. Calculation are then performed to determine, accurately, the tracer concentration which would be measured in an ideal tracer decay experiment.

These concentrations are then used to compute the interzone flows. Clearly, these should be the same as the flows chosen originally.

Errors are introduced into the concentrations, and the effect on the solution is explored.

CHAPTER 1

REVIEW OF NATURAL VENTILATION FUNDAMENTALS

1-0. REVIEW OF NATURAL VENTILATION FUNDAMENTALS:

1-1. Definitions:

VENTILATION - The introduction of fresh air and the removal of contaminated air.

ADVENTITIOUS VENTILATION or INFILTRATION - The uncontrolled entry of fresh air through openings and cracks in the building structure and fabric.

1-2. Functions of ventilation:

(a) To provide a continuous supply of oxygen for breathing

(b) To remove products of respiration and occupation

(c) To remove contaminants arising from domestic and industrial activities (e.g. Water vapour, noxious gases, heat, etc)

(d) To provide combustion air for fossil fuel appliances (e.g. gas cookers, central heating boilers, etc.)

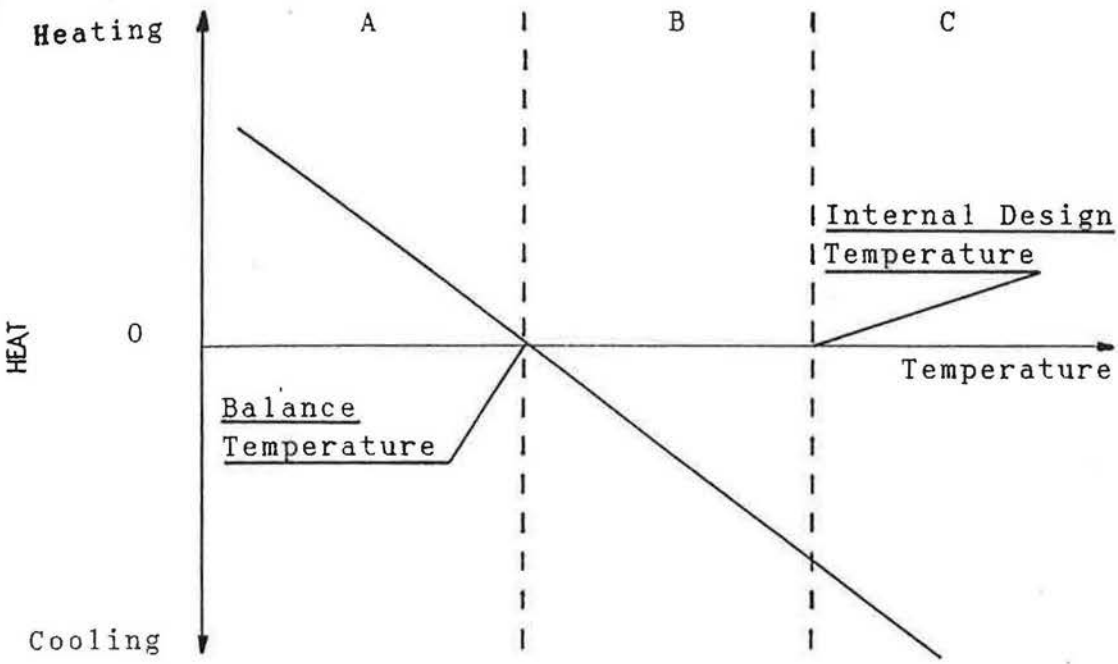
In general, all functions are provided simultaneously, though the provision of combustion air is sometimes provided separately,(e.g. balanced fuel gas appliances). The ventilation requirement is based upon that function which requires most fresh air, and is usually expressed as the minimum rate of fresh air, in litres per second, below which the rate must not fall.

1-3. Ventilation and Heat Loss:

Ventilation has the effect of transferring heat between the inside and the outside of the building. There are three conditions to consider, which may be identified approximately by reference to a graph of building heat transfer against temperature.

Region A	Ventilation rate must be reduced to conserve energy.
Region B	Ventilation rate must be increased to reduce cooling requirement or to control internal temperature.
Region C	Ventilation rate must be reduced to conserve energy.

In the United Kingdom, Region A is of greatest importance, and it can be seen that there is a direct clash between the provision of ventilation for health and safety and the elimination of ventilation for energy conservation.



BUILDING ENERGY BALANCE

1-4. Prediction of Contaminant Concentrations and Minimum Ventilation Rates:

Let F = Flow rate of fresh air into the room

q = Injection rate of contaminant

C_r = Concentration of contaminant in the room

C_s = Concentration of contaminant in the supply air

C_i = Initial concentration of contaminant in the room at the time $t=0$.

Assuming a fully mixed single zone, the equation for C_r

is then:

$$C_r = \left(\frac{FC_s+q}{F+q} \right)(1-e^{-[(F+q)t/V]}) + C_i * e^{-[(F+q)t/V]}$$

The equilibrium concentration is obtained by setting the exponential terms to zero.

If q is injected, then:

$$C_r = \frac{FC_s+q}{F+q}$$

However, if q is generated by direct replacement of a constituent of the air, for example carbon dioxide replacing oxygen, then:

$$C_r = \frac{FC_s+q}{F}$$

1-5. Ventilation requirements:

(a) Oxygen provision and carbon dioxide removal.

Table 1(appendix 1), gives values for an adult male. The carbon dioxide requirement is based on a threshold limit value of 0.5 percent. Notice that the oxygen replacement is much less. Values for an adult female are about 75 percent of those for a male.

(b) Body odour.

The subjective assessment of body odour is highly complex. Two problems are of interest:

(i) The sense of smell fatigues with time. Thus new smells are more noticeable than persistent ones.

(ii) Subjective assessments are affected by the volume of space per person.

Current information and recommendations are still based on work published in 1936 by Yaglou.

(c) Tobacco smoke.

The troublesome products of tobacco smoke are:

(i) The smoke itself and its odour.

(ii) Carbon monoxide, which is toxic.

(iii) Acrolein, which is an irritant.

The current recommended fresh air rate to remove products is 20 m^3 of fresh air per cigarette smoked. The average smoker smokes 1.3 cigarettes per hour, and so the fresh air requirement is 26 m^3 per hour, or 7 litres per second per smoker.

(d) Humidity.

There are numerous activities which generate moisture within buildings. Some typical sources and rates are given in table 2 (appendix 1).

(e) Air for fuel burning appliances.

(i) Open-flued domestic fuel burning appliances.

The typical requirement is 0.8 to 1.1 litres/second for each kilowatt of heat output.

(ii) Flueless appliances.

The typical requirements are given in table 3 (appendix 1).

(f) Other contaminants.

These are evaluated from a suitable criterion for the maximum possible concentration, usually a threshold limit value, or TLV.

1-6. General Recommendations:

The most usual contaminants can be evaluated in terms of a typical room. Diagram 1(appendix 1), shows the result of this for a room of a particular height. The required flow rate, F, is often converted to an air change rate, by means of the equation.

$$n = \frac{F}{V}$$

Where n is the ventilation rate.

CHAPTER 2

THE MEASUREMENT OF NATURAL VENTILATION RATES

SINGLE ZONE THEORY

2-0. THE MEASUREMENT OF NATURAL VENTILATION RATES - SINGLE ZONE THEORY:

The most commonly used means for determining the rate of ventilation for a room or building is the tracer gas technique. The basic concepts of this method have many variations which are used in practice.
In using this technique, a series of assumptions have to be made about how the building is acting in practice with respect to its method of ventilation. As a whole, the building will act as a series of cells, each of which are interconnected and/or connected to the outside air by one or a series of openings. The cells themselves can be of any size, from a whole room to the space between a suspended floor and ceiling. The openings between cells can be equally diverse, varying from doorways or windows to construction joints and holes in the fabric due to the penetration of services. An air flow will occur between these cells through the apertures, when a pressure difference is set up accross them, this can be the result of wind action on the exterior of the building, a temperature differential between cells or a difference in the relative humidity

level. In industrial buildings, it may well be the situation that there is one main cell which is the production area itself. This main cell may be divided into a number of hypothetical cells, which have little effect on the performance in terms of ventilation, of the main cell. Such small cells could be two or three offices, conveniences, and a rest area for the employees.

To determine the ventilation rate, consideration has to be made as to whether the method should be applied to each room within the building individually, and so establish how each of the major cells is reacting in relation to the cells around it and to the outside, or whether the building is assumed to act as one large cell dependant mainly upon the external conditions applied to it.

1. The tracer gas technique:

This is based on the principle of releasing a known gas, called a tracer gas, into the air of a building and then monitoring the concentration of the gas over a period of time. The rate of change in the concentration of the tracer gas in an air space is expressed as follows:

$$V \frac{dC}{dt} = C_0.\bar{v} + G - C.v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}1)$$

Where: V = volume of the ventilated space

C = concentration of the tracer gas in the ventilated space at time t

C_0 = concentration of the tracer gas in the outside air

v = rate at which the air leaves the ventilated space, in volume per unit time

$\bar{v}$ = rate at which the external air enters the ventilated space

G = net rate of generation of tracer gas in the ventilated space

The equation is based upon the assumption that the tracer gas is uniformly mixed throughout the air space in question, and that there is no transfer of the tracer from one part of that space to another during the time of measurement, which means that the incoming air must mix perfectly with the air within the ventilated space, and any generation of the gas in the space must also mix perfectly. Such assumptions cannot be guaranteed in any situation, unless an artificial stirring of the air is introduced during the measurement period. This will, however, affect the overall ventilation rate by creating a much more even pressure throughout the ventilated space. It is generally assumed that after initial mixing to a uniform concentration throughout the space, then this does ventilate at a constant rate throughout.

There are several methods of using a tracer gas to monitor the ventilation rate of a building or room. For the purpose of this research, only the rate of decay method will be considered.

2-2. Rate of decay method:

A fixed quantity of the tracer gas is released into the room or rooms, and mixed evenly throughout to produce as near a uniform concentration within the space as possible. The rate at which the tracer gas concentration decays is a measure of the ventilation rate. The actual rate of decay of the tracer should be exponential in form, as shown in figure 2-2a, because, assuming perfect mixing of the gas, incoming air does not drive out the old air before it, like a piston. In practice, it has been shown that even with good mixing of the external air with the contaminated internal air, one 'air change' in fact only replaces 63 percent of the room air. As mixing is not homogeneous, this rate will tend to be lower or higher,but the extent of this is dependant on the degree of mixing.

In practice, the concentration of tracer is measured over a period of time, either by continuous monitoring, or by taking spot samples at regular intervals. The rate of decay can then be plotted, as shown in figure 2-2a, and the exponential curve of best fit is allocated so as to obtain the ventilation rate. The major problem with the rate of decay method, is obtaining a near uniform concentration of the tracer in the early stages, before the measurement period commences. In a small space, this can be achieved with sufficient accuracy by dispersing the gas into the path of a stream of circulating air, as generated by a fan unit.

2-3. Use of the rate of decay method:

With this method for measuring the ventilation rate, certain simplifications can be made to equation (2-1).

Firstly, the concentration of the gas that is present in the external air (or any other air entering the space under consideration) is negligible, hence the value of tracer gas is zero.

Secondly, none of the tracer is absorbed within the room and there is no source of generation of contaminant within the space, hence, G is also zero. This second approximation is not in practice the actual situation, because some proportion of tracer gas is soluble in water. However, the equation simplifies to:

$$\frac{dC}{dt} = \frac{-v.C}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}2)$$

By integration, this becomes:

$$C = C_{init}.\ e^{-(vt/V)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}3)$$

Where: C_{init} = original concentration of the tracer

By taking the logarithm of this equation, we obtain:

$$Ln\ \frac{C}{C_{init}} = \frac{-v.t}{V} = -I.t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}4)$$

Where: I = rate of air change per unit time

By plotting the change in concentration in its logarithmic form versus time in hours, a straight line of decay is produced, as shown in figure 2-2b. The slope of this straight line is the air change rate per unit time, I. The steeper the slope of the line (or line of best fit), the faster the air change rate, and vice versa.

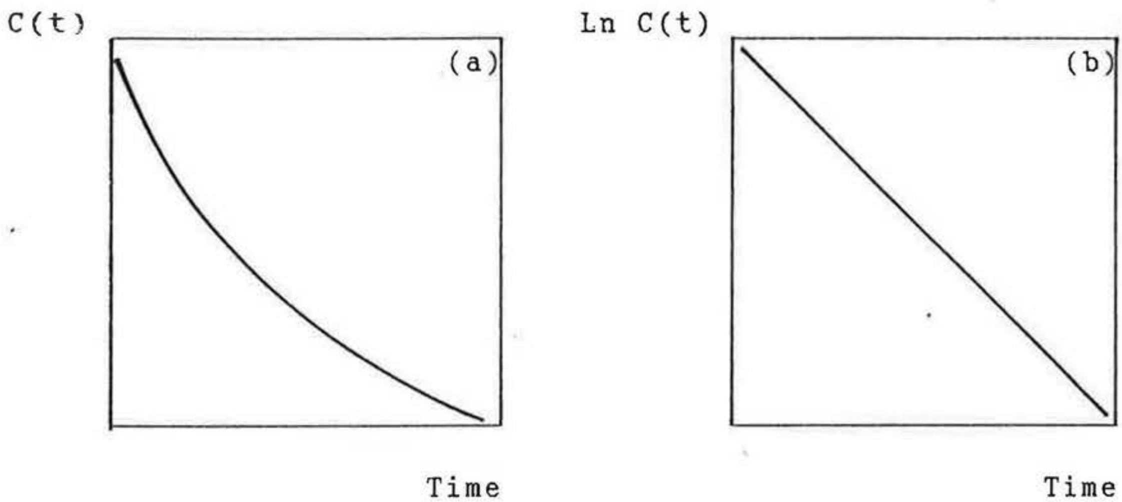


Figure 2-2: Rate of decay of tracer gas with respect to time.

CHAPTER 3

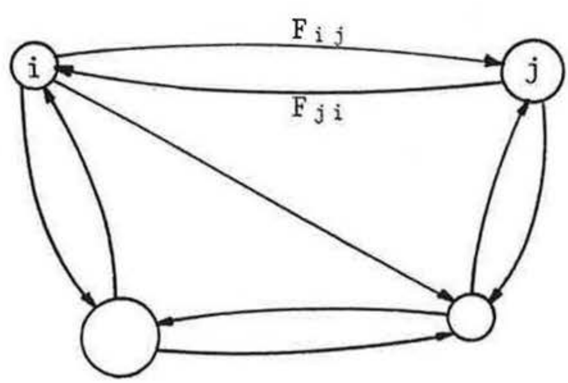
THE MEASUREMENT OF NATURAL VENTILATION RATES AND INTERNAL AIR MOVEMENT

THE MULTIZONE THEORY

3-0. THE MEASUREMENT OF NATURAL VENTILATION AND INTERNAL AIR MOVEMENT - THE MULTIZONE THEORY:

Estimates of air infiltrations in buildings based on the tracer gas measurements have usually assumed the building to be a single perfect mixing chamber in which incoming air is instantaneously and uniformly diffused to all parts of the interior. In fact, some parts of the building exchange air with other parts only very slowly so that actual mixing is far from instantaneous. The theory of the multizone model (Sinden) illustrates this statement.

3-1. The theory of the Multizone Model:



$n + 1$ = Number of zones, zone $n = 0$ representing the outside air

F_{ij} = Flow rate from zone i to zone j

V_i = Volume of zone i

$C_i(t)$ = Contaminant concentration in zone i

Q_i = Contaminant supply rate in zone i

Assumptions: The air in each zone is fully mixed.

Air is transferred between zones instantaneously.

Take a balance of the contaminant concentration in zone j, at time t:

$$V_j.\dot{C}_j(t) = Q_j + \sum_{i=0}^{n} F_{ij}.C_i(t) - \sum_{k=0}^{n} F_{jk}.C_j(t)$$

Note: F_{jj} = 0. Also, since $C_j(t)$ is a constant in the second summation:

$$V_j.\dot{C}_j(t) = Q_j + \sum_{i=0}^{n} F_{ij}.C_i(t) - C_j(t) \sum_{k=0}^{n} F_{jk}$$

The total flow into any zone must balance that out of the zone (conservation), and so, for zone j:

$$\sum_{i=0}^{n} F_{ij} = \sum_{k=0}^{n} F_{jk} = S_j \quad \ldots\ldots\ldots\ldots\ldots(3\text{-}1)$$

To solve the set of equations, it is easier to simplify by choosing specific cases first.

3-11. Case 1 : Zero contaminant injection:

This corresponds to all versions of the tracer gas decay experiment.

$Q_j = 0$, for all j.

The equation becomes:

$$V_j.\dot{C}_j(t) = \sum_{i=0}^{n} F_{ij}.C_i(t) - C_j(t).S_j \quad \text{........(3-2)}$$

Thus:

$$V_0.\dot{C}_0(t) = -S_0.C_0(t)+F_{10}.C_1(t)+F_{20}.C_2(t)+...+F_{n0}.C_n(t)$$

$$V_1.\dot{C}_1(t) = F_{01}.C_0(t)-S_1.C_1(t)+F_{21}.C_2(t)+...+F_{n1}.C_n(t)$$

$$V_2.\dot{C}_2(t) = F_{02}.C_0(t)+F_{12}.C_1(t)-S_2.C_2(t)+...+F_{n2}.C_n(t)$$

$$V_n.\dot{C}_n(t) = F_{0n}.C_0(t)+F_{1n}.C_1(t)+F_{2n}.C_2(t)+...-S_n.C_n(t)$$

In matrix form:

$$V \,.\, \dot{\underline{C}}(t) = F \,.\, \underline{C}(t) \quad \text{..........................(3-3)}$$

Where: $\underline{C}(t) = \sum_{i=0}^{n} C\ (t)$; $\dot{\underline{C}}(t) = \sum_{i=0}^{n} \dot{C}\ (t)$

$$V = \begin{bmatrix} V_0 & 0 & 0 & \cdots\cdots & 0 \\ 0 & V_1 & 0 & \cdots\cdots & 0 \\ 0 & 0 & V_2 & \cdots\cdots & 0 \\ . & . & . & \cdots\cdots & . \\ . & . & . & \cdots\cdots & . \\ 0 & 0 & 0 & \cdots\cdots & V_n \end{bmatrix}$$

$$F = \begin{bmatrix} -S_0 & F_{10} & F_{20} & \cdots\cdots & F_{n0} \\ F_{01} & -S_1 & F_{21} & \cdots\cdots & F_{n1} \\ F_{02} & F_{12} & -S_2 & \cdots\cdots & F_{n2} \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ F_{0n} & F_{1n} & F_{2n} & \cdots\cdots & -S_n \end{bmatrix}$$

Note that:

(i) Matrix V is a diagonal matrix with non-negative elements;

(ii) In matrix F, all row sums and all column sums are zero, because of the conservation equation.

The solution to equations (3-3) may be written in the form:

$$C_0(t) = X_0 \cdot e^{\lambda t}$$
$$C_1(t) = X_1 \cdot e^{\lambda t}$$
$$\vdots$$
$$C_n(t) = X_n \cdot e^{\lambda t}$$

Substituting this solution back into the equations gives:

$$V_0.\lambda.X_0.e^{\lambda t} = -S_0.X_0.e^{\lambda t} + F_{10}.X_1.e^{\lambda t} + \ldots + F_{n0}.X_n.e^{\lambda t}$$

$$V_1.\lambda.X_1.e^{\lambda t} = F_{01}.X_0.e^{\lambda t} - S_1.X_1.e^{\lambda t} + \ldots + F_{n1}.X_n.e^{\lambda t}$$

$$\vdots$$

$$V_n.\lambda.X_n.e^{\lambda t} = F_{0n}.X_0.e^{\lambda t} + F_{1n}.X_1.e^{\lambda t} + \ldots - S_n.X_n.e^{\lambda t}$$

Cancelling $e^{\lambda t}$, and rearranging:

$$\begin{aligned}
-(S_0+V_0.\lambda)X_0 + F_{10}.X_1 + \ldots\ldots + F_{n0}.X_n &= 0 \\
F_{01}.X_0 - (S_1+V_1.\lambda)X_1 + \ldots\ldots + F_{n1}.X_n &= 0 \quad \ldots (3\text{-}4) \\
&\vdots \\
F_{0n}.X_0 + F_{1n}.X_1 + \ldots\ldots -(S_n+V_n.\lambda)X_n &= 0
\end{aligned}$$

This is a set of n linear equations and n unknowns:

X_0, X_1, ..., X_n.

Note that it is a set of homogeneous equations, that is, the R.H.S is zero. This means that:

(i) There is no unique solution, and all solutions can only be determined to a ratio (that is to within an arbitrary constant).

(ii) One solution is: $X_0 = X_1 = \ldots\ldots\ldots = X_n = 1$, provided that $\lambda = 0$

This can be seen by substituting these values in the equations, and remembering from equation (3-1) that:

$$S_j = \sum_{i=0}^{n} F_{ij}$$

(iii)Other solutions will exist if the determinant of the coefficients of X is zero.

The determinant is zero if:

$$\begin{vmatrix} -(S_0+V_0.\lambda) & F_{10} & \ldots & F_{n0} \\ F_{01} & -(S_1+V_1.\lambda) & \ldots & F_{n1} \\ \ldots. & \ldots.. & \ldots & . \\ \ldots. & \ldots.. & \ldots & . \\ F_{0n} & F_{1n} & \ldots & -(S_n+V_n.\lambda) \end{vmatrix} = 0 \qquad (3\text{-}5)$$

If this determinant is expanded, we obtain a polynomial in λ, of power n+1. There are therefore, n+1 possible values of λ, and for a value of λ, there must be a set of values for X_0, X_1, ..., X_n.

We have already found one value of λ, and one set of X's, namely:

$$\lambda = 0 \quad , \quad X_0 = X_1 = \ldots.. = X_n = 1 \qquad (\text{or } \underline{X} = 1)$$

The λ's are called the eigenvalues of the coefficient matrix, and the sets of X's are called the eigenvectors. To find all the eigenvalues and their corresponding eigenvectors, first obtain the eigenvalues from equation (3-5), and then substitute each eigenvalue back into equations (3-4), and obtain the eigenvector. It is convenient to label the eigenvalues and their eigenvectors as:

$$\lambda_0 \ , \quad \underline{X}_0$$

$$\lambda_1 \ , \quad \underline{X}_1$$

$$\lambda_k \ , \quad \underline{X}_k$$

$$\lambda_n \ , \quad \underline{X}_n$$

Where:

$$\underline{X}_0 = \begin{bmatrix} X_{01} \\ X_{02} \\ X_{03} \\ \\ X_{0n} \end{bmatrix} \quad ; \quad \underline{X}_k = \begin{bmatrix} X_{k1} \\ X_{k2} \\ X_{k3} \\ \\ X_{kn} \end{bmatrix}$$

We now have alternative solutions to equation (3-3) which if using vector notation, can be expressed in the general form:

$$\underline{C}(t) = \underline{X}_0 \cdot e^{\lambda_0 t} \ , \quad \underline{C}(t) = \underline{X}_1 \cdot e^{\lambda_1 t} \qquad \text{etc.}$$

Other solutions may be formed by linear combinations. In other words, multiply the first solution by a constant a_0, the second by a_1, up to a_n, and add the results, which, if using vector notation, can be expressed in the general form:

$$\underline{C}(t) = \sum_{k=0}^{n} a_k \cdot \underline{X}_k \cdot e^{\lambda_k t} \qquad \text{..................(3-6)}$$

Equation (3-6) is the general solution to equation

(3-3), in which λ_k and X_k are the eigenvalues and eigenvectors of the matrix. $(F - \lambda . V)$, and a_k are a set of constants. The values of a_k are found by substituting initial conditions in equation (3-6). Then substituting (3-6) into (3-3) gives:

$$\lambda . \underline{V} . X = F . \underline{X} \quad \text{OR} \quad 0 = (F - \lambda . V) . \underline{X} \quad \ldots\ldots\ldots\ldots(3\text{-}7)$$

<u>Properties of the eigenvalues and their eigenvectors</u>:

The format of the matrices V and F causes the eigenvalues and eigenvectors to have certain features.

(a) One eigenvalue is zero and the eigenvector has equal components.

$$\lambda_0 = 0 \quad ; \quad \underline{X}_0 = (1, 1, \ldots\ldots, 1)$$

(b) The other eigenvalues and eigenvectors may be real or complex. However, complex eigenvalues and eigenvectors always occur in conjugate pairs.

Suppose $\quad \lambda = \alpha + i\beta \quad$ and $\quad \underline{X} = \underline{\gamma} + i\,\underline{\delta}$

Where α and $\underline{\gamma}$ are the real parts, and β and $\underline{\delta}$ are the imaginary parts, and $i^2 = -1$. Substituting in equation

(3-7) gives:

$$(\alpha + i\beta)V(\underline{\gamma} + i\underline{\delta}) = F(\underline{\gamma} + i\underline{\delta})$$

$$V[\alpha\underline{\gamma} - \beta\underline{\delta} + i(\alpha\underline{\gamma} + \beta\underline{\delta})] = F(\underline{\gamma} + i\underline{\delta})$$

Real part : $V(\alpha\underline{\gamma} - \beta\underline{\delta}) = F\underline{\gamma}$

Imaginary part: $V(\alpha\underline{\gamma} + \beta\underline{\delta}) = F\underline{\delta}$

Substituting the complex conjugates ($\lambda = \alpha - i\beta$), and ($\underline{X} = \underline{\gamma} - i\underline{\delta}$), produces exactly the same result. Note that, since the first eigenvalue λ_0 is real, then if there are an even number of zones there must be at least one more real λ and $\underline{X}$.

(c) All eigenvalues, except $\lambda_0 = 0$, must have negative real parts. This must be so, because as time progresses, the concentrations must approach a steady value. If any of the λ's were positive, then the concentration in at least one zone would become infinite A mathematical proof is given by Sinden.

3-12. Case 2 , One zone representing the outside:

Let zone zero represent the outside air, and let it be assumed that there is no contaminant present in the outside air. Then:

$$V_0 = \text{"infinite"}$$

and $C_0(t) = \dot{C}_0(t) = 0$

Also, for the case of contaminant decay, the concentra_ tions in all other zones must tend to zero, i.e. $C_i(\text{infinite}) = 0$, and hence, $a_0 = 0$. Thus the equation for $C_0(t)$ can be deleted as being redundant, and all terms involving $C_0(t)$ disappear. The equations thus become:

$$V_1.\dot{C}_1(t) = -S_1.C_1(t) + F_{21}.C_2(t) + \ldots. + F_{n1}.C_n(t)$$

$$V_2.\dot{C}_2(t) = F_{12}.C_1(t) - S_2.C_2(t) + \ldots. + F_{n2}.C_n(t)$$

$$V_n.\dot{C}_n(t) = F_{1n}.C_1(t) + F_{2n}.C_2(t) + \ldots. - S_n.C_n(t)$$

OR: $V.\dot{\underline{C}}(t) = F.\underline{C}(t)$

Where:

$$\underline{C}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \\ \\ C_n(t) \end{bmatrix} \qquad \dot{\underline{C}}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \\ \\ C_n(t) \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 & 0 & 0 \ldots\ldots & 0 \\ 0 & V_2 & 0 \ldots\ldots & 0 \\ 0 & 0 & V_3 \ldots\ldots & 0 \\ . & . & . \ldots\ldots & . \\ . & . & . \ldots\ldots & . \\ 0 & 0 & 0 \ldots\ldots & V_n \end{bmatrix}$$

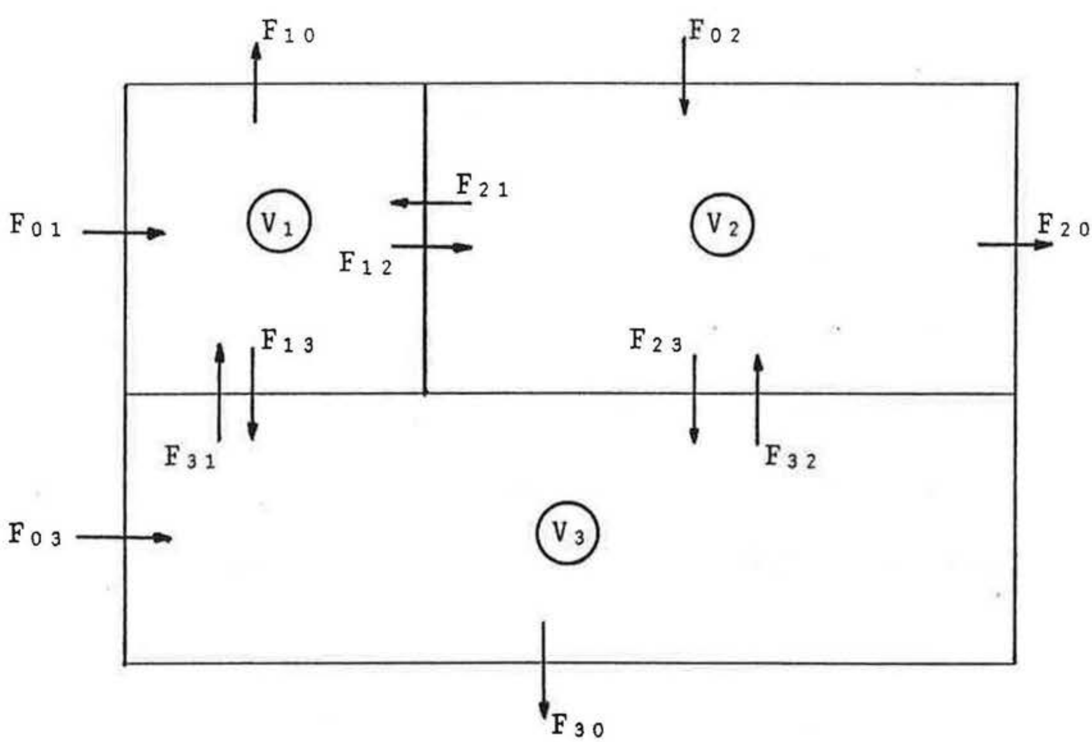
$$F = \begin{bmatrix} -S_1 & F_{21} & F_{31} & \cdots\cdots & F_{n1} \\ F_{12} & -S_2 & F_{32} & \cdots\cdots & F_{n2} \\ F_{13} & F_{23} & -S_3 & \cdots\cdots & F_{n3} \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ F_{1n} & F_{2n} & F_{3n} & \cdots\cdots & -S_n \end{bmatrix}$$

And the solution is:

$$\underline{C}(t) = \sum_{k=1}^{n} a_k . \underline{X}_k . e^{\lambda_k t}$$

Note that $\lambda_0 = 0$, and $\underline{X}_0 = (1,1,\ldots\ldots,1)$ no longer appear in the solution because $a_0 = 0$. The largest eigenvalue to appear in the solution (called the dominant eigenvalue) is thus negative.

3-2. Three zone building connected to outside:



3-21. Redundant case:

The total number of eigenvalues of the coefficient matrix is always equal to the number of zones. However, some or all of the eigenvalues may have the same value. When this happens, it is no longer possible to obtain a solution by the eigenvalue method. The implications of this can be worked out for two or three zone cases.

The eigenvalues will be the roots of a cubic equation:

$$\lambda^2 + a.\lambda^3 + b.\lambda + c = 0$$

Which is obtained from:

$$\begin{vmatrix} -(S_1+V_1.\lambda) & F_{21} & F_{31} \\ F_{12} & -(S_2+V_2.\lambda) & F_{32} \\ F_{13} & F_{23} & -(S_3+V_3.\lambda) \end{vmatrix} = 0$$

Expanding the determinant and substituting $r_i = S_i / V_i$, $f_{ij} = F_{ij} / V_j$, it can be shown that:

$$a = r_1 + r_2 + r_3$$

$$b = r_1 r_2 + r_2 r_3 + r_3 r_1 - f_{12} f_{21} - f_{23} f_{32} - f_{13} f_{31}$$

$$c = r_1 r_2 r_3 - r_1 f_{23} f_{32} - r_2 f_{13} f_{31} - r_3 f_{12} f_{21} - f_{12} f_{23} f_{31} - f_{13} f_{32} f_{21}$$

From elementary algebra, by substituting $\lambda = x-(a/3)$, and thus changing the variable to x, the cubic equation becomes:

$$x^3 + p . x + q = 0$$

Where: $p = b - \frac{a^2}{3}$ and $q = c - \frac{a.b}{3} + \frac{2a^3}{27}$

The cubic equation has repeated roots when:

$$4 . p^3 + 27 . q^2 = 0$$

Where:

$$p = -\{f_{12}f_{21}+f_{23}f_{32}+f_{31}f_{13}+\frac{1}{6}[(r_1-r_2)^2+(r_2-r_3)^2+(r_3-r_1)^2]\}$$

Since all f_{ij} and all r_i are real non-negative numbers, the p is either negative or null, and hence it is possible to find real values of q which satisfy the condition for repeated roots. Thus, there must be some combinations of r_i and f_{ij} which give repeated roots. A particular case occurs when $p = q = 0$. In this case, by substitution, we have:

$$b = a^2/3 \qquad \text{and} \qquad c = a^3/27$$

Substituting for b and c in the equation for λ shows that in this case all three roots are equal and are given by:

$$\lambda = -a/3$$

Obviously, $p = 0$ when $r_1 = r_2 = r_3$ and $f_{21} = f_{32} = f_{31} = 0$. The proof of this statement is given in chapter 4-1(i.e. Application of the redundant case to a three zone building connected to outside).

Repeated eigenvalues can occur in buildings with larger numbers of zones than two or three zone cases. The condition $r_1 = r_2 = r_3 = \ldots\ldots = r_n$ with some $f_{ij} = 0$ is likely to cause this to happen. The physical significance of a repeated eigenvalue in the two or three zone cases is that one zone in the system does not receive a flow from any other zone, except zone 0 which is the outside.

3-22. The non-redundant case:

As for the redundant case, the eigenvalues will be the roots of a cubic equation:

$$\lambda^3 + a\,.\lambda^2 + b\,.\lambda + c = 0$$

Which is obtained from:

$$\begin{vmatrix} -(S_1+V_1.\lambda) & F_{21} & F_{31} \\ F_{12} & -(S_2+V_2.\lambda) & F_{32} \\ F_{13} & F_{23} & -(S_3+V_3.\lambda) \end{vmatrix} = 0$$

Giving:

$$-(S_1+V_1\lambda)\begin{vmatrix} -(S_2+V_2\lambda) & F_{32} \\ F_{23} & -(S_3+V_3\lambda) \end{vmatrix} - F_{21}\begin{vmatrix} F_{12} & F_{32} \\ F_{13} & -(S_3+V_3\lambda) \end{vmatrix}$$

$$+F_{31}\begin{vmatrix} F_{12} & -(S_2+V_2\lambda) \\ F_{13} & F_{23} \end{vmatrix} = 0$$

Which can then be writen as follows:

$$-(S_1+V_1\lambda)[(S_2+V_2\lambda)(S_3+V_3\lambda)-F_{32}F_{23}]+F_{21}[F_{12}(S_3+V_3\lambda)+F_{32}F_{13}]$$
$$+ F_{31}[F_{12}F_{23}+(S_2+V_2\lambda)F_{13}] = 0$$

The solution of this equation, as mentioned above, is a cubic equation of the form:

$$\lambda^3 + a\,.\lambda^2 + b\,.\lambda + c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}2a)$$

In order to find the roots of this cubic equation, the iteration method is used, hence equation (3-2a) becomes:

$$\lambda = \sqrt[3]{-(a.\lambda^2 + b.\lambda + c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}2b)$$

The first step consists in replacing λ in the cube root by an arbitrary value which should be as close to the expected solution as possible. By doing so, a new value of λ will be found and, in turn, replaced in the cube root. This operation must be repeated untill two following values of λ are found to be the same, or very close to each other, according to the degree of precision required.

Once a value of λ has been found, equation (3-2a) can be factorised in order to obtain a quadratic equation for which the roots are easily found.

As mentioned in the previous paragraph, complex solutions are very likely to occur.

From there, to find the eigenvectors, substitute each eigenvalue in turn in equation (3-4), giving:

$$-(S_1+V_1\lambda)X_1 + F_{21}X_2 + F_{31}X_3 = 0$$

$$F_{12}X_1 - (S_2+V_2\lambda)X_2 + F_{32}X_3 = 0$$

$$F_{13}X_1 + F_{23}X_2 - (S_3+V_3\lambda)X_3 = 0$$

In matrix form, we would have:

$$\begin{bmatrix} -(S_1+V_1\lambda) & F_{21} & F_{31} \\ F_{12} & -(S_2+V_2\lambda) & F_{32} \\ F_{13} & F_{23} & -(S_3+V_3\lambda) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From which we can calculate the eigenvectors for each value of λ.

Once the eigenvectors are found, replace the values obtained in equation (3-6):

$$C_1(t) = a_1.X_{11}.e^{\lambda_1 t} + a_2.X_{21}.e^{\lambda_2 t} + a_3.X_{31}.e^{\lambda_3 t}$$
$$C_2(t) = a_1.X_{12}.e^{\lambda_1 t} + a_2.X_{22}.e^{\lambda_2 t} + a_3.X_{32}.e^{\lambda_3 t}$$
$$C_3(t) = a_1.X_{13}.e^{\lambda_1 t} + a_2.X_{23}.e^{\lambda_2 t} + a_3.X_{33}.e^{\lambda_3 t}$$

from which we obtain the coefficients a_i, after having substituted $C_i(t)$ by its boundary conditions, in the equations.

Once the values of the coefficients are obtained, substitute the different values for a_i, F_{ij}, and V_i in equation (3-3) in order to find $C_i(t)$:

$$V_1.\dot{C}_1(t) = -(S_1+V_1\lambda)C_1(t) + F_{21}C_2(t) + F_{31}C_3(t)$$

$$V_2.\dot{C}_2(t) = F_{12}C_1(t) - (S_2+V_2\lambda)C_2(t) + F_{32}C_3(t)$$

$$V_3.\dot{C}_3(t) = F_{13}C_1(t) + F_{23}C_2(t) - (S_3+V_3\lambda)C_3(t)$$

CHAPTER 4

FINDING THE CONCENTRATIONS FROM KNOWN FLOWS

4-0. FINDING THE CONCENTRATIONS FROM KNOWN FLOWS:

It has been decided to work on a three rather than a two zone building, because the two zone model does not give complex eigenvalues, unlike the three zone case. On another hand, the three zone model is quite sufficient for the purpose of this research.

This chapter is the application of the method stated in chapter 3.0, to a three zone building connected to the outside.

Data was created by means of computer programmes both for the redundant and the non-redundant cases. These data tables give the results which would be obtained if an ideal tracer gas decay experiment was carried out. Tracer decay may be expressed in terms of time-constants

$$t_c = -\frac{1}{\lambda_1}$$

Normal experimental durations are typically 2 hours maximum, and λ_1 is usually between -1 and -4, i.e. t_c is thus from 0.25 to 1 hour.

Theoretically, decay curves contain no useful information after approximately $2t_c$, i.e. after 0.5 to 2 hours.

Assuming sampling at 1 minute intervals, typical number of data points per zone is 30 to 120.

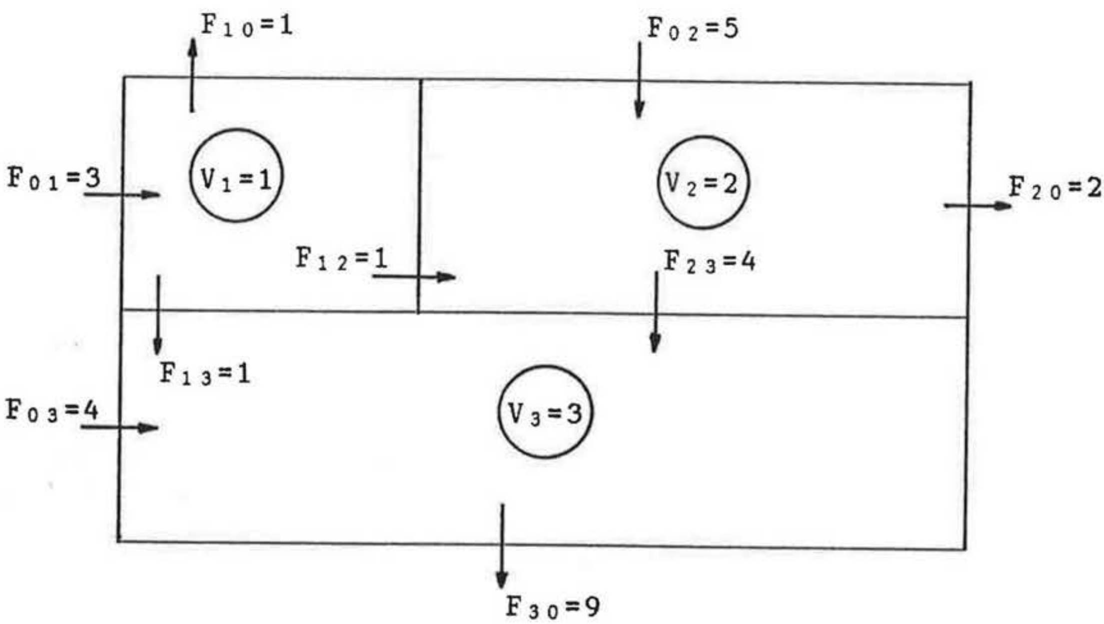
Thus, it is reasonable to assume 55 data points over a time interval of $2.75t_c$. Therefore, data points occur at intervals of $2.75t_c/55$, or $t_c/20$.

4-1. Three zone redundant case:

The volumes chosen in this case are ratios rather than real values, and their choice is arbitrary and could be modified. Once the volumes are specified, the values for the flows have to be attributed so as to obtain:

$$r_1 = r_2 = r_3 = r$$

Which as demonstrated in chapter 3-21, is the condition to meet for redundancy.



$F_{32} = F_{21} = F_{31} = 0$

$S_1 = 3 = 1 + 1 + 1 = 3$

$S_2 = 5 + 1 = 4 + 2 = 6$

$S_3 = 4 + 4 + 1 = 9 = 9$

$r_1 = 3/1 = 3;\ r_2 = 6/2 = 3;\ r_3 = 9/3 = 3$

or: $r_1 = r_2 = r_3 = r = 3$

Rewrite the equations as:

$$\dot{C}_1(t) = -\frac{S_1}{V_1} \cdot C_1(t) + \frac{F_{21}}{V_1} \cdot C_2(t) + \frac{F_{31}}{V_1} \cdot C_3(t)$$

$$\dot{C}_2(t) = \frac{F_{12}}{V_2} \cdot C_1(t) - \frac{S_2}{V_2} \cdot C_2(t) + \frac{F_{32}}{V_2} \cdot C_3(t)$$

$$\dot{C}_3(t) = \frac{F_{13}}{V_3} \cdot C_1(t) + \frac{F_{23}}{V_3} \cdot C_2(t) - \frac{S_3}{V_3} \cdot C_3(t)$$

Substituting r=S/V and eliminating terms which are zero:

$$\dot{C}_1(t) = -r \cdot C_1(t)$$

$$\dot{C}_2(t) = \frac{F_{12}}{V_2} \cdot C_1(t) - r \cdot C_2(t)$$

$$\dot{C}_3(t) = \frac{F_{13}}{V_3} \cdot C_1(t) + \frac{F_{23}}{V_3} \cdot C_2(t) - r \cdot C_3(t)$$

Try: $C_1(t) = a_1 \cdot e^{\lambda t}$

$$\dot{C}_1(t) = a_1 \cdot \lambda \cdot e^{\lambda t} = \lambda \cdot C_1(t)$$

Therefore:

$$\lambda \cdot C_1(t) = -r \cdot C_1(t) \quad \text{and} \quad \lambda = -r$$

Substitute in the equation for $C_2(t)$:

$$\dot{C}_2(t) = \frac{F_{12}}{V_2} \cdot a_1 \cdot e^{\lambda t} + \lambda \cdot C_2(t)$$

$$\dot{C}_2(t) - \lambda \cdot C_2(t) = \frac{F_{12}}{V_2} \cdot a_1 \cdot e^{\lambda t}$$

As before, multiply by $e^{-\lambda t}$, and integrate to give:

$$C_2(t) = \frac{F_{12}}{V_2} \cdot a_1 \cdot t \cdot e^{\lambda t} + a_2 \cdot e^{\lambda t}$$

Substitute in the equation for $\dot{C}_3(t)$:

$$\dot{C}_3(t) - \lambda . C_3(t) = [(\frac{F_{13}}{V_3} . a_1 + \frac{F_{23}}{V_3} . a_2) + \frac{F_{23}}{V_3} . \frac{F_{12}}{V_2} . a_1 . t] e^{\lambda t}$$

As before, multiply by $e^{-\lambda t}$, and integrate to give:

$$e^{-\lambda t} . \dot{C}_3(t) - \lambda . e^{-\lambda t} . C_3(t) = (\frac{F_{13}}{V_3} . a_1 + \frac{F_{23}}{V_3} . a_2) + \frac{F_{23}}{V_3} . \frac{F_{12}}{V_2} . a_1 . t$$

$$e^{-\lambda t} . C_3(t) = \left(\frac{F_{13}}{V_3}.a_1 + \frac{F_{23}}{V_3}.a_2\right)t + \frac{1}{2} . \frac{F_{23}}{V_3}.\frac{F_{12}}{V_2}.a_1.t^2 + a_3$$

From which:

$$C_3(t) = e^{\lambda t} \left[a_3 + t\left(\frac{F_{13}}{V_3}.a_1 + \frac{F_{23}}{V_3}.a_2\right) + \frac{1}{2}.\frac{F_{23}}{V_3}.\frac{F_{12}}{V_2}.a_1.t^2\right]$$

Substituting values:

$$C_1(t) = a_1 . e^{-3t}$$

$$C_2(t) = a_2 . e^{-3t} + \frac{a_1}{2} . t . e^{-3t}$$

$$C_3(t) = a_3 .e^{-3t} + \left(\frac{a_1}{3} + 4\frac{a_2}{3}\right)t.e^{-3t} + \frac{a_1}{3} . t^2 .e^{-3t}$$

For initial conditions $C_1(0)=1$, $C_2(0)=0$, $C_3(0)=0$, we have $a_1=3$, $a_2=a_3=0$, and:

$$C_1(t) = e^{-3t}$$

$$C_2(t) = \frac{1}{2} . t . e^{-3t}$$

$$C_3(t) = \frac{1}{3} . t . e^{-3t} + \frac{1}{3} . t^2 . e^{-3t}$$

From which we obtain:

$$\dot{C}_1(t) = -3 \, . \, e^{-3t}$$

$$\dot{C}_2(t) = -1.5 \, . \, (1-3t) \, . \, e^{-3t}$$

$$\dot{C}_3(t) = [(-t^2) - \frac{1}{3} t + \frac{1}{3}] \, . \, e^{-3t}$$

As mentionned in the introduction to this chapter, in order to facilitate the calculations of the concentrations and their differentials, but, most of all, to create data (Table 4-1.) which gives the results of an ideal tracer decay experiment. The computer programme is given in appendix 2.

Also, to illustrate this data, the decay curves for each of the three zones over a time interval of $2.75t_c$ (Graph 4-1.) were drawn by means of a computer programme also given in appendix 2.

Table 4-1. "DATA FOR THE REDUNDANT CASE"

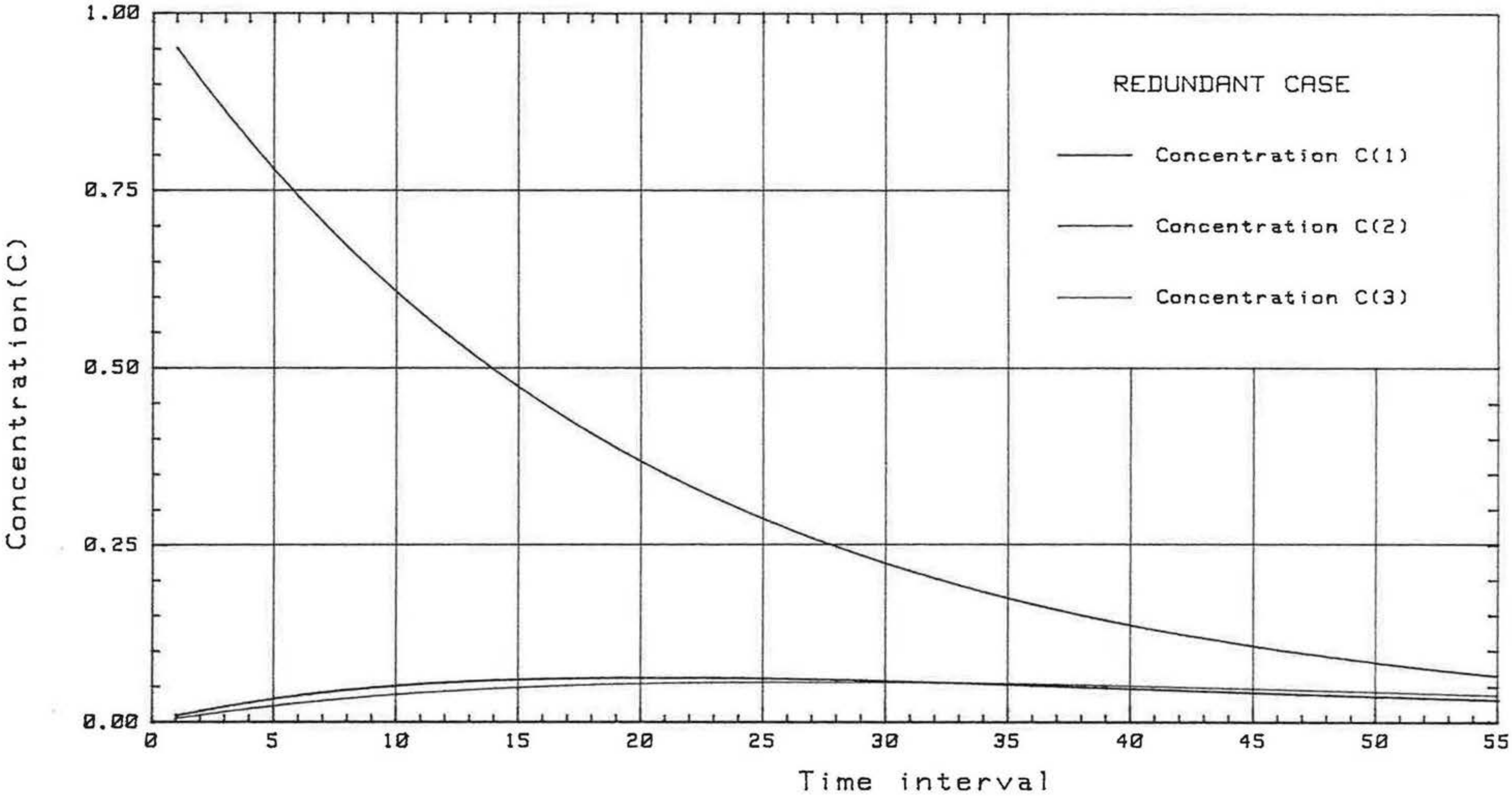
J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
01	0.951229	-2.853688	0.007927	0.451834	0.005373	0.311528
02	0.904837	-2.714512	0.015081	0.407177	0.010389	0.290553
03	0.860708	-2.582124	0.021518	0.365801	0.015062	0.270406
04	0.818731	-2.456192	0.027291	0.327492	0.019407	0.251077
05	0.778801	-2.336402	0.032450	0.292050	0.023436	0.232559
06	0.740818	-2.222455	0.037041	0.259286	0.027163	0.214837
07	0.704688	-2.114064	0.041107	0.229024	0.030602	0.197900
08	0.670320	-2.010960	0.044688	0.201096	0.033764	0.181731
09	0.637628	-1.912884	0.047822	0.175348	0.036664	0.166315
10	0.606531	-1.819592	0.050544	0.151633	0.039312	0.151633
11	0.576950	-1.730849	0.052887	0.129814	0.041722	0.137667
12	0.548812	-1.646435	0.054881	0.109762	0.043905	0.124397
13	0.522046	-1.566137	0.056555	0.091358	0.045872	0.111805
14	0.496585	-1.489756	0.057935	0.074488	0.047635	0.099869
15	0.472367	-1.417100	0.059046	0.059046	0.049205	0.088569
16	0.449329	-1.347987	0.059911	0.044933	0.050591	0.077884
17	0.427415	-1.282245	0.060550	0.032056	0.051804	0.067793
18	0.406570	-1.219709	0.060985	0.020328	0.052854	0.058275
19	0.386741	-1.160223	0.061234	0.009669	0.053750	0.049309

J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
20	0.367879	-1.103638	0.061313	0.000000	0.054501	0.040875
21	0.349938	-1.049813	0.061239	-0.008748	0.055115	0.032952
22	0.332871	-0.998613	0.061026	-0.016644	0.055602	0.025520
23	0.316637	-0.949910	0.060689	-0.023748	0.055968	0.018558
24	0.301194	-0.903583	0.060239	-0.030119	0.056223	0.012048
25	0.286505	-0.859514	0.059688	-0.035813	0.056372	0.005969
26	0.272532	-0.817595	0.059049	-0.040880	0.056424	0.000303
27	0.259240	-0.777721	0.058329	-0.045367	0.056385	-0.004969
28	0.246597	-0.739791	0.057539	-0.049319	0.056261	-0.009864
29	0.234570	-0.703711	0.056688	-0.052778	0.056058	-0.014400
30	0.223130	-0.669390	0.055783	-0.055783	0.055783	-0.018594
31	0.212248	-0.636744	0.054831	-0.058368	0.055440	-0.022463
32	0.201897	-0.605690	0.053839	-0.060569	0.055035	-0.026022
33	0.192050	-0.576150	0.052814	-0.062416	0.054574	-0.029288
34	0.182684	-0.548051	0.051760	-0.063939	0.054061	-0.032274
35	0.173774	-0.521322	0.050684	-0.065165	0.053500	-0.034996
36	0.165299	-0.495897	0.049590	-0.066120	0.052896	-0.037468
37	0.157237	-0.471711	0.048481	-0.066826	0.052252	-0.039702
38	0.149569	-0.448706	0.047363	-0.067306	0.051573	-0.041713
39	0.142274	-0.426822	0.046239	-0.067580	0.050863	-0.043512
40	0.135335	-0.406006	0.045112	-0.067668	0.050124	-0.045112

J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
41	0.128735	-0.386205	0.043984	-0.067586	0.049360	-0.046523
42	0.122456	-0.367369	0.042860	-0.067351	0.048574	-0.047758
43	0.116484	-0.349452	0.041740	-0.066978	0.047769	-0.048826
44	0.110803	-0.332409	0.040628	-0.066482	0.046948	-0.049738
45	0.105399	-0.316198	0.039525	-0.065875	0.046112	-0.050504
46	0.100259	-0.300777	0.038433	-0.065168	0.056265	-0.051132
47	0.095369	-0.286107	0.037353	-0.064374	0.044408	-0.051632
48	0.090718	-0.272154	0.036287	-0.063503	0.043645	-0.052012
49	0.086294	-0.258881	0.035237	-0.062563	0.042675	-0.052280
50	0.082085	-0.246255	0.034202	-0.061564	0.041803	-0.052443
51	0.078082	-0.234245	0.033185	-0.060513	0.040928	-0.052510
52	0.074274	-0.222821	0.032185	-0.059419	0.040053	-0.052487
53	0.070651	-0.211954	0.031204	-0.058287	0.039179	-0.052380
54	0.067206	-0.201617	0.030242	-0.057125	0.038307	-0.052196
55	0.063928	-0.191784	0.029300	-0.055937	0.037439	-0.051941

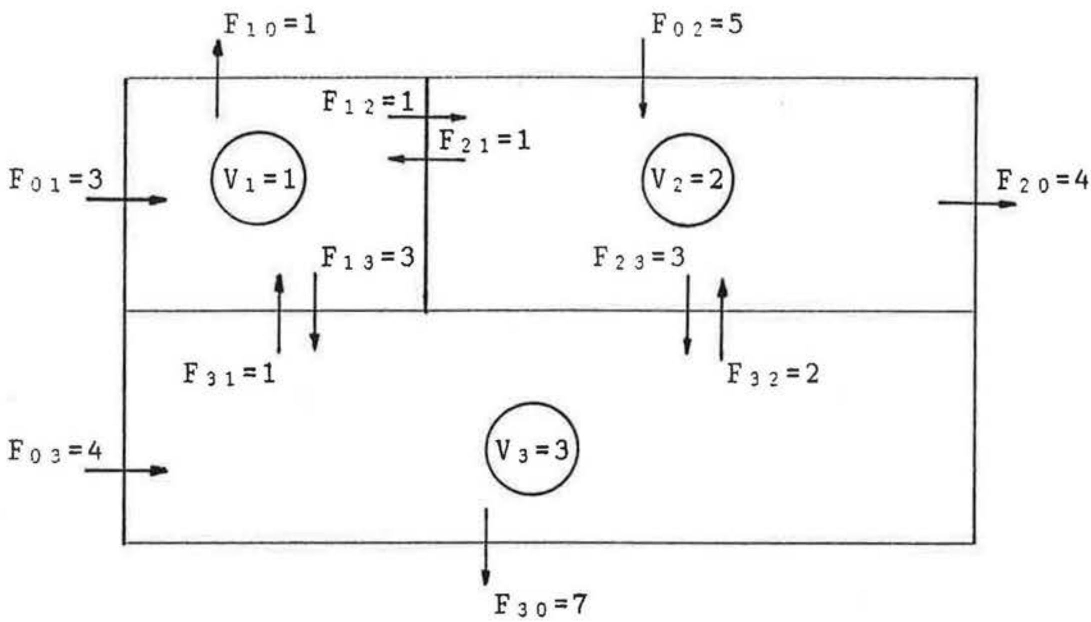
GRAPH 4-1.

CONCENTRATION DECAY CURVES



4-2. Three zone non-redundant case:

As for the redundant case, the volumes of the different zones are expressed as ratios, and their choice is arbitrary.



$$S_1 = 3 + 1 + 1 = 1 + 1 + 3 = 5$$

$$S_2 = 5 + 1 + 2 = 4 + 3 + 1 = 8$$

$$S_3 = 4 + 3 + 3 = 7 + 1 + 2 = 10$$

$$r_1 = \frac{S_1}{V_1} = \frac{5}{1} = 5$$

$$r_2 = \frac{S_2}{V_2} = \frac{8}{2} = 4$$

$$r_3 = \frac{S_3}{V_3} = \frac{10}{3}$$

Calculation of the eigenvalues:

Rewrite the equations as:

$$\begin{vmatrix} -(S_1+V_1.\lambda) & F_{21} & F_{31} \\ F_{12} & -(S_2+V_2.\lambda) & F_{32} \\ F_{13} & F_{23} & -(S_3+V_3.\lambda) \end{vmatrix} = 0$$

Giving:

$$-(S_1+V_1\lambda)[(S_2+V_2\lambda)(S_3+V_3\lambda)-F_{32}F_{23}]+F_{21}[F_{12}(S_3+V_3\lambda)+F_{32}F_{13}]$$
$$+F_{31}[F_{12}F_{23}+(S_2+V_2\lambda)F_{13}] = 0$$

When substituting the different values in the equation, we obtain:

$$-(5+\lambda)[(8+2\lambda)(10+3\lambda)-6]+[(10+3\lambda)+6]+[3+3(8+2\lambda)] = 0$$

Giving:

$$6\lambda^3 + 74\lambda^2 + 285\lambda + 327 = 0$$

Using the iteration method, we have:

$$\lambda = \sqrt[3]{-\frac{1}{6}(327 + 285\lambda + 74\lambda^2)}$$

The arbitrary value chosen for λ is zero. After a few iterations, the value of the first eigenvalue was found to be:

$$\lambda_1 = -2.086572257$$

As specified in chapter 3-22, λ_1 may be factorised in order to obtain a quadratic equation for which the roots are easily found. The roots of this quadratic equation will correspond to the two remaining eigenvalues.

After factorisation, we obtain:

$$(\lambda + 2.086572257)(6\lambda^2 + 61.48056646\lambda + 156.7163557) = 0$$

Giving:

$$\lambda + 2.086572257 = 0$$

or, $6\lambda^2 + 61.48056646\lambda + 156.7163557 = 0$

Hence:

$$\lambda_1 = -2.086572257$$

$$\lambda_2 = -4.763331203$$

$$\lambda_3 = -5.483429873$$

Calculation of the eigenvectors:

As mentionned in chapter 3-22, substitute λ in equation (3-4):

$$\begin{bmatrix} -(5+\lambda) & 1 & 1 \\ 1 & -(8+2\lambda) & 2 \\ 3 & 3 & -(10+3\lambda) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or:

$$-(5+\lambda)X_1 + X_2 + X_3 = 0$$

$$X_1 - (8+2\lambda)X_2 + 2X_3 = 0$$

$$3X_1 + 3X_2 - (10+3\lambda)X_3 = 0$$

The solution to this set of linear equations is:

$X_1 = 1$

$X_2 = (11+2\lambda) / (10+2\lambda)$

$X_3 = (2\lambda^2+18\lambda+39) / (10+2\lambda)$

or, in a more generalised form:

$\underline{X} = [\ 1\ ;\ (11+2\lambda)/(10+2\lambda)\ ;\ (2\lambda^2+18\lambda+39)/(10+2\lambda)]$

<u>Calculation of the coefficients a_1, a_2, a_3:</u>

Substitute the eigenvectors in equation (3-6):

$$C_1(t) = a_1.X_{11}.e^{\lambda_1 t} + a_2.X_{21}.e^{\lambda_2 t} + a_3.X_{31}.e^{\lambda_3 t}$$

$$C_2(t) = a_1.X_{12}.e^{\lambda_1 t} + a_2.X_{22}.e^{\lambda_2 t} + a_3.X_{32}.e^{\lambda_3 t}$$

$$C_3(t) = a_1.X_{13}.e^{\lambda_1 t} + a_2.X_{23}.e^{\lambda_2 t} + a_3.X_{33}.e^{\lambda_3 t}$$

Giving:

$$C_1(t) = a_1.e^{\lambda_1 t} + a_2.e^{\lambda_2 t} + a_3.e^{\lambda_3 t}$$

$$C_2(t) = a_1\frac{(11+2\lambda)}{(10+2\lambda)}.e^{\lambda_1 t} + a_2\frac{(11+2\lambda)}{(10+2\lambda)}.e^{\lambda_2 t} + a_3\frac{(11+2\lambda)}{(10+2\lambda)}.e^{\lambda_3 t}$$

$$C_3(t) = a_1\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)}e^{\lambda_1 t} + a_2\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)}e^{\lambda_2 t} + a_3\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)}e^{\lambda_3 t}$$

Applying the boundary conditions, we can write:

$$C_1(0) = a_1 + a_2 + a_3 = 1$$

$$C_2(0) = a_1\frac{(11+2\lambda)}{(10+2\lambda)} + a_2\frac{(11+2\lambda)}{(10+2\lambda)} + a_3\frac{(11+2\lambda)}{(10+2\lambda)}$$

$$C_3(0) = a_1\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)} + a_2\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)} + a_3\frac{(2\lambda^2+18\lambda+39)}{(10+2\lambda)}$$

When substituting the eigenvalues in these equations, we obtain:

$$a_1 + a_2 + a_3 = 1$$

$$1.171619153\ a_1 + 3.112657039\ a_2 - 0.034276175\ a_3 = 0$$

$$1.741808591\ a_1 - 2.875988244\ a_2 - 0.449153697\ a_3 = 0$$

from which:

$$a_1 = 0.152386767$$

$$a_2 = -0.047502221$$

$$a_3 = 0.895115441$$

The equations for the concentrations thus become:

$$C_1(t)=0.152386767\ e^{\lambda_1 t} - 0.047502221\ e^{\lambda_2 t} + 0.895115441\ e^{\lambda_3 t}$$

$$C_2(t)=0.178539254\, e^{\lambda_1 t} - 0.147858122\, e^{\lambda_2 t} - 0.030681133\, e^{\lambda_3 t}$$

$$C_3(t)=0.265428579\, e^{\lambda_1 t} - 0.136615829\, e^{\lambda_2 t} - 0.40204441\, e^{\lambda_3 t}$$

From which we can write:

$$\dot{C}_1(t)=-0.317966002\, e^{\lambda_1 t} - 0.046962846\, e^{\lambda_2 t} - 4.908302748\, e^{\lambda_3 t}$$

$$\dot{C}_2(t)=-0.372535053\, e^{\lambda_1 t} + 0.431065548\, e^{\lambda_2 t} + 0.168237842\, e^{\lambda_3 t}$$

$$\dot{C}_3(t)=-0.553835909\, e^{\lambda_1 t} - 0.650746439\, e^{\lambda_2 t} - 0.475713725\, e^{\lambda_3 t}$$

As for the redundant case, data (Table 4-2.) was created by means of a computer programme given in appendix 2. This data corresponds to the results of an ideal tracer decay experiment carried out on a time interval of $2.75t_c$.

To illustrate these results, the decay curves (Graph 4-2.) were plotted using a computer programme given in appendix 2.

Table 4-2. "DATA FOR THE NON-REDUNDANT CASE"

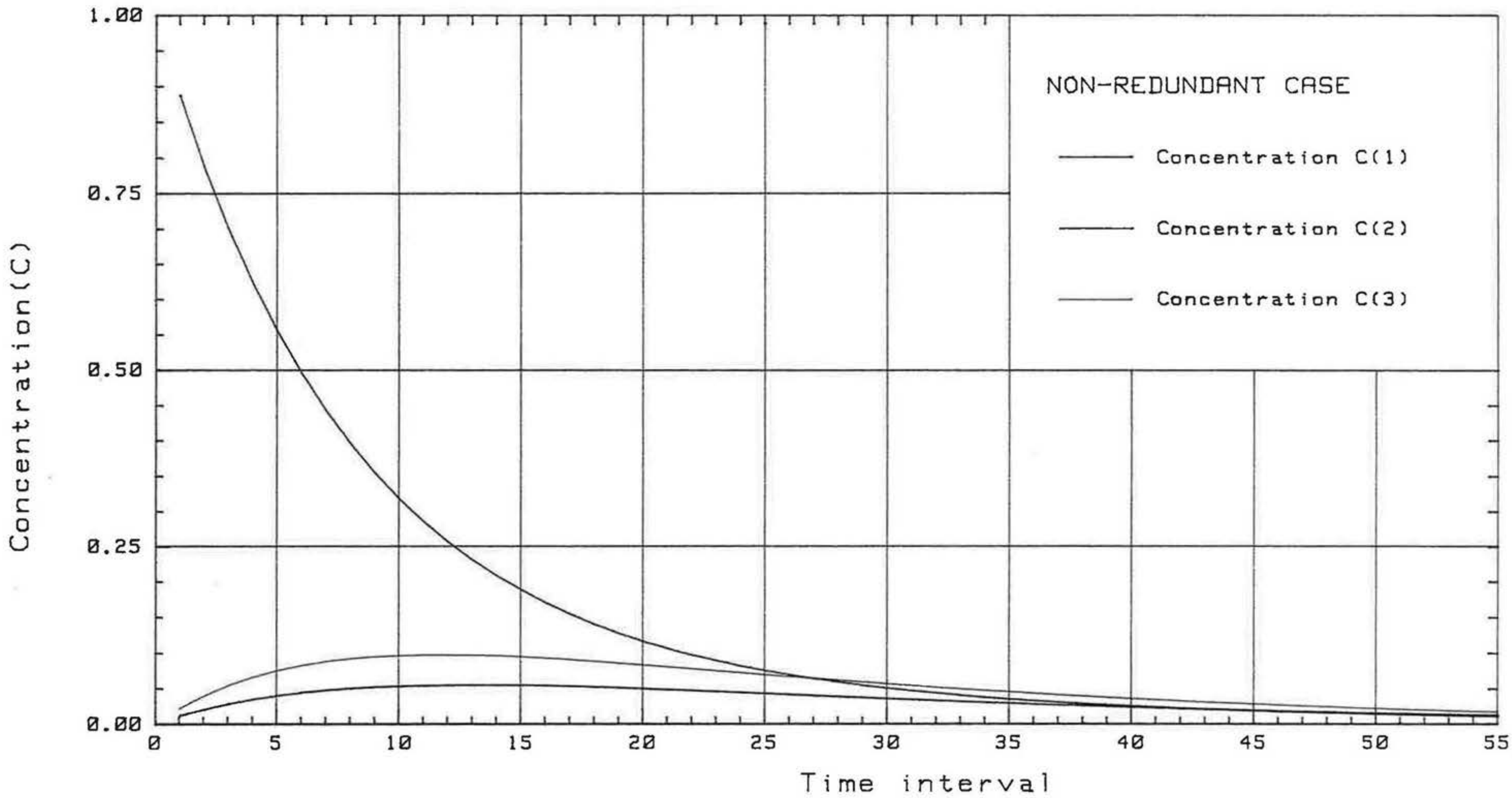
J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
01	0.887475	-4.404534	0.011020	0.421481	0.021822	0.825753
02	0.788331	-3.881608	0.020278	0.352822	0.039770	0.676042
03	0.700939	-3.422306	0.027998	0.292868	0.054392	0.547631
04	0.623870	-3.018812	0.034376	0.240596	0.066163	0.437701
05	0.555871	-2.664272	0.039583	0.195100	0.075497	0.343797
06	0.495841	-2.352680	0.043774	0.155577	0.082751	0.263778
07	0.442816	-2.078766	0.047081	0.121318	0.088235	0.195782
08	0.395951	-1.837913	0.049625	0.091693	0.092217	0.138186
09	0.354502	-1.626072	0.051508	0.066147	0.094929	0.089580
10	0.317817	-1.439692	0.052824	0.044187	0.096572	0.048734
11	0.285326	-1.275659	0.053651	0.025377	0.097318	0.014583
12	0.256524	-1.131243	0.054062	0.009332	0.097317	-0.013804
13	0.230973	-1.004050	0.054118	-0.004288	0.096696	-0.037230
14	0.208284	-0.891980	0.053873	-0.015784	0.095567	-0.056398
15	0.188118	-0.793192	0.053376	-0.025423	0.094022	-0.071914
16	0.170177	-0.706071	0.052668	-0.033438	0.092145	-0.084306
17	0.154198	-0.629199	0.051785	-0.040037	0.090003	-0.094029
18	0.139950	-0.561335	0.050759	-0.045404	0.087657	-0.101480
19	0.127232	-0.501388	0.049617	-0.049698	0.085155	-0.107002

J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
20	0.115865	-0.448400	0.048384	-0.053063	0.082542	-0.110889
21	0.105693	-0.401534	0.047081	-0.055624	0.079852	-0.113399
22	0.096579	-0.360053	0.045724	-0.057491	0.077116	-0.114751
23	0.088400	-0.323309	0.044330	-0.058761	0.074360	-0.115137
24	0.081051	-0.290736	0.042912	-0.059519	0.071605	-0.114719
25	0.074437	-0.261836	0.041481	-0.059839	0.068867	-0.113640
26	0.068476	-0.236171	0.040047	-0.059788	0.066163	-0.112019
27	0.063096	-0.213357	0.038618	-0.059423	0.063502	-0.109961
28	0.058231	-0.193057	0.037201	-0.058795	0.060896	-0.107553
29	0.053825	-0.174975	0.035802	-0.057947	0.058350	-0.104872
30	0.049829	-0.158850	0.034426	-0.056917	0.055871	-0.101982
31	0.046199	-0.144454	0.033076	-0.055740	0.053464	-0.098938
32	0.042894	-0.131585	0.031755	-0.054444	0.051130	-0.095785
33	0.039882	-0.120067	0.030467	-0.053054	0.048874	-0.092563
34	0.037130	-0.109744	0.029213	-0.051593	0.046695	-0.089305
35	0.034614	-0.100479	0.027995	-0.050080	0.044594	-0.086037
36	0.032307	-0.092152	0.026813	-0.048529	0.042571	-0.082783
37	0.030191	-0.084657	0.025669	-0.046956	0.040626	-0.079560
38	0.028244	-0.077900	0.024563	-0.045373	0.038758	-0.076385
39	0.026452	-0.071800	0.023495	-0.043789	0.036965	-0.073269
40	0.024799	-0.066283	0.022465	-0.042213	0.035246	-0.070223

J	C(1,J)	DC(1,J)	C(2,J)	DC(2,J)	C(3,J)	DC(3,J)
41	0.023271	-0.061285	0.021472	-0.040652	0.033599	-0.067253
42	0.021858	-0.056751	0.020516	-0.039113	0.032022	-0.064366
43	0.020548	-0.052630	0.019597	-0.037601	0.030513	-0.061566
44	0.019332	-0.048878	0.018714	-0.036118	0.029071	-0.058856
45	0.018203	-0.045457	0.017866	-0.034670	0.027692	-0.056238
46	0.017152	-0.042331	0.017052	-0.033257	0.026375	-0.053712
47	0.016172	-0.039471	0.016272	-0.031883	0.025117	-0.051280
48	0.015258	-0.036850	0.015524	-0.030549	0.023916	-0.048940
49	0.014404	-0.034443	0.014807	-0.029256	0.022771	-0.046691
50	0.013606	-0.032230	0.014121	-0.028004	0.021678	-0.044533
51	0.012858	-0.030191	0.013465	-0.026794	0.020636	-0.042463
52	0.012158	-0.028309	0.012837	-0.025626	0.019642	-0.040480
53	0.011500	-0.026570	0.012236	-0.024500	0.018695	-0.038580
54	0.010883	-0.024961	0.011662	-0.023415	0.017793	-0.036763
55	0.010303	-0.023469	0.011114	-0.022371	0.016933	-0.035025

GRAPH 4-2.

CONCENTRATION DECAY CURVES



CHAPTER 5

FINDING THE FLOWS FROM KNOWN CONCENTRATIONS

5-0. FINDING THE FLOWS FROM KNOWN CONCENTRATIONS:

This chapter discusses the inverse problem actually encountered in practice: how to deduce the values of the flows F_{ij} from the observed concentrations $C_j(t)$, i.e. how to obtain all the interzone flows.

5-1. Statement of equations used:

The previous chapters discussed the calculations of the concentrations from known flows.

To find the flows from the observed concentrations, the same differential equation as 3-2 in section 3-11 will be used, considering zero contaminant injection:

$$V_j.\dot{C}_j(t) = \sum_{i=0}^{n} F_{ij}.C_i(t) - C_j(t).S_j$$

However, in each system of n chambers communicating with each other and with the outside, there are n(n+1) flows F_{ij} (regarding the outside as a zone of infinite volume, there are n+1 chambers in all, and each receives a flow from the n others). The n(n+1) flows are, however, not independent because they must satisfy the n conservation equations:

$$\sum_{i=0}^{n} F_{ij} = \sum_{i=0}^{n} F_{ji}$$

for $j=1,\ldots\ldots,n$

Note: $F_{jj} = 0$

There are not n+1 of these because the conservation equation for the (n+1)st zone (the outside), is already implied by the first n equations.

To determine the F_{ij}, then, we need exactly n^2 further equations.

In the cases discussed previously, i.e. three zone buildings (Sections 4-1 & 4-2), we thus need nine equations.

5-2. Selection of the nine equations:

Examination of the decay curves (Graphs 4-1 & 4-2) shows that the concentrations decay at approximately the same rate after about $2t_c$, thus it would be pointless to use the concentrations beyond this point, because the equations would be linear, but not independent of each other.

It was therefore decided to select the equations in the first two time constants of the experiment, $2t_c$.

The accuracy in the final results, i.e. F_{ij}, will depend on the way the equations are spaced; many arrangements are possible.

The concentrations being available within $2t_c$, it was decided to select a set of nine equations evenly spaced within this period of time. This is case no 1. From the data tables 4-1 & 4-2, the following evenly spaced time intervals were selected: 6,10,14,18,22,26,30,34,38.

However, within the first time constant, the gradients of the decay curves 5-1 & 5-2 change rapidly, therefore, another set of equation was selected within this period of time, t_c. This is case no 2. The following equally spaced time intervals were selected from the data tables 4-1 & 4-2: 2,4,6,8,10,12,14,16,18.

Finally, a third case was studied, using a set of nine equations selected between t_c and $2t_c$ in Tables 4-1 & 4-2. This set of equations was chosen at the following time intervals: 22,24,26,28,30,32,34,36,38.

5-3. Error analysis:

For each case, the number of decimals was reduced after each calculation, and the effect of reducing the number of decimal places was then analysed.

5-31. Condition number of the matrix:

A calculation in which small relative changes in the input quantities produces large relative changes in the output is said to be "Ill-Conditioned", and the number which measures the degree of ill-conditioning is called the "Condition Number,K".

Any non-singular matrix A possesses a condition number which is large when $A\underline{x}=B$ is ill-conditioned and of order unity for a very well-conditioned system.

The condition number of a matrix may be expressed as follows:

$$K = \|A\| \cdot \|A^{-1}\|$$

Where $\|A\|$ and $\|A^{-1}\|$ are suitable norms of the respective matrices.

The norm of a square matrix A is a non-negative number denoted by $\|A\|$ which satisfies:

(i) $\|A\| > 0$ if $A = 0$, and $\|A\| = 0$ if $A = 0$

(ii) $\|CA\| = |C| . \|A\|$ for any scalar C

(iii) $\|A+B\| < \|A\| + \|B\|$

(iv) $\|AB\| < \|A\| . \|B\|$

The norms selected for the error analysis in this case, are the row and the column sum norms, where:

Row sum norm, $\|A\|_1 = \max_{j} \sum_{i} |a_{ij}|$

Column sum norm, $\|A\|_\infty = \max_{i} \sum_{i} |a_{ij}|$

Note: Kr is the condition number related to the row sum norm.

Kc is the condition number related to the column sum norm.

5-32. R.M.S. error in the final flows:

The condition number is a measure of the sensitivity of the matrix (that is the sensitivity of the nine equations forming the matrix) to rounding and/or small relative changes in the equations. It is <u>not</u> a measure of the errors in the final flows associated with rounding.

However, as we are concerned with the effect of rounding errors, it was decided to determine the root mean square (R.M.S.) error in the final flow rates.

The R.M.S. error is related to the original flow rates used to determined the concentrations, and may be defined as follows:

$$\text{R.M.S. Error} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} |e_i|^2}$$

Where: n = number of flows (n=12)

e_i = $F_i - F_i'$

F_i = original flow rate

F_i' = flow rate determined from the concentrations.

5-4. Results:

The flows were all determined by means of a computer programme given in appendix 3.

Case no 1 uses nine equations selected within $2t_c$, that is at the following time intervals in table 4-1 & 4-2: 6,10,14,18,22,26,30,34,38.

Case no 2 uses nine equations evenly distributed within t_c, i.e. at time intervals 2,4,6,8,10,12,14,16,18.

Case no 3 uses nine equations equally spaced between t_c and $2t_c$, at time intervals 22,24,26,28,30,32,34,36,38.

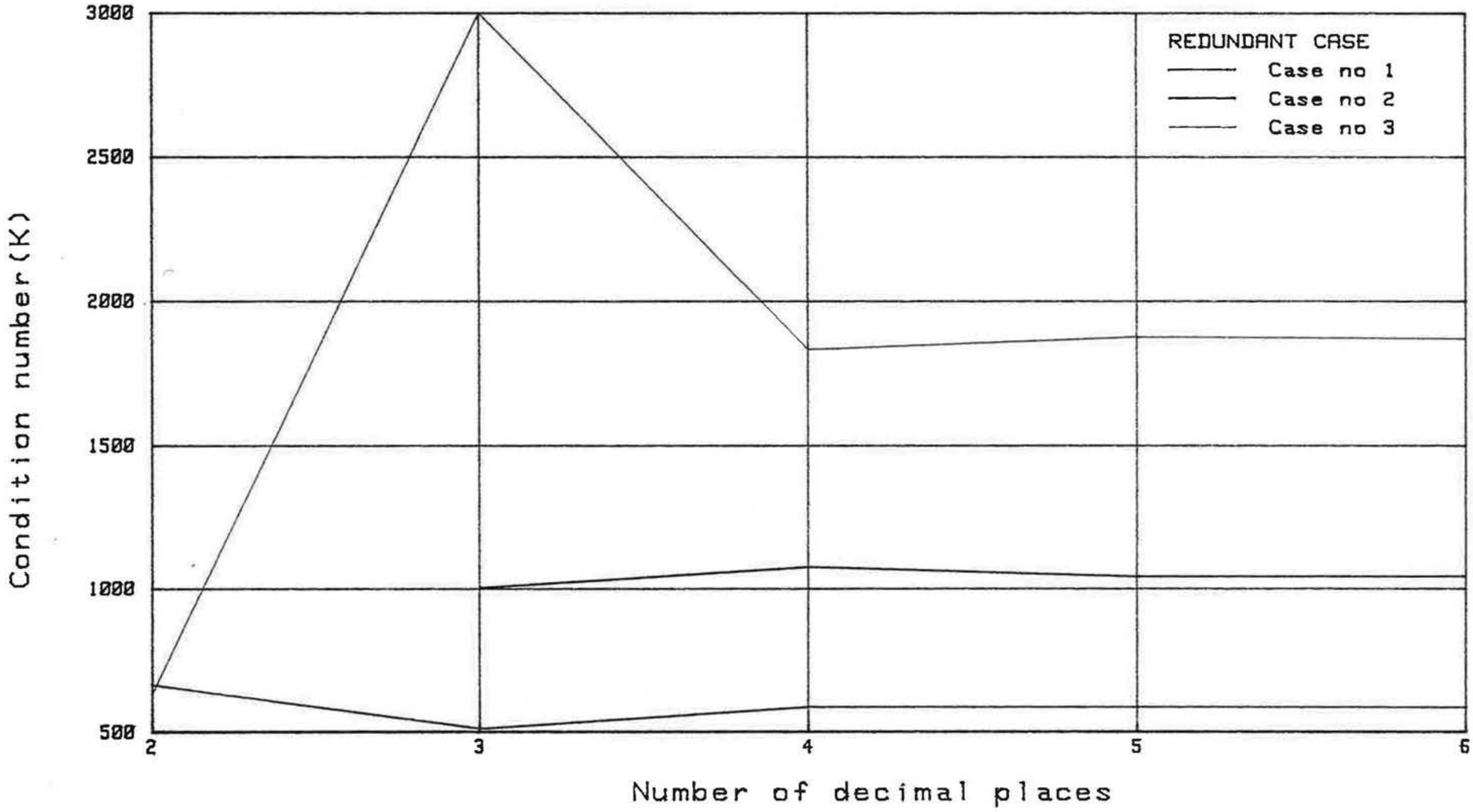
Table 5-1. REDUNDANT CASE

No of Decim.	F(0,1)	F(0,2)	F(0,3)	F(1,0)	F(1,2)	F(1,3)	F(2,0)	F(2,1)	F(2,3)	F(3,0)	F(3,1)	F(3,2)	Kr	Kc	R.M.S. error (%)
*	3.000000	5.000000	4.000000	1.000000	1.000000	1.000000	2.000000	0.000000	4.000000	9.000000	0.000000	0.000000	1*	1*	0.00
CASE No 1															
6	2.999983	5.000041	4.000233	0.999926	1.000023	1.000057	2.001116	0.000258	3.999184	8.999184	-0.000235	0.000494	584	865	0.05
5	3.000000	5.000150	3.997730	0.999993	1.000309	0.999699	2.005865	0.000000	4.006530	8.992023	0.000000	0.011936	585	866	0.49
4	2.998962	4.993014	3.986430	1.001335	0.999179	0.998536	1.945654	-0.043844	4.059757	9.031416	0.044033	-0.030726	587	869	3.23
3	2.964683	4.884840	3.935601	1.012646	0.979174	1.007840	1.521392	-0.119675	3.821843	9.251086	0.154652	-0.640454	509	750	25.68
2	2.765690	4.363636	2.744681	0.937534	1.090909	0.925532	5.224003	-1.828452	2.968085	3.712471	2.016736	0.909091	662	1074	203.53
CASE No 2															
6	3.000084	4.999690	3.999464	0.999947	1.000010	1.000045	2.004969	0.000386	3.995792	8.994321	-0.000468	0.001448	1040	2200	0.25
5	2.999323	4.997554	3.997961	0.999687	1.000122	1.000186	2.028391	-0.002357	3.986026	8.966760	0.003029	0.014384	1038	2196	1.40
4	3.012615	4.988187	4.005173	1.002484	0.999468	0.998112	1.734080	0.061081	4.192660	9.269411	-0.073632	0.000166	1075	2276	12.58
3	3.000425	5.140467	3.901025	0.984187	1.002837	1.013731	2.102426	0.135963	3.272461	8.955303	-0.135632	-0.632454	1001	2077	28.99
CASE No 3															
6	3.000000	4.997779	4.000449	1.000578	0.999295	1.000126	1.989976	0.000000	3.998141	9.007674	0.000000	-0.008958	1868	2796	0.46
5	3.003805	4.989561	4.003191	1.000587	0.996696	1.001442	1.984419	-0.022492	3.980257	9.011551	0.017412	-0.044073	1877	2809	1.75
4	3.000000	5.010894	3.925781	1.016445	1.004241	0.979314	1.680900	0.000000	4.349901	9.239329	0.000000	0.015666	1831	2738	15.50
3	2.647832	5.456981	6.547951	0.126904	1.171561	1.808631	14.765649	1.733903	-7.811247	-0.239789	-1.274639	2.059763	3007	4626	580.94
2	2.133333	5.000000	4.785714	0.880952	1.000000	1.285714	3.214286	2.500000	-1.714286	7.823810	-1.466667	-2.000000	630	920	202.85

*[illegible]

GRAPH 5-11.

CONDITION NUMBER VARIANCE



GRAPH 5-12.

R.M.S. ERRORS IN FINAL FLOWS

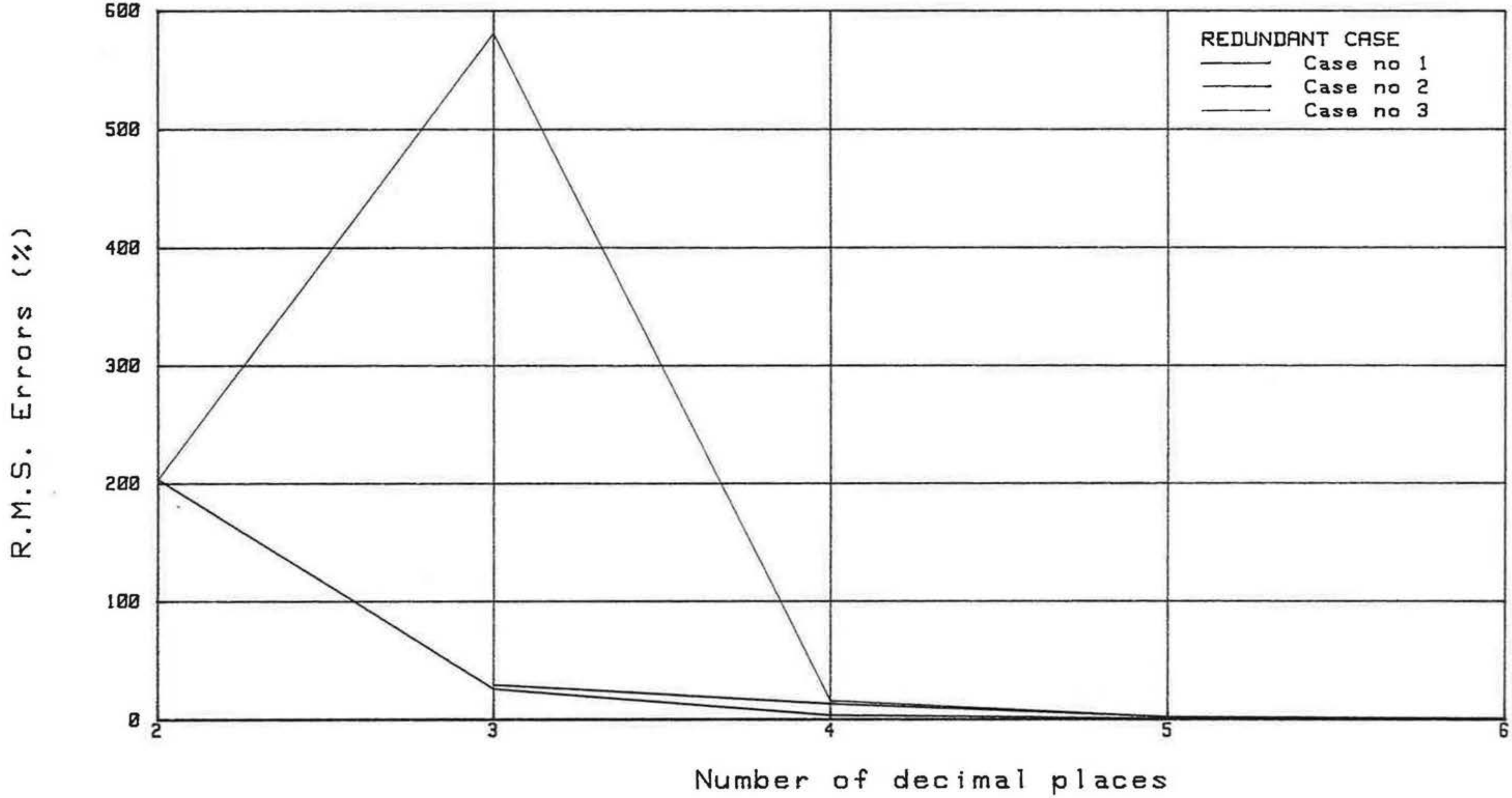
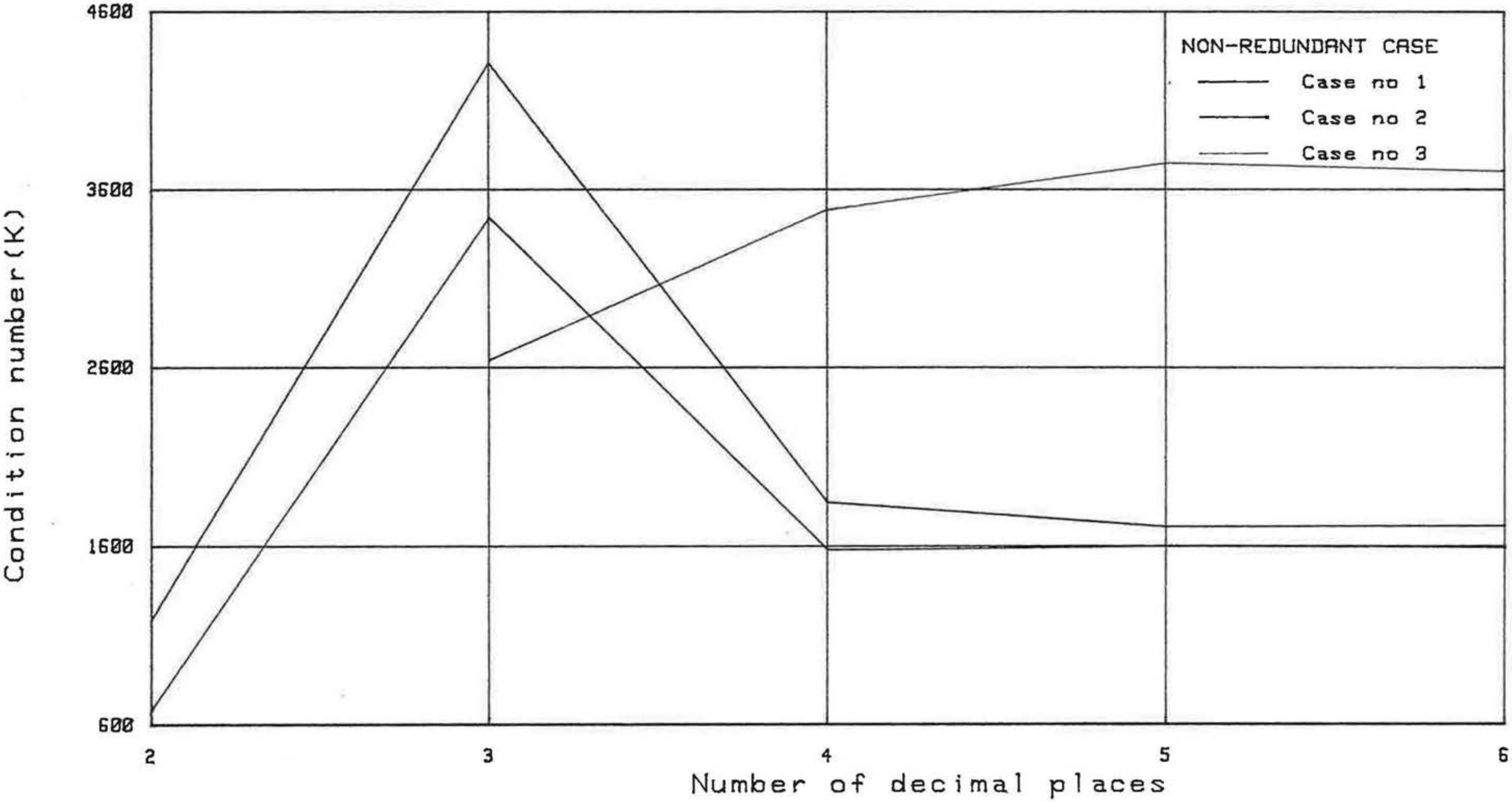


Table 5-2. NON-REDUNDANT CASE

No of Decim.	F(0,1)	F(0,2)	F(0,3)	F(1,0)	F(1,2)	F(1,3)	F(2,0)	F(2,1)	F(2,3)	F(3,0)	F(3,1)	F(3,2)	Kr	Kc	R.M.S. error (%)
*	3.000000	5.000000	4.000000	1.000000	1.000000	3.000000	4.000000	1.000000	3.000000	7.000000	1.000000	2.000000	1^{+}	1^{+}	0.00
CASE No 1															
6	2.998930	4.997227	3.999708	0.999734	1.000189	3.000031	3.989698	1.002635	3.000705	7.006432	0.998389	1.995623	1713	1712	0.39
5	2.997906	4.992314	4.025217	1.002972	1.000331	2.996529	4.045933	1.004941	2.931628	6.966532	0.996985	1.989857	1703	1702	2.70
4	2.941308	4.833735	3.730275	0.965963	1.007396	3.024788	2.080987	1.133835	1.767096	7.729480	0.923004	1.767096	1843	1841	46.31
3	2.673511	3.851244	6.171429	1.170819	1.064938	2.742857	6.071381	1.757117	-2.657143	5.444084	0.557886	0.255173	4312	4345	201.60
2	-1.200000	9.538462	5.600000	1.384615	0.615385	2.800000	17.769231	11.000000	-2.000000	-5.215385	-5.000000	16.615385	1183	1292	792.96
CASE No 2															
6	3.000066	4.998133	3.999135	0.999950	1.000040	3.000012	3.993959	0.999843	3.001990	7.003426	1.000093	1.997618	1584	1968	0.23
5	3.000161	5.004524	3.983556	0.999519	0.999938	3.000499	3.974517	0.998790	3.038135	7.014204	1.001005	2.006980	1595	1982	1.49
4	2.978258	5.086318	4.006190	1.001445	0.998521	2.999439	4.200093	1.046608	2.965616	6.869228	0.974539	2.127477	1570	1949	8.43
3	1.907179	4.961282	4.444339	1.002397	1.007447	2.981446	2.806709	3.376450	1.761893	7.503694	-0.292340	1.976324	3439	4241	99.68
2	3.656738	1.65E+15	1.941176	0.636110	1.064516	3.282353	4.94E+15	-0.676596	7.164706	-3.29E+15	2.002837	3.29E+15	676	866	......
CASE No 3															
6	3.013134	5.030586	3.980793	1.003603	0.993949	3.004523	4.063250	0.966226	3.049022	6.957660	1.022715	2.053963	3729	2758	3.45
5	3.009432	4.925715	3.851906	0.951921	1.014222	3.037771	3.464077	0.974084	3.374140	7.371055	1.020397	1.872365	3793	2806	22.62
4	3.188044	4.635286	5.071461	1.273887	1.057667	2.721570	6.336610	0.502099	0.250257	5.284294	1.362981	1.396013	3488	2565	123.09
3	4.524164	-.635417	8.739130	0.921052	2.359375	2.347826	5.426572	-3.122677	-9.043478	6.280254	4.226766	-8.463542	2643	2021	534.88

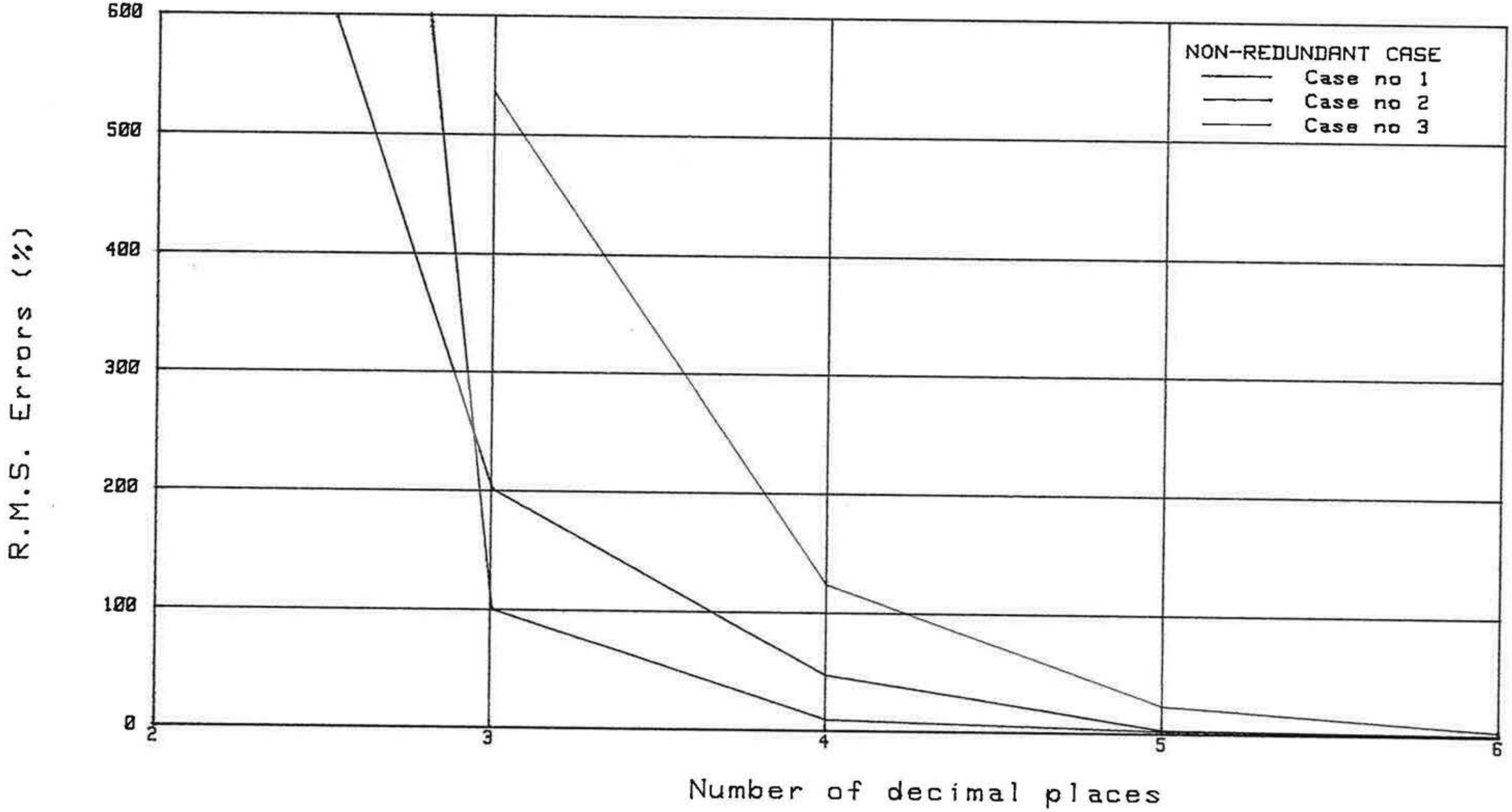
GRAPH 5-21.

CONDITION NUMBER VARIANCE



GRAPH 5-22.

R.M.S. ERRORS IN FINAL FLOWS



DISCUSSION

AND

CONCLUSIONS

6-0. DISCUSSION AND CONCLUSIONS:

Graph 5-11:

The reduction in the sensitivity of case 3 to small changes in the concentrations is not as expected. It was predicted that the condition number would increase with the reduction of decimal places. However, it can be seen that the condition number decreases when reducing the number of decimals from 3 to 2.

From this graph, case 1 seems to give the most precise results, as its condition number is the smallest.

The shape of the curves for cases 1 and 2 are fairly similar, even though case 2 gives a least satisfactory result.

Graph 5-12:

An unexpected behaviour of case 3 can be seen, that is the percentage error suddenly drops between 3 and 2 decimal places. The two other cases are subject to an increasing root mean square error when the number of decimal places is reduced.

Graph 5-21:

As in graph 5-11, the reduction in the sensitivity of case 3 to small changes in the concentrations is not as

expected as it was predicted that the condition number would increase with the reduction of decimal places.
The same behaviour can be observed for cases 1 and 2.
The graph shows that case 2 gives the best results.

Graph 5-22:

All three cases give the expected results, that is, the R.M.S. error increases when the number of decimal places is reduced.
Of the three cases, case 2 is the most satisfactory, i.e. the error is least important.

A big difference can be observed between graphs 5-11 and 5-21. The non-redundant case gives worse answers than the redundant case.
The same can be observed from graphs 5-12 and 5-22.
When comparing graphs 5-11 and 5-12, it can be seen that there is no direct correlation between the condition number and the R.M.S. error.
Examination of graphs 5-21 and 5-22 proves to give the same result.

The condition number as determined in this project is not a good measure of the errors in the calculation of the final flows.

To determine the final flows, a set of nine linear equations must be selected within one time constant. Results given by the equations evenly distributed within $2t_c$ or between t_c and $2t_c$ are not satisfactory.

For the particular flow rates chosen to determine the initial concentrations, the redundant case gives more precise results than the non-redundant case.

When introducing rounding errors in the concentrations and their differentials, substantial changes in the final flows can be registred. Experimental errors are expected to be larger than rounding errors and therefore, they would produce greater errors in the computed flows.

RECOMMENDATIONS FOR FURTHER WORK

7-0. RECOMMENDATIONS FOR FURTHER WORK

The present dissertation only explores the effect of the position of the equations in the data set, and for each choice, assesses the effect of reducing the number of decimal places.

The nine equations used to determine the flow rates are selected from nine different time intervals in the data set, however, three equations can be made out of the results obtained from only one time interval. It would be useful to explore the effect of choosing only three time intervals to create the set of equations needed in order to calculate the flows.

The effect of reducing the number of decimal places in both the concentrations and their differentials has been shown, and it would be useful to observe the effect of introducing rounding errors in the concentrations alone, and not in their differentials, or vice-versa.

In the experimental phase, the tracer gas could be injected in a different zone, or in more than one zone, and the effect on the concentrations observed.

More than one tracer gas could be injected in the model, and their behaviour could be studied.

In the present report, it can be observed that the condition number related to the row or column norms does not produce any conclusion regarding the precision of the final flow rates. It would therefore be useful to try and use different norms to determine the condition number of the matrix.

Two possible norms are:

- The Euclidean norm: $\|A\|_E = \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} |a_{ij}|^2}$

- The l_2 norm: $\|A\|_2 = \sqrt{\text{Largest eigenvalue of } AA'}$

  Where A' is the transpose of A.

The research could be extended to a bigger multizone model.

APPENDIX 1

TABLE 1. OUTSIDE AIR REQUIREMENTS FOR RESPIRATION.

Activity (Adult male)	Metabolic rate, M	Requirements for respiration; O_2 concentration of 16.3% in expired air.	requirements to maintain room CO_2 at 0.5%assuming 0.04% CO_2 in fresh air.*
	W	litres/s	litres/s
seated quietly	100	0.1	0.8
light work	160 to 320	0.2 to 0.3	1.3 to 2.6
moderate work	320 to 480	0.3 to 3.5	2.6 to 3.9
heavy work	480 to 650	0.5 to 0.7	0.3 to 0.5
very heavy work	650 to 800	0.7 to 0.9	5.3 to 6.4

* The rate of production of CO_2 in terms of metabolic rate M is $40 \times 10^{-6}M$ litres/s where M is in watts.

FIGURE 2-a. "Air supply rate for odour removal"
school children

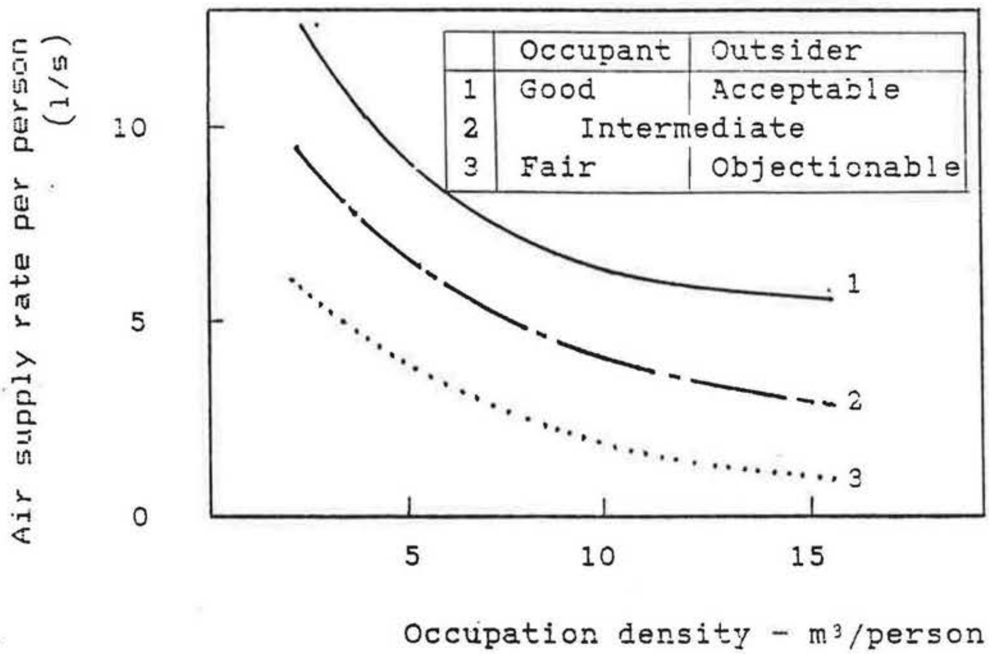


FIGURE 2-b. "Air supply rate for odour removal"
adults

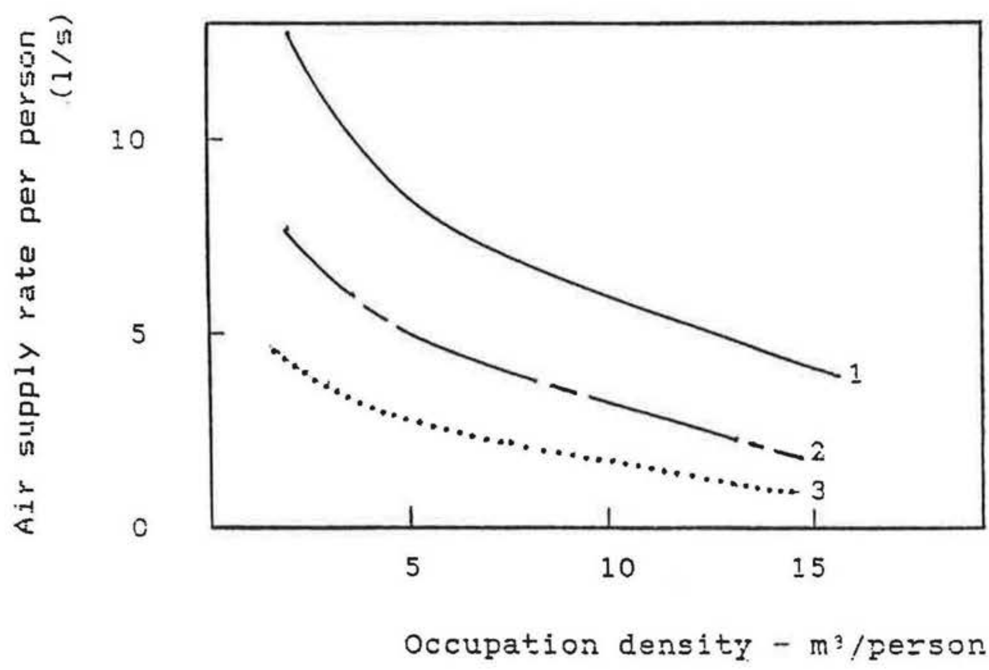


TABLE 2. TYPICAL MOISTURE EMISSION RATES

(a) Rates fixed by the nature of the process	Moisture emission rate
Occupation :	
Sleeping adult	0.04 kg/h
active adult	0.05 kg/h
Flueless combustion :	
natural gas	0.16 kg/h per kW
premium grade kerosene	0.10 kg/h per kW
liquefied petroleum gas	0.13 kg/h per kW
(b) Rates estimated , but subject to variation due to occupants' lifestyle	
Cooking	3.0 kg/day
Bathing, dishwashing, etc.	1.0 kg/day
Clothes washing	0.5 kg/day
Clothes drying	5.0 kg/day

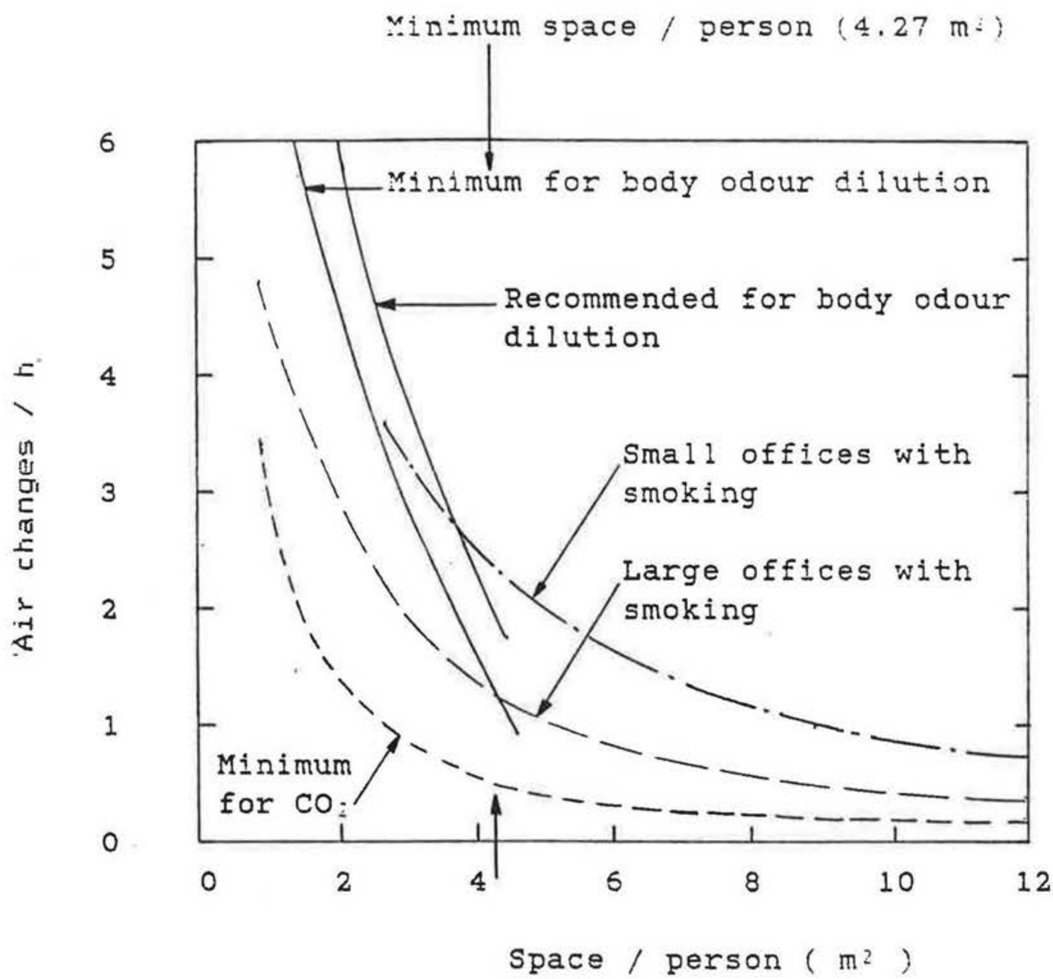
TABLE 3. AIR SUPPLY RATES REQUIRED FOR FLUELESS APPLIANCES

Fuel	Basis	Air supply rate
		litres/s per kW*
Natural gas	CO_2 < 0.5 %	5.4
Liquefied petroleum gas	CO_2 < 0.5 %	6.6
Premium grade kerosene	CO_2 < 0.5 %	6.8
Premium grade kerosene	CO_2<5.0 p.p.m.	1.8+
Premium grade kerosene	CO_2<1.0 p.p.m.	9.0+

* Input rating of the appliance.

\+ These figures are based upon the maximum level of sulphur, of 0.06% allowed by BS 2869. In many cases sulphur levels will be lower than this, in which case the required air supply rates may be reduced in proportion.

DIAGRAM 1. AIR REQUIREMENTS FOR PEOPLE IN A ROOM 2.7 M HIGH



NOTE. Smoking is based on 26 m^3/h per smoker. Small offices may contain all smokers, large offices are assumed to reflect the adult population, i.e. half smokers. Minimum space is as specified in the Offices, Shops and Railway Premises Act.

APPENDIX 2

PROGRAMME 1

"CALCULATION OF THE CONCENTRATIONS"

(THREE ZONE REDUNDANT CASE)

"CALCULATION OF THE CONCENTRATIONS FOR A THREE ZONE REDUNDANT CASE".

```
10  OPTION BASE 1
20  REM "REDUNDANT CASE",PRG02
30  DIM C(3,55),DC(3,55)
40  FOR J=1 TO 55
50  T=J/60
60  C(1,J)=EXP(-3*T)
70  C(2,J)=.5*T*EXP(-3*T)
80  C(3,J)=(1/3)*T*EXP(-3*T)+(1/3)*T*T*EXP(-3*T)
90  DC(1,J)=-3*EXP(-3*T)
100 DC(2,J)=.5*(1-3*T)*EXP(-3*T)
110 DC(3,J)=((-T*T)-(1/3)*T+(1/3))*EXP(-3*T)
120 NEXT J
130 ASSIGN @F_1 TO "RES02"
140 FOR J=1 TO 55
150 OUTPUT @F_1 USING "ZZ,XX,#";J
160 OUTPUT @F_1 USING "6(MZ.DDDDDD)";C(1,J),DC(1,J),
    C(2,J),DC(2,J),C(3,J),DC(3,J)
170 NEXT J
180 ASSIGN @F_1 TO *
190 PRINT TAB(30);"DATA FOR THE REDUNDANT CASE"
```

```
200  PRINT

210  PRINT

220  PRINT TAB(14);"C(1,J)";TAB(25);"DC(1,J)";TAB(36);
     "DC(2,J)";TAB(47);"C(3,J)";TAB(69);"DC(3,J)";

230  PRINT

240  PRINT

250  FOR J=1 TO 55

260  PRINT USING "8X,ZZ,X,#";J

270  PRINT USING "6(MZ.DDDDDD,XX)";C(1,J);DC(1,J);C(2,J)
     ;DC(2,J);C(3,J);DC(3,J)

280  NEXT J

290  END
```

PROGRAMME 2

"CALCULATION OF THE CONCENTRATIONS"

(THREE ZONE NON-REDUNDANT CASE)

"CALCULATIONS OF THE CONCENTRATIONS FOR THE THREE ZONE NON-REDUNDANT CASE"

```
10  OPTION BASE 1
20  REM NON-REDUNDANT CASE,PRG01
30  DIM C(3,55),DC(3,55),Lam(3),A(3),X(3,3)
40  DISP "ENTER VALUE FOR LAMBDA(1)";
50  INPUT Lam(1)
60  DISP "ENTER VALUE FOR LAMBDA(2)";
70  INPUT Lam(2)
80  DISP "ENTER VALUE FOR LAMBDA(3)";
90  INPUT Lam(3)
100 DISP "ENTER VALUE FOR X(1,1)";
110 INPUT X(1,1)
120 DISP "ENTER VALUE FOR X(1,2)";
130 INPUT X(1,2)
140 DISP "ENTER VALUE FOR X(1,3)";
150 INPUT X(1,3)
160 DISP "ENTER VALUE FOR X(2,1)";
170 INPUT X(2,1)
180 DISP "ENTER VALUE FOR X(2,2)";
190 INPUT X(2,2)
200 DISP "ENTER VALUE FOR X(2,3)";
```

```
210  INPUT X(2,3)
220  DISP "ENTER VALUE FOR X(3,1)";
230  INPUT X(3,1)
240  DISP "ENTER VALUE FOR X(3,2)";
250  INPUT X(3,2)
260  DISP "ENTER VALUE FOR X(3,3)";
270  INPUT X(3,3)
280  DISP "ENTER VALUE FOR A(1)";
290  INPUT A(1)
300  DISP "ENTER VALUE FOR A(2)";
310  INPUT A(2)
320  DISP "ENTER VALUE FOR A(3)";
330  INPUT A(3)
340  PRINTER IS 701
350  PRINT TAB(30);"NON-REDUNDANT CASE"
360  PRINT
370  PRINT
380  PRINT TAB(20);"EIGENVALUES :"
390  PRINT
400  PRINT TAB(20);"Lam(1)";TAB(40);"Lam(2)";TAB(60);
     "Lam(3)"
410  PRINT TAB(20);Lam(1);TAB(40);Lam(2);TAB(60);Lam(3)
420  PRINT
```

```
430  PRINT
440  PRINT TAB(20);"EIGENVECTORS :"
450  PRINT
460  PRINT TAB(20);"FOR LAMBDA(1) :"
470  PRINT TAB(20);"X(1,1)";TAB(40);"X(1,2)";TAB(60);
     X(1,3)
480  PRINT TAB(20);X(1,1);TAB(40);X(1,2);TAB(60);X(1,3)
490  PRINT
500  PRINT TAB(20);"FOR LAMBDA(2) :"
510  PRINT TAB(20);"X(2,1)";TAB(40);"X(2,2)";TAB(60);
     "X(2,3)"
520  PRINT TAB(20);X(2,1);TAB(40);X(2,2);TAB(60);X(2,3)
530  PRINT
540  PRINT "FOR LAMBDA(3) :"
550  PRINT TAB(20);"X(3,1)";TAB(40);"X(3,2)";TAB(60);
     "X(3,3)"
560  PRINT TAB(20);X(3,1);TAB(40);X(3,2);TAB(60);X(3,3)
570  PRINT
580  PRINT
590  PRINT TAB(20);"COEFFICIENTS :"
600  PRINT
610  PRINT TAB(20);"A(1)";TAB(40);"A(2)";TAB(60);"A(3)"
620  PRINT TAB(20);A(1);TAB(40);A(2);TAB(60);A(3)
630  FOR J=1 TO 55
```

```
640  T=-J/(20*Lam(1))

650  FOR I=1 TO 3

660  C(I,J)=A(1)*X(1,1)*EXP(Lam(1)*T)+a(2)*X(2,I)*EXP
     (Lam(2)*T)+A(3)*X(3,I)*EXP(Lam(3)8*T)

670  DC(I,J)=+Lam(1)*A(1)*X(1,I)*EXP(Lam(1)*T)+Lam(2)*
     A(2)*X(2,1)*EXP(Lam(2)*T)+Lam(3)*A(3)*X(3,I)*EXP(T
     *Lam(3))

680  NEXT I

690  NEXT J

700  ASSIGN @F_1 TO "RES01"

710  FOR J=1 TO 55

720  OUTPUT @F_1 USING "ZZ,XX,#";J

730  OUTPUT @F_1 USING "6(MZ.DDDDDD)";C(1,J),DC(1,J),
     C(2,J),DC(2,J),C(3,J),DC(3,J)

740  NEXT J

750  ASSIGN @F_1 TO *

760  PRINT

770  PRINT

780  PRINT TAB(30);"DATA FOR THE NON-REDUNDANT CASE"

790  PRINT

800  PRINT

810  PRINT TAB(14);"C(1,J)";TAB(25);"DC(1,J)";TAB(36);
     "C(2,J)";TAB(47);"DC(2,J)";TAB(58);"C(3,J)";
     TAB(69);"DC(3,J)";

820  PRINT
```

```
830  PRINT

840  FOR J=1 TO 55

850  PRINT USING "8X,ZZ,X,#";J

860  PRINT USING "6(MZ.DDDDDD,XX)";C(1,J);DC(1,J):C(2,J)
     ;DC(2,J);C(3,J);DC(3,J)

870  NEXT J

880  PRINTER IS 1

890  END
```

PROGRAMME 3

"PLOTTING OF DECAY CURVES"

"PLOTTING OF DECAY CURVES".

```
 10  DIM C(3,55),DC(3,55)

 20  GINIT

 30  CLEAR SCREEN

 40  DISP "DO YOU WANT TO PLOT ON (S)CREEN OR
     (P)LOTTER?";

 50  INPUT P$

 60  IF P$="P" THEN GOTO 90

 70  PLOTTER IS CRT,"INTERNAL"

 80  GOTO 100

 90  PLOTTER IS 705,"HPGL"

100  GRAPHICS ON

110  X_gdu_max=100*MAX(1,RATIO)

120  Y_gdu_max=100*MAX(1,1/RATIO)

130  LORG 6

140  FOR I=-.3 TO .3 STEP .1

150  MOVE X_gdu_max/2.5+I,Y_gdu_max

160  LABEL "CONCENTRATION VARIANCE"

170  NEXT I

180  PRINTER IS 705

190  GSEND "VS19;"

200  PRINTER IS 1
```

```
210  DEG
220  LDIR 90
230  CSIZE 3.5
240  MOVE 0,Y_gdu_max/2
250  LABEL "Concentration (C)"
260  LORG 4
270  LDIR 0
280  MOVE X_gdu_max/2,.07*Y_gdu_max
290  LABEL "Time interval"
300  VIEWPORT .1*X_gdu_max,.98*X_gdu_max,.15*Y_gdu_max,
     .9*Y_gdu_max
310  FRAME
320  WINDOW 0,55,0,1
330  AXES 1,.05,0,0,5,5,3
340  FOR I=0 TO 34
350  MOVE I,1,0
360  DRAW I,.987
370  NEXT I
380  FOR I=0. TO .45 STEP .05
390  MOVE 55,I
400  DRAW 54.6,I
410  NEXT I
420  FOR I=0 TO 35 STEP 5
```

```
430  MOVE I,0
440  DRAW I,1.0
450  NEXT I
460  FOR I=40 TO 50 STEP 5
470  MOVE I,0
480  DRAW I,.5
490  NEXT I
500  MOVE 0,.25
510  DRAW 55,.25
520  MOVE 0,.5
530  DRAW 55,.5
540  MOVE 0,.75
550  DRAW 35,.75
560  CLIP OFF
570  LORG 2
580  CSIZE 2.5
590  MOVE 41,.800
600  LABEL "Concentration C(1)"
610  MOVE 41,.700
620  LABEL "Concentration C(2)"
630  MOVE 41,.600
640  LABEL "Concentration C(3)"
```

```
650  LORG 6
660  FOR I=0 TO 55 STEP 5
670  MOVE I,-.01
680  LABEL USING "#,K";I
690  NEXT I
700  LORG 8
710  FOR I=0 TO 1 STEP .25
720 MOVE -.5,I
730  LABEL USING "#,Z.DD";I
740  NEXT I
750  REM "READ VALUES FROM FILE"
760 DISP "ENTER FILE NAME";
770  INPUT File_name$
780  DISP File_name$
790  IF File_name$="RES01" THEN GOTO 820
800  IF File_name$="RES02" THEN GOTO 820
810  GOTO 770
820  ASSIGN @F_1 TO File_name$
830  CSIZE 3
840  IF File_name$="RES01" THEN GOTO 860
850  IF File_name$="RES02" THEN GOTO 890
860  MOVE 49,.900
```

```
870  LABEL "NON-REDUNDANT CASE"
880  GOTO 910
890  MOVE 49,.900
900  LABEL "REDUNDANT CASE"
910  FOR J=1 TO 55
920  ENTER @F_1 USING "ZZ,XX,#";I
930  ENTER @F_1 USING "6(MZ.DDDDDD)";C(1,J),DC(1,J),
     C(2,J),DC(2,J),C(3,J),DC(3,J)
940  NEXT J
950  ASSIGN @F_1 TO *
960  PENUP
970  FOR I=1 TO 3
980  IF I=1 THEN PEN 3
990  IF I=2 THEN PEN 2
1000 IF I=3 THEN PEN 4
1010 MOVE 37,(.9-(I*.100))
1020 DRAW 40,(.9-(I*.100))
1030 PENUP
1040 FOR X=1 TO 55
1050 PLOT X,C(I,X)
1060 NEXT X
1070 PENUP
1080 PEN 0
```

```
1090  NEXT I
1100  END
```

APPENDIX 3

"CALCULATION OF FLOWS FROM KNOWN CONCENTRATIONS".

```
10  REM "REVERSE PROCEDURE",PRG 03
20  OPTION BASE 1
30  DIM C(3,55),DC(3,55),A(9,9),B(9,1),K(9),D(9,9),
    E(9,1),F(4,4),A_inv(9,9),A_rs(9),A_invrs(9)
40  REM "READ VALUES FROM FILE"
50  DISP "ENTER FILE NAME";
60  INPUT File_name$
70  DISP File_name$
80  IF File_name$="RES01" THEN GOTO 110
90  IF File_name$="RES02" THEN GOTO 110
100 GOTO 50
110 ASSIGN @F_1 TO File_name$
120 FOR J=1 TO 55
130 ENTER @F_1 USING "ZZ,XX,#";I
140 ENTER @F_1 USING "6(MZ.DDDDDD)";C(1,J),DC(1,J)
    C(2,J),DC(2,J),C(3,J),DC(3,J)
150 NEXT J
160 ASSIGN @F_1 TO *
170 PRINTER IS 701
180 IF File_name$="RES01" THEN GOTO 200
190 GOTO 220
200 PRINT "REVERSE PROCEDURE USING DATA FOR NON
```

```
    REDUNDANT CASE"
210 GOTO 240
220 IF File_name$="RES02" THEN GOTO 230
230 PRINT "REVERSE PROCEDURE USING DATA FOR REDUNDANT
    CASE"
240 PRINTER IS 1
250 REM "SELECT TIME INTERVAL"
260 FOR I=1 TO 9
270 PRINT "ENTER CODE FOR TIME INTERVAL NUMBER";I
280 INPUT K(I)
290 NEXT I
300 PRINTER IS 701
310 PRINT
320 PRINT
330 PRINT "CODE FOR TIME INTERVAL 1:";K(1)
340 PRINT "CODE FOR TIME INTERVAL 2:";K(2)
350 PRINT "CODE FOR TIME INTERVAL 3:";K(3)
360 PRINT "CODE FOR TIME INTERVAL 4:";K(4)
370 PRINT "CODE FOR TIME INTERVAL 5:";K(5)
380 PRINT "CODE FOR TIME INTERVAL 6:";K(6)
390 PRINT "CODE FOR TIME INTERVAL 7:";K(7)
400 PRINT "CODE FOR TIME INTERVAL 8:";K(8)
410 PRINT "CODE FOR TIME INTERVAL 9:";K(9)
```

```
420 PRINTER IS 1
430 REM "SET UP MATRIX A"
440 FOR I=1 TO 3
450 A(1,I)=C(I,K(1))
460 A(2,(I+3))=C(I,K(2))
470 A(3,(I+6))=C(I,K(3))
480 A(4,I)=C(I,K(4))
490 A(5,(I+3))=C(I,K(5))
500 A(6,(I+6))=C(I,K(6))
510 A(7,I)=C(I,K(7))
520 A(8,(I+3))=C(I,K(8))
530 A(9,(I+6))=C(I,K(9))
540 NEXT I
550 PRINT A(*)
560 REM "CORRECT SIGNS"
570 FOR I=1 TO 7 STEP 3
580 A(I,1)=-A(I,1)
590 A((I+1),5)=-A((I+1),5)
600 A((I+2),9)=-A((I+2),9)
610 NEXT I
620 REM "SET UP MATRIX B"
630 B(1,1)=DC(1,K(1))
```

```
640  B(2,1)=DC(2,K(2))
650  B(3,1)=DC(3,K(3))
660  B(4,1)=DC(4,K(4))
670  B(5,1)=DC(5,K(5))
680  B(6,1)=DC(6,K(6))
690  B(7,1)=DC(7,K(7))
700  B(8,1)=DC(8,K(8))
710  B(9,1)=DC(9,K(9))
720  PRINT B(*)
730  REM "SELECT NUMBER OF DECIMALS FOR MATRIX A"
740  DISP "SELECT NUMBER OF DECIMALS FOR MATRIX A";
750  INPUT L
760  PRINTER IS 701
770  PRINT
780  PRINT
790  PRINT "NUMBER OF DECIMALS SELECTED FOR MATRIX A
     IS :";L
800  PRINT
810  REM "SELECT NUMBER OF DECIMALS FOR MATRIX B"
820  DISP "SELECT NUMBER OF DECIMALS FOR MATRIX B";
830  INPUT M
840  PRINT "NUMBER OF DECIMALS SELECTED FOR MATRIX B
     IS :";M
```

```
850  PRINTER IS 1
860  REM "TRUNCATE VALUES IN MATRICES A AND/OR B"
870  FOR I=1 TO 9
880  X=B(I,1)*10^M
890  Z=INT(X)
900  B(I,1)=Z/10^M
910  FOR J=1 TO 9
920  X=A(I,J)*10^L
930  Z=INT(X)
940  A(I,J)=Z/10^L
950  NEXT J
960  NEXT I
970  PRINT A(*)
980  PRINT B(*)
990  REM "CALL SUBROUTINE Ludsht"
1000 MAT D=A
1010 MAT E=B
1020 MAT A_inv=INV(A)
1030 MAT A_rs=RSUM(A)
1040 MAT A_invrs=RSUM(A_inv)
1050 Max_ra=0
1060 Sum_ra=0
```

```
1070  Max_ca=0
1080  Sum_ca=0
1090  Max_ria=0
1100  Sum_ria=0
1110  Max_cia=0
1120  Sum_cia=0
1130  FOR I=1 TO 9
1140  FOR J=1 TO 9
1150  Sum_ra=Sum_ra+ABS(A(I,J))
1160  Sum_ria=Sum_ria+ABS(A_inv(I,J))
1170  Sum_ca=Sum_ca+ABS(A(I,J))
1180  Sum_cia=Sum_cia+ABS(A_inv(I,J))
1190  NEXT J
1200  PRINT Sum_ra,Sum_ria,Sum_ca,Sum_cia
1210  IF Sum_ra>Max_ra THEN Max_ra=Sum_ra
1220  IF Sum_ria>Max_ria THEN Max_ria=Sum_ria
1230  IF Sum_ca>Max_ca THEN Max_ca=Sum_ca
1240  IF Sum_cia>Max_cia THEN Max_cia=Sum_cia
1250  Sum_ra=0
1260  Sum_ria=0
1270  Sum_ca=0
1280  Sum_cia=0
```

```
1290 NEXT I
1300 Condr=Max_ra*Max_ria
1310 Condc=Max_ca*Max_cia
1320 CALL Ludsht(D(*),E(*),9,1)
1330 PRINT "RESULT:"
1340 PRINT
1350 PRINT
1360 PRINT USING "6(DD.DDDDDD,XX)";E(1,1);E(2,1);E(3,1)
1370 PRINT
1380 PRINT USING "6(DD.DDDDDD,XX)";E(4,1);E(5,1);E(6,1)
1390 PRINT
1400 PRINT USING "6(DD.DDDDDD,XX)";E(7,1);E(8,1);E(9,1)
1410 S1=E(1,1)
1420 F(3,2)=E(2,1)
1430 F(4,2)=E(3,1)
1440 F(1,2)=S1-F(3,2)-F(4,2)
1450 F(2,3)=E(4,1)*2
1460 S2=E(4,1)*2
1470 F(4,3)=E(6,1)*2
1480 F(1,3)=(E(5,1)-E(4,1)-E(6,1))*2
1490 F(2,4)=E(7,1)*3
1500 F(3,4)=E(8,1)*3
```

```
1510  S3=E(9,1)*3
1520  F(1,4)=(E(9,1)-E(7,1)-E(8,1))*3
1530  F(2,1)=(E(1,1)-E(4,1)*2-E(7,1)*3)
1540  F(3,1)=(E(5,1)*2-E(2,1)-E(8,1)*3)
1550  F(4,1)=(E(9,1)*3-E(6,1)*2-E(3,1))
1560  PRINTER IS 701
1570  PRINT
1580  PRINT
1590  PRINT "FINAL RESULT:"
1600  PRINT
1610  PRINT
1620  PRINT TAB(4);"F(0,1)";TAB(16);"F(2,1)";TAB(27);
      "F(3,1)"
1630  PRINT
1640  PRINT USING "6(MZ.DDDDDD,XX)";F(1,2);F(3,2);F(4,2)
1650  PRINT
1660  PRINT
1670  PRINT TAB(4);"F(0,2)";TAB(16);"F(1,2)";TAB(27);
      "F(3,2)"
1680  PRINT
1690  PRINT USING "6(MZ.DDDDDD,XX)";F(1,3);F(2,3);F(4,3)
1700  PRINT
1710  PRINT
```

```
1720  PRINT TAB(4);"F(0,3)";TAB(16);"F(1,3)";TAB(26);
      "F(2,3)"

1730  PRINT

1740  PRINT USING "6(MZ.DDDDDD,XX)";F(1,4);F(2,4);F(3,4)

1750  PRINT

1760  PRINT

1770  PRINT TAB(4);"F(1,0)";TAB(16);"F(2,0)";TAB(27);
      "F(3,0)"

1780  PRINT

1790  PRINT USING "6(MZ.DDDDDD,XX)";F(2,1);F(3,1);F(4,1)

1800  PRINT

1810  PRINT

1820  PRINT "CONDITION NUMBER OF THE MATRIX"

1830  PRINT

1840  PRINT Condr

1850  PRINT Condc

1860  PRINTER IS 1

1870  PRINT "DO YOU WANT TO TRY A NEW TRUNCATION:Y/N?"

1880  INPUT R$

1890  IF R$="Y" THEN GOTO 730

1900  PRINT "DO YOU WANT TO TRY A NEW TIME INTERVAL:Y/N?"

1910  INPUT R$

1920  IF R$="Y" THEN GOTO 250
```

```
1930 END
1940 SUB Ludsht(A(*),B(*),N,M)
1950 Baddta=(N<=0) or (M<=0)
1960 IF Baddta=0 THEN 2000
1970 PRINT FNLin$(2);"ERROR IN SUBPROGRAM Ludsht."
1980 PRINT "M=";M;"  N=";N;FNLin$(2)
1990 PAUSE
2000 GOTO 1950
2010 OPTION BASE 1
2020 ALLOCATE Btemp(N),Xtemp(N),Ips(N)
2030 CALL Decomp(N,A(*),Ips(*))
2040 FOR J=1 TO M
2050 FOR I=1 TO N
2060 Btemp(I)=B(I,J)
2070 NEXT I
2080 CALL Solve(N,A(*),Btemp(*),Xtemp(*),Ips(*))
2090 FOR I=1 TO N
2100 B(I,J)=Xtemp(I)
2110 NEXT I
2120 NEXT J
2130 SUBEND
2140 SUB Solve(N,Lu(*),B(*),X(*),Ips(*))
```

```
2150 Baddta=(N<=0)
2160 IF Baddta=0 THEN 2200
2170 PRINT FNLin$(2);"ERROR IN SUBPROGRAM Solve"
2180 PRINT "N=";N;FNLin$(2)
2190 PAUSE
2200 GOTO 2140
2210 OPTION BASE 1
2220 Ip=Ips(1)
2230 X(1)=B(Ip)
2240 FOR I=2 TO N
2250 Ip=Ips(I)
2260 Sum=0
2270 FOR J=1 TO I-1
2280 Sum=Sum=Lu(Ip,J)*X(J)
2290 NEXT J
2300 X(I)=B(Ip)-Sum
2310 NEXT I
2320 Ip=Ips(N)
2330 IF Lu(Ip,N)<>0 THEN 2370
2340 PRINT FLNin$(2);"ERROR IN SUBPROGRAM Solve."
2350 PRINT "DIVISION BY ZERO DETECTED.";FLNin$(2)
2360 PAUSE
```

```
2370  X(N)=X(N)/Lu(Ip,N)
2380  FOR I=N-1 TO 1 STEP -1
2390  Ip=Ips(I)
2400  Sum=0
2410  FOR J=I+1 TO N
2420  Sum=Sum+Lu(Ip,J)*X(J)
2430  NEXT J
2440  IF Lu(Ip,I)<>0 THEN 2480
2450  PRINT FLNin$(2);"ERROR IN SUBPROGRAM Solve."
2460  PRINT "DIVISION BY ZERO DETECTED.";FLNin$(2)
2470  PAUSE
2480  X(I)=(X(I)-Sum)/Lu(Ip,I)
2490  NEXT I
2500  SUBEND
2510  SUB Decomp(N,A(*),Lu(*),Ips(*))
2520  Baddta=(N<=0)
2530  IF Baddta=0 THEN 2580
2540  PRINT FNLin$(2);"ERROR IN SUBPROGRAM Decomp."
2550  PRINT "N=";N;FNLin$(2)
2560  PAUSE
2570  GOTO 2520
2580  OPTION BASE 1
```

```
2810  Idxpiv=I
2820  NEXT I
2830  IF Big<>0 THEN 2870
2840  PRINT FNLin$(2);"ERROR IN SUBPROGRAM Decomp."
2850  PRINT "MATRIX IS MACHINE SINGULAR.";FNLin$(2)
2860  PAUSE
2870  IF Idxpiv-K=0 THEN 2910
2880  J=Ips(K)
2890  Ips(K)=Ips(Idxpiv)
2900  Ips(Idxpiv)=J
2910  Kp=Ips(K)
2920  Pivot=Lu(Kp,K)
2930  FOR I=K+1 TO N
2940  Ip=Ips(I)
2950  Em=-Lu(Ip,K)/Pivot
2960  Lu(Ip,K)=-Em
2970  FOR J=K+1 TO N
2980  Lu(Ip,J)=Lu(Ip,J)+Em*Lu(Kp,J)
2990  NEXT J
3000  NEXT I
3010  NEXT K
3020  Kp=Ips(N)
```

```
3030  IF Lu(Kp,N)<>0 THEN 3070
3040  PRINT FNLin$(2);"ERROR IN SUBPROGRAM Decomp."
3050  PRINT "MATRIX IS MACHINE SINGULAR.";FNLin$(2)
3060  PAUSE
3070  SUBEXIT
3080  SUBEND
3090  DEF FNLin$(X1)
3100  X=INT(X1+.5)
3110  IF X=0 THEN RETURN CHR$(13)
3120  Eol$=CHR$(13)&CHR$(10)
3130  IF X<0 THEN Eol$=CHR$(10)
3140  ALLOCATE R$[X*LEN(Eol$)]
3150  R$=""
3160  FOR I=1 TO X
3170  R$=R$&EoL$
3180  NEXT I
3190  RETURN R$
3200  FNEND !
```

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