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THERMAL BEHAVIOUR OF A HEAT EMITTING DEVICE IN UNSTEADY STATE

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Abstract

A "detailed" model of a heat emitter in hot water circulation has been developed considering, two phases (liquid and metal), the non linear heat transfer and the enthalpy transport by the fluid. A reduced order state model (order two or three) has been then formed, linearising the initial "detailed" model and using a model reduction technique developed in our laboratory which is especially based on modal analysis. The simulated results from both the models for a step input of the inlet temperature have been presented and discussed. The extension of the work in order to identify the parameters of the reduced order state model has been highlighted and further possibilities have been discussed. Finally, this study should be able to define an optimum methodology for the experimental characterisation of fluid circulation emitting devices

1 - Introduction

The analysis of the dynamic thermal behaviour in buildings is usually complex as the buildings are continuously subjected to variable sollicitations and as the thermal behaviour of each component is mutually coupled. In such a case, the dynamic thermal behaviour of heat emitting devices become extremely difficult. Furthermore, in modern installations heat emitting devices have to work in unsteady state due to the use of efficient thermal control systems, to maintain internal thermal conditions within the comfort zone and to minimise energy expenditure.

In such cases, dynamic modelling of the thermal behaviour of heat emitting devices become important and quite often one is compelled to propose simplified analysis due to the necessity of :

- rapid simulations
- optimisation in the design phase
- control or regulation

Model reduction technique is one of these methods widely used.

In litterature one finds that a large number of studies had been done in order to analyse the steady state performances of heat emitting devices commonly used in domestic applications. These have led to the development of standard test conditions, AFNOR[1] for measuring the output of emitters and to the definition of intrinsic parameters characterizing the steady state output under standard conditions. (K, n).

A few autors such as NIBEL et al., STEPHAN [2,3] have tried to develop reduced dynamic thermal models for heat emitters. These models considered the emitter to be divided into a few number of nodes (3 or 4). Each node being regarded to contain a well-mixed liquid / solid phase. External boundary conditions being defined by the well known law of emission in steady state, $P = K\Delta T^n$. Others do not consider an overall emission but take into account radiative and convective quantities. These models have proven to be quite efficient not far from steady state but, experiments have showed that these modellings are not capable of taking into account precisely the dynamic behaviour (they can give rise to errors which can reach the order of 10 to 50 %).

However, the autors of this paper consider that the knowledge of the tranfer function of the device could be practically obtained only by a dynamic state identification procedure but not by a static one. If a procedure of the kind " black-box" to be investigated the form of the model should be considered known before hand. The methodology developed in our center preserves the maximum knowledge of the physical model in defining a reduced form.The main object of this paper is then to develop such a form which should be simple, efficient and sufficiently reduced, for a heat emitting device in hot water circulation.

In the first part of this paper, we will develop a detailed model based on finite volume discretisation. This model will be able to take into account :

- two phases (solid and liquid),
- non linear heat transfer,
- bulk transport

Then , after having linearised the initial model, we will present in the second part the method of reduction of the order of the initial model. This method which is based on work published at CENERG(the center of energy of the Ecole Des Mines de Paris) especially by BACOT and NEIRAC [4,5] contains the following concepts:

- modal analysis
- model reduction techniques

In our case, the inherent problem of the modelling is the presence of the bulk transport in the energy equation which introduces non symetrical heat exchanges. However, EL KHOURY and NEVEU [6,7] have recently shown that the modal analysis could be extended to such cases.

The simulated results from the detailed and reduced models will then be compared and discussed.

After analysing the method, we will conclude by discussing the research work planed for the near future.

2 - Presentation of the model

2.1 Development of the equation

The emitter is considered to be comprised of a series of parallel conduits, one of which is analysed for the development of equations (elemental conduit). The main assumptions made at this state are the following:

i) two distinct temperatures are considered for water and metal.

ii) both solid and liquid phases are homogeneous and isotrop

iii) the flow in the elemental conduit is one-dimensionel, piston type, and constant.

iiii) the axial conduction of water is neglected compared to the bulk transport and that of the metal is not so.

v) physical properties are independent of the temperature

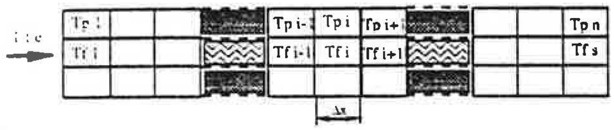

Fig. 1 : finite volume discretisation of the emitter

The energy balances at each node points of the elemental volumes lead to a system of non linear first order differential equations in temperature.

For the i^{th} node point

$$\begin{cases} M_f C_{pf} \dfrac{dT_{fi}}{dt} = \dot{m}_f\, C_{pf}\, (T_{fi-1} - T_{fi}) - h_i\, P\Delta x\, (T_{fi} - T_{pi}) \\ M_p C_{pp} \dfrac{dT_{pi}}{dt} = h_i P\Delta x (T_{fi} - T_{pi}) - k_1 (T_{pi} - T_a)^n + (T_{pi-1} - T_{pi}) \dfrac{\lambda S}{\Delta x} \\ \qquad - (T_{pi} - T_{pi+1}) \dfrac{\lambda S}{\Delta x} \end{cases} \quad (2.1.1)$$

with $k_1 = \dfrac{K}{N/2}$

with boundary and initial conditions as follows :

$T_f(0, t > 0) = T_{fe}$

$$\left.\frac{\partial T_p}{\partial x}\right) + \alpha (T_p - T_a) = 0 \qquad \text{at} \qquad x = 0$$

$$\left.\frac{\partial T_p}{\partial x}\right) + \beta (T_p - T_a) = 0 \qquad \text{at} \qquad x = L$$

$T_f(x, 0) = T_p(x, 0) = T_a$

In the first approach this system of equation is solved by NEPTUNIX preserving its inherant non-linearity.

2.2 State variable solution of the linearised model

The system of equations (2.1.1) has been linearised with respect to temperature as follows :

$$K (T - T_a)^n = K \Delta T^n \sim K (\overline{\Delta T})^{n-1} \Delta T \sim h_c \Delta T$$

where $\overline{\Delta T}$ represents the average value of ΔT within the dynamic temperature range considered. This linearisation can be affected without much difficulty as the value n , in most cases, is very close to 1 (it normally does not exceed the value 1.3).

It can be seen that the system (2.1.1) can be written in matrix form :

$$C \frac{dT(t)}{dt} = AT(t) + BU(t) \qquad (2.2.1)$$

where :

C The matrix of capacitance, diagonal contains the capacities of all node points. It is positive definite. (NxN)

A the matrix of conductances, diagonally dominant, contains the conductive terms between node points. (NxN)

$$\sum_{j=1}^{n} a_{ij} \leq 0 \qquad \forall\ i,j$$

B The command matrix, contains the sollicitations at node points. (NxP)

T(t) The temperature vector, contains the sollicitations at node points for the emitter, it is customany to write the equation of observation as follows :

$$Y(t) = JT(t) \qquad (2.2.2)$$

J The observation matrix, selects the variable to be observed. (QxN)

Y(t) The observation vector, contains the variable to be observed. (Q x 1)

In our case Y(t) is a scalar, and the variable observed is the temperaure of the metal in the middle. Now the system (2.2.1) and (2.2.2) could be written in the state variable form as :

$$\begin{cases} \dot{T}(t) = C^{-1}A\ T(t) + C^{-1}\ B\ U(t) \\ Y(t) = J\ T(t) \end{cases} \qquad (2.2.3)$$

As it is, the resolution of the system (2.2.3) is complex, but expressing the system with respect to the basis of eigen vectors of $C^{-1}A$ (we note that the eigen values are distinct, real and negative).

Using the linear transformation T(t) = P X(t), where P (det [P] ≠ 0) is the matrix of transformation containing the eigen vectors of $C^{-1}A$, the system (2.2.3) is written in basis of eigen vectors of $C^{-1}A$ (modal basis).

$$\begin{cases} \dot{X}(t) = F\,X(t) + G\,U(t) \\ Y(t) = H\,X(t) \end{cases} \qquad (2.2.4)$$

where :

$F = P^{-1}\,C^{-1}\,A\,P = \lambda_i\,\delta_{ij}$ is diagonal

$G = P^{-1}\,C^{-1}\,B$

$H = J\,P$

$X(t)$ is the state vector, contains the modes. (N x 1)

With respect to this basis, the solution is simple as F is diagonal, and the general solution is :

$$\begin{cases} X(t) = e^{Ft}\,X(0) + \int_{\tau=0}^{t} e^{F(t-\tau)}\,G\,U(\tau)\,d\tau \\ Y(t) = H\,X(t) \end{cases} \qquad (2.2.5)$$

It should be noted that the modes are independant as the matrix F is diagonal, and the order is "N" which is the number of nodes.

2.3 Reduction of the state variable model expressed in the modal basis.

We have adopted the methodology of reduction developed in our laboratory in the field of building heat transfer. The solution described in (2.2.5) is considered to be analogous to N first order linear filters subjected to different sollicitations and the observed variable is a linear combination of the responses of all the filters.

Thus each filter is represented by the equation :

$$\begin{cases} \dot{x}_i(t) = \lambda_i\,x_i(t) + \sum_{j=1}^{P} g_{ij}\,u_j(t) \\ y(t) = \sum_{j=1}^{N} h_i\,x_i(t) \end{cases} \qquad (2.2.6)$$

Note $y(t)$ is a scalar

Observing the system (2.2.6) it is seen that each filter is entirely characterized by a characteristic time $\tau_i = -1/\lambda_i$; and a gain g_{ij} for each sollicitation, by intelligently selecting h_i's to be unity.

In this case the impulse response of the i^{th} filer with respect to j^{th} sollicitation, could be written as :

$$x_i(t) = g_{ij}\,e^{-t/\tau_i} \qquad (2.2.7)$$

Then the energy (sense mathematic) contained in each mode with respect to the j^{th} sollicitation, E_{ij} is given by :

$$E_{ij} = \int_0^{\infty} x_i(t)^2\,dt = \frac{1}{2}\,\tau_i\,g_{ij}^2 \qquad (2.2.8)$$

and normalised later ($\bar{E}_{ij}$)

According to the nature of our system, the characteristic times (τ_i) are quite close to each other. Thus, the filters having almost the same τ_i values are first agregated conserving the mode with the hightest energy $\dot{E}_{ij}$, and then elliminating the low energy modes with respect to a threshold value (E_c) pre-defined. As a result the initial state model could be written in a form of a reduced state model of the order $r \ll N$.

$$\begin{cases} \dot{\tilde{X}}(t) = F^{*}\,\tilde{X}(t) + G^{*}\,U \\ \tilde{Y}(t) = H^{*}\,\tilde{X}(t) \end{cases} \qquad (2.2.9)$$

Where F^{*}, G^{*}, H^{*} are matrices of reduced order.

3 - Results and Discussion.

We are presenting the simulated results for both the models (detailed and reduced) for step type variation of the inlet fluid temperature from 20°C to 80°C, for two types of emitters actually existing in domestic applications.

- a heavy emitter with tubular columns made out of cast iron. (K=7.09 ; n = 1.27)
- a light emitter with two corrugated pannels made out of steel.(K=8.65 ; n = 1.29)

In these simulations we have considered the ambient temperature to be constant.

For each case, two flow rates (50 l/h and 150 l/h) actually found in real life applications have been envisaged.

The figures 1 and 2, corresponding to the case of the heavy emitter represent the evolution of the adimensional metal temperature (at the mid point) as a function of time.The figures 1(a) and 1(b) are related to a flow rate of 50 l/h, figures 2(a) and 2(b) are related to a flow rate of 150 l/h.

The figures 3 and 4, corresponding to the case of the light emitter represent the evolution of the adimensional metal temperature (at the mid point) as a function of time.The figures 3(a) and 3(b) are related to a flow rate of 50 l/h, figures 4(a) and 4(b) are related to a flow rate of 150 l/h.

nota : the adimensional temperature is defined as the ratio of the temperature over the amplitude of the step.

The first remarque that we make is that the effect of the linearisation is very little even though the operating temperature range (20°C to 80°C) has been considered.

The following remarque is that the reduction of the order of the initial detailed model appears to be very efficient. In fact it is observed that the difference between the results of the two models does not exceed few percents. Even though we have presented the results for a rise of inlet temperature, the decreasing case of inlet temperature is quite similar.

The important feature to be noted is that modal analysis leads to a very simple form (order 2 or 3). With respect to a reduced model obtained by nodal discretisation (well mixed liquid / solid model) with the same order, precision obtained in our case is very much better. By contrast, the modes x_1 and x_2 can not be physically linked with the actual temperatures of the emitter. Only the observed variables have a physical sense.

Concerning the influence of the flow rate, in the procedure developed, the flow rate is not considered explicitely as a sollicitation like inlet and ambiant temperatures but, as an internal parameter which could change. An aspect very important concerns the influence of the flow rate on the reduced parameters. Studies on this aspect are in the process of being carried out.

4 - Conclusion.

A "detailed" model of an emitter in hot water circulation has been developed considering : two phases (liquid and solid), non linear heat tranfer at the external boundary of the emitter and the bulk transport inside the emitter.
Considering the linearisation to be valid within the working temperature ranges, a reduced state model (order two or three) has been formed.

The comparison of the two models for a wide range of sollicitations has shown that:

- the linearisation does not cause much error in the initial "detailed" model and hence the initial model is more or less unaltered

- in comparison to simple models obtained by nodal discretisation (simply dividing the emitter into two or three nodes) based on the concept of well mixed liquid / solid phase, the reduced model presented in this paper preserves a great deal of the knowledge included in the initial "detailed" model

- it was noted that the reduced model could simulate almost with the same precision as the initial "detailed" model, which is the most important and remarquable feature of this model. However, with respect to the nodal models, it suffers from the deficiency that the modes can not be physically interpreted.

If we had shown that such a form for the reduced model was appropriate in the case of systems with fluid transport, it is not realistic, in real problems, to presume to obtain the parameters caracterizing real devices by modal analysis.

In practice, it appears more realistic and efficient in accessing these parameters by an identification procedure in dynamic state.

Thus, we are in the process of carrying out activities in this study. These activities could be broadly divided into three phases:

- In the first phase we are investigating theoritically the possibility of obtaining a reduced model by a technique of identification.

- In the second phase, we are trying to analyse the influence of the flow rate on the parameters of the reduced model. The first results seem to be quite encouraging. The ideal solution would be to obtain the main parameters as a function of the flow rate.

- If this phases are successful, we plan to extend the procedure of identification based on experiments on real devices. The ultimate objective of this work is to define precisely an optimum experimental methodology capable of caracterizing any kind of emitter in dynamic state.

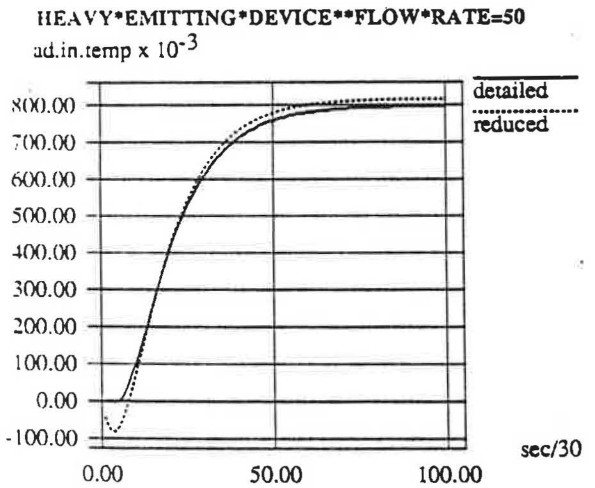

figure 1(a)

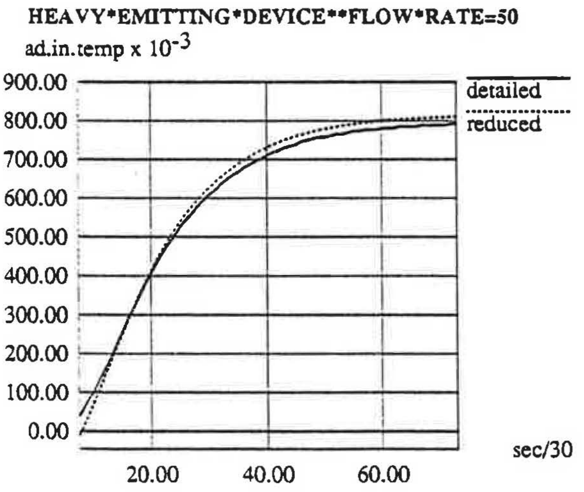

figure 1(b)

HEAVY*EMITTING*DEVICE**FLOW*RATE=150
add.in.temp x 10^{-3}

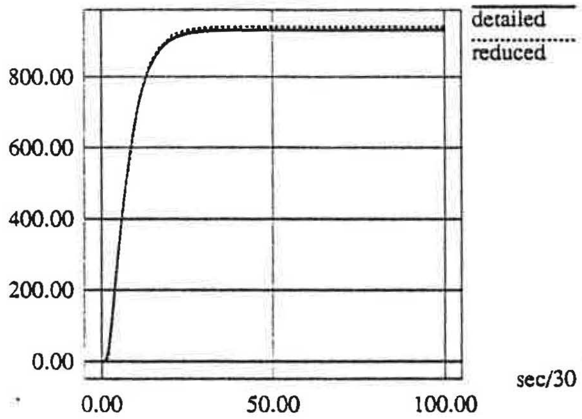

figure 2(a)

HEAVY*EMITTING*DEVICE**FLOW*RATE=150
ad.in.temp

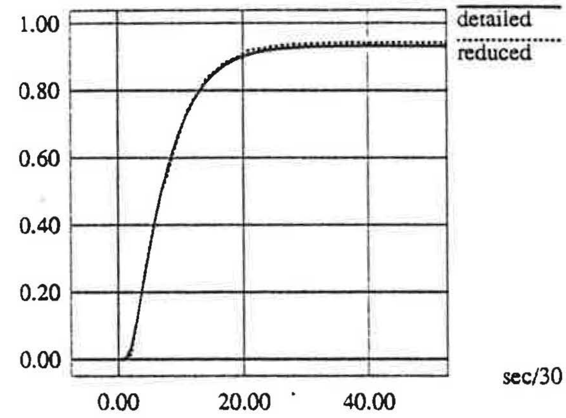

figure 2(b)

LIGHT*EMITTING*DEVICE**FLOW*RATE=50
ad.in.temp x 10^{-3}

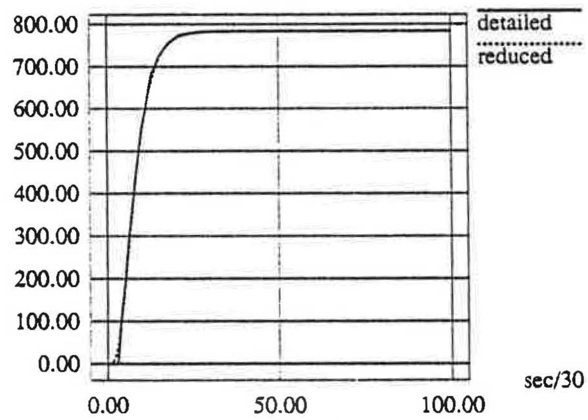

figure 3(a)

LIGHT*EMITTING*DEVICE**FLOW*RATE=50
ad.in.temp x 10^{-3}

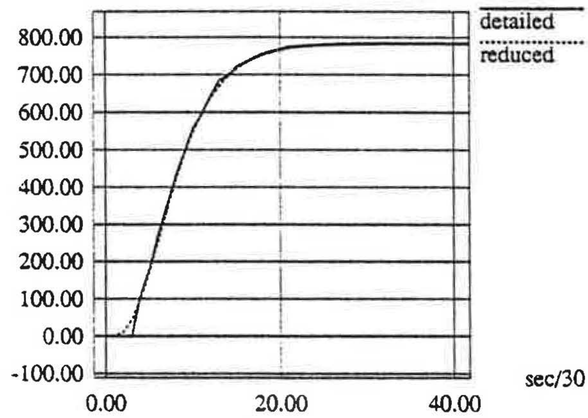

figure 3(b)

LIGHT*EMITTING*DEVICE**FLOW*RATE=150
add.in.temp x 10^{-3}

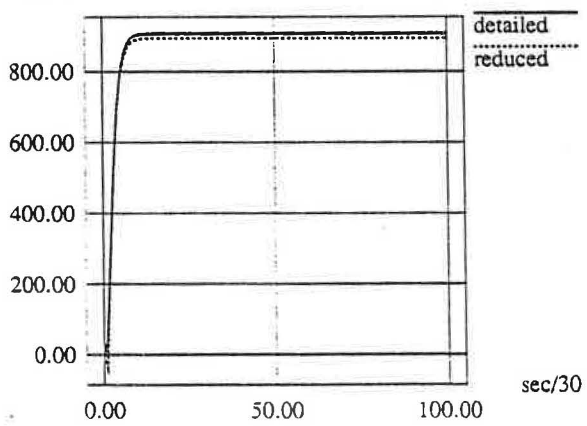

figure 4(a)

LIGHT*EMITTING*DEVICE**FLOW*RATE=150
add.in.temp

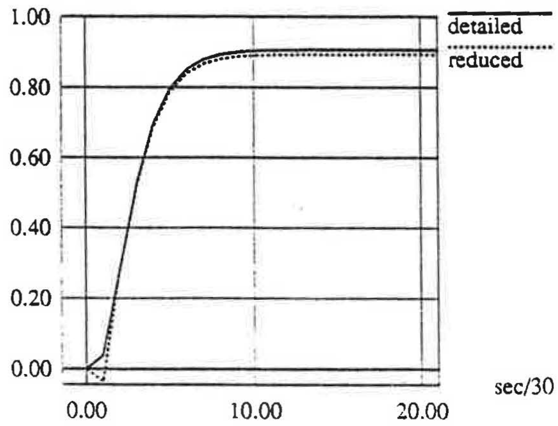

figure 4(b)

Nomenclature.

N	: total number of nodes (i.e. N/2 in solid and N/2 in liquid)
T_{fi}	: fluid temperature at the node point i
T_{pi}	: metal temperature at the node point i
T_{fe}	: inlet fluid temperature
T_{fs}	: outlet fluid temperature
T_a	: ambient temperature
C_{pf}	: specific heat of the fluid
C_{pp}	: specific heat of the metal
Δx	: length of an element
$\dot{m}_f$	: mass flow rate of the fluid
M_f	: mass of the fluid contained in an element
M_p	: mass of the metal contained in an element
h_i	: heat transfer coefficient between metal and fluid
K, n	: factors characterising the emitter
P	: perimeter of the section
L	: length of the emitter

subscripts.

f	: fluid
p	: metal

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