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# A TRANSIENT MODEL FOR ANALYZING THE HYGROTHERMAL BEHAVIOR OF BUILDING CONSTRUCTIONS

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## ABSTRACT

While advanced models for combined heat and moisture transfer have been available in the community of building scientists within the last two decades, such models have not yet become an item in the toolbox of consultants, building designers or manufacturers of building components.

Moisture dimensioning among the practitioners still takes place by rules of thumb or at best by use of the steady state Glaser method or modifications thereof.

Originally developed in a doctoral study (Pedersen 1990) the model, MATCH (Moisture and Temperature Calculations for Constructions of Hygroscopic Materials), comprises the common features of university models for one dimensional heat and moisture transfer. For instance, the model has an exact description of moisture influence on the thermal behavior of constructions by taking into account the transfer of latent heat. The model also provides a description of hysteresis in the moisture retention properties of the materials.

MATCH has recently been rewritten into a user friendly format. Running on a PC, it provides practitioners with a sophisticated tool for transient calculation of the coupled transport phenomena.

The paper introduces the theory behind the program and describes how latent heat and hysteresis effects are taken into account. Further, comparisons are given between predictions with the proposed model and measurements and between the model and the Glaser method.

## THEORY

### Selection of the driving potentials

Many models that have dealt with combined heat and moisture transfer have used a set of driving potentials for the transport phenomena consisting of the temperature and the moisture content in the materials (as a percentage by weight, for instance). The needed material properties have been the thermal and moisture diffusivities together with diffusivities that described the interactions between the transfer of heat and mass. The theoretical background behind such models was described by for instance Philip & de Vries 1957 and Luikov 1966. Such models are quite convenient for the study of moisture transport in single materials. The diffusivities needed for these models may easily be found from time series of moisture distribution in a transient experiment.

In a composite construction, however, the moisture content of the material is not as well suited as a choice of the driving potential for the transport since it does not change in a continuous manner over the material interfaces. Further, the moisture diffusivity may show some sign of hysteresis due to hysteresis in the moisture retention properties.

For this reason, a model was developed in which the vapor pressure was used as the driving potential for the vapor flux while the suction pressure was used for the liquid flux. In the following the combined problem is set up by observing each of the elementary transfer processes: Heat conduction (Fourier), vapor diffusion (Fick) and unsaturated liquid flow (Darcy). First the transport equation is written (a), then the corresponding balance equation (b). Some of the coupled phenomena between the elementary processes are taken into account by adding source terms to the right hand side of the balance equations.

Thermal equations:

$$q = -\lambda(T,u)\frac{\partial T}{\partial x} \tag{1a}$$

$$(\rho c_p)(T,u)\frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x} + S_q \tag{1b}$$

Vapor oriented equations for moisture transfer:

$$g_v = -\delta_p(u)\frac{\partial p}{\partial x} \tag{2a}$$

$$\rho_0\frac{\partial u}{\partial t} = \rho_0\xi(u)\frac{\partial\left(\frac{p}{p_s(T)}\right)}{\partial t} = -\frac{\partial g_v}{\partial x} + S_v \tag{2b}$$

Liquid oriented equations for moisture transfer:

$$g_l = K(u)\frac{\partial P_{suc}}{\partial x} \tag{3a}$$

$$\rho_0\frac{\partial u}{\partial t} = \rho_0\Xi(u)\frac{\partial P_{suc}}{\partial t} = -\frac{\partial g_l}{\partial x} + S_l \tag{3b}$$

where:

| | |
|---|---|
| $q$ | Conducted heat flux [W/m²] |
| $\lambda$ | Thermal conductivity as a function of temperature and moisture content [W/(m·K)] |
| $T$ | Temperature [K] |
| $x$ | One dimensional coordinate [m] |
| $\rho c_p$ | Effective product of density and specific heat, considering the ice/water content [J/(m³·K)] |
| $t$ | Time [s] |
| $S_q$ | Source term [W/m³] |
| $g_v$, $g_l$ | Fluxes of vapor and liquid [kg/(m²·s)] |
| $\delta_p$ | Vapor permeability as a function of moisture content [kg/(m·s·Pa)] |
| $p$ | Vapor pressure [Pa] |
| $p_s$ | Saturation vapor pressure as a function of temperature [Pa] |

$\rho_0$ Dry density [kg/m$^3$]
$u$ Moisture content [kg of moisture / kg of dry material]
$\xi$ Vapor oriented moisture capacity (slope of the sorption curve) [kg/kg]
$S_v$, $S_l$ Source terms [kg/(m$^3$·s)]
$K$ Hydraulic conductivity as a function of moisture content [kg/(m·s·Pa)]
$P_{suc}$ Suction pressure (negative value of hydraulic pressure) [Pa]
$\Xi$ Liquid oriented moisture capacity (slope of the suction curve) [kg/(kg·Pa)]

The source term $S_q$ in equation 1b may represent radiative and convective contributions at the boundaries and contributions from the conversion of latent heat when moisture evaporates or condenses. The term $S_v$ in equation 2b for the vapor transfer may represent a convective contribution at a boundary or it may be the derivative of the liquid flux. Similarly, the derivative of the vapor flux may be inserted for $S_l$ in equation 3b.

To clarify the terminology: The sorption curve gives the relation between the relative humidity in the pores or the environment of a material and its moisture content. The suction curve describes the relation between suction pressure of the water in the pores and the moisture content. Both of these relations are encompassed by the term "retention curves". Such curves give the relation between the quantity being kept track of in the continuity equation - here the moisture content - and the driving potentials for the transport - the pressures.

Though equations 2 and 3 appear to express the same thing they are not always equally well suited. Equations 2 are specially suited for the description of what goes on in the hygroscopic region of moisture content (i.e. when the relative humidity is less than 98%). Below the so-called critical moisture content (the limit at which a continuous liquid phase exists through the pores of the material) the source term of equation 2b becomes negligible. Equations 3 are better used to describe what goes on in the over-hygroscopic region. Here the slope of the sorption curve is very steep so there is not much resolution left of the relative humidity, $p/p_s$, to indicate the moisture content, though a large amount of moisture may still be stored in the material.

The temperature dependence on the moisture transport comes in by calculating the temperature distribution first. It will affect the distribution of the saturation vapor pressure which is used in equation 2b.

Numerical procedure

The combined problem is treated by recursively solving each of the transient equations for heat, vapor and liquid moisture transfer. In the numerical formulation the partial derivatives of the transport equations 1a through 3a are substituted by finite differences. For instance, the thermal resistance between a layer *i* and a layer *i+1* is calculated as the sum of the resistances of half the layer *i* and half the layer *i+1*. The balance equations 1b through 3b are written for finite control volumes. In the following, the discretized versions of equations 1, 2 and 3 will not be written but will be referenced by their original differential counterparts.

The set of equations 1 may easily be joined to formulate an implicit solution of the temperature profile. The amount of vapor condensing or evaporating in the control volume is taken explicitly from the former time step and multiplied by the heat of evaporation to yield the source term $S_q$. Often, the explicit portion will not become dominating and the calculation may therefore proceed with time steps as large as 1 hour - the interval that many weather data are stored with. A check is always performed to see that the temperatures inside a construction do not vary more than at the boundaries, and the time interval is calculated all over with smaller time steps when that is the case.

Eliminating the vapor flux, $g_v$, between equations 2a and 2b may result in an implicit procedure for calculating the vapor pressures. When doing so, the liquid moisture transport is incorporated by adding as an explicitly given source term, $S_v$, the negative value of the derivative of the liquid flux from the former time step. This will result in a distribution of vapor pressures which will be regarded as an intermediate distribution.

By using equations 3 in a similar way, an intermediate distribution of suction pressures will be calculated using the old derivative of the vapor flux in the source term, $S_l$.

The intermediate distributions of vapor and suction pressures are used together with the transport equations 2a and 3a to form a resulting moisture flux, $g = g_v + g_l$, which may be used in the moisture balance for control volume *i*:

$$\rho_{0,i}\frac{\Delta u_i}{\Delta t} = g_{i-1-i} - g_{i-i+1} \tag{4}$$

In this way the moisture content is updated, and the retention curves may be used to give the final new pressures. The result is to have a semi-implicit scheme which has proven to always work stably with time steps of one hour, even when small layers of permeable materials with a low moisture capacity (like mineral wool) are being calculated. The procedure works succesfully no matter which of the two moisture transporting mechanisms that dominates.

Suction pressure and the hydraulic conductivity vary several decades with changes in moisture content and are therefore not very well suited for a discretized solution. Instead, a transformation of Darcy's law is used:

$$g_l = K\frac{\partial P_{suc}}{\partial x} = K\cdot P_{suc}\frac{\partial}{\partial x}\ln(P_{suc}) \tag{5}$$

The natural logarithm of the suction pressure and the new "material parameter" $K{\cdot}P_{suc}$ vary much less than the parameters they originated from.

In some constructions it may be relevant to let the liquid moisture transport out of the calculation. Thus, only equations 2 have to be solved implicitly with respect to the vapor pressure. This vapor pressure is still regarded as an intermediate result which is used to set up the moisture balance. The new vapor pressure may then be found from the moisture content. If this procedure is not followed, the non-linearity of the problem would not guarantee the overall moisture balance to be kept.

Hysteresis

An empirical procedure was developed to describe the path followed in the retention curves when hysteresis occurs between curves followed when moisture content is steadily increasing (absorption) or decreasing (desorption). The procedure will be introduced for hysteresis in the sorption curve though equally valid for the suction curve as well.

To a given relative humidity correspond a moisture content $u_a$ on the absorption curve and $u_d$ on the desorption curve and moisture capacities the two places $\xi_a$ and $\xi_d$ (slopes of the curves). If the history of moisture content shows a decline (over the two most recent time steps calculated) the following value for $\xi$ is used in equation 2b:

$$\xi = \frac{(u-u_a)^2\,\xi_d + 0.1\,(u-u_d)^2\,\xi_a}{(u_d-u_a)^2} \tag{6}$$

where u is the actual moisture content, which may be somewhere between $u_a$ and $u_d$.

A similar equation is used when the so-called scanning curve expresses a situation with increasing moisture content.

Figure 1 shows the path followed in the sorption diagram with the proposed model. Comparison of calculation results with and without the hysteresis model has been performed for constructions that were exposed to climates with natural variations. It showed that even as good results could be obtained when the sorption properties were described by an average curve between absorption and desorption (Pedersen 1990). The reason is that small diurnal variations in moisture content bring the conditions close to the middle of the area between the two curves - even when the long term trend is one of drying or wetting.

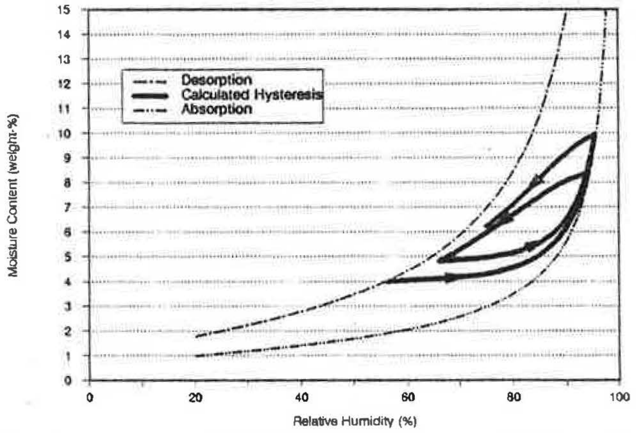

Fig. 1 Scanning curves followed in the sorption diagram for cellular concrete during interchanging periods of drying and wetting.

This may be different in the laboratory where conditions are kept stable and hysteresis may therefore still play an important role when material properties are determined.

## MATERIAL PROPERTIES

Material properties have not been measured particularly for use with MATCH. Instead, such properties have been gathered from the literature. This is by no means a sign of ignoring the importance of having good material properties and a fair amount of work still needs to be done in this basic research field. Andersson (1985) compared measured and calculated moisture profiles in several specimens and also determined their basic properties. She found that even small changes in the description of for instance a sorption curve could cause significant deviations in the calculated results.

In order to facilitate an easy access to good material properties a collection of these have been gathered in a file which is used by MATCH. Thus, the user can easily assign all the relevant parameters by simply identifying a material by its name. Of course, if the user has an idea of having better parameters, it is still possible to incorporate these by simply adding a new material with the new values to the material properties library.

First of all, the usual thermal properties for the materials are listed in the library. They comprise absorptance and emissivity (for use at the boundaries), dry density, heat capacity, thermal conductivity and corrections to the thermal conductivity due to the content of ice and moisture and due to the temperature. There is also a figure for the freezing point depression. Over an interval from 0°C down to the freezing point depression the thermal capacity of a material is increased to take into account the thermal effects of the formation or melting of ice.

The vapor permeability of the material is described by a constant value for relative humidities up to 60%. From here it increases linearly up to another fixed value at 98% RH. Finally, when the over-hygroscopic region is entered the permeability decreases linearly with the moisture content to the value 0 at total saturation.

The sorption isotherms are described by an analytical expression used by Hansen 1986 in a catalogue of sorption isotherms for most building materials. The expression used is:

$$u = u_h \left(1 - \frac{\ln\phi}{A}\right)^{-\frac{1}{n}} \qquad (7)$$

In this expression $\phi$ is the relative humidity [-] and $u_h$, A and n are parameters that describe the curve. Sometimes these parameters are given for both an absorption and a desorption curve.

The hydraulic conductivity is a negligibly small figure below the critical moisture content. Above this limit its natural logarithm increases linearly with moisture content up to the point of capillary saturation (maximum free water intake).

In the hygroscopic region the suction curve is derived from the sorption curve by use of Kelvin's equation. The rest of the curve is described by merging a series of hyperbolic expressions each describing smaller fractions of the curve. Only a few measurements of suction can be found in the literature. Instead, some of the curves have been established from the pore size distribution of the materials.

## ENVIRONMENT

The primary scope of MATCH is to use it in the analysis of constructions which separate an indoor from an outdoor climate. The outdoor climate will usually be hourly values of temperature, dewpoint, solar radiation and wind velocity taken from a weather file, for instance a test reference year, of a particular location. These values are used in a determination of the radiative heat exchanges and both the heat and vapor transfer coefficients at the outer surface.

The interior conditions are given by a temperature together with a humidity indicator that are kept fixed for a period of a month at a time. The indicator for humidity may either be a fixed relative humidity or a fixed humidity generation expressed as an increase in vapor concentration of the indoor over the outdoor air. The inside surface coefficients are kept fixed.

For special purposes a special format of the environmental data file may be used which holds hourly values of temperature and relative humidity on both sides of the construction. This is particularly functional when using data that have been measured.

## VALIDATION

As with models for building thermal load analysis, moisture transfer models are difficult to completely validate because they incorporate too many interrelated processes. Each of the elementary processes may be checked against analytical solutions, but this is usually not possible for the combined problem when the material properties vary with for instance the moisture content. Further, it is a problem that moisture content and distribution are difficult to accurately measure and that the material properties for moisture may vary largely even if specimens are from the same batch of material. Clarke (1985) on energy simulation tools states that: "Ultimately, it is only by comparing model predictions with the corresponding results from actual buildings in use that a model's usefulness as a predictive agent can be ascertained". The same could have been written about moisture transfer models

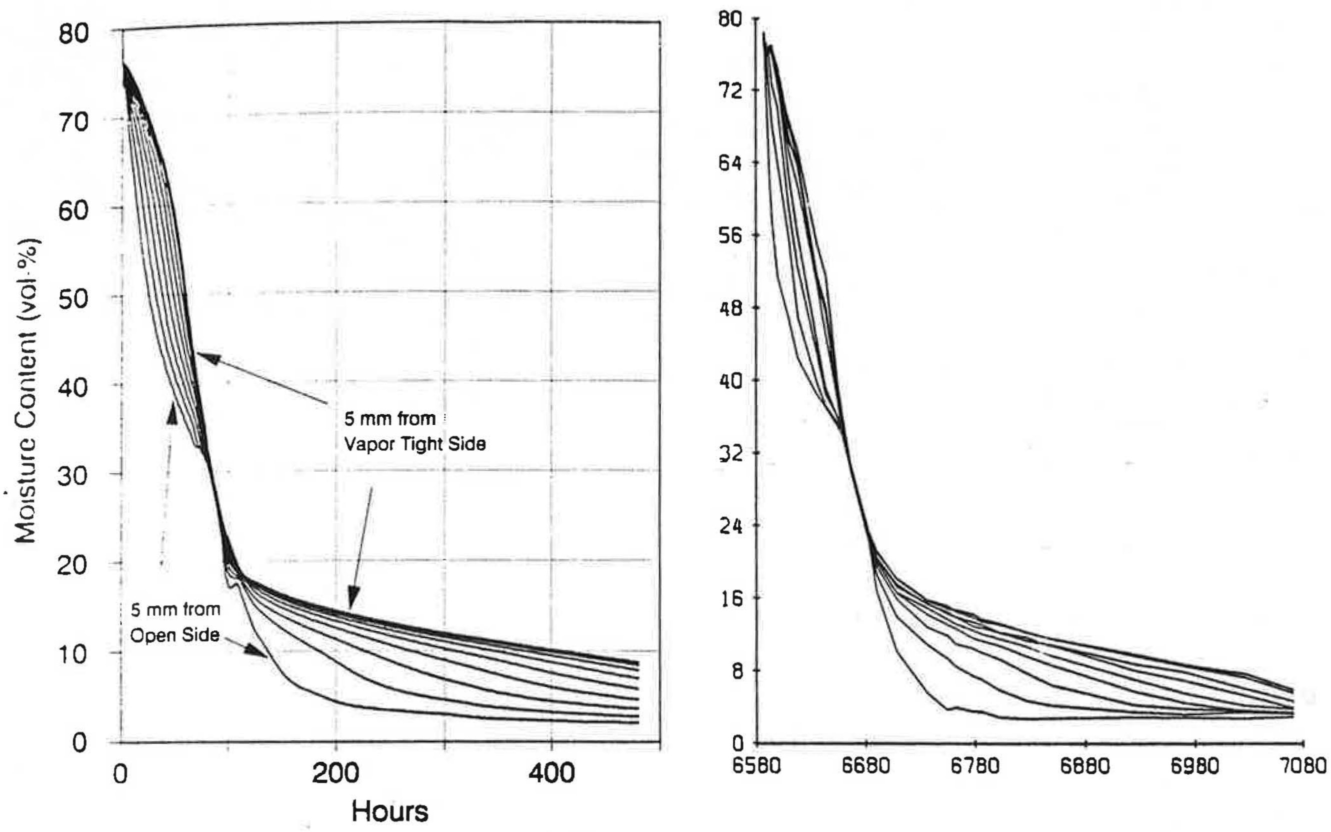

Fig. 2 Calculated (left) and measured changes of moisture content with time for the points measured.

Several projects are going on at the Technical University of Denmark and at the Danish Building Research Institute to study the moisture performance of low slope roofs. MATCH will be used to compare with results from these experiments. Earlier comparisons with such experiments have been shown in Pedersen (1990). There are also some comparisons with accurate laboratory measurements on cellular concrete and with measurements of heat fluxes in constructions insulated with wet mineral wool.

### Drying of cellular concrete

Nielsen (1974) made extensive investigations of moisture transport in cellular concrete using gamma ray equipment. In one test, a 50 mm high and 121 mm wide cylinder which was sealed at all sides except for one of the end surfaces, had been totally saturated under vacuum. Then it was allowed to dry out under exposure to air at a constant velocity. The moisture diffusivity for the very same specimen was reported by Nielsen and recalculated into the parameters used by MATCH. A calculation was performed using these parameters and the information of the environment in the room which was measured by Nielsen.

Figure 2 shows the calculated and the measured course of the moisture content in 9 points of the material (from Pedersen 1990). The overall agreement is quite satisfactory. A small deviation from a smooth curve is seen in the calculation around a moisture content of 18 vol-%. This is at the critical moisture content where the liquid transfer suddenly ceases. It was also found that the solution was somewhat sensitive to having a fine grid close to the open surface. It is necessary to get a good description of the steep gradients encountered in moisture content and its effect on the hydraulic conductivity.

### Latent heat

Measurements of temperature and heat flux are usually more accurate than the measurements of potentials and fluxes of moisture. The latent heat effects on thermal measurements may even be used to estimate the vapor transport properties of the materials (see for instance Kumaran 1987). Here predictions of heat flux will be compared with field measurements on wet fiber glass in a flat roof performed by Hedlin (1988) to indicate that the transfer of heat and moisture is calculated correctly.

The heat flux was measured at the bottom of a fiber glass cylinder with a known moisture content which was wrapped in polyethylene and inserted in similar materials in a roof. When the temperatures varied in diurnal cycles, the moisture condensed on the heat flux transducer in the daytime and re-evaporated in the night. This movement and the heat flows were simulated with MATCH using the measured temperatures at the boundaries and compared with the measurements. The material properties of the fiber glass were estimated from knowledge of its density. Figure 3 shows the result for a three day period of a specimen which held 0.25 vol-% moisture. As it is seen, the agreement is good. The deviation between measured and calculated heat flows over longer periods varied between 2 and 16% for specimens which held different amounts of moisture. The heat flow that would have occurred if the specimen were dry is also shown with a dashed line for reference. It is seen that the latent transfer may give an appreciable contribution.

Similar results were obtained from tests with flat roofs with plywood decks in a Large Scale Climate Simulator. Both of these experiments are described further in (Pedersen 1990).

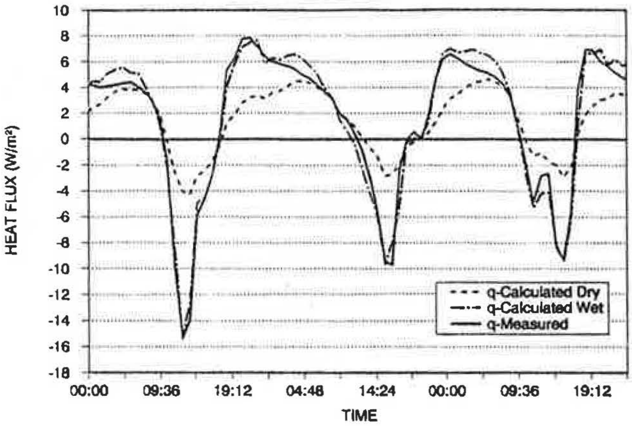

Fig. 3 Measured wet and calculated wet and dry heat flows for a fiber glass insulated roof construction with 0.25 vol-% moisture. Upward heat flow is positive.

COMPARISON WITH THE GLASER METHOD

The Glaser method (Glaser 1959), which is originally a graphical method, calculates moisture transfer as steady state diffusion after a steady state thermal calculation has been done. If the vapor pressures found exceed the saturation values in some points of the construction, condensation takes place and corrections are made to find the true distribution of vapor pressure. However, while steady state may be an acceptable approximation for the temperature profile it hardly ever occurs for the moisture profile. The Glaser method does not treat building materials as being hygroscopic i.e. showing their moisture capacity by gradually absorbing moisture at a steadily increasing vapor pressure. Usually this absorbing process will not get into equilibrium (it may take months) before the boundary conditions change. With the Glaser method materials are merely wet or dry. Further, vapor diffusion is the only moisture transport mechanism taken into account with the Glaser method.

Nevertheless, the Glaser method is still the primary method of dimensioning used in engineering practice side by side with rules of thumb which, of course, do not work with new constructions or materials. Therefore, there is a need for an easy-to-use-tool which circumvents some of the major drawbacks of the Glaser method.

Calculation of a flat roof

In the following, as an example, calculations will be shown for the following unventilated flat roof construction exposed to the Danish climate: Dark roofing felt, 12.5 mm plywood, 200 mm mineral wool, polyethylene vapor retarder and gypsum ceiling. Typically, a first calculation would employ the Glaser method with the same outdoor temperature as for designing heat loss. In Denmark this is -12°C and the result is seen from Figure 4. It says that 6.1 g/m² will condense in the construction per month. If there are no means to say if this moisture may dry out in other periods the construction should be condemned.

Of course, using this result alone would be very much on the safe side. The next step will therefore be to refine the calculation - for instance by making similar calculations for every month in the year using the mean surface temperature of the outer surface (which has to be calculated by some means if the solar radiation should be taken into account). Some moisture may be dried out in the summer months if it is assumed that the plywood is saturated. By comparing the potential drying with the condensation that takes place in winter, a better judgment should be possible. The result is shown in Figure 5 together with the result of an hour by hour calculation with MATCH.

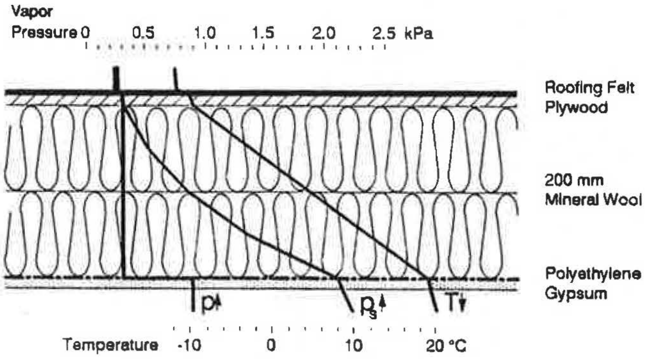

Fig. 4 The Glaser method applied to a flat roof construction exposed to an outdoor temperature of -12°C. 6.1 g/(m²·month) condenses in the plywood.

It is seen that MATCH predicts higher rates of moisture accumulation in winter. This is due to the fact that the plywood absorbs moisture hygroscopically. Since the plywood absorbs moisture at a vapor pressure different from saturation, say RH = 80%, the vapor pressure difference across the vapor retarder is larger than assumed with the Glaser method. Hence, the Glaser method is actually on the unsafe side. In summer, MATCH predicts considerably larger drying rates than the Glaser method. This is because MATCH makes an hour by hour evaluation where there are short but large peaks of downward vapor pressure gradients when the roof heats up on summer days. Since vapor pressure does not increase linearly with temperature such peaks do not show up when the average temperature of a full month is being used.

More clever applications of the Glaser method therefore make a vapor pressure weighted average of the outdoor temperature. Or, instead of making calculations month by month, the hours of the year are divided into smaller periods where the number of hours with certain intervals of the outer temperature are used. In this way the downward drying would be calculated as being very powerful - too powerful indeed. Now the moisture dries out of the construction at too fast a rate because saturation is assumed to prevail in the wood. Therefore, this will also be a solution which is too much on the unsafe side.

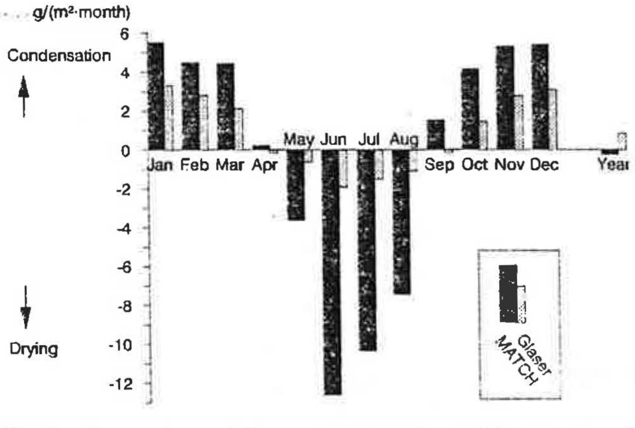

Fig. 5 Comparison of the net results of monthly and annual moisture movement when the Glaser method and MATCH are applied to analyze the roof from Figure 4.

An investigation of several types of flat roofs using MATCH was performed by Korsgaard and Pedersen (1990). That investigation took advantage of the possibility with MATCH to follow not only the total moisture flow in and out of the constructions. It was also shown how the moisture redistributed within the construction in response to changes in the environment.

## CONCLUSION

With small PC's it is possible to make numerical models for simulation of combined heat and moisture transfer in building constructions which have the same capabilities as the models used 10 and 20 years ago on main frame computers at the universities. Today as then, such models will never be more accurate than the material properties they use.

Since the field of moisture calculation is not at a very advanced level among designers of building constructions, it would be advantageous if models like the one presented in this paper were brought into use. Such models give results which are superior to those obtained with the traditional Glaser method because they consider the important effect of moisture capacity of hygroscopic materials and because they, by default, work with small time steps and therefore may incorporate the effect on vapor pressure of even short temperature peaks. Further, they are able to consider liquid moisture transport at the same time as vapor diffusion.

MATCH, which is described in this paper, uses an implicit formulation of the heat, vapor and liquid moisture flow which works stably in all ranges of moisture content with a time step equal to the one the weather data come with. Such data may come from a test reference year as it has long been the standard for building energy simulation programs. The material properties required for MATCH are stored in a file where they may easily be retrieved for use by the program. In these ways the program makes it possible to perform a more advanced moisture calculation at less work than with the Glaser method.

## ACKNOWLEDGMENT

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