ALGORITHMS
FOR MULTI-DIMENSIONAL HEAT TRANSFER IN BUILDINGS

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ABSTRACT

A first order correction to uni-directional heat transfer is proposed, so that multi-dimensional heat transfer effects can be accounted for with only a moderate increase in storage and CPU time requirements. The model has been implemented into the ESP building energy simulation program and is shown to be able to predict the order of magnitude of changes due to corner effects and thermal bridges. These effects are shown to be non-negligible even in full scale buildings, especially if one is interested in an accurate prediction of internal surface temperatures. As expected, the effects are particularly significant in scaled test cells, for which use of the algorithm results in an improved agreement with some experimental results.

INTRODUCTION

The use of comprehensive building energy simulation programs has considerably increased during the last ten years. The major building energy simulation programme in the European Community is ESP (Energy Simulation Program), developed by ESRU in Strathclyde University, Glasgow, Scotland, which has been chosen as the simulation program for the European cooerative effort for passive solar systems (PASSYS).

Although ESP (Clarke 1985) can be used, in principle, to model two and three-dimensional heat tranfer in building elements, in most cases heat transfer in building elements (walls, partitions, roofs etc.) is modelled as uni-directional, the enormous increase in storage and CPU time requirements being seldom justified in terms of increase in accuracy.

The uni-directionality assumption can lead to errors in the vicinity of the interface between two dissimilar building elements, where lateral temperature gradients may be appreciable. These phenomena can lead to deviations in the predicted loads and internal surface temperatures (which are important in comfort and condensation calculations). Although these effects do not justify switching to the full three dimensional equations, it is possible to obtain satisfactory results using simplified methods at a very small increase in cpu and storage requirements. This is particularly important for the PASSYS programme, in which

a lot of effort is invested in describing the behaviour of scaled models, in which thermal bridge effects are more important than in full-scale buildings, due to the increased perimeter-to-surface area ratio.

In previous publications (Hassid 1989, 1990) methods have been proposed to correct uni-directional heat transfer algorithms, to account for heat transfer accross thermal bridges between parallel elements.

In this work, an algorithm is proposed, which can account for several two-dimensional heat transfer phenomena without deviating essentially from the simplicity of the one-dimensional calculation. The algorithm will be introduced within the ESP programme, so that comparisons of heat transfer calculations with and without multi-dimensional heat transfer can be made.

THEORY

The problem of heat transfer around a right-angled corner admits an analytical solution, which is based on a Schwartz-Christoffel transformation of the flow domain (Carslaw and Jaeger 1959) :

$$Q_b = \lambda \frac{2}{\pi} \left[\frac{d_2}{d_1}\tan^{-1}(\frac{d_1}{d_2}) + \frac{d_1}{d_2} \tan^{-1}(\frac{d_2}{d_1}) + \ln(\frac{d_2^2 + d_1^2}{4d_2 d_1}) \right] \quad (1)$$

where Q_b is the corner heat transfer (ie the heat transfer per unit length aditional to the one calculated from a uni-directional algorithm, λ is the conductivity and d_2 and d_1 are the thicknesses of the sides of the corner. For the particular case that $d_2 = d_1$, Eq. (1) gives :

$$Q_b = 0.559 \lambda \quad (2)$$

Equation (1) can be easily generalised to the case of two walls which are homogeneous, but have different thermal conductivities, when the borderline between then two constructions is the line joining the internal and the external corner:

$$Q_b = \frac{2}{\pi}\left[\lambda_1 \frac{d_2}{d_1}\tan^{-1}(\frac{d_1}{d_2}) + 2\lambda_2 \frac{d_1}{d_2} \tan^{-1}(\frac{d_2}{d_1}) \right] +$$

$$+ \frac{1}{\pi}\left[\lambda_1 \ln(\frac{d_2^2 + d_1^2}{4d_2^2}) + \lambda_2 \ln(\frac{d_2^2 + d_1^2}{4d_1^2}) \right] \quad (3)$$

The above equations have some usefull properties. One of them is that contrary to what is assumed sometimes, the solution of taking the lateral dimension of the wall equal to average between its internal and its external dimensions is an not satisfactory and leads to an overestimate of heat transfer from a wall.

In the approach chosen in this work, heat flow in a corner between two homogeneous constructions is simulated using the electrical analogue in Figure 1, in which walls A and B are represented by two conductors. Flow through the corner is represented by two resistors with conductance $(\lambda_A + \lambda_B)/2$ and $3(\lambda_A + \lambda_B)/2$, respectively, with additional resistors connecting the inside, midplane and outside nodes of the resistors representing the walls. A property of trhe proposed network is that in the particular case of a corner between two homogeneous walls of equal width, one obtains, approximately, Eq. (2).
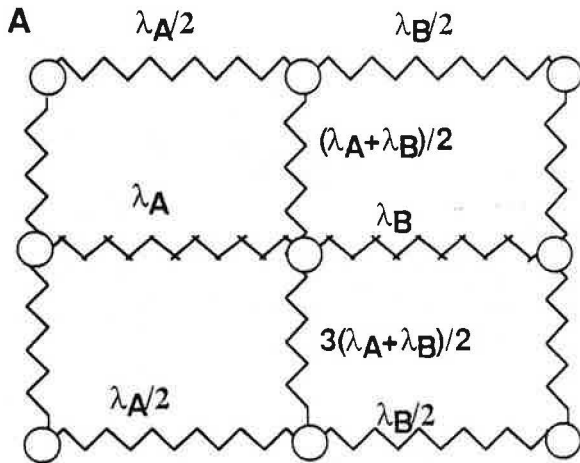

Figure 1. Corner Electric Equivalent

For corners bewteen multi-layer walls, one can divide the corner into layers and nodes, ina way similar that constructions are divided into layers in ESP and draw a balance around each node, lateral heat transfer being taken into account through the electrical analogue of Figure 1. For example, the heat balance in a node communicating with in the midplanes of two homogeneous layers would be (See Fig. 2):

$$\rho cwd\frac{T_N - T_N^-}{\Delta t} = (\lambda_A + \lambda_B)\left[(2w-d)\frac{T_{N+1} - T_N}{d} - (2w+d)\frac{T_N - T_{N-1}}{d}\right]$$

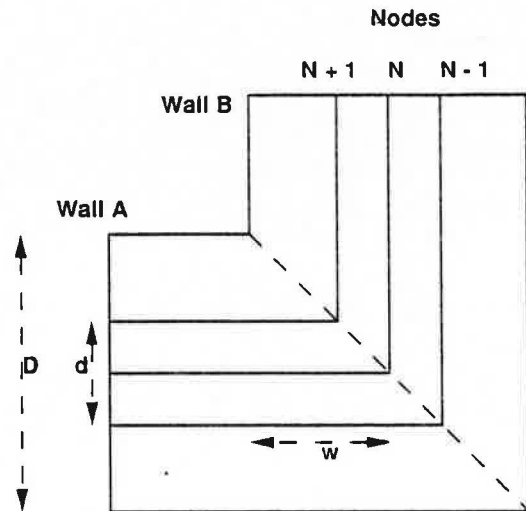$$+ \frac{\lambda_A d}{D}(T_{AN} - T_N) + \frac{\lambda_B d}{D}(T_{BN} - T_N) \qquad (4)$$

Fig. 2. Schematic representation of corner construction

where :

T_N	is the temperature at node N
T_{AN}, T_{BN}	are the temperatures at the corresponding nodes of the adjacent surface constructions A and B.
d	is the thickness of the layer whose midpoint is node N
D	is the total thickness of the construction
w	is the width of the corner construction at node N
T_N^-	is the value of T_N at time t - Δt

Note that:

a. The inner surface node does not communicate with the internal surface air by convection or the internal surface temperatures by radiation; it only communicates with the internal surface nodes of the adjacent wallls by conduction.
b. The outer surface (node 1 in ESP) communicates by radiation and convection to the outer environments of both constructions A and B, the convection coefficient and radiation intensity assumed equal to the one of these walls. Of course, this assumption can be questioned, but it is the only one that can be made without needing to consider multi-dimensional radiative and convective heat transfer.
c. Contrary to the Crank-Nickolson center differnced scheme used for the plane constructions by ESP, a forward differnceing scheme , for numerical stability reasons.

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IMPLEMENTATION IN ESP

The algorithm developped in the previous section has been implemented in ESP (Clarke, Hand & Strachan 1990). The algorithm, which can be invoked in the programme if required and consists of several stages:

a. For each building zones, all "line" constructions are identified and characterised by their two vertices and the surfaces adjacent to them.

b. If the angle of the construction is convex inside, the program invoques the algorithm described in the previous section. If the angle is 180° or more, then only intra-construction heat transfer is taken into account.

c. The properties of the layers of each line construction are determined from the layers of the adjacent constructions.It is assumed that the internal nodes of each construction are at the same plane. The matrices describing heat transfer in the corner constructions and the cross-construction heat transfer conductances are calculated.

d. At each time interval, the temperatures at the nodes of the line constructions and cross-construction heat transfer rates are calculated prior to the zone matrix inversion of ESP.

The program with the corner heat transfer requires, for 5000 line construction nodes, a 20 % increase in memory requirements and a 50-80 % increase in CPU time requirements, in comparison to Version 6.27.

SAMPLE RESULTS

Presented in Tables 1 and 2 are the results from a sample run based on Test 1 (See Figure 3) in the ESP Handbook (Clarke et al 1990), run with and without the algorithm presented here. One can see that as a result of accounting of two-dimensional heat transfer with the algorithm presented in this work, the predicted heating energy requirements are shown to be approximately 20 % higher and the predicted peak heating 10 % higher, for a full scale building.This figure does not account for the thermal bridges around the windows and doors.

A much larger difference can be observed for scaled test-cells, like the PASSYS Test Cell of Strathclyde University, Glasgow, Scotland (Strachan, 1990 - See Figure 4). In this run , the heat input in the cell is considered a casual gain and the predicted results are shown in Figure 5 and compared with the measured internal temperature variations. The version of ESP with the two-dimensional algorithm described in this work results in some improvement in comparison with the original ESP 6.27 Version. For example, the maximum predicted temperature is 35°C, versus 38°C without the 2-D algorithm

and 30°C the measured result. The above improvement, though, is consistent with the analytical result for a similar test cell whose wall thickness is 40cm and further improvement is impossible without accounting for the particular details of the corner construction.

In Figure 6, the internal surface temperature variation' of two adjacent surface constructions of test1 is shown: Surafce 1, an external wall and Surface 8, a wall over the ground. It is seen that in addition to influencing the values of the internal surface temperature, the algorithm predicts a much lower surface temperature at the line joining them. This phenomenon is very important if one is interested in condensation calculations.
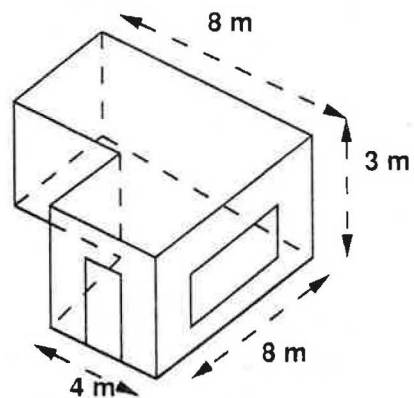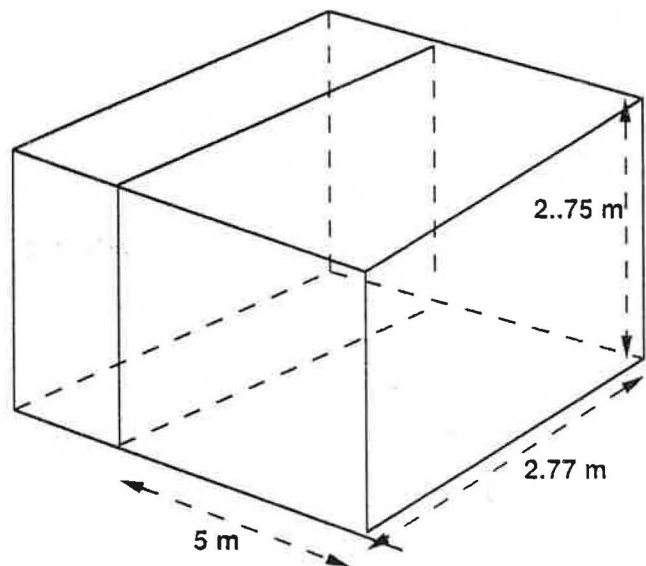

Fig. 3. Test 1 of ESl

Fig. 4. PASSYS Test Cell

Table 1 Output from test1 in ESP handbook (without 2-D algorithm)

```
Period simulated from day 22 of month  2 to  day  7 of month  3
    Zone Mx Air tmp Mn Air tmp    Max heat   Max cool  Heating  Cooling
         (deg. C)   (deg. C)       (KW)       (KW)     (Kwhrs)  (Kwhrs)
      1    21.50      13.98         1.697      0.000     68.3      0.
         @13.50-2-3  @ 6.50-5-3  @ 9.50-24-2 @ 0.50-22-2
All zones :
Max. Temp. =     21.5 in Zone  1 on day  2 of month  3 at 13.50 hrs
Min. Temp. =     14.0 in Zone  1 on day  5 of month  3 at  6.50 hrs
Max. Heat. =      1.7 in Zone  1 on day 24 of month  2 at  9.50 hrs
Max. Cool. =      0.0 in Zone  1 on day 22 of month  2 at  0.50 hrs
    Total heating requirements =     68.30 (Kwhrs)
    Total cooling requirements =      0.00 (Kwhrs)
```

Table 2 Output from test1 in ESP handbook (with 2-D algorithm)

```
Period simulated from day 22 of month  2 to  day  7 of month  3
    Zone Mx Air tmp Mn Air tmp    Max heat   Max cool  Heating  Cooling
         (deg. C)   (deg. C)       (KW)       (KW)     (Kwhrs)  (Kwhrs)
      1    21.19      13.60         1.871      0.000  ·  81.2      0.
         @13.50-2-3  @ 6.50-5-3  @ 9.50-24-2 @ 0.50-22-2
All zones :
Max. Temp. =     21.2 in Zone  1 on day  2 of month  3 at 13.50 hrs
Min. Temp. =     13.6 in Zone  1 on day  5 of month  3 at  6.50 hrs
Max. Heat. =      1.9 in Zone  1 on day 24 of month  2 at  9.50 hrs
Max. Cool. =      0.0 in Zone  1 on day 22 of month  2 at  0.50 hrs
    Total heating requirements =     81.21 (Kwhrs)
    Total cooling requirements =      0.00 (Kwhrs)
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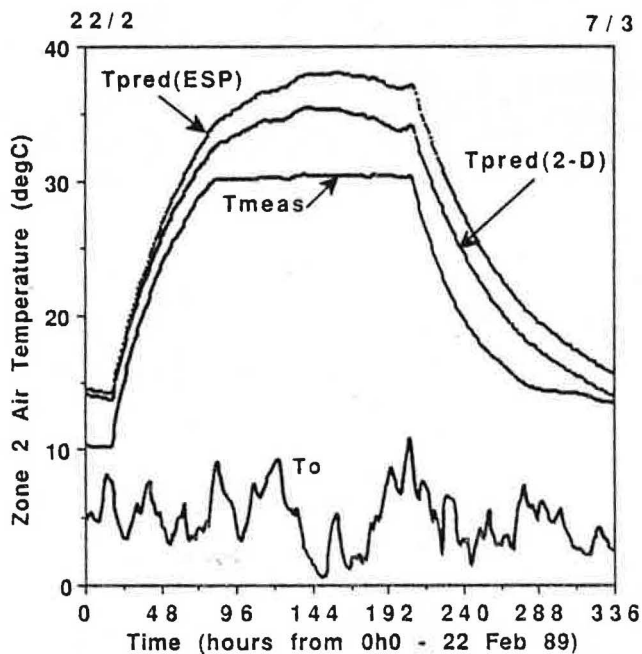

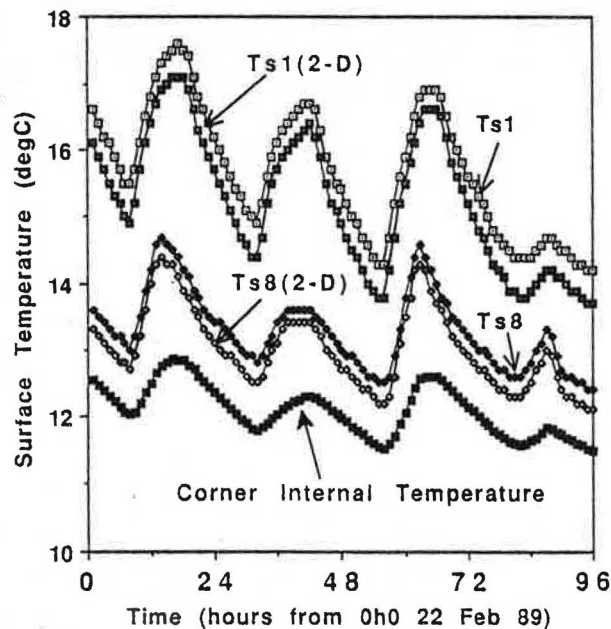Fig. 5. Predicted and measured internal temperature in PASSYS cell.

Fig. 6. Predicted internal surface temperature in test1

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DISCUSSION AND CONCLUSIONS

A model has been proposed, which can give a first order correction to uni-directional heat transfer so that multi-dimensional effects can be accounted for. The model has been implemented into the ESP building energy simulation program and is shown to be able to predict the order of magnitude of changes due to multi-dimensional heat transfer, while keeping the basic simplicity of the uni-directional heat transfer algorithms used by most building heat transfer simulation programs and with only a moderate increase in storage and CPU time requirements.

Two-dimensional and thermal bridge heat transfer effects are shown to be non-negligible even in full scale buildings, especially if one is interested in an accurate prediction of internal surface temperatures, as well as heating requirements and internal air temperatures. As expected, the effects are particularly significant in scaled test cells.

Of course, the model has several limitations, the most important one being that it cannot account for the particular details of the corner construction between two surfaces. To fully account for this effect, one has to introduce the particular details of the corner construction, solve numerically the relevant two dimensional heat transfer equations and attempt to simulate the results using an electrical analogue similar to Figure 1, proper consideration being given to the thermal storage properties, too. Such work is currently under way.

Acknowledgement

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