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MODELLING FOR BUILDING ENVIRONMENTAL CONTROL

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Summary

In this paper, different techniques for modelling building thermal systems are investigated. These consist of advanced stochastic time series analysis using correlation techniques and the more usual deterministic approach based on physical analysis. A test cell is used as the basis for comparing and contrasting the two approaches. It is shown that since building thermal behaviour can be subject to considerable stochastic influences, the former approach offers good potential for describing system behaviour; this is of importance in certain applications, such as control. Evaluation of the deterministic approach shows it is capable of handling stochastic effects but only with difficulty. However, the latter approach offers the ability to evaluate differing designs prior to the building construction.

1 INTRODUCTION

Since the earliest times mankind has sought shelter from the inclemencies of the natural environment. Initially this was achieved by the use of caves and tree hollows, but with time, this has progressed to the stage where use is now made of elaborate purpose-built structures in which artificial environments are provided by means of complex heating, ventilating and air-conditioning (HVAC) systems (building services). Until recently, control was effected by means of classical analogue techniques only. However, the advent of the microprocessor has offered building services engineers the ability to carry out digital control by computer. Although computer control methods have been utilised for a number of years in areas such as process control, the introduction of the techniques to building services systems is relatively recent. This application to the built sector is popularly terms "BEMS" - Building Energy Management Systems, and the phrase "intelligent buildings" has been coined to describe those in which a BEMS has been installed.

The "high-tech" image conveyed by BEMS can, however, give a false impression. While benefits in terms of energy savings have been substantial, this has often resulted from a better, more detailed, understanding of climate/building/HVAC plant behaviour which a BEMS can offer, by virtue of the large quantity of data recorded and the ease with which it can be analysed. In many commercially-available BEMS, the conventional control

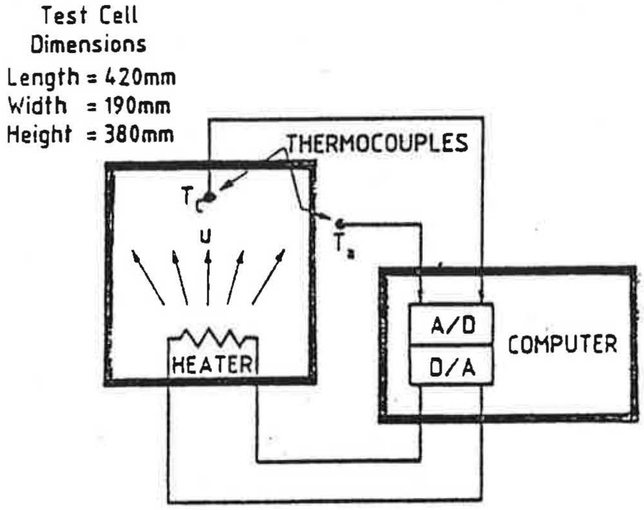


Figure 1: Computer control of a test cell

(i) the heating element was enclosed within a metal canister to introduce a larger dead-time;

(ii) steel plates were placed on the floor of the cell enclosure in an attempt to slow down the dynamics, and

(iii) an electric extract fan was inserted in one wall of the cell to introduce large magnitude stochastic disturbances.

Figure 2(a) shows the open-loop step response of the original cell and Figure 2(b) the response for the modified cell. The remainder of the discussion in this paper relates to the modified test cell.

3 STOCHASTIC MODELLING

The test cell may be represented by a mathematical model of the form (see for example Box and Jenkins [7], Norton [8] and Ljung [9]):

$$A\left(z^{-1}\right)T_c(t) = z^{-\ell 1}B_1\left(z^{-1}\right)u(t) + z^{-\ell 2}B_2\left(z^{-1}\right)T_a(t) + C\left(z^{-1}\right)e(t) \qquad (1)$$

where

$$A\left(z^{-1}\right) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na} \qquad (2)$$

$$B_1\left(z^{-1}\right) = b_{10} + b_{11} z^{-1} + b_{12} z^{-2} + \cdots + b_{1,nb} z^{-nb} \qquad (3)$$

$$B_2\left(z^{-1}\right) = b_{20} + b_{21} z^{-1} + b_{22} z^{-2} + \cdots + b_{2,nb2} z^{-nb2} \qquad (4)$$

$$C\left(z^{-1}\right) = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{nc} z^{-nc} \qquad (5)$$

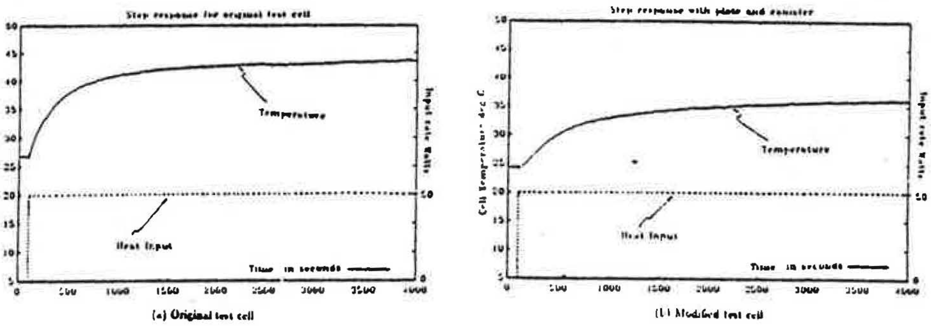


Figure 2: Step responses of the test cell

The term z^{-1} is the delay operator ($z^{-1}f(t) = f(t-T)$ where T is the sampling interval in seconds), $T_c(t)$ is the cell internal air temperature at time t, $u(t)$ is the heat input rate to the cell, $T_a(t)$ is the ambient air temperature, $e(t)$ is a white noise process to represent stochastic effects, ℓ_1, ℓ_2 are the dead times of the system and $na, nb, nb2$ and nc are orders of the respective polynomials.

An open-loop step response was carried out and values of T_c, T_a and u were recorded every second for three hours. The correlation between the step input u and the cell temperature T_c gave the dead time ℓ_1 as approximately 60 seconds. The value of T_a remained essentially constant and hence was ignored in the modelling process, as discussed in Virk et al [3]. To represent this dead time by an integer number of sampling intervals, the sampling rate can be chosen to be any factor of 60 seconds. Tests could therefore be carried out for values of 10, 20 and 60 seconds; the faster the sampling rate the greater the processing power required for on-line modelling. In view of this, a rate of 60 seconds was chosen giving a dead time of one sampling interval.

The test cell was subjected to a pseudo random binary sequence (PRBS) heating input, having values 0 or 1, at a sampling interval of one minute for a total time of ten hours. Readings of T_c and u were recorded for off-line identification purposes. These sequences are shown in Figure 3. For effective identification, such data must in general be normalised for the removal of trends, means, cyclic and seasonal effects (Box and Jenkins [7]). For the data in Figure 3 the means were removed giving a model of the form :

$$A\left(z^{-1}\right) T_c(t) = z^{-1} B\left(z^{-1}\right) u(t) + C\left(z^{-1}\right) e(t) + d_0 \qquad (6)$$

where d_0 is a D.C. term. The next stage in the identification procedure involved assuming a value for the order of the polynomials na, nb and nc and determining an estimate of the model parameters using the Identification Toolbox in PC-MATLAB. This model was validated by comparing predicted and measured values giving the errors (residuals). The squares of these can be summed to give an indication of model quality. As the orders are increased the model quality improves which is indicated by a reduction in the sum of the squares of the residuals. Repeating this procedure for several orders permits the plotting of errors against model order. A sudden change in slope indicates the correct model order. These orders were found to be $na = 3, nb = 2, nc = 2$. Using these orders a least squares

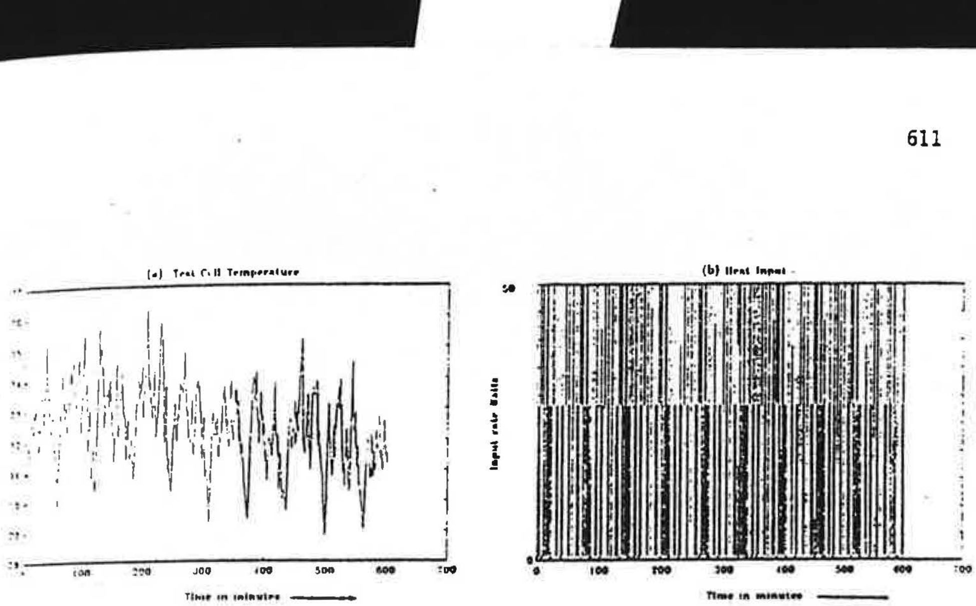


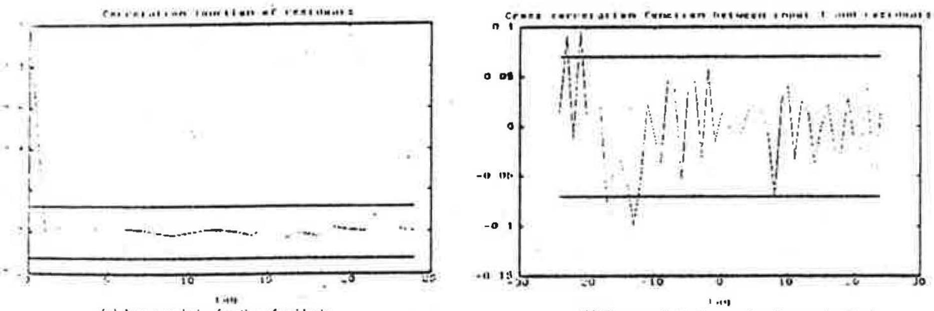
Figure 3: Test cell input/output data series

estimate yields an ARMAX model for the test cell as:

$$\left(1 - 2.03z^{-1} + 1.25z^{-2} - 0.22z^{-3}\right) T_c(t) = z^{-1}\left(0.74 - 0.34z^{-1} - 0.33z^{-2}\right) u(t) + \left(1 - 1.13z^{-1} + 0.16z^{-2}\right) e(t) - 0.04 \quad (7)$$

3.1 Model Validation and Discussion

MATLAB allows the models to be validated by performing auto-correlations on the residuals and cross-correlating the residuals with the input. These are shown in Figures 4(a)



(a) Auto-correlation function of residuals

(b) Cross-correlation between heat input and residuals

Figure 4: Correlation results

and 4(b), where both are seen to be satisfactory. In addition, the model can be used to predict the cell temperature, giving forecasts which may be compared with the measured values. This comparison is shown in Figure 5, where it can be seen that for the majority of the time forecasted and measured values agree to well within 1.0°C. Although this would be adequate in most cases, in practice more accurate forecasts could be achievable, since an air change rate of 400 ach is an exceedingly large stochastic effect. Even so, such large effects can be accommodated if sensors and instrumentation are installed to measure the

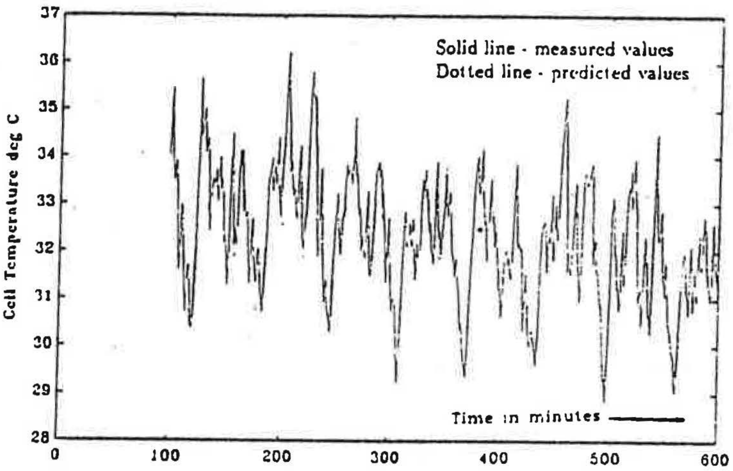


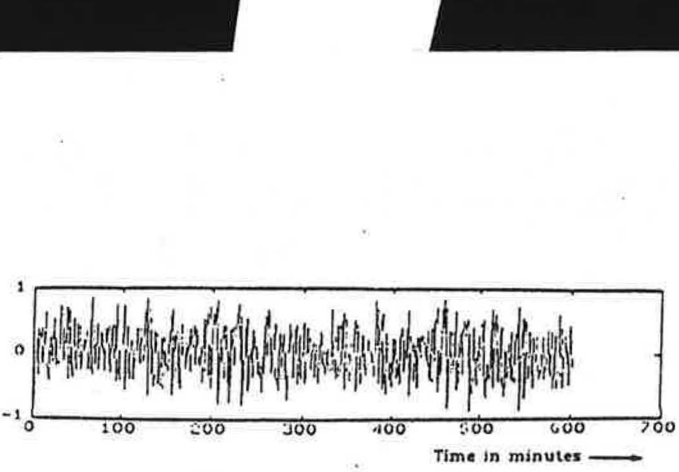
Figure 5: Model validation

disturbances. This is a similar finding to that in the physical modelling discussed in section 4. In our case, the disturbance was implemented by a PRBS on/off input to the electric extract fan. Were this to be treated as an additional known system input and modelled, then better forecasts are possible. This two-input case can be, and was, analysed, and resulted in a model that predicted cell temperatures to within 0.5°C. The prediction errors for both these cases are shown in Figure 6; these clearly show the improvements. Sums of the $(\text{errors})^2$ over the range 100 mins - 600 mins for single input and two-input cases were found to be 332.3 and 162.9 respectively.

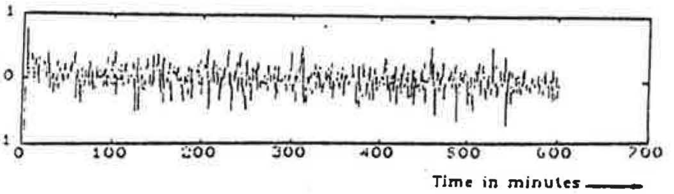
The off-line modelling process described can be modified for on-line use by employing recursive identification methods (see for example Norton [8] and Ljung [9]). The main objective for doing this would be to develop and implement advanced control techniques for use in building energy management systems. As shown above, the models can accurately predict the thermal behaviour. It is clear that if such a model is used as part of a control algorithm better performance is possible together with reduction in energy consumption. This is because the forecasts permit reductions in the overshoots and undershoots from set points that occur in current PID systems. Research to render this on-line approach viable for building services control is being actively pursued, Virk et al [4], [5], [10], Virk and Loveday [11], Loveday et al [12], [13].

4 PHYSICAL (DETERMINISTIC) MODELLING

At this stage it is useful to show, in general terms, the relationship between the stochastic modelling method, discussed in section 3, and the deterministic modelling method. Both represent different aspects of what is termed system identification - the technique for ob-



(a) Heater input modelled



(b) Heater and fan inputs modelled

Figure 6: Prediction errors

taining a model which describes the behaviour of a system for subsequent use in design or for control purposes. This relationship is illustrated in Figure 7. Method 1, which has already been described, finds particular use in control engineering where the model produced is often used in the design of the controller. Here, stochastic (noise) effects are accounted for. Method 2 and Method 3, which are essentially the same as one another, can be used to model systems which are stochastic, but the quality of system description is dependent on the amount of random influence present in the real system. If this is small, the model produced by Method 2 may be adequate for control engineering purposes. Otherwise, Methods 2 and 3 offer a technique for evaluating component and system designs. The choice of method is therefore determined by the nature of the system to be modelled, and by the subsequent use to which the model is put.

With respect to the modelling of buildings, Method 1 represents a relatively new approach. Method 2 and Method 3 usually comprise the well-known techniques for describing thermal behaviour. At its simplest, this can constitute the simple steady state approach:

$$Q = UA\,(t_i - t_o) \tag{8}$$

where Q is the rate of flow of heat (Watts) through a constructional element of area A (m^2), t_i and t_o are, respectively, internal and external temperatures (K), and U is the U-value ($Wm^{-2}K^{-1}$) of the element (Markus and Morris [14]). At its most complex, it involves solution of the three-dimensional transient conduction equation, which, for no internal heat generation, may be expressed:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\nabla^2 T \tag{9}$$

Here k, ρ and C_p are, respectively, the thermal conductivity ($Wm^{-1}K^{-1}$), the density ($kg\ m^{-3}$) and the specific heat capacity ($J\ kg^{-1}\ K^{-1}$), of the building constructional ele-

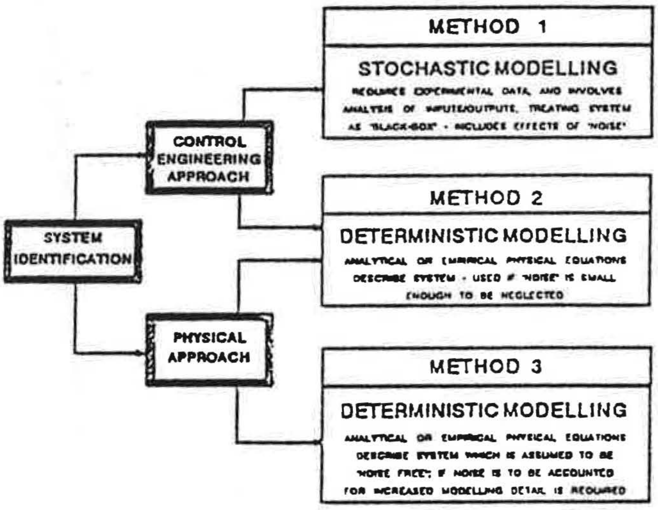


Figure 7: Overview of system identification

ment, T the temperature (K) and ∇^2 the Laplacian operator, defined as:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{10}$$

Between these static and fully dynamic extremes exist a number of simplified approaches. Solution of equation 9 is often restricted to one dimension which permits more practical techniques such as the response factor method (Kusada [15]; Kimura [16]) to be developed. Another technique is the "admittance procedure" (Millbank and Harrington-Lynn [17]), in which the internal temperature of a zone is determined from an assumed sinusoidal external temperature variation; this requires the assignment of mean and peak temperature values. The procedure defines the parameters of admittance Y ($Wm^{-2}K^{-1}$), time lag ϕ (hours) and decrement factor f (*op cit*) which arise as a result of heat storage effects within the building fabric. It is less rigorous than other techniques but offers a first step towards the dynamic analysis of a building design combined with being relatively straightforward to use.

The use of such deterministic models as those described above, or ones in the form of well-known computer programmes such as ESP, SERI-RES, assumes the building/climate system to be noiseless, that is, free of random disturbances. However, buildings are in reality subject to significant stochastic influences such as natural ventilation effects (from opening and closing windows), solar gain fluctuations, occupancy and appliance usage variations. The effect that such disturbances have on system performance will vary from case to case but, nevertheless, handling any such disturbance in a deterministic model presents additional difficulties which are now illustrated.

4.1 Deterministic Modelling of Test Cell

The test cell system described in section 2 was modelled using the admittance procedure in the form of a modified version of the computer programme BRE-ADMIT written by the UK Building Research Establishment (Bloomfield, 1985). This programme is designed for single-zone modelling of building structures, so in order to adequately model the test cell, the following modifications were made.

(i) The minimum dimensions of the cuboid that could be modelled were reduced to $0.1m \times 0.1m \times 0.1m$ high. Though this will affect the values of surface resistance the original programme values were retained since this is to be a comparative study based on changes in ventilation rate.

(ii) The maximum permissible value of internal temperature was raised from 25°C to 100°C, and that of day-time/night-time ventilation rate from 50 ach to 500 ach.

(iii) The thermal conductivity range was increased to permit inclusion of the value for steel ($60Wm^{-1}K^{-1}$).

The above modifications constituted a new programme entitled "BREAMOD2" which was used to model the test cell. Since the cell was located inside a laboratory the absence of solar radiation was treated by setting to zero the solar absorptivity values of the modelled cell surfaces. The laboratory air temperature (the internal temperature in "BREAMOD2") was modelled by setting the mean value to 28°C, with a swing (maximum minus mean) of 0.5°C and a value of one cycle per day (24 hours). The "heating plant" output was set at 15 Watts continuous operation, this value being found to produce the correct order of magnitude for results of internal air temperature by off-setting the effects of other approximations; the value is consistent with the intermittent operation of the 50 Watt heater in the actual test cell. An effect of the canister which enclosed the heater was to reduce radiant emission. This was modelled by setting the convective:radiative emission ratio to 0.9:0.1. Figure 8 illustrates the cell arrangement for modelling purposes. Table 1 gives the values of the relevant thermophysical properties and Table 2 the derived quantities as obtained from the "BRE-ADMIT THERMAL FACTORS" sub-routine. These show the cell to be an extremely lightweight structure, even with the steel plates, and to have negligible thermal storage. This confirmed results obtained from step response tests carried out on the actual cell. Therefore the steel plates do not sufficiently slow the dynamics, and other materials will need to be employed in future work.

The chief stochastic disturbance in the actual test cell system was the operation of the electric extract fan driven by a pseudo random binary sequence. This produced a cell air change rate estimated to be 400 ach when the fan was on. A background infiltration rate of 2 ach was assumed when the fan was off. Since "BRE-ADMIT" (and hence "BREAMOD2") is a relatively simple simulation model, this disturbance can be handled only coarsely. Hence simulations were performed for four values of ventilation rate: 2, 5, 200 and 400 ach, and the results compared. Simulations showed that the small casual heat gain from the fan motor had a negligible effect on results, and so this gain was ignored.

Figure 9 shows the effect, on cell internal temperature, of varying the ventilation rate. These results are in good agreement with the measured temperatures in the actual test cell (see Figure 3(a)), thus validating the simulation model. Ventilation rate is a stochastic disturbance, and is seen to have a significant effect upon simulated cell temperature. This

Table 1: Cell thermophysical properties

Surface	No. of layers	Width (mm)	Density $(kg\ m^{-3})$	Conductivity $(W\ m^{-1}\ K^{-1})$	Sp. heat capacity $(J\ kg^{-1}\ K^{-1})$
1,2,3,4	1	5	105	0.047	1507
5	2:				
	first	5	105	0.047	1507
	second	5	105	0.047	1507
6	3:				
	first	1	7854	60	434
	second	5	105	0.047	1507
	third	5	105	0.047	1507

Table 2: Derived quantities from "THERMAL FACTORS"

Surface	U-value $(W\ m^{-2}\ K^{-1})$	Y-value $(W\ m^{-2}\ K^{-1})$	f	ϕ $(hours)$
1,2,3,4	3.492	3.492	1	0
5	2.546	2.546	1	0
6	2.546	2.554	1	0

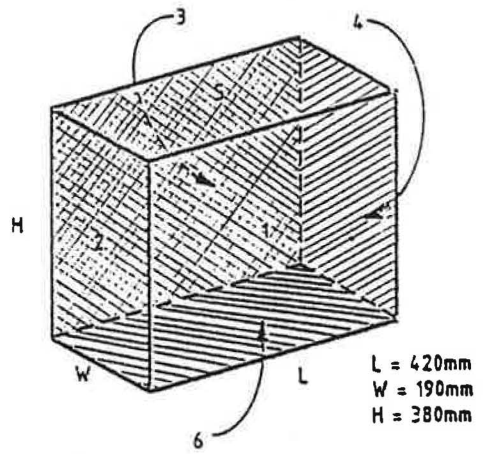


Figure 8: Illustration of the test cell as simulated

is because the size of the disturbance is very large (for demonstration purposes). However, even small variations in infiltration rates, such as from 2 ach to 5 ach, affect the cell temperature by about 0.3°C. Prediction of the cell internal temperature at any given time is therefore difficult, because this particular model is unable to handle ventilation rate changes over small time steps. To deterministically model this system adequately it would be necessary for the model to cope with infiltration data over a finer time resolution - provided that such data were available. Thus model complexity must be increased. A similar approach would be needed if other disturbances such as occupancy changes and other casual gains variations were to be modelled. This would require more detailed data on these variations and this might be both difficult and expensive to obtain.

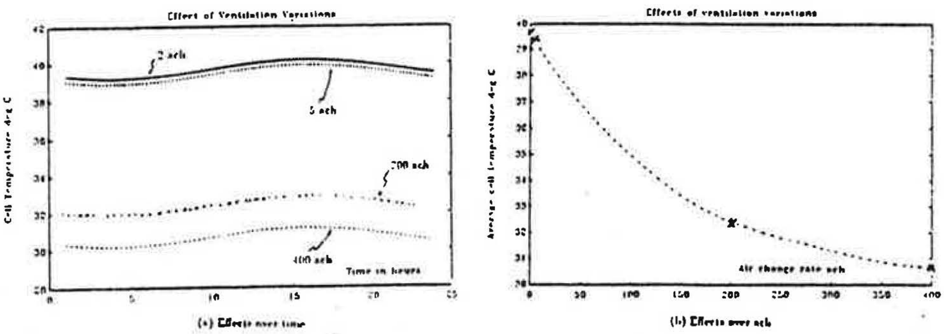


Figure 9: Effects of ventilation variations

5 CONCLUSIONS

The stochastic approach to system modelling has been described. Its application to a test cell subject to random disturbances results in a good mathematical description of that system which incorporates stochastic effects within a relatively compact model. The deterministic approach has also been described and applied to the same system. It has been shown that to model stochastic effects accurately, the complexity of the models needs to be increased.

Since building thermal behaviour can be subject to significant random influences, the stochastic approach in some respects offers better potential. This is particularly the case if some form of on-line model implementation is contemplated. An example of this is on-line control via a building energy management system (BEMS). Here, due to the parsimony of the model, implementation would require less computer power with reduced computation time. However, since such a model is drawn from input/output data, a building must already be constructed and functioning to obtain the model. The deterministic approach, while producing models which might prove too unwieldy for on-line implementation purposes, does offer the advantage of being usable at the pre-construction stage of a building for the evaluation of different designs. However, a further use might be to assess the sensitivity of, for example, a zone temperature in response to variation in a stochastic variable (occupancy, ventilation, casual gains). This could aid the selection of the important variables to be monitored prior to developing a stochastic model for on-line control purposes, and since this is dependent upon the type, design and location of a building, it could be carried out at the design stage. The authors are currently investigating these aspects.

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Cheung/8

DISCUSSION

JIANG Y. (China)

1. What rule should we use to determine the time step ? It is very important for your time series model.

2. What does fan input mean ? During your test, is the fan turned on and off sometimes ? The air flow rate is a non-linear parameter, I believe that the RAMA model (stochastic model) is not very suitable for this kind of non-linear influence. If you use your model to control a VAV system there may be some problems.

ANSWER :

1. The sampling time depends on the dominant time constants and dead times of the system; therefore tests need to be performed on the building to establish these characteristics. Also, a lower limit that the sampling rate must exceed for adequate representation is provided by the Sampling Theorem (twice the highest important frequency component in the analogue signals being considered). In practice, for good performance, much faster rates are used; 10-40 times faster than the fastest frequency have been used in various applications.

2. The fan input is introduced to represent occupancy and other stochastic effects into our test-cell work; it is an on-off input of random form, hence it is on sometimes and off at other times. The pattern can be chosen to represent various occupancy, daily and seasonal patterns; in our work a white noise distribution was applied to demonstrate the technique.

   Although a linear analysis is assumed, it is unknown at this stage whether such an assumption is adequate for use in the modelling and control of buildings. We accept that buildings are nonlinear systems, but linearised approximations may still be valid for control purposes.

LARET L. (France)

In your conclusions, when you compare stochastic and deterministic models, the comparison can be different if you use very compact deterministic physical models where some parameters are identified in line. With the physical model formulation, you naturally take into account some essential properties, as for instance energy conservation. These constraints can be very important and difficult to introduce in stochastic models.

ANSWER :

In the comparison of deterministic and stochastic models, we make the point that a high integrity, complex, deterministic model would be more cumbersome to operate on-line than would a compact stochastic model identified from input/output data. In your research, you have developed very compact deterministic models, and of course these would be easy to operate on-line because of their compactness. However, as we understand it, your compact models are derived from much more complex deterministic models which are assumed to represent the actual behaviour of a real system. Even though it might be possible to alter your compact model on-line, its format, quality and content must be dependent upon the initial complex deterministic model from which it was derived, and this cannot easily self-adapt to possible changes in the physical system. Since no simulation model can fully and accurately represent real building thermal behaviour, or changes in that behaviour, we identify stochastic models from the actual system data itself. Complex and compact deterministic models have, however, an important role to play in the design of buildings and their controller algorithms as discussed in our paper, but their self-adaptive properties are subject to constraints as outlined above.

We don't agree with your comment that properties such as energy conservation are difficult to introduce in stochastic models. In our work (reported elsewhere), the stochastic model is used as the basis of a controller, the operation of which can be arranged so as to reduce energy consumption (and hence increase energy conservation). The stochastic model itself is simply a mathematical description of the thermal behaviour of the physical system as it stands (inclusive of random effects) and the model can self-adapt to form a modified description, should that system change. In a similar way, a physical model formulation is also simply a mathematical description of system behaviour. Energy conservation is a quality resulting from system performance rather than being a modellable property.

DEGUNDA N. (Switzerland)

Suppose heating with a heating coil. Assume that the temperature of the heating medium is varying (perhaps because of capacity problems during heating up and cold morning). Should the model follow these changing parameters ? Don't you think you need a supervisory level to detect illegal situations ?