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Subtask 1: Room Air and Contaminant Flow

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LOW REYNOLDS NUMBER EFFECTS IN SINGLE-ROOM AIR FLOW

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ABSTRACT

The influence of the Reynolds number on low speed flow (essentially incompressible flow) is discussed. A first set of literature is reviewed to identify effects of low Reynolds number on flows in rooms. Methods of incorporating low-Reynolds-number effects in existing turbulence models are surveyed.

CONTENTS

1. How the Reynolds number influences incompressible flow

The following three examples show Reynolds number effects of different character. Classical flow situations are illustrated by graphs in which some flow parameters are shown as functions of the "Reynolds number."

According to these examples it is proposed to classify Reynolds number effects by the length scale used in the definition of Re (Reynolds number)

1.1. Drag of cylinders and spheres

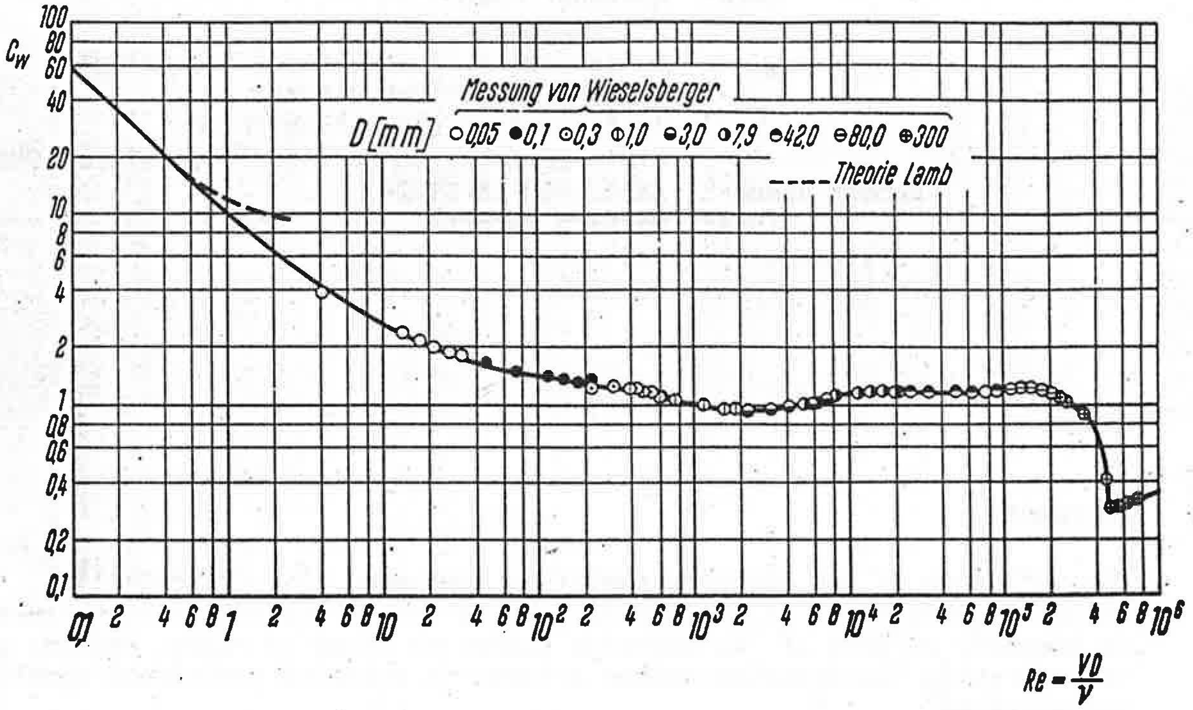
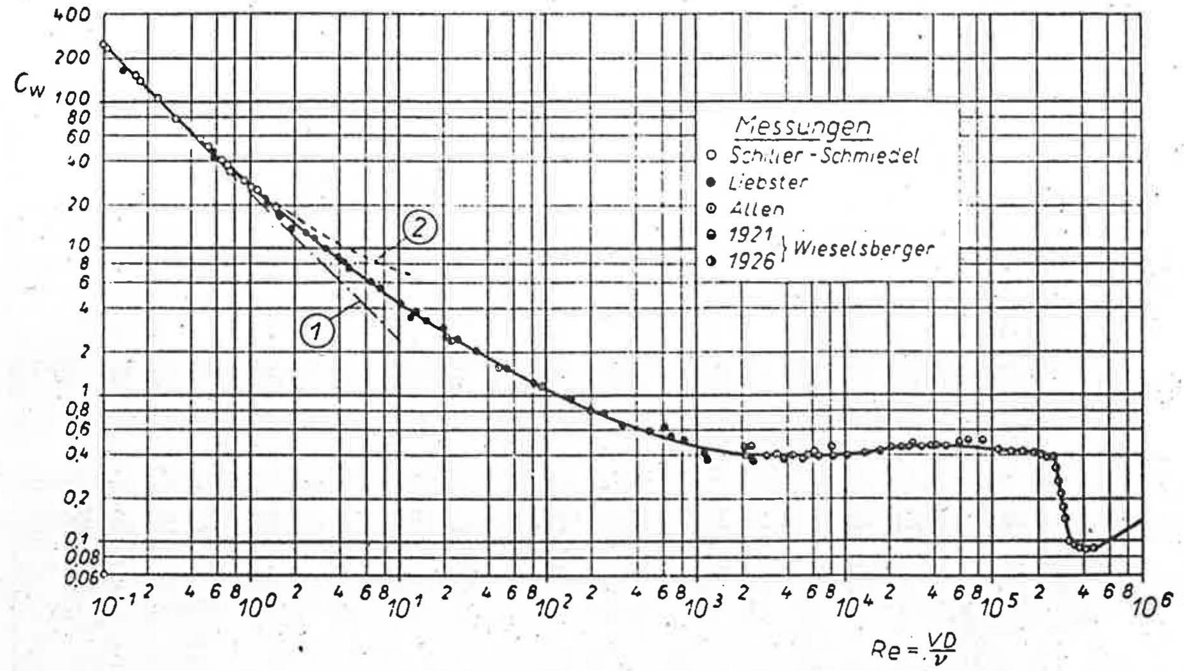


Fig. 1 Drag coefficient (Cw) of circular cylinder as function of Reynolds number (based on diameter D).

In the examples of figures 1 and 2 the entire flow patterns along with the relevant aerodynamic coefficients depend on the magnitude of a **global** Reynolds number. The reference length is some fixed typical dimension of the overall geometry. This length is a constant for a given geometry (and not an independent variable).

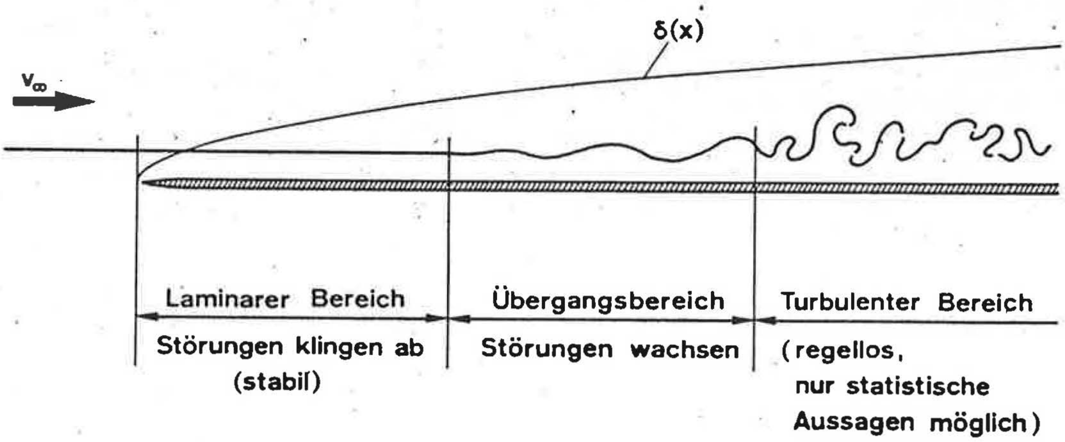
The dependence of the friction coefficient on Reynolds number in pipe flow is of the same type.



<u>Fig. 2</u> Drag coefficient (Cw) of a sphere as function of Reynolds number (based on diameter D).

1.2. Streamwise boundary layer development and growth

The growth of a boundary layer from the leading edge of a flat plate or from the stagnation point on an airfoil through a laminar region to transition and eventually to turbulent separation may be plotted as a function of the surface length x (Fig. 3).



<u>Fig. 3</u> Boundary layer thickness, delta(x), and type of flow (laminar, transitional, turbulent) on a flat plate with sharp leading edge. Transition occurs at 300,000 < Re_x < 3,000,000.

Non-dimensional quantities, such as the local skin friction coefficient, are normally plotted as a function of Re(x), the Reynolds number based on the surface length x. This is illustrated in Fig. 4.

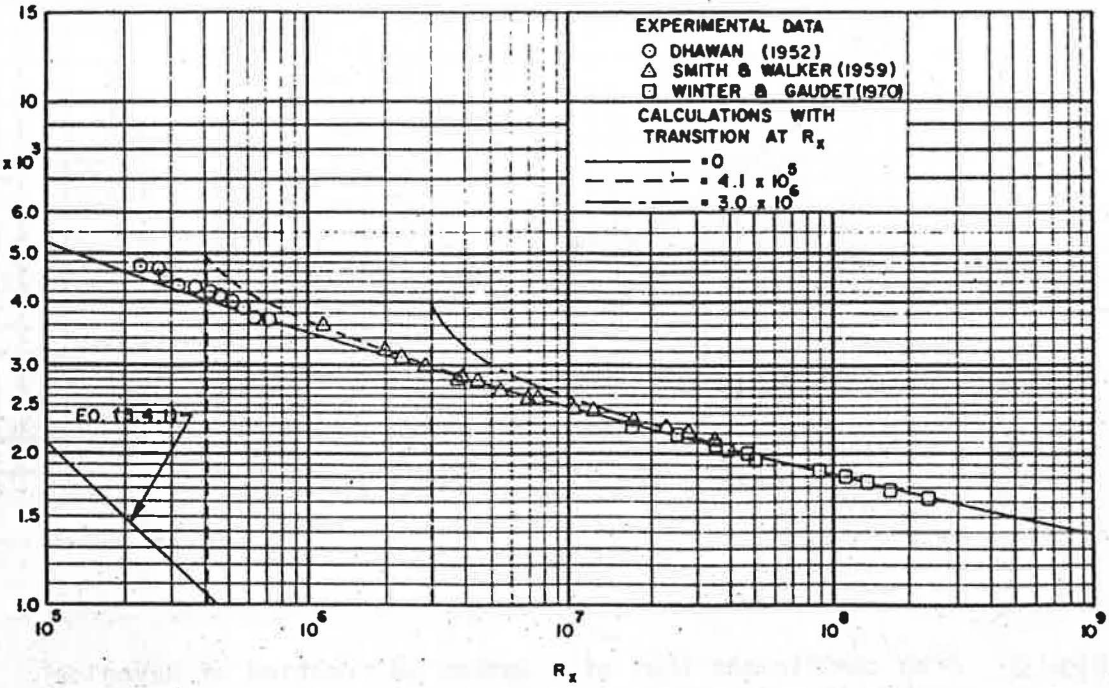


Fig. 4 Local skin-friction coefficient on a smooth flat plate.

Here the Reynolds number represents the **independent variable x**, measured along the surface. If the definition of x has a physical meaning, i.e., if the origin of the boundary layer is at x=0, then low values of this Reynolds number refer to early stages of the boundary layer, where it is still laminar.

The Reynolds number Re(delta), formed with the displacement- or momentum-thickness of the boundary layer, is correlated with Re(x) for a given stream-wise pressure distribution. It is a non-dimensional measure of the "thickness" of the boundary layer. Re(delta) has a different value for each station along the surface.

1.3. Profile across a turbulent boundary layer, normal to the wall

In dimensionless boundary layer profiles the distance, y, from the wall is often expressed as a Reynolds number with the length y and the reference velocity, u(τ), the so-called friction velocity. This Reynolds number is usually called y+, as in Fig. 5, where the profile is shown both in a linear and in a logarithmic y-scale.

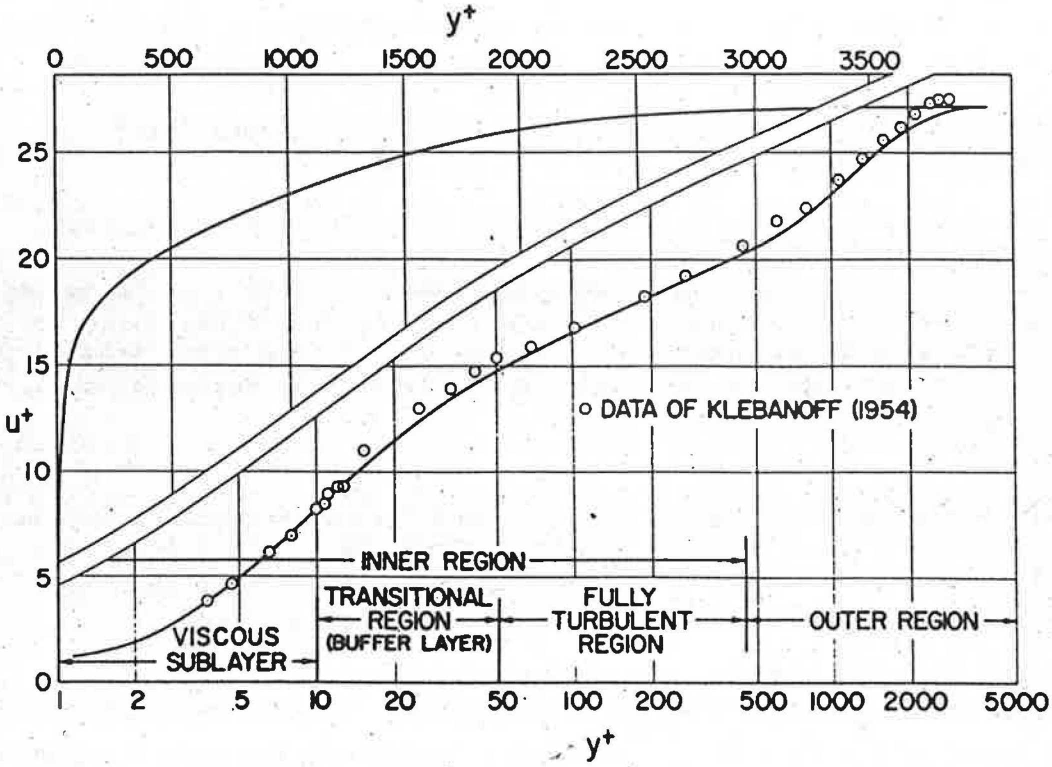


Fig. 5 Semi-logarithmic and linear plots of mean velocity distribution across a turbulent boundary layer with zero pressure gradient. The linear plot is included to show a true picture of the thickness of various portions.

Here the Reynolds number, y+, represents the **independent variable y**, measured normal to the surface. Low values of this parameter refer to the inner layers of the profile, in particular to the viscous sublayer.

It is important to distinguish between low values of y+, as in the viscous sublayer close to the wall within a turbulent boundary layer and low values of Re(delta), which are associated with thin laminar layers, or situations with generally low velocity levels or small geometric dimensions or both.

1.4. Conclusions: Classification by type of length scale

The above examples suggest three types of Reynolds numbers according to the reference lengths used in their definitions:

- o The reference length is L, a typical general dimension of the flow geometry. This leads to a **global Reynolds number**, specific for a particular test or simulation run.

- o The reference length is x, the streamwise distance along a surface, or delta, the boundary layer thickness. The resulting Reynolds number relates to the local character or type of a particular boundary layer. Therefore it can be called a **boundary layer Reynolds number.**

- o The reference length is y, the normal distance from the surface. This parameter is an independent variable across the boundary layer profile. If the employed reference velocity is $u(\tau)$ this Reynolds number is called y+ and has universal character. It delimits the different portions within the profile (Fig. 5) and quantifies what means "very close" to the wall. For clarity, this Reynolds number is simply called **"y-plus"** (y+).

Typical values for these three types of Reynolds numbers depend on the reference velocities and actual lengths, of course. For a kinematic viscosity of air of 1.5E-5 m2/s, some numerical examples are given for each class:

The global Reynolds number:
for an entire room, say for u_{ref}= 0.25 m/s and L = 10 m Re = 167 000
for an inlet diffuser, with u_{ref}= 1.0 m/s and D = 0.1 m Re = 7 000

The boundary layer Reynolds number:
The transition of a flat-plate boundary layer from laminar to turbulent occurs at a Reynolds number of not more than Re(x) = 3 000 000
the corresponding momentum-thickness Reynolds number is approximately Re(theta) = 1000

The y-plus in the viscous sublayer is of the order of y+ = 10
The edge of the turbulent boundary layer can be at values as high as y+ = 10 000

A comparison of "room air flow" values of the global Reynolds number with Re(x) values for transition already hints, that the boundary layers on smooth walls are not necessarily turbulent but might be laminar or transitional if other flow disturbing influences are excluded.

2. Definition of the Reynolds number

In this section it will be demonstrated how the Reynolds number is obtained by making the momentum equation non-dimensional.

In the partial differential equations for momentum transport the **Boussinesq** approximation is applied so that the influence of variable density appears only in the buoyancy term. In all other terms ρ is replaced by a constant reference density ρ_r.

In the sense that the fluid velocities are much lower than the speed of sound, - i.e., when the Mach number is close to zero, - the equations are considered **incompressible.**

2.1. The Momentum equation

The equations that govern two-dimensional, incompressible flow are made non-dimensional by some reference quantities. When a transport equation has terms that express different types of transport, such as convection and diffusion, it can be expected that for each such type of transport, other than convection, a new dimensionless parameter or variable will emerge. In the case of the momentum equation, these parameters express the relative magnitudes of the different forces acting on the infinitesimal control volume as contributions to the overall balance.

The steady-state **momentum equation** in z-direction is:

$$\rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} - \rho g + \rho\nu\left(\frac{\partial^2 w}{\partial y^2} + \ldots\right) \qquad (1)$$

According to Boussinesq, the density, ρ, on the left-hand side and in the viscous term is replaced by the constant reference density ρ_r.

Before normalizing the equation, the hydrostatic pressure, p_h is introduced as

$$p_h = p_o - \rho_r gz \qquad (2)$$

where p_o is the absolute (atmospheric) pressure at z = 0.

Now, a term $\rho_r g$ is subtracted and added to the right-hand side of (1), leaving its value unchanged. Note the differentiation of p_h with respect to z.

$$\rho_r v \frac{\partial w}{\partial y} + \rho_r w \frac{\partial w}{\partial z} = - \frac{\partial(p - p_h)}{\partial z} - (\rho - \rho_r)g + \rho_r \nu(\frac{\partial^2 w}{\partial y^2} + \ldots) \quad (3)$$

This equation contains variables of **three** basic units of dimensions, i.e., units for LENGTH, TIME, and MASS. Therefore it should be possible to make it non-dimensional by introducing **three** reference quantities. Together, they define scales for length, time, and mass:

Reference quantities:	Length	L
	Velocity	U
	Density	ρ_r

Equation (3) is now divided by $\rho_r U^2$ and multiplied by L.

The new independent variables are: $y' = y/L$, $z' = z/L$.

The new dependent variables are:

Velocity $w' = w/U$; pressure $P = \dfrac{(p - p_h)}{\rho_r U^2}$.

The familiar **pressure coefficient** is defined as $C_p = 2P$ (4)

The momentum equation (3) becomes non-dimensional in the new variables. The new viscosity term has a dimensionless parameter which is the reciprocal of the

Reynolds number, $Re = \dfrac{UL}{\nu}$ (5)

The new buoyancy term is:

$$\frac{(\rho - \rho_r)gL}{\rho_r U^2} = \frac{\Delta\rho}{\rho_r} \cdot \frac{gL}{U^2} \quad (6)$$

The coefficient of thermal expansion, β, is defined by

$$\beta = - \frac{1}{\rho} (\frac{\partial\rho}{\partial T})_p \quad \text{with} \quad \frac{\Delta\rho}{\rho_r} \approx - \beta \, \Delta T \quad (7)$$

This may be inserted in eq. (6). With a reference temperature difference, ΔT_r, the buoyancy term is then

$$- \beta \, \Delta T_r \cdot \frac{gL}{U^2} \cdot \frac{\Delta T}{\Delta T_r}$$

The parameter that expresses the importance of buoyancy is the

$$\text{Archimedes Number,} \qquad Ar = \beta\, \Delta T_r \frac{gL}{U^2} \qquad (8)$$

The inverse of the last group in (8), $U^2/(gL)$, is equal to the Froude number, Fr. The Froude number expresses the importance of gravity relative to dynamic pressure. If the $\beta\, \Delta T$ of (7) is back-substituted in (8) the following expression results:

$$\frac{\Delta\rho}{\rho_r} \frac{gL}{U^2} \qquad (\ = \text{Richardson number, } Ri\ )$$

For the case of a stratified flow with a density difference $\Delta\rho$ across a fluid layer of thickness L with a typical velocity U, this group is called the Richardson number [TURN73]. If density and velocity fluctuations are highly correlated, buoyancy can have a large effect [BRAD76]. For instance, if the density increases upward (heavy fluid on top of light fluid) the flow is "unstable" and the density-velocity correlation can convert potential energy into turbulent kinetic energy, and vice-versa. A convenient parameter for quantifying this effect is the Richardson number. There is another form of the Richardson number, defined in terms of gradients, which is even more appropriate for the investigation of flow stability. The above expression for Ri is equivalent to the Archimedes number and, in fact, amounts to an Archimedes number applied to the special case of density-stratified flow.

In conclusion then, the three variables that describe the importance of pressure-, buoyancy-, and laminar friction-forces are C_p , Ar , and Re, respectively. They express the different effects by ratios relative to the dynamic pressure, $\frac{1}{2} \rho\, U^2$, of a flow with the reference velocity U. Note that the influence of laminar viscosity is proportional to 1/Re, i.e., low values of Re indicate high viscosity.

2.2. Natural convection

What is the Reynolds number for a flow that is driven by natural or free convection only ?

In this case no reference velocity is available to form a Reynolds number. But one would still like to quantify the effect of viscous forces somehow.

The dimensionless numbers derived from the momentum equation are proportional (~) to ratios of forces:

$$\mathrm{Re} \sim \frac{\text{dynamic pressure}}{\text{viscous stress}}, \qquad \mathrm{Ar} \sim \frac{\text{buoyancy}}{\text{dynamic pressure}} \tag{9}$$

So the product of these numbers should have some meaning for free convection because the dynamic pressure drops out:

$$\mathrm{Re} \,.\, \mathrm{Ar} \sim \frac{\text{buoyancy}}{\text{viscous stress}} = \frac{\beta\ \Delta T\ g\ L^2}{\nu\ U} \tag{10}$$

But this combination cannot be the relevant parameter since the reference velocity, U, still appears in it. Natural convection is certainly affected by heat **convection** and heat **conduction**. The ratio of these two energy transport mechanisms is

$$\frac{\text{heat convection}}{\text{heat conduction}} \sim \frac{\rho\ c_p\ U\ \Delta T}{k\ \Delta T\ /\ L} \tag{11}$$

Where c_p and k are the specific heat at constant pressure and the thermal conductivity, respectively.

In natural convection heat is fed to the plume or to a vertical thermal boundary layer by conduction and carried away in a vertical direction by convection. Turbulent and laminar conduction together thus balance the vertical convection of heat (see numerator of (11)). Expression (11) can be considered to be the ratio of total conduction (i.e., molecular plus turbulent) to molecular conduction alone.

In a similar way, the buoyancy in (10) balances total frictional forces (laminar plus turbulent stress), and (10) expresses a ratio of effective or total friction to solely laminar friction.

So in natural convection on a vertical wall each of the expressions (10) and (11) are a measure of turbulence, expression (10) with respect to friction, and (11) with respect to thermal conduction.

The Rayleigh number, which is the characteristic parameter for free convection, is now formed by multiplication of (10) and (11). In this combination the velocity is cancelled. As a consequence of the nature of the two factors, (10) and (11), the square root of the product has almost the meaning of a Reynolds number.

Rayleigh number,

$$\mathrm{Ra} = \frac{\beta\ \Delta T\ g\ L^3 \rho\ c_p}{\nu\ k} \tag{12}$$

The Rayleigh number can be factored into two other non-dimensional parameters

$$\mathrm{Ra} = \mathrm{Gr}\ .\ \mathrm{Pr} \tag{13}$$

where

Grashof number, $$Gr = \frac{\beta \, \Delta T \, g \, L^3}{\nu^2} \qquad (14)$$

Prandtl number $$Pr = \frac{\rho \, c_p \, \nu}{k} \qquad (15)$$

One other relation may now also be verified:

$$Gr = Ar \, Re^2 \qquad (16)$$

It is concluded that no global Reynolds number exists for free convection. However, low Rayleigh numbers are also an indication for laminar flow. The critical value for transition to turbulence on vertical walls is around $Ra = 10^8$.

Boundary layer Reynolds numbers become meaningful in free convection when the maximum velocity in the profile is used as velocity scale.

A detailed discussion on this topic is contained in [BEJA84] where the method of scale analysis is applied to natural convection. A theory for natural convection along heated vertical surfaces by George and Capp [GEOR79] provides interesting ideas on wall functions for thermal boundary layers.

3. Evidence of Reynolds number effects in air flows

All air velocities in a room with forced ventilation and isothermal walls would scale with the inlet jet velocity if the flow were independent of Reynolds number. But P. V. Nielsen has shown by experiments on forced ventilation [NIEL88] that the maximum air velocity in the occupied zone is proportional to the air exchange rate, n, only at high values of n (Fig. 6).

This indicates a Reynolds number dependence at lower flow rates. Nielsen attributes the deviation from a linear relationship to effects in three regions where the local flow depends on Reynolds number:

a) The air terminal device,
b) the wall jet along the ceiling, and
c) recirculating zones.

These effects seem to be governed by the global Reynolds number, but also by the boundary layer Reynolds number (region b).

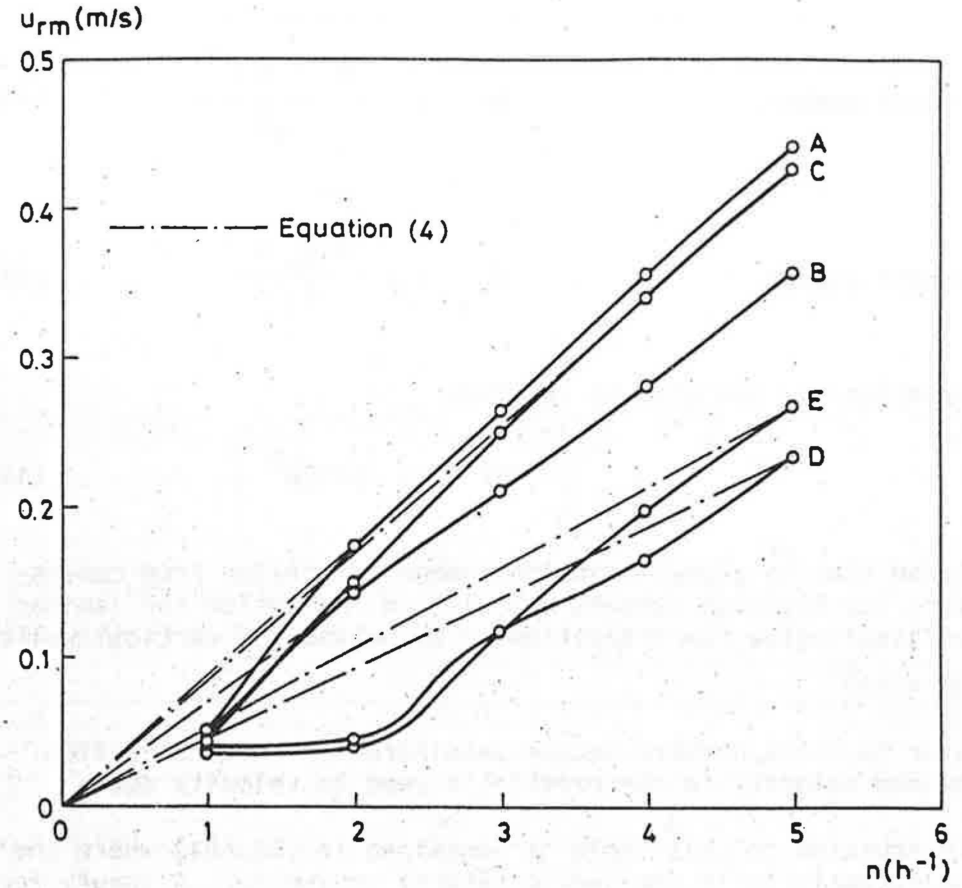


Fig. 6 Maximum velocity in the occupied zone as a function of the air exchange rate. The tests were performed for five different air terminal devices (A to D)

Jones and Launder have commented on the importance of low Reynolds number effects in 1973. Quoting from their paper [LAUN73]: "One of the most important and least understood aspects of turbulence is that which occurs when the local Reynolds number of the turbulence is low..... The presence of the wall ensures that over a finite region of the flow, however thin, the turbulence Reynolds number is low enough for molecular viscosity to influence directly the processes of production, destruction, and transport of turbulence".

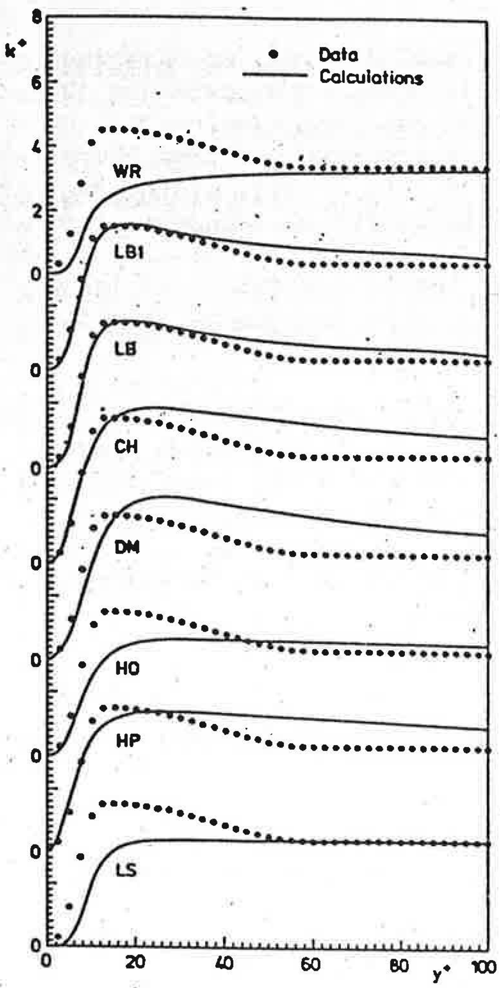
This statement refers to low values of y+ and of the turbulence Reynolds number. The latter contains the square root of the turbulent kinetic energy, k, as a velocity scale. Various expression for such Reynolds numbers exist in the literature.

Cebeci and Smith [CEBE74] report that Coles [COLE62] has measured a dependence of mean velocity profiles in an incompressible boundary layer outside the sublayer at zero pressure gradient on momentum-thickness Reynolds number, Re_θ. If this parameter is below

$$Re_\theta' \approx 6000$$

the non-dimensional velocity distribution of Coles is affected by Reynolds number. The data of Bradshaw [BRAD82] shows influences mainly in the region of the "law of the wake," i.e., the outer part of Coles wall function, that scales with the boundary layer thickness. When simulation codes employ the logarithmic wall function only, care should be taken that the off-wall mesh point is not too far out (not in the outer region of Fig. 5).

Bradshaw et al. have made extensive conditionally sampled hot-wire measurements of low Reynolds number flows [BRAD82]. They conclude that the turbulence length scale is proportional to the boundary layer thickness, i.e., independent of Reynolds number, for Re_θ above approximately 5000. Above that limit the law of the wake also becomes constant.



Many methods of numerical flow field simulation with turbulence models rely on the application of the wall functions that relate surface boundary conditions to points in the fluid away from the boundaries and thereby avoid the problem of modeling the direct influence of viscosity. In a review article [RODI85] on near-wall and low Reynolds number flows Rodi et al. caution that this procedure is restricted to situations in which the Reynolds number is sufficiently high for the viscous effects to be unimportant or where the first grid point from the wall is located at a high enough y+ value. In room air flow predictions these conditions can usually not be fulfilled.

It should be noted that there is also an outer limit for the location of the first mesh point if the logarithmic law of the wall is to be used: Chen [CHEN88] recommends to place the first node between y+ = 40 and 130. At larger distances the influence of the law of the wake is felt ("outer region" in Fig. 5), which is normally not programmed into the "wall functions" because the local boundary layer thickness, δ, is not known to flow field simulation codes. Experimental evidence of the range of the logarithmic law may be found, e.g., in [BRAD82].

Fig. 7 Measured and calculated turbulent kinetic energy profiles in the near-wall region [RODI85]

In [RODI85] it is further admitted that "Experimental information pertaining to near-wall and low Reynolds number turbulence is rather limited and the data suffer from uncertainties arising from probe interference effects and the determination of the wall shear stress that provides the characteristic velocity and length scales".

Figure 7 is reproduced from [RODI85]. It shows a comparison of eight different low Reynolds number modifications of the k-ε model with experimental data. Although the computation with the standard model is not shown, the difficulty of numerically predicting the near-wall layer is well documented. The graph presents turbulent kinetic energy profiles in a flat plate boundary layer.

Conclusions of this section are:

- More experimental evidence of the effect must be collected, in particular for room air flow.
- A numerical study on representative room air flows or existing information from the literature should answer the following questions: What is a typical momentum thickness Reynolds number, and what are the physical distances from walls for the range where the first mesh point should be located ($40 < y+ < 130$). If the computational grid starts in this range viscous effects become negligible.
- If the computational grid penetrates the transitional or viscous layer on a solid surface (Fig. 5) the turbulence model itself should be corrected for low Reynolds number.
- Situations which require special attention with regard to Reynolds number effects include: Boundary layers near transition, heat transfer, buoyancy, and three-dimensional effects. Less important for room air flow are: Accelerated flows with re-laminarization, mass transfer, curved or spinning surfaces, and rotating-frame-of-reference forces. (Problems with separation or high turbulence-to-mean-flow ratios are not discussed here)

4. Modifications to the k-ε turbulence model for low Reynolds number effects

This section is not yet complete. More discussions with experts of the field and a thorough and continuing literature search are necessary.

A very comprehensive survey on existing turbulence models for near-wall flows was published by Patel, Rodi, and Scheurer in 1985 [RODI85]. It reviews and evaluates eight different methods of correction and compares their performance with the standard k-ε model. All these models multiply the constants of the classical high Reynolds number model with factors which are functions of R_T and/or R_y or y+.

Where

R_T is a turbulence Reynolds number containing the square of the turbulent kinetic energy, k, and the dissipation rate, ε, and

$R_y = k^{\frac{1}{2}} y/\nu$, is the non-dimensional distance from the wall.

The functions approach unity in regions of high Reynolds number flow. Some models also use additional terms in the k- and ε-transport equations.

In a Lecture Series of the University of Karlsruhe in 1988, G. Scheurer [SCHE88] mentions only one of the eight models examined by him, Rodi, and Patel three years earlier: The Lam-Bremhorst model [LAM81]. His reasons for this selection are not known to me.

Chen [CHEN88] has proposed a modification to the law of the wall to provide a better transition from the fully turbulent region to the viscous sublayer. He fits an algebraic function of y+ across the buffer layer (Fig. 5). This results in better agreement with most of the measured data at low Reynolds number.

5. Conclusions

The main conclusion has two parts:

(1) Low Reynolds number and near-wall corrections are necessary in regimes of appreciable influence of molecular viscosity.

(2) Preliminary studies indicate that the relevant viscosity parameters (Reynolds numbers) are low enough in room air flow situations to fulfill the above premise.

A summary of additional considerations follows:

o Near-wall problems will quite often arise in combination with natural convection boundary layers because there the velocities induced by buoyancy are small. Therefore, it is recommended to apply low-Reynolds-number corrections to the simulation of free or mixed convection.

o Low Reynolds number "devices" already provided by the producers of a software package should be carefully tested and understood and should only be used where applicable.

o The User's Guide (an Annex product) should include easy-to-read sections on how to deal with the problem of low Reynolds numbers.

o Low Reynolds number methods should be validated against existing measured data or new measurements.

- If modifications of the standard k-ε model are programmed, the computed results should be compared with identical computations without the correction. At least for one typical case.

- The following criteria might guide the selection of models:
  (1) Simplicity,
  (2) Consistency with the standard turbulence model, i.e., the new model "contains" the classical model,
  (3) Experimental confirmation at least with regard to turbulent jet spreading rates, flat plate boundary layers, and heated-cavity flows,
  (4) Simple boundary condition for the dissipation rate ε.

- Other limitations of two-equation turbulence models that possibly need evaluation:
  (1) The assumption of isotropic turbulence in boundary layers (an anisotropic k-ε model is proposed in [YOSH87]),
  (2) Wall functions in three dimensions,
  (3) Aerodynamic modelling of the fine detail of inlet devices with geometric dimensions orders of magnitudes below the scales of the ventilated room, and velocities much higher.

6. References

AKBA86
Akbari H., Mertol A., Gadgil A., Kammerud R., Bauman F.
Development of a turbulent near-wall temperature model and its application to channel flow,
Waerme- und Stoffuebertragung, Vol. 20, No. 3, p. 189-201, 1986.

BEJA84
Bejan Adrian, (Kakac S., Aung W., Viskanta R., editors)
The method of scale analysis: Natural convection in fluids,
Natural Convection, Fundamentals and Applications,
p. 75-94, 1984.

BRAD82
Murlis J., Tsai H.M., Bradshaw P.
The structure of turbulent boundary layers at low Reynolds numbers,
Journal of Fluid Mechanics, JFM, Vol. 122, p. 13-56, 1982.

BRAD76
Bradshaw P., Woods J.D.
Geophysical turbulence and buoyant flows,
Topics in Applied Physics, Vol. 12, Turbulence, Ed. P. Bradshaw,
pp. 171-192, Springer Verlag, New York 1976.

CEBE74
Cebeci Tuncer, Smith A.M.O.
Analysis of turbulent boundary layers,
Academic Press, 1974.

CHEN88
Qingyan Chen
Indoor air flow, air quality and energy consumption of buildings, Ph.D. Thesis, 1988.

CHEN90
Qingyan Chen
Construction of a low-Reynolds-number k-epsilon model,
PHOENICS Journal, Vol. 3, No. 3, pp.288-329, Oct. 1990.

CHEN90
Chen Qingyan, Moser A., Huber A.
Prediction of buoyant, turbulent flow by a low Reynolds number k-epsilon model,
ASHRAE Winter Meeting, Atlanta, GA, February, 1990, Vol. 96, No. 1, 1990 . AT90-2-2(3366)

COLE62
Coles D.
The turbulent boundary layer in a compressible fluid,
Rep. R-403-PR, Rand Corp. Santa Monica, California, 1962.

DAVI89
Davidson Lars, Contributors: Olsson E., Hedberg P.
Numerical simulation of turbulent flow in ventilated rooms,
PhD Thesis, School of Mech. Engineering, Chalmers University of Technology, Göteborg, 1989.

DAVI89
Davidson L., Fontaine J.R.
Calculation of the flow in a ventilated room using different finite-difference schemes and different treatments of the walls,
2n World Congr. on Heating, Vent, & Refrig., Sarajevo, 1989.

ELHA80
Elhadidy Mohamed Abdel-Meguid
Applications of a Low-Reynolds-Number Turbulence Model and Wall Functions for Steady and Unsteady Heat-Transfer Computations,
Ph.D. Thesis, p. 1-2,73-76, 1980 . HTS/80/7

GEOR79
George William K. Jr., Capp Steven P.
A theory for natural convection turbulent boundary layers next to heated vertical surfaces,
Int. J. Heat and Mass Transfer, Vol. 22, p. 813-826, 1979.

LAM81
Lam C.K.G., Bremhorst K.
A modified form of the k-epsilon model for predicting wall turbulence,
Transactions of the ASME, Vol. 103, p. 456-460, 1981.

LAUN73
Jones W.Launder B.E.
The calculation of low-Reynolds-number phenomena with a two-equation model of turbulence,
Int. J. of Heat and Mass Transfer, Vol. 16, p. 1119-1130, 1973.

NAGA87
Nagano Y., Hishida M.
Improved form of the k-epsilon model for wall turbulent shear flows,
Trans. of the ASME, Journal of Fluids Engineering, Vol. 109, p. 156-160, 1987.

NIEL88
Heiselberg Nielsen Peter V.
Flow conditions in a mechanically ventilated room with a convective heat source,
9th AIVC Conference, Gent, Belgium, 12-15 Sept. 1988, Vol. 3, No. 37, p. 1-14, 1988 . Poster 37

RODI85
Patel V.C., Rodi W., Scheuerer G.
Turbulence models for near-wall and low Reynolds number Flows: A review,
AIAA Journal, Vol. 23, No. 9, p. 1308-1319, 1985.

RODI89
Rodi W.
Recent developments in turbulence modelling,
Refined Flow Modelling and Turbulence Measurements, p. 3-17, 1989.

SCHE88
Scheurer G.
Numerische Berechnung turbulenter Stroemungen in Forschung und Praxis: 4. Das k-epsilon Turbulenzmodell,
Hochschulkurs 1988, Universitaet Karlsruhe, 26.-28. Sept. 1988, p. 4/1-4/28, 1988 . 210

TURN73
Turner J.S.
Buoyancy effects in fluids,, 1973.

YOSH87
Nisizima Shoiti, Yoshizawa Akira
Turbulent channel and Couette flows using an anisotropic k-epsilon model,
AIAA Journal, Vol. 25, No. 3, p. 414-420, 1987.