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An Experiment for Airflow Determination by Quadratic Programming

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During the past decade a multitude of tracer gas techniques to measure ventilation rates in buildings have been developed. These techniques have proven very useful for a building which may be treated as a single zone. However, as the need to understand more complex buildings increases, it has become necessary to develop more advanced mathematical methods.

In buildings with mechanical ventilation there are spaces with substantial pressure differences, which bring exfiltration, infiltration and transferred air between the rooms. For such buildings we have converted a multiple cell theory to a quadratic programming problem, and developed a computer programme, MCSPID for airflow identification. Today MCSPID is in practical use to simultaneously determine flow rates for air supply, exhaust air, transferred air, infiltration and exfiltration with a single tracer gas. The purpose of this paper is to describe an application in a seven room building, where all airflows are determined in an active tracer gas experiment.

Multiple Cell Model

The basic theory is probably well known. The mixing in each room or cell is supposed to be perfect and instantaneous, and the airflows are constant. In that case the flow model can be represented in matrix form as:

$$V * \dot{c}(t) = Q * c(t) + p(t)$$

where c(t) = tracer gas concentration vector

V = diagonal volume matrix

p(t) = tracer gas supply vector

Q = flow matrix

The flow matrix Q contains the interflows between the cells, except for the diagonal elements, which are the entire flow rate from each cell. If the flow system consists of n cells, both the flow and volume matrix have dimension n*n and the concentration and gas supply vector have n*1. There are n volumes and n^2 flows to identify, in all n(n+1) model parameters.

measurements proceed during m sampling inter- length T_s. If Q_{ii} is the total air flow from cell i, and Q_{in} are the air flows to cell i from cells 1, 2...n, mentary mass balance during sample t_k, cell i is:

$$V_{ii} * \dot{c}_i(t_k) - Q_{i1} * c_1(t_k) .... + Q_{ii} * c_i(t_k) .... - Q_{in} * C_{n(t_k)} = p_i(t_k)$$

A column-vector x_i can be formed for the volume and airflows (model parameters), a matrix A_i for concentrations, and a column-vector p_i for tracer gas supply. Each row in A_i and p_i contains concentration and gas supply during one sample.

$$x_i = [V_{ii}, -Q_{i1} + Q_{ii}, ... -Q_{in}]^T$$

$$A_i = [\dot{c}_i(t_k), -c_1(t_k), ... c_i(t_k), ... -c_n(t_k)]$$

$$p_i = [p_i(t_k)]$$

The mass balances for cell i are then simplified to:
$A_i * x_i = p_i$

To avoid derivatives and reduce measurement noise, Bjorn Hedin has derived a method for integrating the mass balance equation (Hedin 1989). If we however remain at the momentary expression, the time derivatives are calculated with an iterative method, using old model parameters:

$$\dot{c}_i(t) = D * [c_i(t + T_s) - c_i(t)]$$

$$D = V^{-1} * Q * [exp(V^{-1} * Q * T_s) - 1]^{-1}$$

As we are interested in identifying airflows between all cells, the model is extended for a complete multiple cell system. The column-vectors for model parameters and gas supply are extended to include all cells, and a new matrix A of concentrations is established, where $A_i(i = 1...n)$ are located into the diagonal.

$$A = \begin{pmatrix} A_1 & 0 & & 0 \\ 0 & A_2 & & 0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & A_n \end{pmatrix}$$

$$x = (x_1, x_2, ... x_i, ... x_n)^T$$

$$p = (p_1, p_2, ... p_i, ... p_n)^T$$

Then the mass balance for a multiple cell becomes:
$A * x = p$

As the dimension of A_i is (n+1) * m, the dimension of A becomes n(n+1)*nm. The vector x contains all n(n+1) model parameters, and receives the dimension

$n(n+1)*1$. The vector p contains all tracer gas flow rates (for n cells during m sampling intervals), and has the dimension $mn*1$.

A Linear Programming Problem

If the number of sampling intervals is exactly one more than the number of cells a solution will be possible, which however will not give any proper model parameters. Considerably more sampling intervals are necessary, which result in an overdetermined equation system. The solution will be the volumes and airflows which minimize the norm $||r||$ of the residuals r.

$$||r|| = Ax - p$$

If the solution is to correspond to a flow system it remains to consider physical constraints, such as volumes and interflows, which have to be positive or zero. The same is also true for infiltration and exfiltration, which are the negative sum of each row and column of Q. The last constraint is expressed with the vector G, whose elements are +1, -1 or 0.

$$x \geq 0 \qquad\qquad Gx \geq 0$$

Lars Jensen first formulated a multiple cell model with linear constraints during Roomvent 1987 (Jensen 1988). He chose a linear norm, where $||r|| = \Sigma r_i$ and applied a linear programming method to determine the airflows.

A Quadratic Programming Problem

Another procedure used for identification is the method of least squares, where the model parameters are selected by minimizing the sum of the squared error. The norm is accordingly:

$$||r|| = \Sigma r_i^{\,2} = r^T r = (Ax-p)^T(Ax-p)$$
$$= p^T p - p^T Ax - x^T A^T p + x^T A^T A x$$

The constant term $p^T p$ has no influence on x at minimizing. The expression is further simplified by:

$$C = -2A^T p \qquad B = A^T A$$

Instead of $||r||$ we have received the following quadratic equation to minimize:

$$f(x) = C^T x + x^T B x$$

which still has the constraints $Gx \geq 0$ and $x \geq 0$.

The identification of an air flow system has thus become a problem of minimizing a quadratic equation under certain constraints. This is a quadratic programming problem, which is solved by Lemke's complementary pivot algorithm. This method for identification of multiple cell systems was introduced by Bjorn Hedin at the AIVC conference in Finland 1989 (Hedin 1989), and applied by Lars Jensen in his programme MCSPID.

One important benefit of converting the model to a quadratic equation is that the dimensions of the matrixes are not dependent on the number of samples. The vector C^T has the dimension $n(n+1)*1$, and the matrix B has $n(n+1)$, and they are therefore only dependent on the number of cells. The influence of measurement errors decreases as the number of samples increases. Therefore it is possible to identify a large airflow system on personal computers, almost independent of the number of measurements, by converting the multiple cell model to a quadratic programming problem.

Building and Experiment

Tracer gas measurements for airflow determination by quadratic programming have been performed in our experimental building Minilab (Figure 1), which consists of seven rooms and has mechanical air supply and exhaust air. The air condition unit is completed with a damper for recirculated air. Each room forms a cell and is furnished with a fan to assure a perfect and instantaneous mixing. The main air supply and exhaust air ducts make another two cells, but with small volumes. The size of each room is 24.3 m^3 or 29.4 m^3. Some rooms are connected with ducts and fans for transferred air, which increases the pressure difference and brings infiltration and exfiltration.

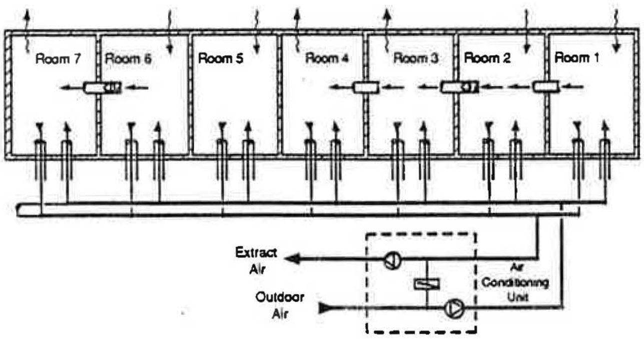

Figure 1: Minilab

Airflows Measured With Tracer Gas

The measurements are performed with constant tracer gas supply during about one hour in each cell. The throttle valve for gas supply is transferred to a personal computer, where an input file to the programme MCSPID is established. Quadratic programming takes general physical constraints into consideration. There are however further constraints for each special experiment, such as fixed parameters, which are to be stated in the input file. The volumes are fixed, and we may also specify where transferred air is impossible, as well as infiltration or exfiltration.

The output file contains a matrix with calculated air flow rates and volumes. The result from the Minilab building is to be found in Figure 2. Because of numerical problems there are small differences in total flow rates. Recirculated air is 39%.

	From Room 1	From Room 2	-From Room 3	From Room 4	From Room 5	From Room 6	From Room 7	From Air Supply	From Exhst Air	Infil-tra-tion	Total
To Room 1								12.3		1.1	13.4
To Room 2	8.9							4.7		4.7	18.3
To Room 3		15.6						4.2			19.8
To Room 4			9.1					4.0			13.1
To Room 5								7.4		4.4	11.8
To Room 6								16.1		4.2	20.3
To Room 7						12.6		8.3			20.9
To Air Supply									62.6	97.3	159.9
To Exhaust Air	3.7	2.8	4.4	13.0	11.2	7.7	18.0	103.1			163.9
Exfiltration			6.3				2.9		101.3		110.5
Total	12.6	18.4	19.8	13.0	11.2	20.3	20.9	160.1	163.9	111.7	

Figure 2: Airflow rates (l/s) measured with tracer gas

Airflows Measured with Orifice Plates

All ducts for air supply, exhaust air and transferred air are furnished with an orifice plate, where the airflow rate is measured with an u-tube (Figure 3). The measuring range is 9-25 l/s and the uncertainty in measurement is 7.5 %, according to the manufacturer. Values in brackets are outside measuring range, and infiltration and exfiltration are estimated. It appears from the figures, that the measurements with tracer gas and orifice plates differ max 15% (when inside measuring range).

	From Room 1	From Room 2	From Room 3	From Room 4	From Room 5	From Room 6	From Room 7	From Air Supply	Infil-tra-tion	Total
To Room 1								11.1	(0.9)	12.0
To Room 2	9.0							(3.9)	(4.7)	17.6
To Room 3		15.0						(3.5)		18.5
To Room 4			8.9					(3.0)	(0.5)	12.4
To Room 5								?	?	10.7
To Room 6								14.9	(5.7)	20.6
To Room 7						14.4		(3.0)		17.4
To Exhaust Air	(3.0)	(2.6)	(3.9)	12.4	10.7	(6.0)	15.3			
Exfiltration			(5.7)				(2.2)			
Total	12.0	17.6	18.5	12.4	10.7	20.4	17.5			

Figure 3: Air flow rates (l/s) measured with orifice plates

Comparison Between Modelled and Measured Concentrations

Beyond the airflow rate estimations, modelled concentrations are calculated with estimated air flow rates and actual tracer gas supply. The deviation between the model and the measurements, is tabulated in Figure 4. The most substantial deviation is for room 7, where the standard deviation is 11 ppm.

	Mean dev.	Standard dev.	Max. dev.	Min. dev.
Room 1	-0.46	7.43	23.56	-14.95
Room 2	-1.03	5.04	9.55	-11.49
Room 3	-0.93	7.12	11.79	-24.08
Room 4	-0.85	6.62	14.21	-18.41
Room 5	-0.44	2.77	13.93	-5.33
Room 6	-0.29	5.64	17.13	-10.40
Room 7	1.70	11.25	23.33	-26.14
Air supply	-0.22	2.42	3.84	-9.68
Exhaust air	-0.11	2.81	6.62	-6.79

Figure 4: Deviations between measured and estimated concentration

The concentrations according to the model and as measured, are shown in Figure 5, where it appears that all pairs of curves are almost identical. Tracer gas is first brought into the air supply duct, which causes the first small peak after about 80 min. Then gas is supplied to each of the seven rooms and at last into the exhaust air duct.

According to the theory the concentrations in all cells have to be measured at the same time, but in reality they were measured 30 seconds after one another. Interpolation or Kalmanfilter would have improved the model, which however has not been done.

These measurements took place in a building without complicated spaces, and where perfect mixing was rather easy to achieve. Under such circumstances, it was possible to identify airflow rates by using a method of quadratic programming, which has been proved by this experiment.

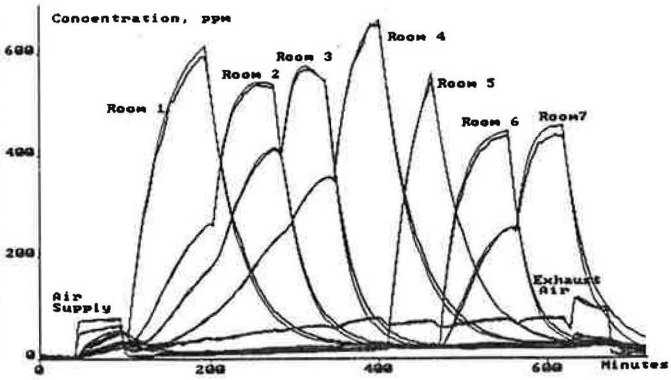

Figure 5: Tracer gas concentration in nine cells

References

1. Hedin, B. Identification methods for multiple cell systems. Proceedings of the 10th AIVC Conference, 1989.
2. Jensen, L.H. Determination of flows and volumes in multiple cell systems. Air Infiltration Review, February 1988.