Power Law Rules – OK?

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Introduction

In air infiltration and ventilation calculations, a power law equation is frequently used to represent the characteristics of flow through openings. Such an equation takes the form

$$Q = k(\Delta P)^n \quad m^3/s \qquad (1)$$

where Q = flow rate (m^3/s)

ΔP = pressure difference across opening (Pa)

k, n = flow coefficients

The flow coefficient, k, is related to the size of the opening and the exponent, n, characterises the type of flow. The flow exponent ranges in value between 0.5 for fully turbulent conditions, to 1.0 for laminar flow. In practice its value for cracks or adventitious openings tends to vary between 0.6 and 0.7. The validity of the power law approach is essentially based on the observed behaviour of flow at pressures in excess of those normally occurring under ambient infiltration conditions. This is because normal pressure differences (typically 0–5 Pa) are very low and it is difficult to make accurate field measurements in this range. However, there is some evidence to suggest that at these lower pressures, air flow is more accurately represented by a quadratic equation of the form

$$\Delta P = \alpha Q + \beta Q^2 \quad (Pa) \qquad (2)$$

where α and β are flow coefficients.[1] Another preference of the quadratic form is that the equation is dimensionally correct throughout the flow regime, whereas the power law equation is not.

The object of this article is to compare the performance of these two flow representations against a small set of measurement data using the concepts outlined in the Air Infiltration and Ventilation Centre's Calculation Techniques Guide.[2] A particular aspect of the approach presented is that in each case an identical solution technique, flow network and pressure field has been applied, thus enabling a direct comparison of each of the flow representations to be made. The tests were by no means exhaustive and further evaluation is recommended.

Test Data and Flow Network

The test data were taken from a small subset of the data used in the AIVC's model validation exercise.[3] It relates to a fairly tight single family dwelling located on the edge of a small estate. A summary of essential data items is presented in Table 1. In terms of the power law equation, the flow characteristics of the building are given as

$$Q = 0.0168\,(\Delta P)^{0.71} \qquad (3)$$

Using the calculated flow rates at 30 Pa and 50 Pa, the equivalent quadratic equation is

$$\Delta P = 101.31Q + 310.14Q^2 \qquad (4)$$

On a 0–50 Pa plot (Figure 1a) the flow characteristics of each equation appear to be almost identical. However, in the critical 0 – 5 Pa regime (Figure 1b), the difference in flow prediction is substantial. At 1 Pa the quadratic flow prediction is 40% below the power law value. Clearly it is inevitable that ventilation and infiltration predictions will also differ substantially.

Building height: 8m
Volume of shielding: 386 m^3
Shielding conditions: see text
Flow characteristics: see text

Run No.	Inside/outside Temp. Diff. (°C)	Wind (m/s)	Measured air change rate (h^{-1})
1	7.5	3.84	0.11
2	13.0	3.35	0.09
3	26.8	2.19	0.20
4	26.2	4.43	0.16
5	28.1	5.63	0.20
6	29.9	4.69	0.19
7	29.1	5.68	0.18
8	32.2	5.23	0.20
9	33.9	5.99	0.20
10	33.5	6.30	0.21
11	37.5	3.84	0.19
12	40.5	5.23	0.21
13	40.6	5.86	0.21
14	17.5	1.07	0.10

Wind speed is measured on site at a level of 18m above ground.

Table 1: Summary of Test Data

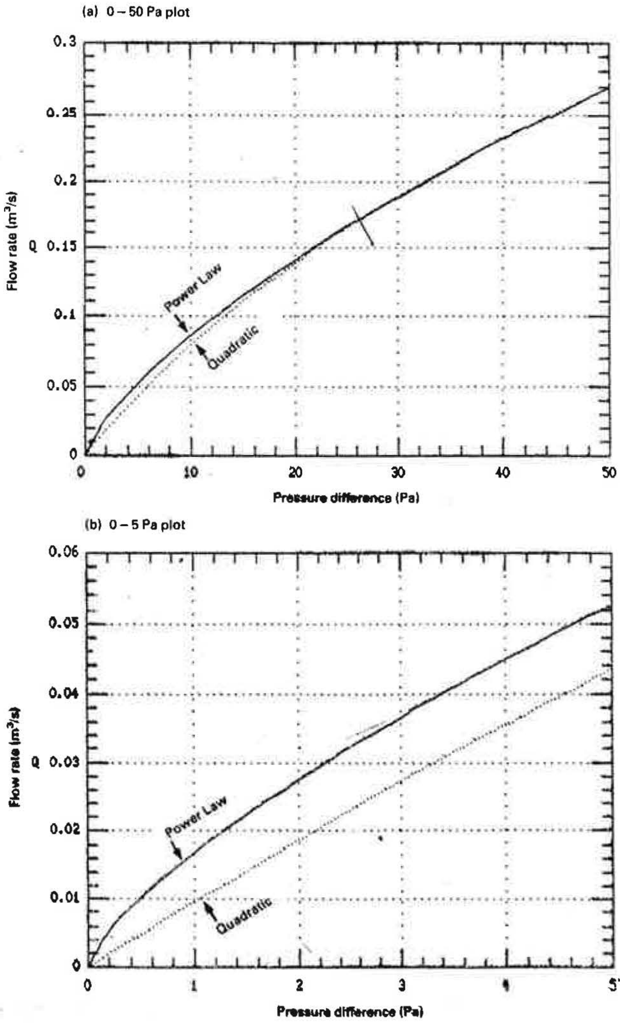

Figure 1: Comparison of power law and quadratic flow rates

Numerical Approach

The building was essentially approximated by a 'single zone' square structure in which the leakage was distributed uniformly about the exposed surface of the heated volume, i.e. external walls and top floor ceiling. Discrete flow paths were introduced on each face of the building as indicated in Table 2.

Equations (1) and (2) were applied to each flow path to yield

(i) Power Law

$$\sum_{i=1}^{j} k_i \, |p_i - p_{int}|^{n_i} \left(\frac{p_i - p_{int}}{|p_i - p_{int}|} \right) = 0 \qquad (5)$$

where k_i = flow coefficient of the i'th flow path

n_i = flow exponent of the i'th flow path

p_i = external pressure acting on the i'th flow path

p_{int} = internal pressure

j = number of flow paths

(ii) Quadratic

$$\sum_{i=1}^{j} \left(-\alpha_i + (\alpha_i^2 + 4\beta_i |p_i - p_{int}|)^{1/2}\right)/2\beta_i \times \frac{(p_i - p_{int})}{|p_i - p_{int}|} \qquad (6)$$

where α_i and β_i = quadratic flow parameters of the i'th path

In both formulations the absolute pressure difference, $|p_i - p_{int}|$, is used in the power law or square root term. The true sign of the flow direction is restored by the last term of the equation.

The pressures driving the air change process were approximated by the wind and pressure equation

$$P_w = \frac{\rho}{2} C_p V^2 \quad \text{(Pa)} \qquad (7)$$

where ρ = air density $\simeq 1.29$ kg/m^3

C_p = wind pressure coefficient (depending on wind direction and shielding)

V = wind speed at building height (m/s)

and the stack pressure equation

$$P_s = -\rho_o g \, 273 \, h \left(\frac{1}{T_e} - \frac{1}{T_i} \right) \quad \text{(Pa)} \qquad (8)$$

where ρ_o = air density at 0°C (~1.29 kg/m^3)

h = height of opening above lowest opening (m)

T_e = outside temperature (K)

T_i = inside temperature

The relevant wind pressure coefficients and height of openings are summarised in Table 2 where the wind pressure data have been taken from Section 6 of the AIVC Calculation Techniques Guide.[2]

Flow path no.	Leakage Site	*Height (m)	Wind pressure coeffic. (urban)	Wind pressure coeffic. (partly sheltered)
1	Front ground floor facade	0.0	−0.3	−0.38
2	Garage – NE facade	0.0	0	0
3	Garage – NW facade	0.0	0	0
4	NW ground floor facade	0.0	0.05	0.1
5	Rear ground floor facade	0.0	0.05	0.1
6	SE ground floor facade	0.0	−0.3	−0.35
7	Front first floor facade	2.6	−0.3	−0.35
8	NW first floor facade	2.6	0.05	0.1
9	Rear first floor facade	2.6	0.05	0.1
10	SE first floor facade	2.6	−0.3	−0.35
11	Ground floor 'flat roof'	1.4	−0.4	−0.45
12	First floor roof	4.0	−0.4	−0.45

*Level given with respect to lowest opening

Table 2: Flow Path Data

Shielding and Wind Speed

Surrounding shielding makes an important contribution to the net wind pressure distribution. For the purposes of this exercise it was assumed that shielding conditions would range between surrounding obstructions equal to the height of building, i.e. adjacent houses, and surrounding obstructions equal to half the height of the building, e.g. fences, walls, shrubbery, etc. In part this is dependent on wind direction but in this study both sets of conditions were applied in an attempt to illustrate the relative significance of this parameter. However, the most realistic shielding condition for this particular set of data was thought to be the 'half height' value. The shielding parameter is transferred to the infiltration calculation via the pressure coefficient, C_p, in equation (7). The applied values are presented in Table 2.

Since the strength of the wind increases with height above ground level, it is imperative that the correct building height windspeed is used in equation (7). Measurements were made within the locality of the building at a height of approximately 18m. Guidelines presented in Section 6 of the Calculation Techniques Guide were used to determine the necessary multiplication or 'wind reduction' factors, necessary to convert the measured wind speed to the 8m building height value. The resultant factors were 0.82 for the sheltered condition and 0.85 for the partly sheltered condition.

Simulations

In a single zone calculation, an internal pressure is evaluated such that the flow into the building is balanced by the out flow (equations 5 and 6). This is achieved by an iteration process in which an arbitrary guess at the internal pressure is successively amended until flow balance is achieved. The data summarised in Tables 1 and 2 were accordingly applied to equations (5) and (6) to produce four sets of infiltration calculations. These were:

i) Power law equation with half height surrounding obstructions

ii) Power law equation with equal height surrounding obstructions

iii) Quadratic equation with half height surrounding obstructions

iv) Quadratic equation with equal height surrounding obstructions

The corresponding results are presented in Figures 2 – 5 respectively.

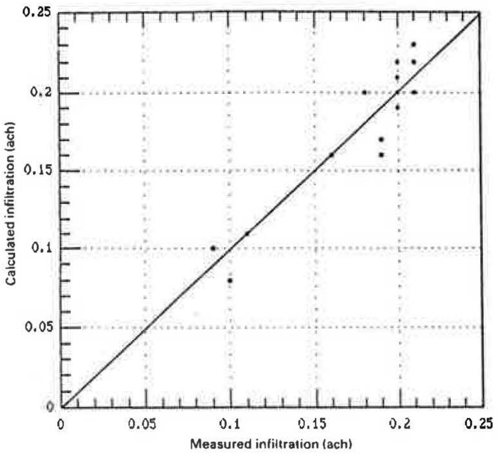

Figure 2: Calculated vs measured air infiltration Power law – 'half height' shielding

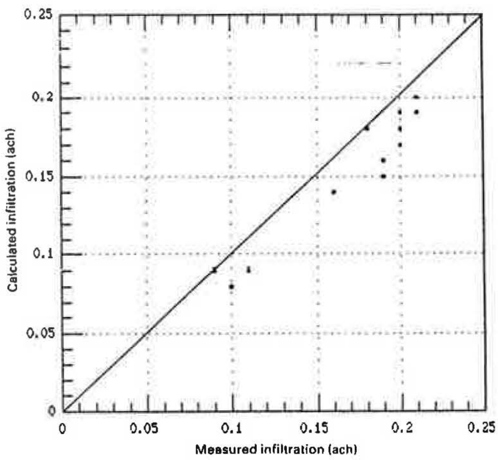

Figure 3: Calculated vs measured air infiltration Power law – 'equal height' shielding

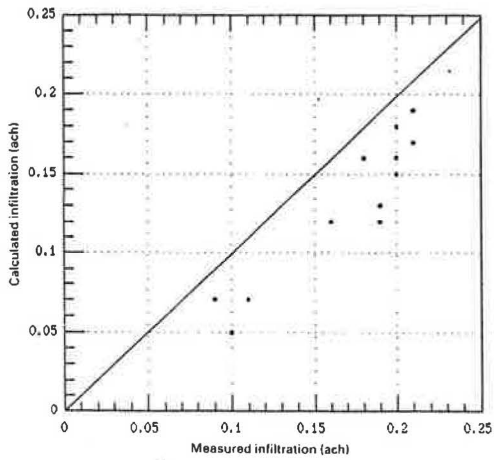

Figure 4: Calculated vs measured air infiltration Quadratic law – 'half height' shielding

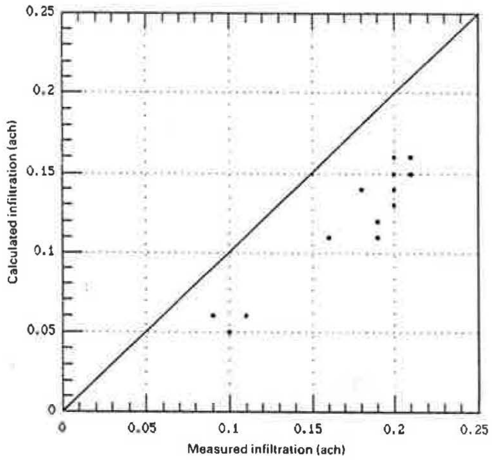

Figure 5: Calculated vs measured air infiltration Quadratic law – 'equal height' shielding

Results

Figure 2 illustrates the comparison between calculated and measured air infiltration using the power law with half height shielding. Calculated values were fairly evenly distributed about the line of perfect agreement and all the calculations were well within ±25% of the measured values. Figure 3 illustrates the power law results for equal height surrounding shielding. Again, all results are within 25% of measurement but they are below the measured infiltration rates. The quadratic law results with half height shielding are presented in Figure 4. These all underestimate the observed values with four of the data points diverging from the measured values by more than 25%. Finally the quadratic law results with equal height of surrounding obstructions is given in Figure 5. In this example all the points underestimate the measured infiltration rate and only four of the data points are within 25% of measurement. By taking an average of all the data points, the mean measured infiltration was 0.18 ach. This compared with a calculated value of 0.18 ach for simulation (i), 0.16 ach for simulation (ii), 0.14 ach for simulation (iii) and 0.12 ach for simulation (iv). The difference in shielding class had approximately a 10% influence on the power law results and a 12% affect on the quadratic results.

Discussion and Conclusions

It would be incorrect to completely rule out the quadratic approach since much also depends on the interpretation of flow path distribution, e.g. the validity of assuming a uniform distribution of air leakage openings, and on the assumptions used to derive the wind pressure field, i.e. shielding conditions and pressure coefficient values. With a sufficiently large database, the wind pressure problem could be eliminated by concentrating on stack driven (low wind speed) air infiltration conditions only. Although insufficient data was available for this exercise, a useful indication is presented in run number 14 (Table 1), where the building height wind speed is less than 1 m/s and the inside/outside temperature difference is 17.5°C. Under these conditions the wind effect is small. In the case of the power law, the infiltration prediction was an acceptable 0.08 ach for each shielding condition (compared with a measured value of 0.10 ach), while the quadratic formulation yielding a somewhat less acceptable 0.05 ach for each condition. Another possibility is that, in terms of steady state flow, the quadratic formulation is indeed correct but that the power law approach fortuitously compensates for the additional influence of turbulent fluctuations at low driving pressures.

Before any firm conclusions can be drawn it is necessary to consider a wider field of data but these preliminary results indicate that the power law approach is the most suitable method.

References

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Note: AIVC Technical Note 11 is now available to non-participating countries. Please see publications list in this Review for details.