A THREE-DIMENSIONAL NUMERICAL STUDY OF SLAB-ON-GRADE HEAT TRANSFER

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ABSTRACT

A detailed, hourly three-dimensional finite difference model was used to conduct parametric studies of heat transfer from slab-on-grade floors. Studies considered effects of geometry, climate, soil properties, and boundary conditions. The results indicate that the widely used F_2 method for heating load calculations may err by 50% or more because its central assumption (that heat transfer is proportional to perimeter length) is erroneous. The ratio of floor area to perimeter length (A/P) was found to be an appropriate length scale for correlating average heat flux from L-shaped and rectangular floors. A simplified analysis of daily averaged heat transfer based on A/P scaling could be the basis for an improved design method useful for both load and energy calculations. Thermal conductivity and ground surface boundary conditions proved to be important parameters affecting heat transfer, while thermal diffusivity and far-field boundary conditions were relatively unimportant. The shading of adjacent soil by a building's shadow caused significant changes in heat transfer rate under some conditions.

INTRODUCTION

The accurate estimation of loads and energy consumption due to thermal interactions between a building and the ground is a difficult task. Factors that complicate this analysis include the multi-dimensional nature of most earth-coupled heat transfer processes, large phase lags caused by soil thermal mass, limited practical ability to model soil thermal properties, and the variability of soil temperature with ground surface conditions. In the past, it was customary to treat this difficult problem in a highly simplified manner because foundation heat transfer processes accounted for a small portion of a building's energy budget. Modern, energy-conscious design and construction techniques, however, have improved the thermal performance of the above-grade envelope to the point that an uninsulated basement might contribute half the heating load on a well-built residential structure. Given the increased importance of earth-coupled processes, the commonly used methods of analysis are less acceptable today than they once were.

Despite the existence of a number of recent, more rigorous approaches to analysis of the slab on grade, the most widespread method applied to this problem is the F_2 equation currently found in the *ASHRAE Fundamentals* (ASHRAE 1989):

$$Q = F_2 \cdot P \cdot (T_i - T_o) \quad (1)$$

According to this model, the rate of heat transfer (Q) from a slab-on-grade floor is a simple function of its perimeter length (P), the indoor-outdoor design temperature difference ($T_i - T_o$), and a proportionality constant (F_2). F_2 is a tabulated function of construction type and climate (heating degree-days). The origins of this model are experimental studies conducted during the 1940s at the National Bureau of Standards* (Dill et al. 1943) and the University of Illinois (Bareither et al. 1948), which showed that heat transfer rates during the heating season are much higher at the slab edge than near its center. Bareither's report notes, however, that Equation 1 is of questionable validity when the ratio of area to perimeter (A/P) of a floor exceeds 12 ft (3.66 m) due to the effect of heat transfer through floor core area. The current F_2 values are based on a more recent two-dimensional finite element study (Wang 1979).

Further evidence of the complexity of earth-coupled heat transfer is revealed by the recent work of Walton (1979), which compared two- and three-dimensional finite difference models of slab and basement heat transfer for a variety of area and perimeter values. Walton's results showed that ostensibly comparable two- and three-dimensional cases had heat transfer rates that differed by as much as 50%. He proposed and tested an approximate method that represents the three-dimensional problem as the sum of two-dimensional Cartesian and cylindrical parts. Conservation of both perimeter and area between the actual foundation and the model determines the sizes of these components. This "rounded rectangle" method agreed with the true three-dimensional model to within 2%.

This paper describes a three-dimensional parametric study of slab-on-grade heat transfer, which gives a new perspective on both the validity of the F_2 method and the relationship between two- and three-dimensional heat transfer discussed in the references cited previously. Parameters considered in the study include floor shape and size, climate, surface and deep ground boundary conditions, soil properties, insulation, and shadowing of the ground by the building. On the basis of these results, an empirical scaling technique is derived that gives good agreement with the detailed model and may be of value in both manual and numerical algorithms.

MODEL DESCRIPTION

A fully detailed model of heat transfer in the soil must recognize that soil is an inhomogeneous porous medium in which transport of energy and mass are intimately coupled. Thermally induced migration of moisture can have a significant effect on heat transfer rates, particularly in situations where large temperature gradients exist (e.g., buried steam distribution lines). In most prior studies of earth-coupled building heat transfer, however, soil has been assumed to be homogeneous with constant properties. This assumption, which considerably simplifies the analysis, is justified on a number of grounds. Most importantly, it has been shown (Eckert and Pfender 1978) that the coupling between heat and mass transfer is quite weak under conditions expected to prevail in the soil near a building. Consequently, effects of moisture on heat transfer can be

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represented through an effective thermal conductivity. From a practical point of view, the distribution of moisture and soil solid matrix properties is unlikely to be known and is of questionable relevance in parametric studies such as the present one. Therefore, the mathematical basis of the model for this study is a boundary value problem on the three-dimensional heat conduction equation:

$$\rho c\, \partial T/\partial t = \nabla \cdot (k \cdot \nabla T) \qquad (2)$$

The following boundary conditions were adopted:

Interior Slab Surface: combined, linearized radiation/convection:

$$Q = h_i \cdot A \cdot (T_{room} - T_{floor}) \qquad (3)$$

where h_i is the linearized radiation/convection coefficient and $(T_{room} - T_{floor})$ is the difference between room air and floor surface temperatures.

Far-field Soil: undisturbed soil temperature distribution. This condition simply requires that soil temperature be a function only of time and the vertical coordinate, *z*:

$$T = T(z, t) \qquad (4)$$

Deep Ground: fixed temperature or zero gradient condition:

$$T(z_{max}) = Constant \text{ or } \partial T/\partial z|_{z\to\infty} = 0 \qquad (5)$$

where z_{max} is the lower boundary depth. The former approximates the effect of a water table, while the latter is the classical semi-infinite medium boundary condition.

Ground Surface: Heat transfer at the earth's surface may be modeled as a specified flux condition:

$$-k\, \partial T/\partial z|_{z=0} = G(t) \qquad (6)$$

where the gradient is determined by an energy balance involving conduction, convection, evaporation, and radiation (Kreith and Sellers 1975), i.e.,

$$G = R_t - q_{cs} - q_{et} \qquad (7)$$

Equation 7 states that the rate of heat conduction into the ground (G) is equal to the rate at which solar and infrared radiation is absorbed at the surface (R_t), less the sum of sensible convective (q_{cs}) and evapotranspiration (q_{et}) fluxes. Solar radiation data are contained in standard weather files. Infrared radiation was estimated by empirical correlations. Convective coefficients were computed hourly from empirical correlations for the turbulent atmospheric boundary layer.

The detailed energy balance of Equation 7, unlike the simplified treatments in some previous models, permits the study of effects caused by ground cover variation. Kusuda (1975), Gold (1967), Gilpin and Wong (1975), and others have shown that surface cover variations can produce ground surface temperature changes of 18°F (10°C) or more.

Evapotranspiration includes all forms of latent energy loss at the surface, including evaporation, convection, and transpiration by vegetation. The actual transpiration rate depends upon both meteorological conditions and the supply of moisture to the ground surface. To determine the degree of saturation of the surface, a soil moisture balance is necessary. The limiting case in which the ground surface is saturated is called "potential evapotranspiration." The potential rate of evapotranspiration is a theoretical maximum that depends only on meteorological factors (incident radiation, wind speed, air temperature, and humidity ratio) and so is more easily calculated. The potential rate was used in this study as a matter of convenience. It should be recognized that this choice tends to exaggerate the effects of latent heat exchanges that were predicted.

A more detailed discussion of models for the right-hand-side flux terms in Equation 7 is given in Bahnfleth (1989). The general approach is similar to that adopted by Speltz and Meixel (1981) in their study of earth-covered roofs.

NUMERICAL METHOD

The boundary value problem described in the preceding section was solved in Cartesian coordinates by a Fortran program implementing the explicit Patankar-Spalding finite difference technique (Patankar 1980). This method has been applied previously in two-dimensional analysis of earth-coupled heat loss, in, for example, the study of Shipp (1979). The three-dimensional domain in the present study was discretized by an irregular grid into as many as 10,000 cells. The time discretization employed a one-hour step consistent with the frequency of TMY weather data entries. Minimum grid spacing was 4 in. (0.1 m) near the ground surface and slab boundaries. Near the outer boundaries of the domain, larger grid spacing was used to reduce computation time. When possible, the number of cells in the model was reduced through the use of symmetry conditions. Typical execution times for rectangular plan shapes without shading were on the order of 5 to 10 hours of microcomputer CPU time, approximately seven annual cycles. L-shaped and shadowing runs took much longer due to their lack of symmetry.

For the purpose of modeling its shadow, the building was presumed to be a flat-roofed, rectangular solid with a single wall height. The shadow cast on the ground was approximated by turning off beam solar radiation to surface cells whose centers were shaded at the beginning of an hour. To determine which cell centers were shaded, the hourly beam radiation direction was computed, and a line in this direction passing through each cell center was tested for intersection with building walls. To obtain a reasonable approximation of the shadow's shape, surface cell dimensions were kept less than or equal to 3.28 ft (1 m). Execution times for shadowing cases were as long as 52 hours of CPU time.

Program parameters assignable at the discretion of the user include:

- domain dimensions and grid spacings
- weather data file (TMY)
- soil and slab properties
- ground surface properties
- slab shape (rectangle or L) and size
- deep ground boundary condition (zero flux or fixed temperature)
- evaporative loss at ground surface (on/off)
- shadowing (on/off)
- building height for shadow calculations.

Parameter values used in this study are discussed in the following section. A listing of the program and a sample input form may be found in Bahnfleth (1989).

TEST PLAN

Climate

The study considered four representative continental United States locations for which TMY weather data are available. Data for these sites appear in Table 1. Minneapolis and Phoenix are typical of the cold and hot extremes of U.S. weather. Philadelphia and Medford are situated in moderate climatic zones having similar mean temperatures but different degree-days. Oregon's coastal location is responsible for the statistically less severe climate of Medford.

Soil Properties

Data gathered by Kersten (1949) and presented graphically by Andersland and Anderson (1978) were the primary source for soil properties. A mid-range set of properties representing a moist soil was used as the base case in most of the simulations. Four other sets representative of both dry (lower conductivity) and wet (higher conductivity) extremes were used to study property effects. These five sets of properties are shown in Table 2. Relationships between sets permitted comparison of independent thermal conductivity and thermal diffusivity

TABLE 1 Test Site Climatic Data

Parameter*		Medford, OR	Minneapolis, MN	Philadelphia, PA	Phoenix, AZ
Latitude	[Deg]	42° 2′	44° 5′	39° 5′	33° 3′
Longitude	[Deg]	122° 5′	93° 1′	75° 2′	112° 0′
Elevation	[m]	396	251	2	340
	[ft]	1299	823	7	1115
T_{mean}	[°C]	11.7	7.0	12.2	21.8
	[°F]	53.1	44.6	54.0	71.2
HDD	[°C]	2735	4636	2855	773
	[°F]	4923	8345	5139	1391
CDD	[°C]	315	506	614	2023
	[°F]	567	911	1105	3641

*T_{mean} = mean air dry-bulb temperature, HDD = heating degree-days, CDD = cooling degree-days.

changes. (For example, in set A, diffusivity [α] remains constant, while conductivity [k] doubles with respect to the base case. In set B, diffusivity is halved with respect to the base case, while conductivity remains constant.) Density (ρ) and specific heat (c) always appear as a product in Equation 1, so they were arbitrarily assigned equal values in the SI system of units as a matter of convenience. (By coincidence, SI values of density and conductivity of soil are of the same order.)

Ground Surface Properties

Surface properties were drawn from a number of sources summarized by Sellers (1965). A surface covered by short grass was assumed. Snow cover was simulated by changes in surface convective and radiative properties. Average solar albedo (reflectivity) values were taken from the extensive measurements of Kung et al. (1964), who derived tables of continental averages as a function of latitude and snow cover from measurements taken from an airplane. Values used in this study were:

- 30-35° north latitude (Phoenix)—snow: 0.191; no snow: 0.172
- 35-40° north latitude (Philadelphia)—0.285/0.165
- 40-45° north latitude (Medford, Minneapolis)—0.379/0.158.

Geiger (1961) and others have found that infrared emissivity is 0.90 or higher for most natural surfaces, including snow and grass, so a value of 0.90 was used in all cases. Surface roughness height values of 0.30 in. (0.0075 m) for short, bare grass and 0.01 in. (0.0003 m) for snow were used in the the calculation of turbulent convection coefficients.

Building Parameters

A number of floor parameters were held constant so attention could be focused on effects of variation in floor size and shape. All floors were 4 in. (0.1 m) thick concrete slabs with thermal conductivity, density, and specific heat of, respectively, 0.54 Btu/h·ft·°F (0.93 W/m·K), 143.6 lbm/ft³ (2300 kg/m³), and 0.156 Btu/lbm·°F (653 J/kg·K) (ASHRAE 1989). Insulation, when specified, was polystyrene board with a thermal conductivity of 0.017 Btu/h·ft·°F (0.029 W/m·K). Floor surface conductances (h_i) for heat transfer to and from the room were, respectively, 1.63 and 1.08 Btu/h·ft²·°F (9.26 W/m² and 6.13 W/m²). Indoor temperature was maintained at a constant 71.6°F (22°C).

Floor plans were either rectangular or L-shaped. Area varied from 1550 to 38,750 ft² (144 to 3600 m²). The majority of runs used either a "residential" size of 1550 ft² (144 m²) or a "commercial" size of 21,800 ft² (2025 m²). Aspect ratio varied from unity (a square) to nine (a long, narrow rectangle). Area to perimeter (A/P) ratios ranged from 7.9 to 49.2 ft (2.4 to 15 m). Four insulation configurations were considered: 1 in. (0.025 m) on the slab edge and under the first 3.28 ft (1 m) of the floor, 1 in. of insulation covering the edge and entire bottom surface of the slab, and 2 in. (0.05 m) of insulation in each of the preceding arrangements.

Parametric Groups

Ninety-three runs were divided into seven groups, each isolating effects of a different parameter or family of parameters:

- Floor and domain geometry
- Deep ground boundary condition
- Climate
- Evapotranspiration
- Shadowing
- Soil properties
- Insulation

The base case to which all parametric studies were compared was an uninsulated slab on a 49 ft (15 m) deep domain with base case soil properties (see Table 2), a specified temperature deep ground condition (equal to annual average dry-bulb), and a potential evapotranspiration ground surface condition. Each series included several area and aspect ratio combinations in order to show the dependence on geometric factors of the effect produced by the parameter under study.

RESULTS

Floor Geometry

The primary objective of this study was to quantify the effects of area and shape on heat transfer rates. To this end, a group of simulations was performed in which only geometric factors were varied. Both rectangular and L-shaped floors were con-

TABLE 2 Soil Property Groups

	Property*	Base Case	A	B	C	D
k	[W/m·K]	1.0	2.0	1.0	0.5	2.0
	[Btu/h·ft·°F]	0.58	1.16	0.58	0.29	1.16
ρ	[kg/m³]	1200	1700	1700	1200	1500
	[lbm/ft³]	75	106	106	75	94
c	[J/kg·K]	1200	1700	1700	1200	1500
	[Btu/lbm·°F]	0.2866	0.4060	0.4060	0.2866	0.3583
α	[m²/s]	6.9×10^{-7}	6.9×10^{-7}	3.5×10^{-7}	3.5×10^{-7}	8.9×10^{-7}
	[ft²/s]	7.4×10^{-6}	7.4×10^{-6}	3.8×10^{-6}	3.8×10^{-6}	9.6×10^{-6}

*k = conductivity, ρ = density, c = specific heat, α = thermal diffusivity.

sidered. Parameters held constant were climate (Medford), domain depth (49.2 ft/15 m), lower boundary condition (fixed temperature equal to average dry-bulb), ground surface condition (no shading, potential evapotranspiration), slab type (uninsulated, 4 in. [0.1 m] thick concrete), and indoor dry-bulb temperature (72°F/22°C).

The results of these simulations show that, in contradiction to conventional wisdom, heat loss per unit of perimeter length has a significant dependence on floor area. The magnitude of this effect is evident in Figure 1, which compares daily averaged heat loss per unit of perimeter length for 39 ft by 39 ft (12 m by 12 m) and 148 ft by 148 ft (45 m by 45 m) floors. Annual amplitude of heat loss appears to be the same for both floors; however, the larger floor shows a mean heat loss that is approximately 8.3 Btu/h·ft (8 W/m) greater than the smaller floor. This example shows that F_2 factors based on heat loss from the smaller of the two floors would systematically underpredict the peak heat loss of the larger floor by roughly 25% and underpredict the minimum by 50%.

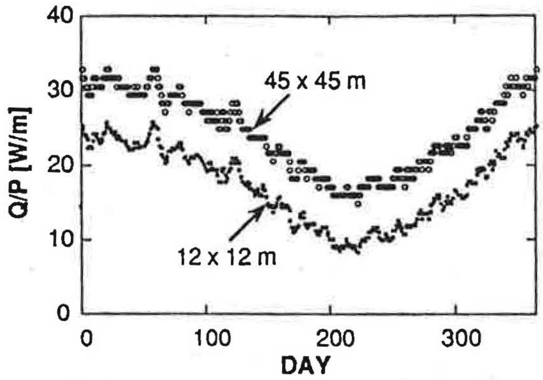

Figure 1 *Daily averaged heat loss per unit perimeter length for large and small slabs, Medford, OR*

The effect of area on mean heat loss is revealed with greater clarity by Figure 2, which gives annual average heat loss as a function of perimeter for 20 runs of varying area and shape. For each of the five areas represented, the case having the smallest perimeter is a square (aspect ratio of one). Larger perimeter for a given area corresponds to increasing aspect ratio. Clearly, the data do not approximate a simple linear function of perimeter. Each floor area group defines a distinct curve on which heat loss increases with perimeter length. However, there is no evidence of an effect due to shape alone. L-shaped floors fall into place according to their perimeter length among rectangles of the same area. (For example, the middle point of the five 21,797 ft² [2025 m²] cases is an L-shaped slab.) The relationship between area and perimeter appears to be the critical factor (although this might not be true for cases such as a floor that completely encloses a courtyard).

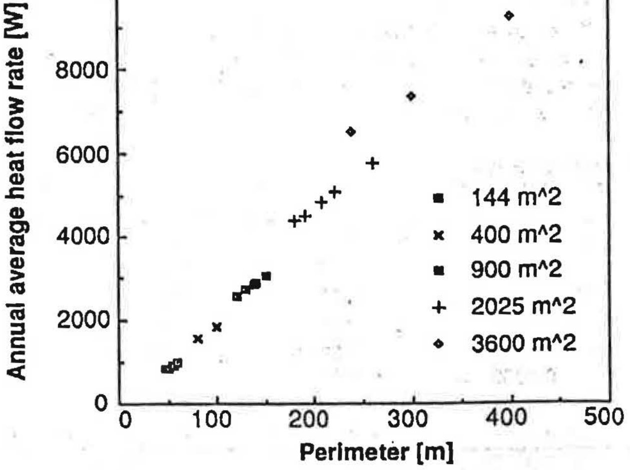

Figure 2 *Heat loss vs. perimeter length for uninsulated floors in Medford, OR, 49 ft (15 m) deep domain*

Attempts to establish a simple functional relationship between heat loss, area, and perimeter yielded the result shown in Figure 3. When the total heat transfer vs. perimeter length results of Figure 2 are replotted as heat loss per unit area (q) vs. the ratio of area to perimeter (A/P), a single curve is obtained. A logarithmic function of the form

$$q = c \cdot (A/P)^d \tag{8}$$

where c and d are empirical constants, gives an excellent approximation to these scaled results. The length scale A/P has physical significance as a measure of the narrowest dimension of a planar shape. For a square of side "L," A/P is equal to $L/4$. In the general case of a rectangle with short side "L" and aspect ratio "μ" (defined ≥ 1), A/P is equal to $L/[2(1 + 1/\mu)]$. An infinite strip of length "L" has an aspect ratio of infinity and an A/P of $L/2$.

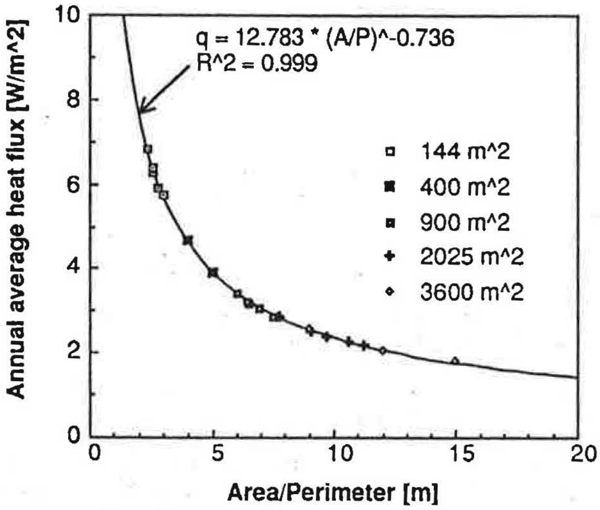

Figure 3 *Heat loss per unit area vs. A/P for uninsulated slabs, Medford, OR, 49 ft (15 m) deep domain*

For a given rectangular area, a square has the largest value of A/P, so for each floor area group in Figure 3, the square case is the rightmost point. Note that there is overlap between the A/P values of the 21,797 ft² (2025 m²) and 38,750 ft² (3600 m²) data, and that the heat flux values for these overlapping cases fall into place on the same curve. As the characteristic width of a slab increases, its average rate of heat loss decreases. This reflects the fact that floors with large A/P have longer average heat flow paths to the ground surface and proportionately more core area than floors with small values of A/P.

The constants "c" and "d" depend on a great many parameters, including the annual average temperature difference, soil properties, domain geometry, and details of foundation design. There is no reason to suppose that the values pertaining to the plotted data set are universal in any sense. The fluctuating component of heat loss may follow a relationship of the same form as Equation 8; however, the constants c and d would not necessarily have the same values as in the expression for the mean component.

It is worthwhile to consider the implications of the heat flux relation (Equation 8) for average whole-floor heat loss. As a consequence of Equation 8, the total heat loss for a floor (Q) has the form

$$Q = c \cdot (A/P)^d \cdot A = c \cdot P^{-d} \cdot A^{1+d}. \tag{9}$$

If d has a value of −1, then Q is independent of area and a linear function of perimeter. Values of d greater than −1 indicate a combined dependence on total area and perimeter. A value of 0 indicates linear dependence on area and independence of perimeter. On physical grounds, one can argue that d must lie between these limiting cases of 0 and −1. If d is greater than zero, Equation 9 implies that an increase in perimeter causes decreased heat loss for a fixed area. A value of d less than −1 implies that heat loss decreases as area increases. Both of these behaviors are implausible.

An extension of this scaling technique to the full, unsteady heat transfer problem can be developed from linear conduction theory, which permits the decomposition of the total heat flux into mean and fluctuating parts:

$$q_{total}(t) = q_{mean} + q_{periodic}(t). \tag{10}$$

If the mean heat transfer component is assumed to be proportional to the difference between the indoor air and outdoor ground surface temperatures and the periodic component is assumed to be a linear function of the difference between the daily averaged and mean ground surface temperatures, then

$$q_{total}(t) = K_1 \cdot (T_{room} - T_{g,mean}) + K_2 \cdot (T_{g,mean} - T_{g,\phi}) \tag{11}$$

where K_1 and K_2 are constant mean and periodic conductances for that floor. $T_{g,\phi}$ is the time-dependent ground surface temperature shifted by a phase lag (ϕ) to account for the effect of soil thermal mass. Ground surface temperature is superior to either air or deep ground temperature as an ambient reference temperature because it more nearly represents soil conditions near a slab. Air temperature may be considerably different than ground temperature as a result of the surface heat balance, while deep ground temperature is essentially invariant (and so, of no value as a reference for the periodic component).

The ground temperature, T_g, may be approximated by a sinusoidal least-squares model:

$$T_g = T_{g,mean} + \Delta T_g \cdot \sin\,[2\pi\,(\text{Day} + \zeta)/365] \tag{12}$$

where ΔT_g is the amplitude of the annual ground temperature cycle, "Day" is the day of the year (1-365), and ζ is the phase shift (in days) of the ground temperature with respect to the calendar. $T_{g,\phi}$ includes the additional phase shift, ϕ:

$$T_{g,\phi} = T_{g,mean} + \Delta T_g \cdot \sin\,[2\pi\,(\text{Day} + \zeta + \phi)/365] \tag{13}$$

The geometric dependence of K_1 and K_2 is approximated by expressions of the same form as Equation 8:

$$K_1 = c_1 \cdot (A/P)^{d_1} \tag{14a}$$

$$K_2 = c_2 \cdot (A/P)^{d_2} \tag{14b}$$

The complete daily averaged heat flux model thus becomes

$$q_{total}(t) = c_1 \cdot (A/P)^{d_1} \cdot (T_{room} - T_{g,mean}) - c_2 \cdot (A/P)^{d_2} \cdot \Delta T_g \cdot \sin\,[2\pi\,(\text{Day} + \zeta + \phi)/365]. \tag{15}$$

Values of the constants c_1, c_2, d_1, and d_2 are determined by a two-stage process. K_1 and K_2 values are calculated for a number of floor A/P values by least-squares approximation of daily averaged heat flux data by Equation 11. The values of c_1, c_2, d_1, and d_2 are obtained by a least-squares fit to Equations 14a and 14b using the K_1 and K_2 values calculated in the previous step. The periodic heat transfer phase lag, ϕ, is relatively insensitive to floor size for a given set of soil and environmental conditions, so an average value can be used in practical application of Equation 15.

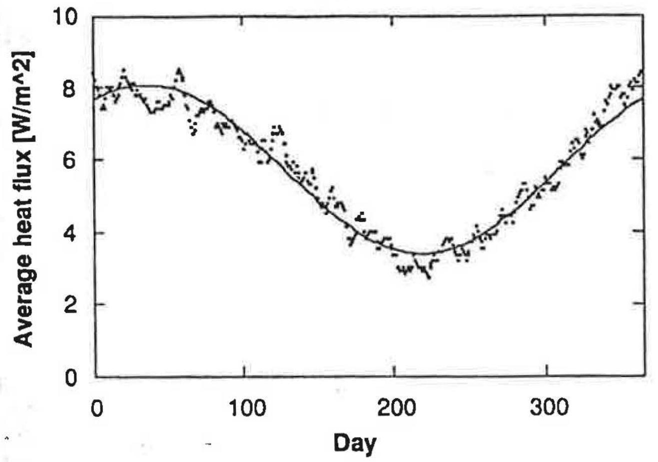

Figure 4 *Daily averaged heat loss model for a 39 ft by 39 ft (12 m by 12 m) floor in Medford, OR*

Figure 4 shows a typical daily averaged heat flux model compared with the predictions of the three-dimensional finite difference model. It is important to note that the simplified model represents a prediction based on the entire set of Medford data, not merely a curve fit to the single set of data shown. Agreement in the mean is nearly exact. The sinusoidal model for the periodic component is slightly in error because the 1979 Medford TMY temperature record is colder in December than in January.

Climate

The effects of climatic variation were evaluated by comparison of model coefficients for the four typical climates. Sixteen cases were considered: four rectangular plans in each of the four climates. A comparison of air and ground temperature models (see Equation 12) is given in Table 3. For the three temperate cases (Medford, Minneapolis, and Philadelphia), the mean ground temperature is depressed 4.3° to 5.2°F (2.4° to 2.9°C) beneath the average dry-bulb, while the amplitude* of the daily averaged ground temperature is within 1.8°F (1°C) of the air temperature amplitude. For Phoenix, however, which has a warm, dry climate with year-round high evapotranspiration potential, the mean ground temperature is a full 10.8°F (6°C) less than mean air temperature and the ground temperature model amplitude is 6.1°F (3.4°C) smaller than the dry-bulb amplitude.

Data summarized by Kusuda and Achenbach (1965) suggest that the average ground temperature depression obtained for Phoenix is larger than is likely to occur naturally. The large amount of precipitation needed to maintain potential evapotranspiration conditions does not occur in this environment.

*Values of amplitude in Table 3 are negative as a result of the form of the model and the choice of representation for phase shift. Only the magnitude is of significance to this discussion.

TABLE 3
Mean, Amplitude, and Phase Shift of Daily Averaged Air and Ground Surface Temperatures (Potential Evapotranspiration)

Location	$T_{air,mean}$ [°C]	$T_{air,mean}$ [°F]	ΔT_{air} [°C]	ΔT_{air} [°F]	ζ_{air} [Days]	$T_{g,mean}$ [°C]	$T_{g,mean}$ [°F]	ΔT_g [°C]	ΔT_g [°F]	ζ_g [Days]
Medford	11.4	52.5	−9.7	−17.5	69.2	8.6	47.5	−9.9	−17.8	73.0
Minneapolis	7.2	45.0	−17.0	−30.6	72.3	4.8	40.6	−16.3	−29.3	72.6
Philadelphia	12.4	54.3	−12.6	−22.7	68.9	9.5	49.1	−12.1	−21.8	68.8
Phoenix	21.9	71.4	−11.7	−21.1	72.2	15.9	60.6	−8.3	−14.9	68.3

However, such a condition can be induced locally by watering and/or shading. Indeed, typical humidity levels in the Phoenix area have risen in recent years as a result of irrigation.

Table 4 gives daily averaged heat flux model coefficients for the four sites. There is excellent agreement between the results for Medford, Minneapolis, and Philadelphia. In all four locations, the time-varying component of heat loss is linearly proportional to perimeter length ($d_2 \approx -1.0$). Phoenix deviates from the other cases in its d_1 value, which indicates a weaker area dependence of the mean heat transfer component than in the other locations. This may be the result of the small Phoenix mean temperature difference. As this difference decreases, the mean influence of area should diminish substantially because core heat transfer rapidly approaches zero. Because the model parameters are little affected by changes in climate, it is reasonable to assume that (1) the mean and amplitude of the ground surface temperature are a suitable representation of climatic influences on floor heat loss and (2) Equation 14 is applicable to arbitrary locations for given soil properties and boundary conditions.

TABLE 4
Daily Averaged Heat Loss Model Coefficients for Climatic Variation Tests

Location	c_1 SI	c_1 English	d_1	c_2 SI	c_2 English	d_2
Medford	0.978	0.172	−0.747	0.713	0.126	−0.999
Minneapolis	0.997	0.176	−0.735	0.759	0.134	−0.999
Philadelphia	1.007	0.177	−0.750	0.765	0.135	−0.995
Phoenix	1.041	0.183	−0.901	0.769	0.135	−0.997

Note: Coefficients d_1 and d_2 are dimensionless and have the same values in both the English and SI systems. The units of c_1 and c_2 depend on the values of d_1 and d_2, respectively. The expressions $c_1\ (A/P)^{d_1}$ and $c_2\ (A/P)^{d_2}$ are conductances with SI units (W/m^2·K) and English units (Btu/h·ft^2·°F).

Boundary Conditions

Three types of boundary effects were considered:

- Potential evapotranspiration vs. no latent loss at the ground
- Lower boundary depth
- Lower boundary condition type (zero flux vs. fixed temperature).

The most significant of these, by far, was the ground surface condition. Lower boundary condition effects, within the range of conditions considered, were not very significant. This is fortunate from the standpoint of simplified model construction, as it suggests that fairly arbitrary choices of lower boundary condition are acceptable. Ground surface conditions, on the other hand, are very important and must be treated carefully.

To determine the effect of ground surface latent heat transfer, seven cases with no evapotranspiration (four in Minneapolis and one each in Medford, Philadelphia, and Phoenix) were compared to an otherwise identical set with potential evapotranspiration. Table 5 summarizes ground surface temperature statistics for the zero evapotranspiration cases. Mean temperatures are higher than average air temperature by 1.8° to 7.2°F (1° to 4°C) and surface temperature amplitude is greater than air temperature amplitude. In contrast, the corresponding potential evapotranspiration cases summarized in Table 3 have mean ground temperatures several degrees lower than air temperature and ground temperature amplitudes that are generally smaller than those of air. The difference in mean ground temperature between the potential and zero evapotranspiration cases varied from a minimum value of 6.3°F (3.5°C) for Minneapolis to a maximum of 18°F (10°C) for Phoenix. The respective differences in surface temperature amplitude for these two cases were 5.0°F (2.8°C) and 8.1°F (4.5°C).

Daily flux models for the two sets of Minneapolis results were compared. Annual minimum (Q_{min}), maximum (Q_{max}), and average (Q_{avg}) heat transfer rate data for 39 ft by 39 ft (12 m by 12 m) uninsulated slabs under potential and zero evapotranspiration conditions are given in Table 6. Due to upward-shifted mean ground temperature, the average loss when evapotranspiration is suppressed is considerably lower than in the analogous potential evapotranspiration case. Seasonal differences in heat loss are analogous to the variations in surface temperatures considered above. Maximum heat loss values, which occur during the winter, differ by less than minimum values, which occur during the summer. The differences in Q_{max} between the potential evapotranspiration and no evapotranspiration cases are −4.2%, −10.6%, −10.8%, and −47.1%, respectively, for Minneapolis, Medford, Philadelphia, and Phoenix. Because evapotranspiration potential increases with air temperature, changes in both mean and extreme heat loss increase with mean air temperature.

In general, the lower soil boundary condition exerted a small influence on ground surface temperature. Because conditions in the soil near the ground surface are relatively insensitive to deep ground conditions, slab-on-grade heat loss does not depend strongly on deep ground conditions unless one or more

TABLE 6
Annual Heat Loss from an Uninsulated 39 × 39 ft (12 × 12 m) Slab in Four Climates

Location	Q_{min} [W]	Q_{min} [Btu/h]	Q_{max} [W]	Q_{max} [Btu/h]	Q_{avg} [W]	Q_{avg} [Btu/h]
Medford	388.8	1326.6	1224.0	4176.5	825.7	2817.4
Minneapolis	403.2	1357.8	1915.2	6534.9	1088.9	3715.5
Philadelphia	302.4	1031.8	1339.2	4569.5	784.3	2676.1
Phoenix	−14.4	−49.1	734.4	2505.9	336.3	1147.5

a) Potential evapotranspiration

Location	Q_{min} [W]	Q_{min} [Btu/h]	Q_{max} [W]	Q_{max} [Btu/h]	Q_{avg} [W]	Q_{avg} [Btu/h]
Medford	−57.6	−196.5	1094.4	3734.2	558.6	1906.0
Minneapolis	86.4	294.8	1843.2	6289.3	885.0	3019.7
Philadelphia	−14.4	−49.1	1195.2	4078.2	561.5	1915.9
Phoenix	−806.4	−2751.2	388.8	1326.6	−236.8	808.0

b) No evapotranspiration

TABLE 5
Mean, Amplitude, and Phase Shift of Daily Averaged Air and Ground Surface Temperatures (Zero Evapotranspiration)

Location	$T_{air, mean}$ [°C]	$T_{air, mean}$ [°F]	ΔT_{air} [°C]	ΔT_{air} [°F]	ζ_{air} [Days]	$T_{g, mean}$ [°C]	$T_{g, mean}$ [°F]	ΔT_g [°C]	ΔT_g [°F]	ζ_g [Days]
Medford	11.4	52.5	−9.7	−17.5	69.2	13.2	55.8	−13.8	−24.8	74.6
Minneapolis	7.2	45.0	−17.0	−30.6	72.3	8.3	46.9	−19.1	−34.3	74.6
Philadelphia	12.4	54.3	−12.6	−22.7	68.9	13.4	56.1	−14.1	−25.4	71.1
Phoenix	21.9	71.4	−11.7	−21.1	72.2	25.9	78.6	−12.8	−23.0	75.2

TABLE 7
Heat Loss Data for Floors in Minneapolis with Fixed-Temperature and Zero-Flux Deep Ground Boundary Conditions

	Fixed Lower Boundary Temperature			Zero Flux Lower Boundary		
Dimensions [m]	Q_{min} [W]	Q_{max} [W]	Q_{avg} [W]	Q_{min} [W]	Q_{max} [W]	Q_{avg} [W]
6 × 24	432.0	2347.2	1303.3	460.8	2361.6	1320.5
12 × 12	403.2	1915.2	1088.9	417.6	1929.6	1107.3
18 × 112	3830.4	12297.6	7653.6	4032.0	12499.2	7923.2
45 × 45	3240.0	9112.5	5895.8	3442.5	9315.0	6162.7

a) SI units

	Fixed Lower Boundary Temperature			Zero Flux Lower Boundary		
Dimensions [ft]	Q_{min} [Btu/h]	Q_{max} [Btu/h]	Q_{avg} [Btu/h]	Q_{min} [Btu/h]	Q_{max} [Btu/h]	Q_{avg} [Btu/h]
19.7 × 78.7	1474.0	8009.0	4447.0	1572.3	8058.1	4505.7
39.4 × 39.4	1375.8	6534.9	3715.5	1424.9	6584.0	3778.3
59.0 × 367.5	13069.9	12297.6	41961.2	13757.8	42649.0	27035.1
147.6 × 147.6	11055.3	31093.1	20117.3	11746.2	31784.1	21028.0

b) English units

Note: Lower boundary temperature in the fixed temperature case is equal to the mean dry-bulb temperature, 7.2°C. Mean ground surface temperature is 4.8°C in both cases (potential evapotranspiration boundary).

TABLE 8
Change in Floor Heat Loss Due to Substitution of Zero-Flux Lower Boundary for Fixed-Temperature Lower Boundary (Data from Table 7)

Dimensions		A/P		ΔQ_{min}			ΔQ_{max}			ΔQ_{avg}		
[m]	[ft]	[m]	[ft]	[W]	[Btu/h]	%	[W]	[Btu/h]	%	[W]	[Btu/h]	%
6 × 24	19.7 × 78.7	2.4	7.9	28.8	98.3	6.7	14.4	49.1	0.61	17.2	58.7	1.3
12 × 12	39.4 × 39.4	3.0	9.8	14.4	49.1	3.6	14.4	49.1	0.75	18.4	62.8	1.8
18 × 112	59.0 × 367.5	7.75	25.4	201.6	687.9	5.3	201.6	687.9	1.6	269.6	920.0	3.5
45 × 45	147.6 × 147.6	11.25	36.9	202.5	691.0	6.3	202.5	691.0	2.2	266.9	919.9	4.5

TABLE 9
Effect of Lower Boundary Depth on Mean Heat Loss for Uninsulated Floors in Medford, OR (Potential Evapotranspiration)

Dimensions		A/P		$Q_{49\ ft\ (15\ m)}$		$Q_{33\ ft\ (10\ m)}$		
[m]	[ft]	[m]	[ft]	[W]	[Btu/h]	[W]	[Btu/h]	Δ%
12 × 12	39.4 × 39.4	3.0	9.8	823.0	2808.2	825.7	2817.4	0.33
15 × 60	49.2 × 196.9	6.0	19.7	3062.7	10450.4	3142.6	10723.0	2.61
30 × 30	98.4 × 98.4	7.5	24.6	2583.9	8816.6	2703.0	9223.0	4.61
23 × 88	75.5 × 288.7	9.1	29.9	5076.1	17320.4	5386.1	18378.1	6.11
45 × 45	147.6 × 147.6	11.25	36.9	4367.9	14934.6	4760.1	16241.8	8.98
30 × 120	98.4 × 393.7	12.0	39.4	7319.3	24974.5	8001.9	27303.6	9.32
60 × 60	196.9 × 196.9	15.0	49.2	6467.2	22067.0	7281.9	24846.9	12.60

special conditions exist. These include a high water table, a sharp change in soil properties, or conditions that maintain a large mean temperature difference between the ground surface and the deep ground (for example, irrigation in an arid climate). In these cases, the ground temperature distribution may lose its close resemblance to the semi-infinite medium distribution (which is driven by the surface boundary condition).

The difference between heat loss with fixed-temperature and zero flux conditions was evaluated by comparing several Minneapolis runs that were identical except for the lower boundary condition. In the fixed-temperature cases, deep ground temperature was set equal to the mean air temperature, 45°F (7.2°C). Because the mean ground surface temperature was 40.6°F (4.8°C), a mean temperature difference of 4.3°F (2.4°C) existed between the upper and lower boundaries in the fixed-temperature case. Consistent with the semi-infinite medium solution, the lower boundary temperature in the zero flux case assumed a value equal to the surface mean.

The lower deep ground temperature of the zero flux cases presented a stronger heat sink to the underside of the floor. Consequently, heat loss in zero flux cases was greater than in the corresponding fixed-temperature cases (Table 7). Table 8 compares results from Table 7 in terms of both the heat transfer rate difference and the percentage difference with respect to the fixed lower boundary case. Absolute differences in mean heat loss are larger than differences in extreme values. Percentage differences in mean heat loss are larger for larger floors. This is consistent with the observation (Bahnfleth 1989) that a floor disturbs the ground temperature to depths comparable to its characteristic length. Note that the percentage change in mean heat loss for the cases shown in Table 8 increases monotonically with A/P.

The effect of domain depth was investigated by comparing results for z_{max} = 33 ft (10 m) with the z_{max} = 49 ft (15 m) Medford results discussed previously. In all cases, the annual average heat loss was greater for z_{max} = 33 ft, with the magnitude of the difference depending on A/P. Table 9 compares the average annual heat loss for several floors as a function of z_{max}. For the smallest area, 1550 ft^2 (144 m^2), there is no appreciable difference between the two cases. As area (and, more particularly, A/P) increases, differences become larger. It appears that the strength of interaction between a floor and the lower boundary is related to the comparative magnitudes of A/P and the lower boundary depth. As size increases, the boundaries

TABLE 10
Heat Loss Data for Varied Soil Thermal Property Cases
(Philadelphia, PA Weather and Potential Evapotranspiration)

Dimensions		Properties	Q_{min}		Q_{max}		Q_{avg}	
[m]	[ft]		[W]	[Btu/h]	[W]	[Btu/h]	[W]	[Btu/h]
12 × 12	39.4 × 39.4	Base	302.4	1031.8	1339.2	4569.5	784.3	2676.1
"	"	A	547.2	1867.1	1987.2	6780.6	1226.6	4185.3
"	"	B	345.6	1179.2	1267.2	4323.9	781.8	2667.6
"	"	C	187.2	638.8	835.2	2849.8	482.9	1647.7
"	"	D	504.0	1719.7	2030.4	6928.0	1227.9	4189.7
6 × 24	19.7 × 78.7	Base	331.2	1130.1	1641.6	5601.4	941.8	3213.6
"	"	A	576.0	1965.4	2419.2	8254.7	1454.0	4961.3
"	"	B	388.8	1326.6	1555.2	5306.6	937.2	3197.9
"	"	C	201.6	687.9	1022.4	3488.6	584.0	1992.7
"	"	D	532.8	1818.0	2476.8	8451.2	1454.9	4964.3
45 × 45	147.6 × 147.6	Base	2227.5	7600.5	6277.5	21419.7	4152.6	14169.3
"	"	A	4252.5	14510.1	9922.5	33857.0	7003.7	23897.6
"	"	B	2632.5	8982.5	6075.0	20728.8	4207.6	14356.9
"	"	C	1215.0	4145.7	3847.5	13128.2	2450.5	8361.5
"	"	D	4252.5	14510.1	10125.0	34547.9	6997.6	23876.8
18 × 112	59.0 × 367.5	Base	2822.4	9630.4	8467.2	28891.3	5443.2	18573.0
"	"	A	5241.6	17885.1	13305.6	45400.6	9018.9	30773.8
"	"	B	3024.0	10318.3	8064.0	27515.5	5463.6	18642.6
"	"	C	1612.8	5503.1	5040.0	17197.2	3232.2	11028.7
"	"	D	5040.0	17197.2	13507.2	46088.5	9026.7	30800.4

of the building-induced disturbance expand and the building's heat loss becomes sensitive to changes in conditions at greater distances.

This investigation of lower boundary condition type effects indicates that they are probably not significant when the domain is deep and the lower boundary temperature is close to the annual mean surface temperature. If the lower boundary temperature exceeds mean surface temperature, the substitution of a zero flux condition will lead to greater heat loss and area dependence. If the opposite is true, decreased heat loss and area dependence result. Such effects could be exaggerated or suppressed as a result of other influences, for example, soil property values.

Soil Properties

The effect of four different combinations of soil thermal conductivity and diffusivity (k and α) on heat transfer from four uninsulated floors with different values of A/P in Philadelphia weather was studied. The standard boundary conditions used in other parametric studies also applied to this group, i.e., potential evapotranspiration at the ground surface and fixed temperature in the deep ground. The previously considered Philadelphia runs with "base case" properties provided a fifth set of results. The five property groups (listed in Table 2) were designed to permit isolation of thermal conductivity and thermal diffusivity effects. Conductivity varied by a factor of two in either direction from the base value. Diffusivity ranged from a factor of 2 smaller to a factor of 1.3 larger than the base value.

The daily averaged heat loss results summarized in Table 10 show that conductivity and diffusivity have very different effects on heat loss. Thermal conductivity strongly influences the mean heat transfer component. For example, Q_{avg} for the 148 ft by 148 ft (45 m by 45 m) floor varies from less than 8530 Btu/h (2500 W) to more than 23,885 Btu/h (7000 W). Thus, a 400% increase in thermal conductivity produces a nearly 300% increase in mean heat loss. Diffusivity, on the other hand, has a negligible effect on the mean transfer. The difference in mean heat transfer rate between the base and set B property cases for a 59 ft by 367 ft (18 m by 112 m) floor is only 68 Btu/h·ft² (20 W), less than 0.5% of the mean.

Diffusivity has a greater, but still small, influence on the annual range of heat transfer. For example, the difference between Q_{max} and Q_{min} for the property set B, 39 ft by 39 ft (12 m by 12 m) floor is 2121 Btu/h·ft² (921.6 W), while the annual range with base case properties is 3538 Btu/h·ft² (1036.8 W). Thus, an increase of approximately 100% in thermal diffusivity causes the annual range to widen by only 12.5%. Thermal conductivity has a larger effect on the annual range. If the base case thermal diffusivity is fixed and conductivity is doubled (as in case A), the annual heat loss range increases by 56.3%. The greater influence of conductivity suggests that, on a daily averaged scale, the floor's heat transfer is quasi-steady.

Soil thermal properties also affect the area dependence of floor heat transfer rates. The nature of such effects can be shown through the use of the A/P model developed previously. Figure 5 shows plots of K_1 and K_2 coefficients as a function of A/P for the cases summarized in Table 10. Curves through the plotted values of K_1 and K_2 are instances of Equations 14a and 14b, respectively. (Each curve is labeled to show its SI values of c_1 and d_1 or c_2 and d_2.) The observations made above concerning conductivity and diffusivity dependence are readily apparent in these plots.

K_1 values for cases with the same conductivity but different diffusivities essentially coincide, indicating the absence of a diffusivity effect on mean heat loss (Figure 5a). Fractional changes in K_1 are comparable to, but smaller than, corresponding fractional changes in thermal conductivity. The area dependence of K_1 increases with increasing conductivity. This is shown by the decrease in magnitude of d_1 (the exponent of A/P) as k becomes larger. Heat loss from the low-gradient core region of the floor grows more rapidly than edge loss when conductivity increases, thus weighting total area more heavily. Table 11 illustrates the increasing contribution of the core with increasing conductivity. For both 39 ft by 39 ft (12 m by 12 m) and 148 ft by 148 ft (45 m by 45 m) cases, the floor center heat transfer varies in direct proportion to conductivity. Maximum edge flux values, however, change by 20% or less in response to twofold increases and decreases in k. Thus, more of the difference in average floor heat loss results from changes in core loss.

The fractional change in K_1 due to a given increase in conductivity grows with increasing A/P. The percentage change in mean heat transfer resulting from an increase of k from 0.6 to 1.3 Btu/h·ft·°F (1 W/m·K to 2 W/m·K) with α fixed (base case vs. set A properties) for a 20 ft by 79 ft (6 m by 24 m) slab (A/P = 7.9 ft [2.4 m]) is 54.4%. When A/P increases to 9.8 ft (3 m), the fractional change increases to 56.4%. Floors with A/P values of 25.4 ft (7.75 m) and 36.9 ft (11.25 m) experience increases of 65.7% and 68.7%, respectively.

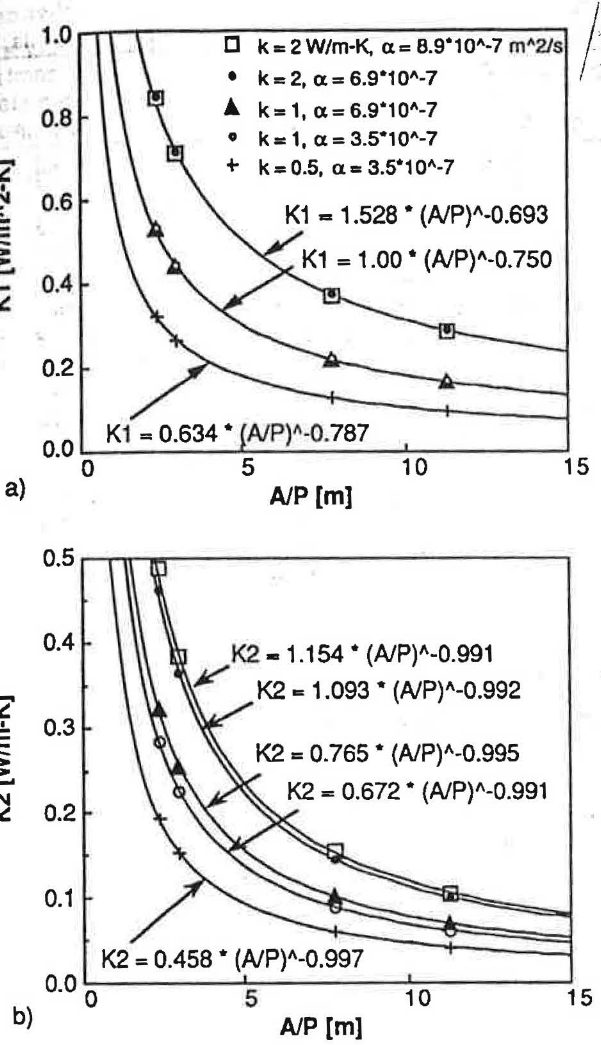

Figure 5 *Influence of soil properties on area dependence of floor heat loss, a) mean b) amplitude*

Figure 5b, which shows K_2 as a function of A/P, demonstrates the subordinate role that thermal diffusivity plays to conductivity in the determination of K_2, and consequently, the periodic component of floor heat transfer. Cases with like conductivity fall much closer together than those with like diffusivity but different conductivities. As in other examples considered above, K_2 for these varied property groups is proportional to $(A/P)^{-1}$, indicating linear dependence on perimeter of the periodic heat transfer component.

Insulation

A limited examination of insulation was conducted to determine its effect on the area dependence of floor heat transfer rates. Restrictions on the range of parameters considered included:

- Minneapolis weather only
- Insulation limited to 1 in. or 2 in. (0.025 or 0.05 m) thicknesses of expanded extruded polystyrene board (k = 0.017 Btu/h·ft·°F [0.029 W/m·K])
- Two configurations: (1) coverage of edge + 3.28 ft (1 m) under the slab perimeter and (2) edge + entire bottom surface of slab.

The effect of insulation was represented in the finite difference model by added surface resistance on the exterior of floor cells rather than by additional finite difference cells. This permitted the use of the same grids employed in uninsulated cases.

Table 12 compares heating, cooling, and net energy consumption and peak heating load for two floor sizes and the five insulation configurations considered in this study. Heating and cooling loads were considered to be energy exchanges occurring when the outdoor dry-bulb was, respectively, below and above the indoor dry-bulb temperature of 72°F (22°C). The 1979 Minneapolis TMY file has 314 days of heating and 51 days of cooling by this definition. When insulation is added, the peak heating load is reduced by a much larger amount than the peak cooling contribution of the floor is decreased. For both floor sizes, the first increment of insulation produces the greatest benefit. The addition of more material increases energy savings, but marginal gains become increasingly smaller. The reduction in contribution to peak heating load due to insulation is greater than the reduction in heating energy. Energy savings and load reductions are smaller in the 148 ft by 148 ft (45 m by 45 m) case than in the 39 ft by 39 ft (12 m by 12 m) case by 7% to 10%. Because of its larger associated core soil mass, the 148 ft by 148 ft slab is less affected than the smaller floor by changes at the slab perimeter. This dif-ference in fractional savings might make it economically feasible to insulate the smaller structure more heavily than the larger.

It was found that floors with either total or partial insulation can be described by the simple A/P-based model developed for uninsulated floors. However, insulation has a significant effect on the model coefficients. Model coefficients for an uninsulated slab and two perimeter insulated cases appear in Table 13. The area dependence of both the steady and periodic heat transfer components increases as insulation is added. As in previous cases, the steady-state component is more strongly affected: the maximum fractional change in d_1 is more than three times greater than the corresponding change in d_2. The periodic heat flux component remains essentially proportional to $(A/P)^{-1}$. These results show that a more uniform floor tem-

TABLE 11
Influence of Thermal Conductivity on January 21 Floor Center and Edge Heat Loss Values for Two Uninsulated Slabs in Philadelphia, PA

	Thermal Conductivity		Center Heat Flux			Max Edge Heat Flux			Floor Average Heat Flux		
	[W/m·K]	[Btu/h·ft·°F]	[W/m²]	[Btu/h·ft²]	Δ%	[W/m²]	[Btu/h·ft²]	Δ%	[W/m²]	[Btu/h·ft²]	Δ%
Base case	1.0	0.6	1.8	0.6	—	69.0	21.9	—	7.9	2.5	—
Set A	2.0	1.2	3.5	1.1	94.4	78.9	25.0	14.4	12.2	3.9	54.4
Set C	0.5	0.7	0.9	0.3	-50.0	55.5	17.6	-19.6	4.7	1.5	-40.5

a) 39 × 39 ft (12 × 12 m)

	Thermal Conductivity		Center Heat Flux			Max Edge Heat Flux			Floor Average Heat Flux		
	[W/m·K]	[Btu/h·ft·°F]	[W/m²]	[Btu/h·ft²]	Δ%	[W/m²]	[Btu/h·ft²]	Δ%	[W/m²]	[Btu/h·ft²]	Δ%
Base case	1.0	0.6	0.7	0.2	—	68.9	21.8	—	2.7	0.9	—
Set A	2.0	1.2	1.4	0.4	100.0	78.8	25.0	14.4	4.5	1.4	66.7
Set C	0.5	0.7	0.4	0.1	-42.9	55.5	17.6	-19.5	1.6	0.5	-40.7

b) 148 × 148 ft (45 × 45 m)

TABLE 12
Influence of Insulation Treatment on Heating and Cooling Energy Requirements for Two Slab Floors in Minneapolis, MN

			Insulation Thickness and Configuration			
		None	1″/3.3 ft (0.025 m/1m)	1″/full (0.025 m/full)	2″/3.3 ft (0.05 m/1m)	2″/full (0.05 m/full)
Heating Energy	[GJ]	31.9	22.5	19.9	19.4	15.5
	[MMBtu]	30.2	21.3	18.9	18.4	14.7
Savings	[GJ]	0	9.5	12.0	12.5	16.5
	[MMBtu]	0	9.0	11.4	11.8	15.6
	[%]	0	29.6	37.6	39.2	51.6
Max Heating Load	[W]	1915.2	1238.4	1080.0	1051.2	806.4
	[Btu/h]	6534.9	4225.6	3685.1	3586.8	2751.6
Reduction	[W]	0	676.8	835.2	864.0	1108.8
	[Btu/h]	0	2309.3	2849.8	2948.1	3783.4
	[%]	0	35.4	43.6	45.1	57.9
Cooling Energy	[GJ]	−2.4	−2.1	−1.9	−1.8	−1.6
	[MMBtu]	−2.3	−2.0	−1.8	−1.7	−1.5
Savings	[GJ]	0	−0.3	−0.5	−0.5	−0.7
	[MMBtu]	0	−0.3	−0.5	−0.5	−0.7
	[%]	0	−13.7	−18.8	−22.4	−31.3
Net Energy Savings	[GJ]	0	9.1	11.6	12.0	15.8
	[MMBtu]	0	8.6	11.0	11.4	15.0
	[%]	0	30.9	39.1	40.6	53.1

a) 39 × 39 ft (12 × 12 m)

			Insulation Thickness and Configuration			
		None	1″/3.3 ft (0.025 m/1m)	1″/full (0.025 m/full)	2″/3.3 ft (0.05 m/1m)	2″/full (0.05 m/full)
Heating Energy	[GJ]	169.1	132.2	122.6	119.2	104.5
	[MMBtu]	160.3	125.3	116.2	113.0	99.0
Savings	[GJ]	0	36.9	46.5	49.9	64.6
	[MMBtu]	0	35.0	44.1	47.3	61.2
	[%]	0	21.8	27.5	29.5	38.2
Max Heating Load	[W]	9112.5	6480.0	5872.5	5670.0	4657.5
	[Btu/h]	31093.1	22110.7	20037.8	19346.8	15892.1
Reduction	[W]	0	2632.5	3240.0	3442.5	4455.0
	[Btu/h]	0	8982.4	11055.3	11746.3	15201.1
	[%]	0	28.9	35.6	37.8	48.9
Cooling Energy	[GJ]	−16.8	−15.3	−14.9	−14.2	−13.7
	[MMBtu]	−15.9	−14.5	−14.1	−13.5	−13.0
Savings	[GJ]	0	−1.5	−1.9	−2.6	−3.2
	[MMBtu]	0	−1.4	−1.8	−2.5	−3.0
	[%]	0	−8.9	−11.5	−15.7	−18.8
Net Energy Savings	[GJ]	0	35.4	44.6	47.3	61.4
	[MMBtu]	0	33.6	42.3	44.8	58.2
	[%]	0	23.3	29.3	31.0	40.3

b) 148 × 148 ft (45 × 45 m)

perature distribution enhances the shape dependence of heat loss. By decreasing heat loss at the floor perimeter, insulation raises the contribution of interior area to total heat loss. Thus, while the floor temperature distribution is "less three dimensional" in the sense of being more uniform over the floor, the total heat transfer rate is *more* shape dependent and three dimensional. Consequently, the predictions of perimeter heat loss factor methods may be especially misleading for well-insulated floors.

TABLE 13
Comparison of Heat Loss Model Coefficients for Three Insulation Thicknesses in Minneapolis, MN (Insulation on Slab Edge and Outer 3.28 ft [1 m] of Slab Bottom)

Insulation	c_1		d_1	c_2		d_2
Thickness	SI	English		SI	English	
Uninsulated	0.997	0.176	−0.735	0.759	0.134	−0.999
1″ (0.025 m)	0.603	0.106	−0.623	0.408	0.072	−0.953
2″ (0.050 m)	0.475	0.084	−0.570	0.308	0.054	−0.921

Note: Coefficients d_1 and d_2 are dimensionless and have the same values in both the English and SI systems. The units of c_1 and c_2 depend on the value of d_1 and d_2, respectively. The expressions $c_1\ (A/P)^{d_1}$ and $c_2\ (A/P)^{d_2}$ are conductances with SI units (W/m²·K) and English units (Btu/h·ft²·°F).

Shadowing

The influence of building shadowing of the ground on floor heat transfer rates is not mentioned in the literature. It might be expected that this effect could contribute to a significant increase in heat loss. This study provides some evidence that this is the case. Because full three-dimensional computations with a refined surface grid are quite time consuming, only a 1550 m^2 (144 m^2) plan was considered. A flat-roofed building with a height of 13 ft (4 m) was assumed for the purpose of calculating shadow length in all cases. Three uninsulated rectangular cases were modeled, all with Medford weather and potential evapotranspiration: square, 20 ft by 79 ft (6 m by 24 m) long north-south and 20 ft by 79 ft long east-west. Two additional 39 ft by 39 ft (12 m by 12 m) square runs, one with no evapotranspiration and one with Phoenix weather and potential evapotranspiration, were performed to obtain information about effects of surface boundary and climate.

Heat loss with shading was greater than without shade throughout the year but particularly during the summer. Averaged over the entire year, the heat transfer from a 39 ft by 39 ft (12 m by 12 m) shaded case in Medford with potential evapotranspiration was 6.5% greater than from the same slab without shade (a difference of 181 Btu/h [53 W]). The greatest monthly average difference, 17.2% (287 Btu/h [84 W]), occurred during July. Figure 6 compares these two cases on a monthly averaged basis for visual clarity. Differences between shade and no shade heat loss for the two 20 ft by 79 ft (6 m by 24 m) runs were only slightly larger than those for the square case (7.3% annually and 19.6% maximum). Orientation did not have a significant effect on heat loss for the cases considered; values for north-south and east-west major axis orientations were virtually identical. In the absence of evapotranspiration, the effect of shade was more pronounced. Annual average heat loss increased by 14% (266 Btu/h [78 W]) for the 39 ft by 39 ft (12 m by 12 m) slab in Medford, while the maximum difference on a monthly average basis grew to 433 Btu/h (127 W).

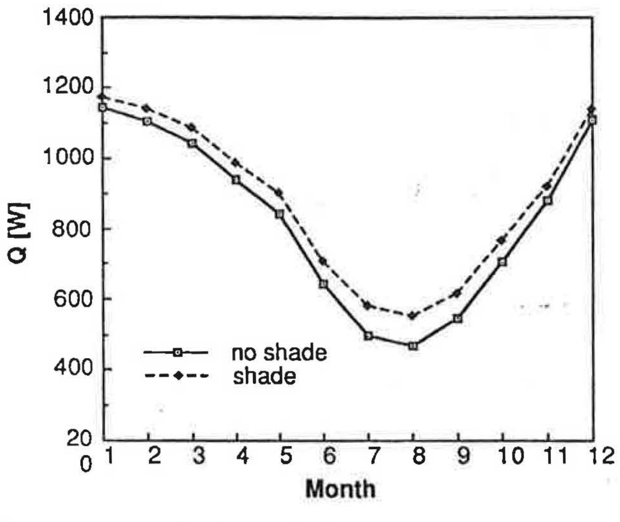

Figure 6 *Effect of building shadow on monthly averaged heat loss from a 39 ft by 39 ft (12 m by 12 m) uninsulated slab in Medford, OR*

The effect of shade in the warmer, sunnier Phoenix climate was both qualitatively and quantitatively different from that in Medford. The mean difference in heat loss resulting from shade was larger, 314 Btu/h [92 W]/27.4% greater than the no-shade case. Also, seasonal differences between shade and no-shade cases were much smaller than in Medford. This difference probably was due to differences in sky conditions. The 1979 Medford TMY weather file contains a long period of time during the winter months when beam solar radiation is quite small due to overcast skies. The Phoenix TMY set, on the other hand, has generally clear skies throughout the year.

CONCLUSIONS

The results of this study point to a number of significant conclusions regarding the estimation of heat transfer from slab-on-grade floors:

- The underlying assumption that heat loss is proportional to perimeter length, the cornerstone of the commonly used F_2 method for floor heating load estimation, is not correct. Consequently, the F_2 method may be in error by 50% or more due to erroneous scaling.
- Floor heat transfer rates are dependent on both shape and size. However, the effects of shape and size correlate in a simple manner with the floor characteristic length, A/P.
- Mean and fluctuating components of floor heat transfer obey different scaling relationships and should be treated separately in simple models. The fluctuating component of heat flux is proportional to $(A/P)^{-1}$, the dependence assumed in the F_2 method. The geometric dependence of the mean component is well approximated by a more general expression of the form $(A/P)^d$. In this study, values of d ranged from -0.57 to -0.90, depending on soil properties, insulation treatment, and, to a lesser extent, climate. It is of particular note that heavily insulated floors showed the greatest deviation from the behavior implicit in the F_2 method.
- An easily applied manual method suitable for both load and energy calculations could be based on the analysis presented in this work. The use of A/P as a scaling parameter could have wider use in computer-based models. For example, heat transfer from any plan shape could be modeled by an equivalent two-dimensional infinite strip having the same A/P. (A multiple-input transfer function model based on the results of this study [Amber 1989] will be the subject of a later paper.)
- Parametric studies of thermal property and boundary condition effects showed that thermal conductivity of the soil and ground surface conditions exert a strong influence on floor heat transfer rates, while thermal diffusivity, far-field boundaries, and deep ground conditions (in general) do not. Consequently, models to be used for accurate estimates should take both soil conductivity and ground surface conditions into account.
- The effect of building ground shadow, a local surface boundary condition effect, has the potential to produce changes of more than 20% in floor heat transfer rate under conditions of high evapotranspiration potential. In other climates, however, this effect could be so small as to be negligible. Designers should be aware of the potential for influencing earth-coupled heat transfer through a combination of shading and ground cover control.

Future work in this area could profitably focus on the experimental verification of these numerical results, extension of this analysis to basements, and the development of improved manual design procedures to replace those currently published by ASHRAE.

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DISCUSSION

Richard C. Alexander, Professor, Montana Tech, Butte: Please elaborate on the boundary conditions used in your model, especially the deep boundary conditions on soil temperature. My students have done similar calculations for a full basement and found large discrepancies with the ASHRAE basement method of estimating heat losses (1989 *Fundamentals*, p. 25.6). One reason is a discrepancy in the boundary values assumed for deep soil temperature. Measurements indicate that the deep soil is warmed under the building. A kind of heat island develops, which reduces heat losses through the floor. In this case, the assumption of a mean annual air temperature or water table temperature as a deep-soil boundary condition under the building is probably incorrect.

W.P. Bahnfleth: Our boundary condition models are discussed in detail in the references to our paper. Our general approach was to be as fundamental as we could and to consider alternatives whenever possible.

We compared zero flux and specified temperature lower boundary conditions at two different depths. We found that differences in heat transfer rates due to lower boundary condition changes depended on the relationship between the size (A/P ratio) of the floor and the depth at which the condition was applied. As might be expected, a shallow lower boundary was found to have a stronger influence than a deep one. Likewise, for a given boundary depth, heat transfer from a floor with a large A/P value was more strongly affected than that from one with a small value. This is due to a correspondence between the A/P ratio of a floor and the depth of penetration of the temperature disturbance ("heat island") that it causes.

This disturbance approaches a limiting size and shape as the depth of the lower boundary increases and the domain becomes a semi-infinite medium for practical purposes. A fixed-temperature surface distorts the disturbance and changes heat transfer relative to the asymptotic case. Real boundary conditions probably lie somewhere between these extremes. The problem in application is that subsurface conditions at a depth of 10 to 15 m are seldom known to the analyst. Even if the building creates a temperature disturbance beneath the surface of the water table, however, a specified temperature condition can be used as long as it is applied at a sufficient depth.

We can only agree with your assessment that the ASHRAE basement method has questionable features. We undertook this work because of our feeling that most existing methods are seriously flawed. The limited number of parameters that are explicitly variable in most models is one of the most serious defects. Lower boundary conditions are among this group but are of less significance than ground surface conditions, which are also implicit in most models.