Technical Note

Summary   The term 'mean radiant temperature' is used in Section A1 of the *CIBSE Guide* in connection with thermal comfort, and in Section A5 in connection with the thermal response of an enclosure. With one amendment, the argument of Section A1 is correct but those of Section A5 are flawed and incomplete. This Note draws attention to the need for three separate measures of radiant temperature in a room: the local value T_{rp} (potentially observable), its space-averaged value (T_{rv}), and the radiant star temperature T_{r*}. ('Mean radiant temperature' might refer to any of these.) A study of room radiant exchange requires explicit consideration of the role of surface emissivity, and of the effect of long-wave energy flow from an internal heat source. It is pointed out that the conductances associated with this exchange (and also the convective exchange) are 'large', while the radiant and convective conductances associated with comfort temperature are 'small'.

Mean radiant temperature in the *CIBSE Guide*

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1   Introduction

Section A1 of Reference 1 deals with environmental criteria for design. The early subsections are devoted to matters of thermal comfort. On p A1–4, resultant temperature, useful for specification of thermal comfort, is defined as the temperature recorded by a thermometer at the centre of a blackened globe 100 mm in diameter. It is given by

$$t_{res} = \frac{t_r + t_{ai}(10\,v)^{1/2}}{1 + (10\,v)^{1/2}} \qquad (A1.1)$$

where t_{ai} is the inside air temperature, t_r is the mean radiant temperature and t_{res} is the dry resultant temperature. v is the air velocity (m s^{-1}); in still air conditions with $v = 0.1$ m s^{-1} or so

$$t_{res} = \tfrac{1}{2}t_r + \tfrac{1}{2}t_{ai} \qquad (A1.2)$$

t_{res} is an observable temperature in a small locality defined by the spherical volume of 100 mm diameter at the particular position it happens to occupy in the room. It follows that both t_{ai} and t_r must here denote local measurable temperatures.

t_{res} is an appropriate measure for thermal comfort: values of t_{res} are relevant for areas in a room that is occupied, and the high values it might assume, say, in front of a large panel radiator or near the ceiling would not be taken to indicate that the room as a whole is too hot for occupation.

t_r is seen as a local quantity in section A1 but its definition is defective in one respect. In section A5 the term 'radiant temperature' is beset by ambiguities of a more fundamental kind. These shortcomings are the subject of this paper.

2   The *Guide* definition of mean radiant temperature

The *Guide*[1] p A1–19 defines mean radiant temperature in the absence of short-wave radiation as

$$T_r^4 = \frac{1}{4\pi} \int_0^{4\pi} T_s^4 \, d\psi \qquad (A1.6)$$

where T_r is the mean radiant temperature, T_s is the absolute surface temperature and ψ is solid angle.

The *Guide* does not formally set up the situation in which T_r is to be determined, but by default, it would appear to refer to a rectangular room whose surfaces are at known temperatures, say $T_1, T_2 \ldots$ and the radiant temperature is to be determined at some given location within the room.

The expression for T_r omits any consideration of surface emissivity and this is discussed in section 3 of this Note. Furthermore, as expressed in equation A1.6, T_r is a *point function*—a quantity whose value can vary from point to point in a room; the radiant temperature to be used in Section A5[1] must be a *space-averaged* quantity in order that it should be combined with the space-averaged air temperature (section 4). Finally, radiant temperature as understood in Section A5[1] does not include the effect of any internal source of long-wave radiation (section 5).

From now on, the radiant temperature at a point will be expressed as T_{rp}; the first subscript denotes 'radiant' as previously and the second denotes that the quantity is that at a *point*.

3   Surface emissivity

Equation A1.6 evaluates T_{rp} without regard to the emissivity of any particular surface. To take an extreme case to illustrate the point, suppose that surface 1 of a rectangular enclosure had a long-wave emissivity ε_1 of zero. Its radiant output into the adjoining space in virtue of its own temperature is accordingly zero and so T_1 will not contribute anything at all to T_{rp}. Surface 1 however will *reflect* all long-wave radiation falling upon it (it absorbs none) and so the effect of integrating over the solid angle corresponding to surface 1 must be to pick up the effect of the temperature of all surfaces in the enclosure *other than that of surface 1*.

Equation A1.6 is thus insufficient as it stands. It has to be reformulated to take account of surface emissivities. This can be done in terms of fourth powers of temperatures but it is rather easier to present the argument using the linearised

difference, as indeed the *Guide*[1] does later. (The linearisation process can be performed as shown in Reference 2). It then becomes convenient to present T_{rp} in terms of the 'black-body equivalent radiant temperatures' T_1', T_2', or in general T_s', of the surface whose measurable temperature is T_s. The T_s' values follow from the T_s values themselves by solution of the simultaneous equations

$$T_1'(+G_{11}) + T_2'(-G_{12}) + T_3'(-G_{13}) + \ldots = T_1 G_1$$
$$T_1'(-G_{21}) + T_2'(+G_{22}) + T_3'(-G_{23}) + \ldots = T_2 G_2$$
$$T_1'(-G_{31}) + T_2'(-G_{32}) + T_3'(+G_{33}) + \ldots = T_3 G_3$$
$$\ldots$$

There are six such equations for a six-surface enclosure.

G_1 (with one subscript) denotes the emissivity conductance $A_1 \varepsilon_1 h_r/(1 - \varepsilon_1)$ between T_1 and its black-body equivalent node T_1' and h_r is the linearised radiant heat transfer coefficient with a value of about 5.7 W m^{-2} K^{-1} at room temperatures.

G_{12} (with two subscripts) denotes the geometric conductance between T_1' and T_2' and is given as

$$G_{12} = G_{21} = A_1 F_{12} h_r = A_2 F_{21} h_r$$

where F_{12} is the view factor from surface 1 to surface 2. G_{11} is the sum of all the radiant conductances linked to T_1':

$$G_{11} = G_1 + G_{12} + G_{13} + \ldots$$

The simultaneous equations follow from consideration of heat balance at each of the black-body equivalent nodes.

The expression for T_{rp} can then be expressed in terms of these black-body equivalent temperatures. The linearised version of equation A1.6 is

$$T_{\text{rp}} = \frac{1}{4\pi} \int_0^{4\pi} T_s' \, d\psi$$

or

$$\frac{1}{4\pi} \sum T_s' \, \delta\psi$$

If surface 1 for example has an emissivity of unity, the value of G_1 becomes infinite and $T_1' = T_1$. If all surfaces have unit emissivities, equation A1.6 becomes correct. Since most building materials have emissivities of around 0.9—not far removed from unity—equation A1.6 may give a satisfactory estimate of T_{rp}. It is wrong in principle however.

4   Space-averaged radiant temperature T_{rv}

From the point of view of comfort, the local value of temperature defined by equation A1.6 (linearised and corrected for emissivity) is a relevant quantity. However, Section A5[1] is concerned with thermal conditions in a room as a whole, that is, with the space-averaged conditions rather than any local value. Clearly we need a space averaged value of T_{rp}:

$$T_{\text{rv}} = \frac{\iiint T_{\text{rp}} \, dx \, dy \, dz}{\iiint dx \, dy \, dz}$$

(The space or volume-averaged air temperature T_{av} is similarly related to the distribution of local values T_{ap} as

$$T_{\text{av}} = \frac{\iiint T_{\text{ap}} \, dx \, dy \, dz}{\iiint dx \, dy \, dz}$$

It is a matter for further enquiry as to how T_{rv} can be estimated, and whether its estimate can be legally combined with the space-averaged or mean air temperature T_{av}. It turns out that T_{rv} can be satisfactorily estimated by the further radiant measure, the radiant star temperature T_{rs}, and furthermore that T_{rs} and T_{av} can be merged—exactly in restricted circumstances—to form the 'rad–air' temperature T_{ra}.

This is not the way, however, in which the *Guide* handles radiation globally in Section A5; mean radiant temperature continues to be seen as a local rather than a global quantity. As p A5–6 has it, '*the mean radiant temperature usually changes with position in the room . . .*'. An attempt is made to combine something like this radiant temperature with the space-averaged air temperature to form environmental temperature. The attempt proves to be invalid for a variety of reasons, but clearly, a combination of a local radiant temperature with a space-averaged air temperature represents a merging of unlike quantities and must be at least dubious.

5   Effect of an internal radiant input

If there is a source of long-wave radiation in an air-free enclosure, some of it falls directly on internal objects such as people and furnishings. If a beam of intensity I from such a source falls on a globe thermometer of radius r and absorptivity α, the absorbed flux is $\pi r^2 \alpha I$. If the walls are at a uniform temperature and are black-body, and the globe is an increment ΔT_{rp} higher, the heat loss to the walls is $4\pi r^2 \varepsilon h_r \Delta T_{\text{rp}}$, where ε is the emissivity of the globe surface. Equating the two quantities, the globe elevation is

$$\Delta T_{\text{rp}} = \frac{I}{4h_r}$$

since the long-wave emissivity and absorptivity are equal. Further, as a result of the linearisation process

$$h_r = 4\sigma T_{\text{mean}}^3$$

where σ is Stefan's constant. So we have

$$\Delta T_{\text{rp}} = \frac{I}{16\sigma T_{\text{mean}}^3}$$

This is in effect equation A1.11 of the *Guide*. It is substantially correct and is appropriate for its purpose—determination of the temperature elevation at some location in a room due to the presence of a local radiant source. The space-averaged value of ΔT_{rp} can be found from the equation in section 4. Values are listed in Reference 3.

Again, however, the presentation of radiant exchange in Section A5 is not consistent with that of Section A1. Among the defects of the analysis in Appendix 1 of Section A5 in which the concept of environmental temperature is explained, no consideration such as that indicated by equation A1.11 is given to estimate the effect of inputting long-wave radiation. (The procedure suggested on p A5–8 to handle radiant input is effectively correct but it conflicts with the content of Appendix 1.)

6 Discussion

The defects of Section A5 lie mainly in its treatment of radiation and a number of features lead to difficulties:

(a) The expression 'mean' always denotes an averaging of some sort. In its definition of 'mean radiant temperature', equation A1.6, the averaging concerned is over all directions, but from just one point in space. The resulting quantity is useful for specification of local comfort conditions. The author would suggest however that the term 'mean' is redundant in this case. An observing head such as a globe must necessarily pick up radiation from all directions, unless steps are deliberately taken to block off radiation from some particular direction, so this form of averaging can be taken for granted. It would be better to describe the measure concerned as the 'local radiant temperature' T_{rp}.

(b) In order to describe both heat exchange and average comfort conditions in the enclosure as a whole we require a measure of radiant temperature that represents a space-averaged value of the distribution of local radiant temperatures over the entire enclosure, the average radiant temperature T_{rv}. T_{rv} is an average of (potentially) observable measures, just as the average air temperature T_{av} is an average of local air temperatures.

(c) In the expression leading to local radiant temperature (*Guide* equation A1.6) no account was taken of surface emissivity.

(d) The direct exchange of radiation between black-body surfaces 1 and 2, say, in a room can be described exactly in terms of the conductance G_{12} defined above and the total exchange by a function of the complete set of G_{jk} conductances. In a rectangular room there are 15 such conductances. The external effect of this network can be represented approximately by an equivalent network in which each node T_1, T_2 etc. is linked to a central node, the 'radiant star' node T_{rs}. Provided that the conductances S_1 between T_1 and T_{rs}, S_2 between T_2 and T_{rs} etc. are suitably chosen, the external effect of the star-based network is closely that of the surface-to-surface network. T_{rs} is of course an equivalent construct and has no physical meaning. If however the radiant input Q_r to the enclosure is taken to act at T_{rs}, the value it generates at T_{rs} provides a convenient *estimate* of the average observable temperature T_{rv} in the enclosure. (It is not an accurate estimate: if the radiant source is in the middle of the room, T_{rv} is around 14% higher than T_{rs}; they are more nearly equal if the source is placed nearer a wall.) It is clearly necessary, however, in dealing with radiant exchange to have both these global measures: the physically meaningful 'average radiant temperature' T_{rv}, which is on the same footing as the average air temperature; secondly the physically meaningless equivalent network quantity the radiant star node T_{rs}, which is useful in that it provides an estimate of T_{rv}.

The author believes that the need for two global radiant measures has not been recognised in the past, either in the *Guide* or elsewhere. There is no problem of this kind over air temperature: the single quantity average air temperature T_{av} does duty as the companion of T_{rv} in connection with a description of average comfort conditions, and as the companion of T_{rs} in network formulations of enclosure heat flows.

(e) As noted above, the appendices of Section A5 of the *Guide*[1], in setting up a global measure of temperature in an enclosure, ignored the temperature elevation due to a radiant source.

(f) Nor does it appear to have been recognised that the node representing comfort or dry resultant temperature is a *low conductance* node, and is on quite a different footing from all the other measures of temperature in an enclosure, all of which are linked by *high conductances*. T_c is not associated with large heat flows—kilowatts. In its formulation of dry resultant temperature however, the *Guide* places t_c on the very *high conductance* path between environmental and air temperatures.

(g) Finally we may note the meanings that have become attached, explicitly or tacitly, to the expression 'mean radiant temperature':

    (i) the quantity that has been termed here 'local radiant temperature' T_{rp}

    (ii) the space-averaged radiant temperature T_{rv}

    (iii) the radiant star temperature T_{rs}

    (iv) the mean surface temperature t_m in *Guide* notation.

The first three of these quantities are useful in their different ways. Mean surface temperature however proves to be a non-quantity, irrelevant to radiant or convective exchange in a room and to the conductive losses from it.

This Note is a plea for clarification of the concepts attached to long-wave radiant exchange in a room as they are expressed in current CIBSE literature. The issues raised only become acute however when surfaces have low emissivities and since building surfaces with few exceptions have high emissivities, the *Guide* methods, operationally speaking, are likely to be adequate for their purpose.

References

1 *CIBSE Guide* Volume A: Design Data (London: Chartered Institution of Building Services Engineers) (1986)

2 Davies M G A thermal circuit for radiant exchange *Building and Environ.* 14 43–46 (1979)

3 Davies M G Design models to handle radiative and convective exchange in a room *Trans. Amer. Soc. Heating Refrig. Air-cond. Eng.* 94(2) 173–195 (1989)

4 Davies M G Room internal heat exchange: new design method *Building Serv. Eng. Res. Technol* 8(3) 47–60 (1987)