Simplified method of combining natural and exhaust ventilation

Yuguo Li

Dept. of Heating and Ventilation
Royal Institute of Technology
Stockholm, Sweden.

Introduction

Mechanical ventilation system have become more popular in recent decades in many countries. These can be in the form of either a simple exhaust ventilation system, a balanced supply/exhaust ventilation system or a me-chanical ventilation system that incorporates an air-to-air heat exchanger. To avoid making the ventilation rate too high and causing excessive energy consumption, and to avoid inadequate ventilation causing poor indoor air quality, it is necessary to predict the total air change rate. So far several mathematical models determining the natural infiltration rate of buildings have been developed. One simple method to estimate the total flow rate, where a mechanical ventilation system exists, is to combine the result of the above mentioned models and the mechanical ventilation rate at the indoor design conditions. This kind of simplified model has been used in some calculations such as the LBL model as used in Sherman et al (1980) and Modera et al (1983).

The problem is determining which combination model is the best. There are different opinions, for example, Modera et al (1987) suggested the quadrature super-position model was most appropriate, but Kiel et al (1987) preferred the linear superposition model. Another problem is that so far there is not a resonable explanation for all these models, especially for the commonly used quadrature superposition model, see Modera et al (1987).

This paper firstly presents an improved theoretical derivation process for the combination models, and then employs the multi-room flow balance model MIX by Li et al (1990) to examine ten combination models. Some realistic physical explanation is also given, mainly on the combination of natural and exhaust ventilation.

General derivation process for combination models

Basic principles

The basic idea of the combination model is to estimate the total air exchange rate Q_{pred} from Q_{nat} and Q_{vent}.

$$Q_{pred} = f(Q_{nat}, Q_{vent}) \tag{1}$$

where

Q_{pred} = predicted total air change rate from the combination model
Q_{nat} = predicted total air change rate due to natural ventilation
Q_{vent} = the air change rate flowing through mechanical ventilation at the indoor design conditions.

This is different from the flow balanced mathematical model which uses the iteration method to calculate the combined pressure conditions resulting from the natural ventilation driving forces and the mechanical ventilation driving forces. Thus the combination model has its own advantages in terms of its simplicity and rapid computation time.

There are two points which should be taken into consideration. One is to define the pressure superposition. Due to the nonlinear interation between the pressure that drive the natural and mechanical ventilation, this is difficult to specify. Most combination models adopted the linear assumption to estimate the predicted pressure difference.

$$\Delta P_{pred} = \Delta P_{nat} + \Delta P_{vent} \tag{2}$$

where

ΔP_{pred} = predicted pressure diffrence
ΔP_{nat} = pressure difference due to natural ventilation
ΔP_{vent} = pressure difference due to mechanical ventilation.

Although the linear addition of the two pressure difference is correct for local pressure at a given leakage site, it is only a rough approximation for a whole building, see Kiel et al (1987). It is also neccessary to choose the relationship between pressure difference and the flow rate. It is known that the better correlated formula are equations (3) and (4).

$$Q_{inf} = f \frac{A_L C}{V} (\Delta P)^n \quad (3)$$

$$Q_{exf} = (1-f) \frac{A_L C}{V} (\Delta P)^n \quad (4)$$

where

A_L = the total leakage area
f = the fraction of A_L on infiltration
$1-f$ = the fraction of A_L on the exfiltration
V = the building interior volume
C = constant
n = exponent, in general, n = 2/3.

A general combination formula

We can get a general combination formula from the above equations and the mass flow balanced equation, through a modified "derivation method", see Kiel et al (1987). From this general combination formula several surrent models can be obtained including the quadrature superposition model.

Consider a building with a mechanical exhaust ventilation system, the air inflow and the air outflow through the building envelope must be balance.

$$Q_{inf} = Q_{exf} + Q'_{vent} \quad (5)$$

where

Q'_{vent} = mechanical air change rate with the influence of weather indued infiltration

Since the Q_{nat} in equation (1) is the air change rate due to infiltration and natural ventilation, in the absence of mechanical ventilation, we can get $Q'_{vent} = 0$ in equation (5).

then

$$Q^0{}_{inf} = Q^0{}_{exf}$$

where

$Q^0{}_{inf}$ = air infiltration in the absence of mechanical ventilation

$Q^0{}_{exf}$ = air exfitration in the absence of mechanical ventilation

In such a case f=0.5

As $Q_{nat} = Q^0{}_{inf}$

Then

$$Q_{nat} = 0.5 \frac{A_L C}{V} (\Delta P_{nat})^n \quad (6)$$

If there is no wind influence and no temperature difference, then the mechanical ventilation system is acting alone. Since the exhaust ventilation system creates underpressure, all leakage sites are infiltrating.

Thus in equation (5), $Q_{exf} = 0$

Equation (5) becomes,

$$Q'_{vent} = Q_{inf}$$

and

$$Q'_{vent} = \frac{A_L \cdot C}{V} \cdot (\Delta P_{vent})^n \quad (7)$$

One of the assumption in these calculations is that

$$Q_{vent} = Q'_{vent} \quad (8)$$

This gives values for the terms Q_{vent} (by equation (8)) and Q_{nat} (by equation (6)) in equation (1).

Considering now the function form in equation (1). Substituting equation (3) and (4) into equation (5), we have the fraction of leakage area active in infiltration.

$$f = (2 - Q_{vent} / Q_{inf})^{-1} \quad (9)$$

Considering $Q_{pred} = Q_{inf}$

Then

$$Q_{pred} = (2 - Q_{vent}/Q_{inf})^{-1} \frac{A_L C}{V} (\Delta P_{pred})^n$$

Considering equation (2), (6), and (7)

$$Q_{pred} = (2 - Q_{vent} / Q_{inf})^{-1} (Q_{vent}^{1/n} + (2Q_{nat})^{1/n})^n \quad (10)$$

Equation (10) is a very important formula and from it several combination models (see Table 1) will now be derived.

Quadrature superposition model

If $Q_{vent} << Q_{nat}$ (i.e. the mechanical ventilation is very weak and the natural ventilation is dominating)

Q_{vent} / Q_{inf} -> 0

and f=0.5 (from equation (9)). Substituting in equation (19) gives

$$Q_{pred} = 0.5\,(Q_{vent}^{1/n} + (2\,Q_{nat})^{1/n})^n \quad (11)$$

If $n = 0.5$

Then $Q_{pred} = 0.5\,(Q_{vent}^2 + (2\,Q_{nat})^2)^{0.5}$ (12)

Saying that $Q_{vent} << Q_{nat}$, is similar to $Q_{vent} << Q_{inf}$.

Equation (11) and (12) can be written as follows.

$$Q_{pred} = (Q_{vent}^{1/n} + (Q_{nat})^{1/n})^n \quad (13)$$

$$Q_{pred} = (Q_{vent}^2 + Q_{nat}^2)^{0.5} \quad (14)$$

If $Q_{vent} >> Q_{nat}$, (i.e. the mechanical ventilation is dominating).

Then Q_{vent} / Q_{inf} -> 1

So $f = 1$ and from equation (10)

$$Q_{pred} = (Q_{vent}^{1/n} + (2Q_{nat})^{1/n})^n \quad (15)$$

If $n = 0.5$

Then

$$Q_{pred} = (Q_{vent}^2 + (2Q_{nat})^2)^{0.5} \quad (16)$$

Table 1. Summary of combining models.

No.1	Models	Exponent	Range
1	$Q_{pred} = (Q_{vent}^2 + Q_{nat}^2)^{0.5}$	0,5	all
2	$Q_{pred} = (Q_{vent}^2 + (2Q_{nat})^2)^{0.5}$	0,5	strong.vent.
3	$Q_{pred} = (Q_{vent}^{1/n} + (Q_{nat})^{1/n})^n$	0,667	all
4	$Q_{pred} = Q_{nat} + Q_{vent}\exp(-Q_{nat}/Q_{vent})$		all
5	$Q_{pred} = ((0.5Q_{vent})^2 + Q_{nat}^2)^{0.5} + Q_{vent}/2$	0,5	all
6	$Q_{pred} = ((0.5Q_{vent})^{1/n} + Q_{nat}^{1/n})^n + Q_{vent}/2$	0,667	all
7	$Q_{pred} = Q_{vent} + Q_{nat}$	1	all
8	$Q_{pred} = 0.5\,(Q_{vent}^2 + (2\,Q_{nat})^2)^{0.5}$	0,5	weak. vent.
9	$Q_{pred} = (Q_{vent}^{1/n} + (2Q_{nat})^{1/n})^n$	0,667	strong.vent.
10	$Q_{pred} = 0.5\,(Q_{vent}^{1/n} + (2\,Q_{nat})^{1/n})^n$	0,667	weak. vent.
11	$Q_{pred} = F(Q_{vent.stack}^{1/n} + (Q_{wind})^{1/n})^n$	0,667	all

As Q_{vent} >> Q_{nat}, euqtions (15) and (16) become

$$Q_{pred} = (Q_{vent}^{1/n} + Q_{nat}^{1/n})^n \qquad (17)$$

$$Q_{pred} = (Q_{vent}^2 + Q_{nat}^2)^{0.5} \qquad (18)$$

Equations (17) and (18) are the same as equations (13) and (14). Thus equations (17) and (18) can be proposed to be used for the whole range, and equation (18) is Sherman's (1980) or Modera's (1983) quadrature superposition, which is used in chapter 22 of ASHRAE (1985). Shaw (1985) used a correction factor F to account for the over prediction of total flow rate by equation (17), and his model produces slightly different results.

Kiel et al (1987) concluded that equations (17) and (18) can only be used for instance of weak mechanical ventilation. This is probably not correct, see their experimental results in Figs. 1 and 2. The figures illustrate that it appears to be the case that the weaker the mechanical ventilation, the larger the error.

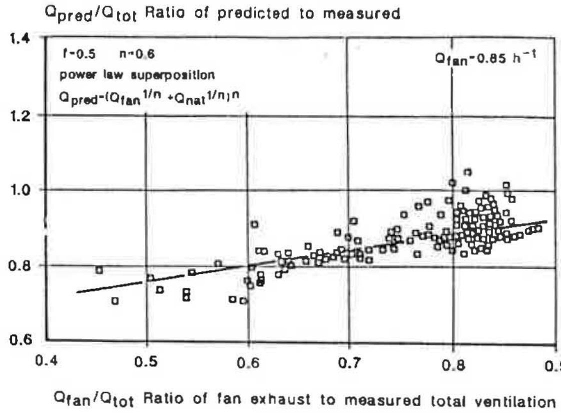

Fig. 1 Comparing predicted rate and measurement, see Kiel et al (1987).

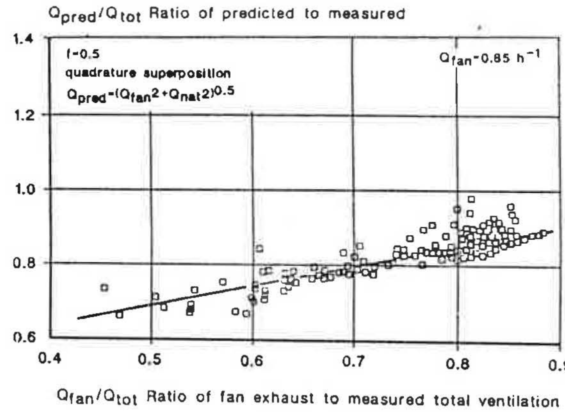

Fig. 2 Comparing predicted rate and measurement, see Kiel et al (1987).

Other superposition models

Substituting Q_{pred} for Q_{inf} in equation (10) gives

$$Q_{pred} = ((0.5Q_{vent})^{1/n} + Q_{nat}^{1/n})^n + Q_{vent}/2 \qquad (19)$$

If $n = 0.5$

$$Q_{pred} = ((0.5Q_{vent})^2 + Q_{nat}^2)^{0.5} + Q_{vent}/2 \qquad (20)$$

Equation (20) is also derived by Kiel et al (1987).

From equation (17), if $n = 1$, the linear superposition model is obstained.

$$Q_{pred} = Q_{vent} + Q_{nat} \qquad (21)$$

Another model is Levin's (1982) exponential model.

$$Q_{pred} = Q_{nat} + Q_{vent} \exp(-Q_{nat}/Q_{vent}) \qquad (22)$$

If λ is denoted for Q_{nat}/Q_{vent}

Equation (18) besomes

$$Q_{pred} = Q_{vent}\sqrt{1+\lambda^2}$$
$$= Q_{vent}\left(1 + \frac{1}{2}\lambda^2 - \frac{1\cdot3}{2\cdot4}\lambda^4 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\lambda^6 - \dots\right) \qquad (23)$$

Equation (22) becomes

$$Q_{pred} = Q_{vent}(\lambda + e^{-\lambda})$$
$$= Q_{vent}\left(1 + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} - \dots\right) \qquad (24)$$

From this mathematical point of view, these two equations give almost the same predicted result, as will be shown.

Comparison and Discussion

The multi-room flow balanced model MIX, see Li et al (1990) was used. The test building was a model building, the same as that used in the above mentioned article, and consisted of 12 rooms and one stairwell. Three of the rooms were considered as kitchens or bathrooms, where an exhaust ventilation system was designed. The relative accuracy and error of the combination models are expressed as follows.

$$R_a = Q_{pred}/Q_{balan} \qquad (25)$$

$$Error = \frac{Q_{pred} - Q_{balan}}{Q_{balan}} \cdot 100\% \qquad (26)$$

Where

Q_{pred} = the calculated rate from the predicted combination models

Q_{balan} = calculated rate from the MIX.

In total 1440 tests were carried out and the main parameters used in calculation are shown in Table 2.

Table 3 shows the calculation results for these 1440 tests. With both the average and the maximum of the relative error, the quarature superposition model and Levin's exponential model appear to be the best ones. For these two models, the average error is less than 5%, and the maximum error is less than 20%. These results agree with the calculation by Modera et al (1987). It is necessary to remember that Levin's exponential model is almost the same as the quadrature superposition model, from the point of view of mathematics.

Figs. 3 and 4 show the total comparison for test groups 1 and 2 respectively. To facilitate easy comparison, a least-squares linear result is shown in the figures. It can seen from these figures that when combining the natural ventilation and the mechanical ventilation, errors partly depend on the leakage configurations, the natural ventilation forces, and so on, and further study on the influence of these factors is expected.

Table 2. Test conditions list (θ_{in} = 20°C, θ_{out} = -25, -15, ..., 20°C, U = 0, 1, 2, 3, .., 9 m/s)

Test No.	k_t-value	θ	door	E. vent
1	0,0002	0	open	strong
2	0,0004	0	open	strong
3	0,0008	0	open	strong
4	0,0002	0	open	weak
5	0,0002	90	open	strong
6	0,0004	90	open	strong
7	0,0008	90	open	strong
8	0,0002	90	closed	weak
9	0,0002	0	closed	strong
10	0,0004	0	closed	strong
11	0,0008	0	closed	strong
12	0,0002	0	closed	weak
13	0,0002	90	closed	strong
14	0,0004	90	closed	strong
15	0,0008	90	closed	strong
16	0,0002	90	closed	weak

Table 3. Comparison of ten combination models.

Model	Average error	order	Max. Error	order	Per.<20% a)	order	No.<20% b)
1	4.87%	2	17.00%	1	100%	1	1440
2	56,1%	9	100%	8	22.1%	7	318
3	10,7%	3	30,4%	3	79,9%	2	1151
4	4,76%	1	19,4%	2	100%	1	1440
5	12,4%	4	34,6%	4	76,9%	3	1108
6	17,9%	5	42,2%	5	61,4%	4	884
7	33,4%	8	63,6%	7	20,4%	8	294
8	21,9%	7	50,1%	6	49,9%	5	718
9	98,3%	10	160%	9	8,0%	9	115
10	20,8%	6	50,1%	6	47,9%	6	690

a) the percentage of the points of which the relative error are less than 20%

b) the number of the points of which the relative error are less than 20%.

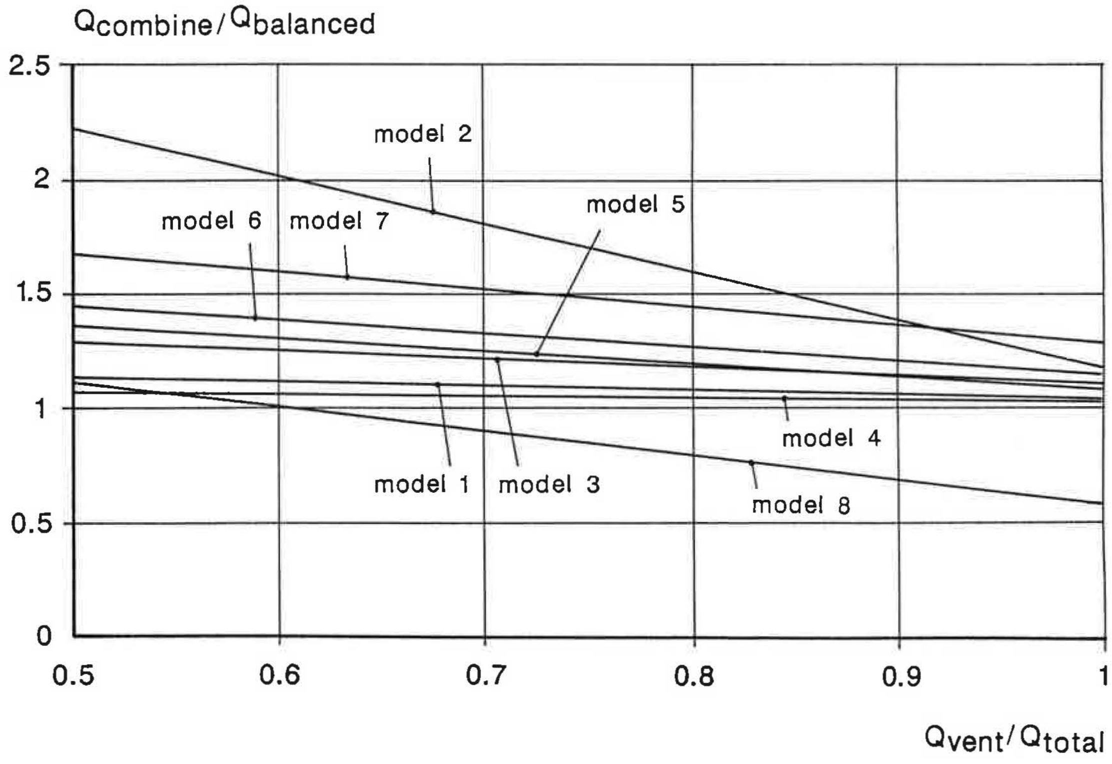

Fig. 3. Comparison of combination models for test data group 1.

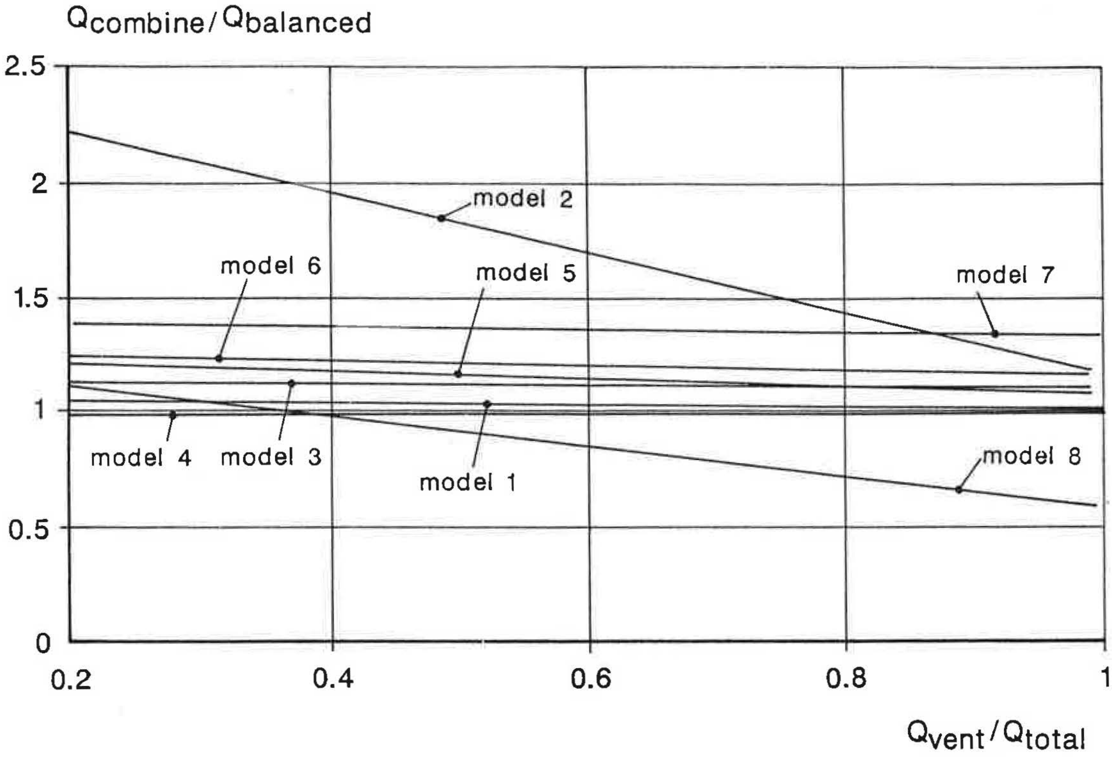

Fig. 4. Comparison of combination models for test data group 2.

Conclusion

The quadrature superposition model and the exponential superposition model give the best estimate of the combined total flow rate from the natural ventilation and mechanical exhaust ventilation. This conclusion agrees with the model recommended by ASHRAE (1985), Modera et al (1987), but disagrees with the results of Kiel et al (1987), where the direct linear addition was found to be the most suitable.

A reasonable general theorectical derivation process of the combination models is given. This is useful to help understand the physical meanings of the combination models. An explanation of the success of the quadrature and exponential superposition model is proposed from the pressure superposition and mathematicl viewpoints.

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