The modelling of multi-room air flow and its application to exhaust ventilation analysis

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Introduction

With the the demand for reducing energy costs and improving air quality in buildings, more attention is being focused on the modelling of infiltration and indoor air flow in multi-room buildings. One characteristic of most existing buildings is that they have floor plans and internal partitioning, which makes the single-cell model redundant. During the last 20 years, several multi-room models have been developed. The Air Infiltration and Ventilation Centre has compared ten mathematical infiltration models and five of them are multi-room models, see Liddament et al (1983). Feustel and Kendon (1985) have also reviewed 15 different multi-room models using a questionnaire. However, these authors have pointed out that most of these programmes are not commonly available to the public, especially the programmes which can be suited to specific needs, such as energy cost analysis and indoor air quality analysis.

The air flow between rooms and between indoor and outdoor is caused by pressure differences, which are affected by wind, thermal buoyancy, the mechanical ventilation system or a combination of these factors. The multi-room building can be considered as a complicated interconnecting system of flow paths. The general method is to present the physical relationship between the pressure difference and the leakage area as the power function. If we assume that there is mass flow balance in every room, we will have a set of nonlinear equations, which can only be solved by iteration. In some programmes for larger openings between rooms, it was not possible to get convergence, see Walton (1982).

The modelling framework described here was carried out to simulate the infiltration, indoor air movement and its interaction with the mechanical ventilation as the first step for studying and indoor air quality. This paper concentrates on the formulation of the model, algorithms for both indoor pressure distribution and ventilation system analysis, and the question of the influence of infiltration on the exhaust ventilation system in multi-room buildigns. The program was named as MIX (*m*ulti-room *i*nfiltration and e*x*filtration).

Multi-room air flow model

The building surfaces and indoor partitions contain openings, either large or small, which permit the air to flow through them whenever there are pressure differences across the opening. The pressure difference may be due to naturally occurring wind, the thermal effects, and the operation of mechanical ventilation.

Wind force and thermal force

When the path of wind is blocked by buildings, the velocity pressure is converted to static pressure. On the windward side a pressure increase is produced, whereas on the leeward side a pressure drop results. The other side could incur pressure increase or decrease, depending on the wind angle and building shape. This static pressure caused by the wind can be represented by equation (1).

$$\Delta P_w = \frac{1}{2} c_p \rho_o U^2 \qquad (1)$$

where

ΔP_w = surface pressure due to the wind, Pa
c_p = wind pressure coefficient
U = mean wind speed at datum level (building height), m/s
ρ_o = the density of outside air, kg/m^3.

Here the pressure produced by the wind is related to the square of the wind velocity. In the present model, the mean wind velocity at building height could be either given or calculated from the meteorological data. If the terrain conditions around the building are known, together with the wind speed at standard conditions, the wind velocity at the building height can be calculated according to the formula

$$U = U_0 \cdot \alpha \cdot (\frac{H}{H_0})\gamma \qquad (2)$$

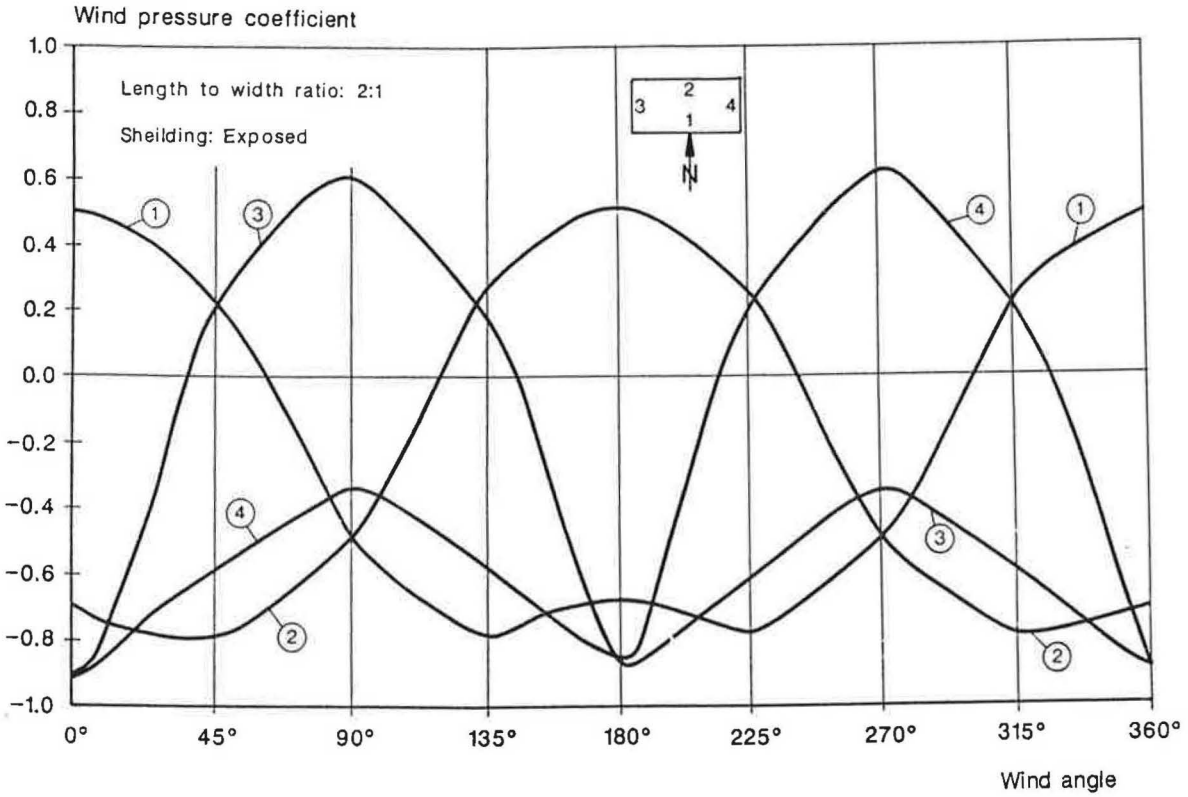


Fig. 1. Wind pressure coefficient data.

where

U_0 = wind velocity at height H_0 in standard conditions, m/s
H = building height, m
H_0 = standard height of wind measurements, 10 m
α, γ = terrain-dependent coefficient and exponent.

The wind pressure coefficient c_p is an empirically determined parameter which is assumed to be independent of wind speed but related to the wind direction, the building shape and the shielding conditions. The range of c_p is between -1.0 and 1.0, where the negative figure expresses the pressure decrease, and the positive figure expresses the pressure increase.

For low building, typically up to 3 stories, c_p can be considered approximately as an average value for each face of the building for different directions, see Liddament (1986). This is shown in Fig. 1.

Another force causing the pressure difference is the thermal effect. Warm air in a building tends to rise because of its low density when inside teperature is higher than that outside and vice versa.

The stack pressure, see Fig. 2, can be calculated.

$$p_1 = p_{01} - \rho_1 g h \tag{3.1}$$

$$p_2 = p_{02} - \rho_2 g h \tag{3.2}$$

$$\Delta p = (p_{01} - p_{02}) + (\rho_2 - \rho_1) g h \tag{3.3}$$

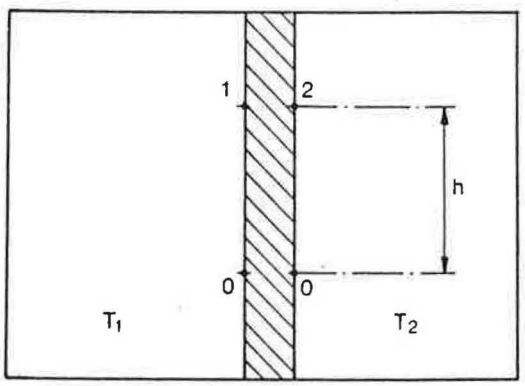


Fig. 2. The stack pressure.

where

p_0 = pressure at datum level h^0, Pa
g = acceleration due to gravity, m/s^2
Δp_s = $(\rho_2 - \rho_1)gh$ is stack pressure at level h.

The total pressure difference distribution for envelope surfaces can be obtained from the sum of the wind pressure and stack pressure. The present model adopts the summation formula described by Lilliengren (1986), see Fig. 3.

$$\Delta p_{tot}(h) = \left(\frac{1}{2} \cdot c_p \cdot \rho_o \cdot U^2 + \Delta\rho g \frac{H}{2} - p_i \right) \cdot \left(1 - \frac{h}{H^*} \right) \quad (4)$$

where

$\Delta\rho$ = the density dofference of the inside and outside air, kg/m^3
H = the building height, m
h = the height considered, m
H^* = neutral level height, m
P_i = the internal pressure, Pa.

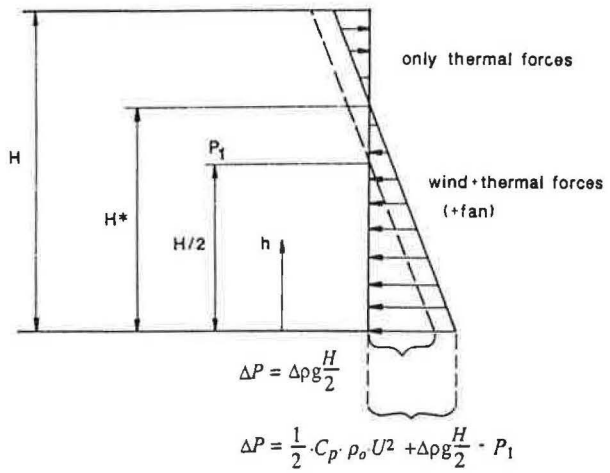


Fig. 3. The pressure across the envelope.

The neutral level height is determined by the following equation,

$$H^* = \left(\frac{1}{2} \cdot C_p \cdot \rho_o \cdot U^2 + \Delta\rho g \frac{H}{2} - P_i \right) / (\Delta\rho \cdot g) \quad (5)$$

For the building envelope, we should separate the exfiltration and infiltration, which are respectively driven by negative and positive applied pressure difference, depending on the neutral level. In the calculation of the building envelope therefore, three cases were considered,

- The neutral level is below the floor ($H^* < 0$)
- The neutral level is between the floor and the ceiling ($0 < H^* < H$)
- The neutral level is above the ceiling ($H^* > H$)

The pressure difference for internal room surface, for example, room i and room j, is

$$\Delta P = P_i - P_j \quad (6)$$

Exhaust ventilation system

For the ventilation systems in residential buildings, an exhaust fan is often the most attractive choice because of its low capital and maintenance costs. Traditionally, it is designed with exhaust from bathrooms or kitchens and not from the living rooms or bedrooms. The mechanical exhaust of air pressure will create an internal pressure drop. This additional pressure imbalance could be calculated to treat ventilation as another flow path in which the pressure versus flow rate relationship (the system characteristic) is specified, that is,

$$\Delta p_e = R_e q_e^2 \quad (7)$$

where

Δp_e = pressure loss in the ventilation system, Pa
R_e = the resistence of the air way, Ns^2/m^8
q_e = volume flow rate, m^3/s.

The fan characteristic is present by the following equation which gives a good agreement in the normal working mode.

$$p_f = C_1 (1 - C_2 q^2) \quad (8)$$

where

p_f = pressure drop created by fan, Pa
q_f = volume flow rate, m^3/s
C_1, C_2 = fan coefficients.

The intersection of the fan charateristic and the system characteristic will give the point of operation, see Fig. 4. When the internal pressure is changed, the operation point in the exhaust ventilation system will be changed, see Fig. 5.

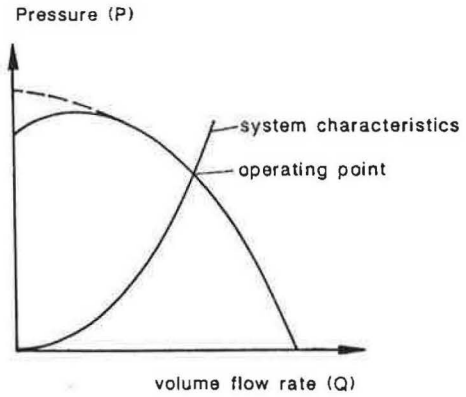
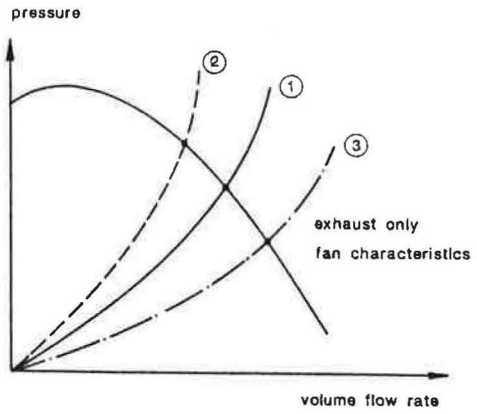


Fig. 4. Fan and system characteristics.



1. Room at atmospheric pressure
2. Room above atmospheric pressure
3. Room below atmospheric pressure

Fig. 5. Room pressure effect on the operating point.

In a ventilation network, the algebraic sum of the pressure drop in any closed mesh should be zero and in any junctions, the mass balance is right. For example, for the exhaust ventilation system illustrated in Fig. 6, the following set of equations can be obtained.

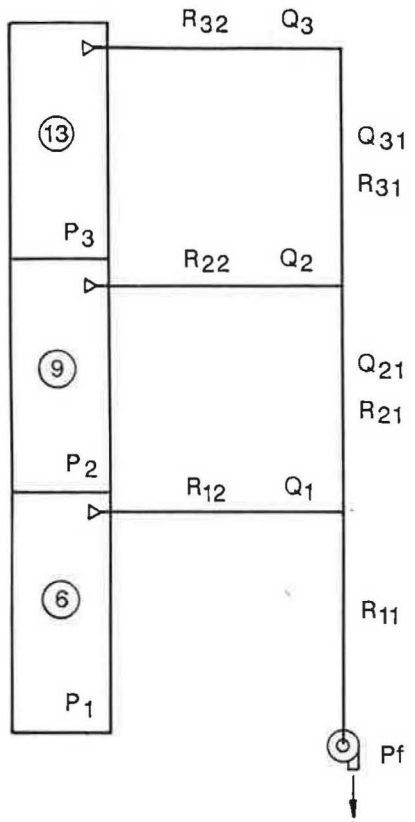


Fig. 6. Exhaust ventilation system.

$$R_{11}(q_1 + q_2 + q_3) + R_{12}q_1^2 - P_f - P_1 = 0$$

$$R_{11}(q_1 + q_2 + q_3) + R_{21}(q_2 + q_3)^2 +$$

$$R_{22}q_2^2 - P_f - P_2 = 0$$

$$R_{11}(q_1 + q_2 + q_3) + R_{21}(q_2 + q_3)^2 + (R_{32}$$

$$+ R_{31})\, q_3^2 - p_f - p_2 = 0 \qquad (9)$$

To solve this set of non-linear equations without too many interations a special method is necessary.

Flow equations

For relatively large openings with flow paths, such as the door opening, the following equation based on the orifice turbulent flow is used,

$$q = C_d \cdot A \cdot \sqrt{2\Delta p/\rho} \qquad (10)$$

where

q = air flow rate, m^3/s
C_d = discharge coefficient
ρ = air density, kg/m^3
Δp = pressure differences across opening, Pa
A = area of opening, m^2.

For a smaller opening, the flow rate, q, is represent by the power law equation

$$q = k_t (\Delta p)^n \qquad (11)$$

where

k_t = flow coefficient, 1 m^3/s at 1 Pa
n = flow exponent
Δp = pressure difference, Pa

The coefficient k_t is related to the leakage area, and the exponent n is close to 2/3 which agrees with experimental measurements and theretical consideration, see Peterson (1980).

Air flows from high pressure to low pressure. Flow of air into a room (inflow) is defined as positive and flow of air out of a room (outflow) is defined as negative. For each room, there must be a mass balance.

$$q_e + \sum q_i = 0 \qquad (12)$$

where

q_e = the air flow rate from the ventilation system
q_i = the air flow rate across the surface i.

The balance of the mass flows in all rooms together can be expressed by the flollowing vector equation.

$$f_i (\boldsymbol{P}) = 0 \quad (i = 1, 2, 3, \ldots , N) \qquad (13)$$

where

$\boldsymbol{P}$ = $(\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \ldots , \boldsymbol{P}_N)$

N = the number of rooms.

This is a system of non-linear equations, and the number of the equation is equal to the number of the unknown indoor pressure (rooms). This set of non-linear equations can be used to determine room pressure using interation method.

The equations used in MIX are summarised in Table 1.

Table 1. Main equations used in MIX.

Object	Formula
Flow through small openings	$q = k_t (\Delta p)^{2/3}$
Flow through large openings	$q = C_d \cdot A \cdot \sqrt{2\Delta p/\rho}$
Flow through ductwork	$\Delta p_e = R_e q_e^2$
Fan flow characteristics	$p_f = C_1 (1 - C_2 q^2)$
Thermal buoyancy	$\Delta p_s = (\rho_o - \rho_i) g h$
Wind pressure	$\Delta P_w = \frac{1}{2} c_p \rho_o U^2$
Mass balance equations	$q_e + \sum q_i = 0$
Algorithm for nonlinear equations	$\boldsymbol{P}_{new} = \boldsymbol{P}_{old} + \lambda \cdot \delta \boldsymbol{P}$

Numeric method

This is a description of the numerical method for the ventilation system simulation and the indoor air pressure distribution. These are the two related parts of the procedure for calculating the air flow model.

Ventilation system

Initially, considering an airway

$$\Delta P_e = Rq^2 \tag{14}$$

where

ΔP_e = pressure drop along the air way
R = resistance of the airway
q = volume flow rate of air.

Thus

$$\frac{d\Delta P_e}{dq} = 2Rq \tag{15}$$

If an initial estimate is made of value the q_0 (it could be obtained from the design data or measuremental data), its error Δq can be determined in the following way.

Approximately, from equation (15)

$$\frac{\Delta(\Delta P_e)}{\Delta q} = 2Rq_0 \tag{16}$$

and it is known from Fig. 7 that

$$\Delta(\Delta P_e) = Rq^2 - Rq_0^2 \tag{17}$$

This means that

$$\Delta q = \frac{Rq^2 - Rq_0^2}{2Rq_0} \tag{18}$$

Now a closed loop m within a ventilation system network is considered, having n branches. The above equation could be modified to obtain a correct volume flow rate of air for the mesh considered, which is,

$$\Delta q_m = \frac{\sum_{i=1}^{n}(R_i q_i^2 - R_i q_{i0}^2)}{\sum_{i=1}^{n} 2R_i q_{i0}} \tag{19}$$

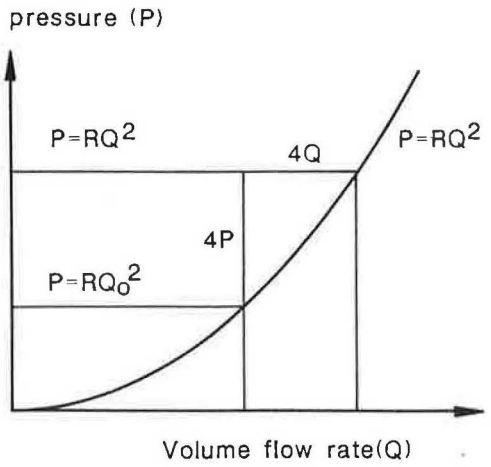


Fig. 7. Numerical analysis of ventilation system.

Since the algebraic sum of the pressure drop in a loop is zero, that is

$$\sum_{i=1}^{n} R_i q_i^2 = 0 \tag{20}$$

So we can obtain

$$\Delta q_m = \frac{\sum_{i=1}^{n}(- R_i q_{i0}^2)}{2\sum_{i=1}^{n} R_i q_{i0}} \tag{21}$$

If the defined pressure drop is positive in the direction of the flow, then taking into consideration the sign of the pressure drop, the room pressure P_m and the fan pressure P_f gives,

$$\Delta q_m = \frac{-(\sum_{i=1}^{n} R_i q_{i0}\, q_{i0} - P_f - P_m)}{\sum_{i=1}^{n}(2R_i q_{i0} - \frac{\delta P_f}{\delta q_f})} \tag{22}$$

where P_f and $\frac{\delta P_f}{\delta q_f}$ can be obtained from the characteristic of the fan at the volume flow rate q_0.

The correct value for the volume flow rate q_m can be obtained from every room, and it is possible to correct the volume flow rates and repeat the above procedure until all the values of q_m are below a prescribed level, for example, $1 \cdot 10^{-4}$ m^3/s.

Therefore, once the internal air pressure distribution is known, the air flow rate from the ventilation system can be calculated.

Internal pressure distribution

When it is decided to use one interactive method, two factors should be considered. One is the convergence; the other is the length of calculation times. To prevent long calculation times, it is necessary to develop fast algorithms and estimate the initial values as near to real velues as possible. This depends on analysis rather than numerics.

The present calculation adopted different ways to estimate initial values in different calculation conditons.

- o If all the flow exponents are replaced with 1, then a linear equation can be obtained. The solution of the linear equation set is the first set of estimated values.
- o If the inter-room openings are larger than the envelope openings, then the internal air being uniform. In these circumstances the result of a single-cell model calculation are used as the initial value of the multi-room model. The more rooms there are, the more efficient the method.
- o If the air flow for different outside air temperature is calculated at the same wind speed and direction with the same sheilding and terrain conditons, the result of the temperature θ_1 can be used as the intial value of temperature θ_2. This is providing that about $\theta_1 - \theta_2 < 10$ °C

Around the neighbourhood of initial values P, each of the fuction fi in equation (13) can be expanded by the Taylor series.

$$fi(P + \delta P) = fi(P) + \sum_{j=0}^{N} \frac{\delta f_i}{\delta P_j} \delta P + O(\delta P^2) + \cdots\cdots \quad (23)$$

By neglecting terms of order δP^2 and higher, a set of linear equations for correction δP is obtained. This makes each fuction fi closer to zero simultaneously. That is

$$\sum_{j=0}^{N} \frac{\delta f_i}{\delta P_j} \delta P = - fi(P) \quad (i = 1, 2, \ldots, N) \quad (24)$$

We can solve this using the Causs-Jordan method.

Then

$$P_{new} = P_{old} + \lambda \cdot \delta P \quad (25)$$

where

λ= relaxing factor ($0 < \lambda < 1$)

The air volume flow rate from the ventilation system needed in equation (13) is calculated using the method described in the previous section.

The process is iterated to convergence which means that both functions f_i and variables P_i have converged.

$$f_i(P_{new}) < \varepsilon$$

$$\frac{P_{new} - P_{old}}{P_{new}} < \varepsilon$$

For a large opening between rooms (for example, when a door is open), the pressure difference is small. This is a potential source of trouble in solving the above linear equation sets using the Newton-Rapson method or its revisions, as described above, where the derivatives $\frac{\delta f_i}{\delta P_j} \cdot \Delta P_{n-1}$ must be calculated. Calculations show that the flows corresponding to small pressure differences can not be casually neglected to avoid division by zero. This is because the flows could be large due to a large leakage area, although the pressure difference is very small. Thus it seems that the lower the pressure difference limit to be neglected, the better. The present calculations chose

$$\Delta P_{limit} = 1 \cdot 10^{-10} \text{ Pa}$$

This is a reasonable value, even for a large opening like an open door (2 m^2). The air flow rate will be less than $1 \cdot 10^{-6}$ m^3/s, which is small enough to neglected. In such a case, less than 10 interactions are required (λ=0.5) for a 13 rooms model calculation. The above discussion could be a solution to the problem proposed by Walton (1982), where even a case with 0.186 m^2 opening between rooms proved impossible to simulate.

An example house

A sudy of the air flow simulation and especially the influence of infiltration on the exhaust ventilation system was done using a example house. The floor plans are shown in Figs 8, 9 and 10. Fig. 11 is a section of the building. There are a total area of 450 m^2 on all three floors, and a volume of 1125 m^3.

A example exhaust ventilation system was designed from rooms 6, 9, 13, which were considered as kitchens or bathrooms, whereas the others were considered as living rooms or bedrooms, see Fig. 6.

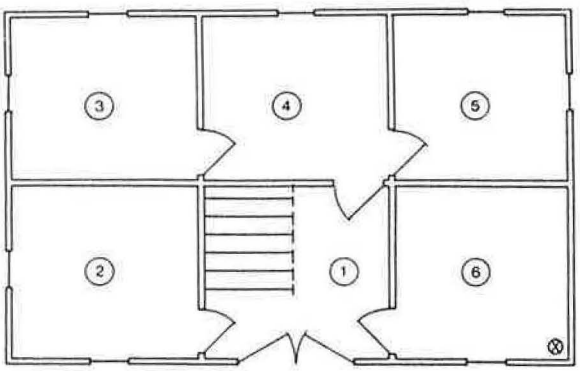


Fig. 8. First floor plan.

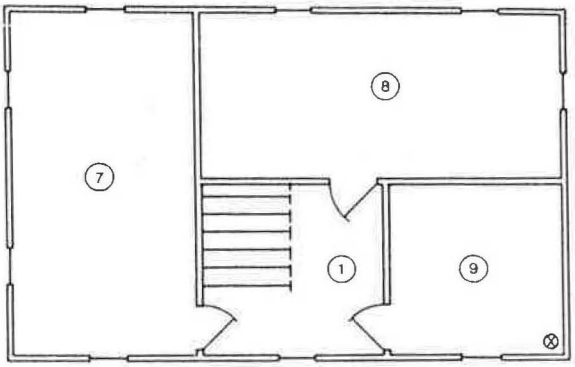


Fig. 9. Second floor plan.

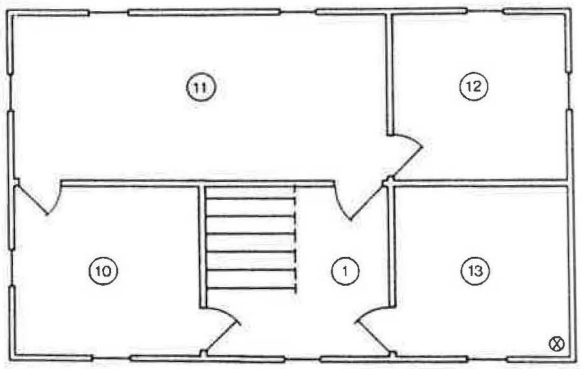


Fig. 10. Third floor plan.

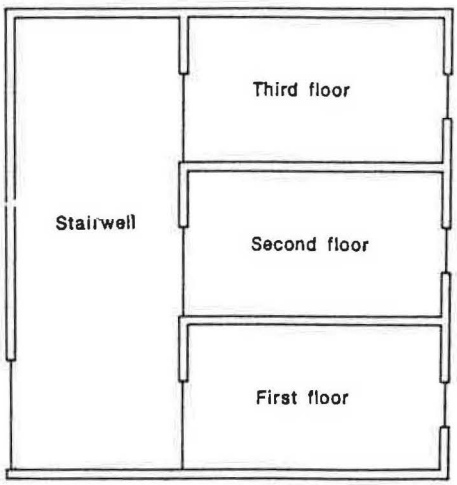


Fig. 11. Section of the example building.

Result and discussions

Underpressure created by the exhaust system

Operation of the mechanical exhaust ventilation system results in the reduction of an internal pressure or underpressure. When all the inter-room doors are open, there is a uniform internal underpressure, see Fig. 12. In this case, the single-cell model can be used to calculated the air change rate accurately and quickly. When the inter-room openings are smaller, the internal pressure distribution will not be completely uniform. Fig. 13 shows the calculated result when the interrom doors are almost closed. Fig. 15 shows the result when the doors are closed completely, k_t as 0.0002 as others. The largest indoor pressure difference between two neighbouring rooms can be as high as 10 P_a in the example house. This non-uniform pressure distribution will obviously affect the infiltration, indoor air movement.

The calculation of a single-cell model showed that when all the door are open and assuming that every face of the building is exposed to the same degree of local shielding and symmetrical leakage conditions, the influence of the symmetrical wind direction on the indoor air pressure and infiltration is small, see Danielson et al (1987). For the multi-room model, however, the result is different. The present calculation shows that if the inter-room flow resistence distribution is unsymmetrical due to the different room sizes, location of doors, and inlet of the mechanical ventilation system, the symmetrical wind direction (θ = 0° and θ = 180° are two examples illustrated in Figs. 13 and14), also causes different indoor pressure distributions and infiltration.

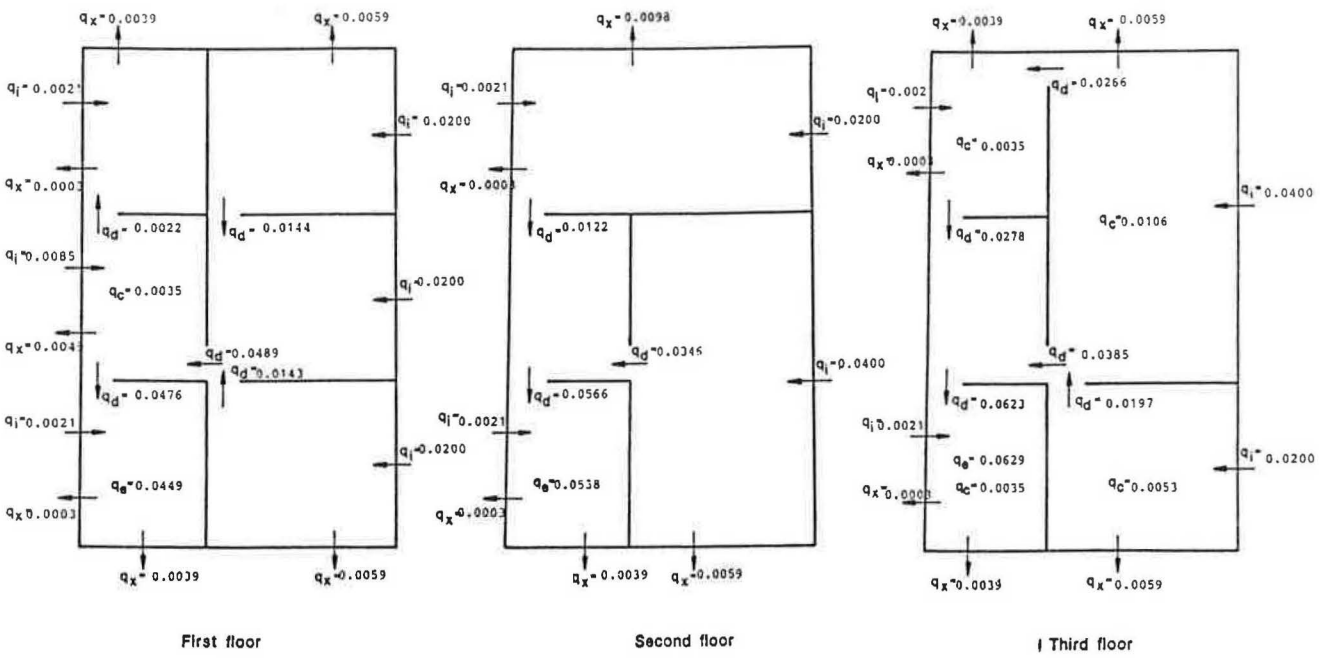


Fig. 12. Infiltration rates, exfiltration rates, ventilation flow rates, and indoor air flow rates, θ_{in} = 20°C, θ = 0°, U = 4 m/s, θ_{out} = -10°C, k_t = 0.0002, all doors are open.

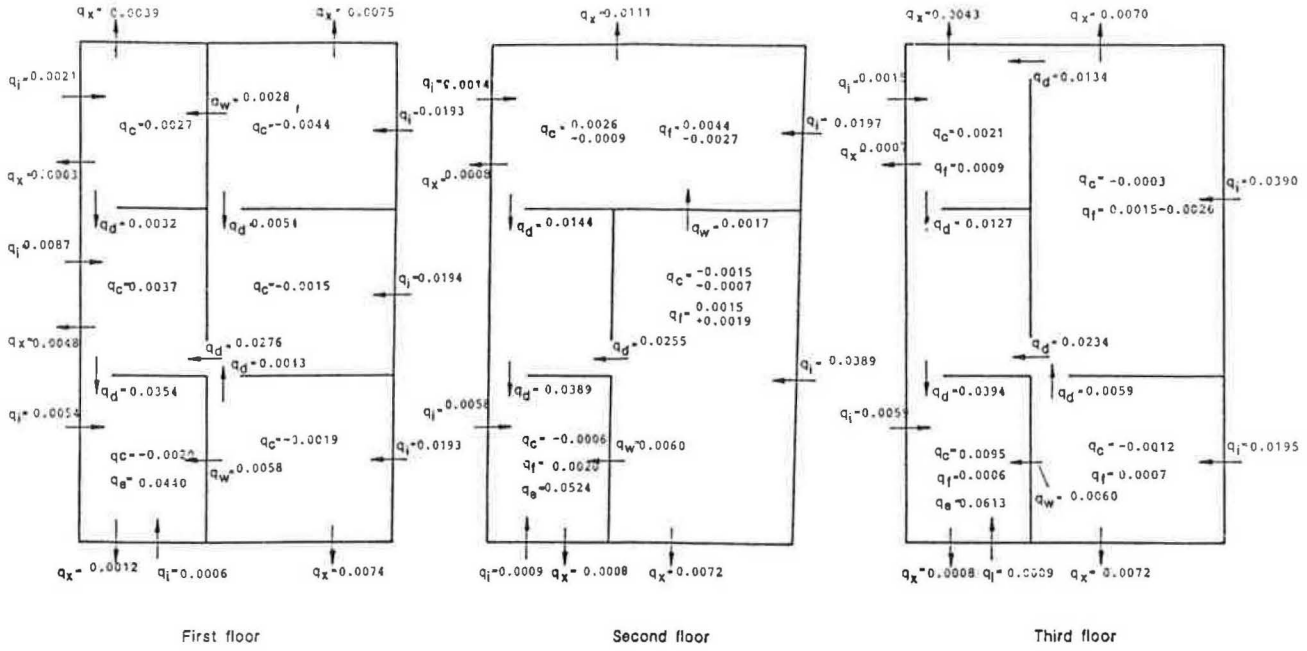


Fig. 13. Infiltration rates, exfiltration rates, ventilation flow rates, and indoor air flow rates, θ_{in} = 20°C, θ = 0°, U = 4 m/s, θ_{out} = -10°C, k_t = 0.0002, all doors are almost closed.

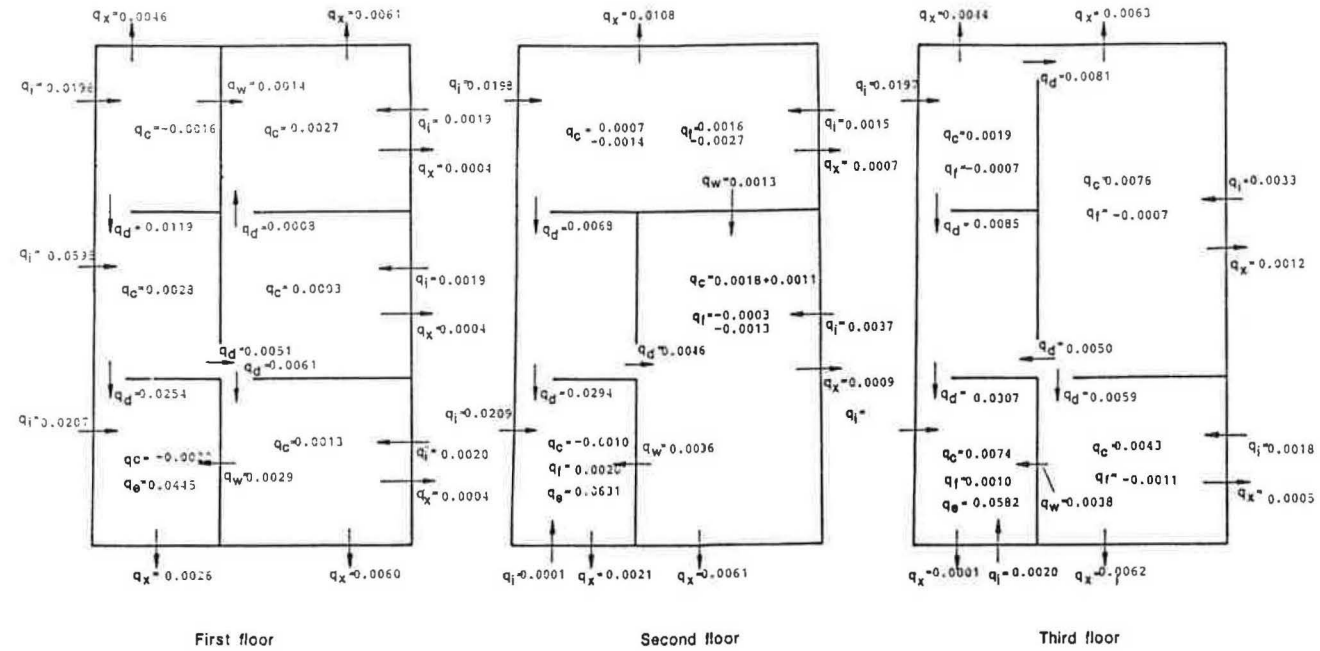


Fig. 14. Infiltration rates, exfiltration rates, ventilation flow rates, and indoor air flow rates, θ_{in} = 20°C, θ = 180°, U = 4 m/s, θ_{out} = -10°C, k_t = 0.0002, all doors are almost closed.

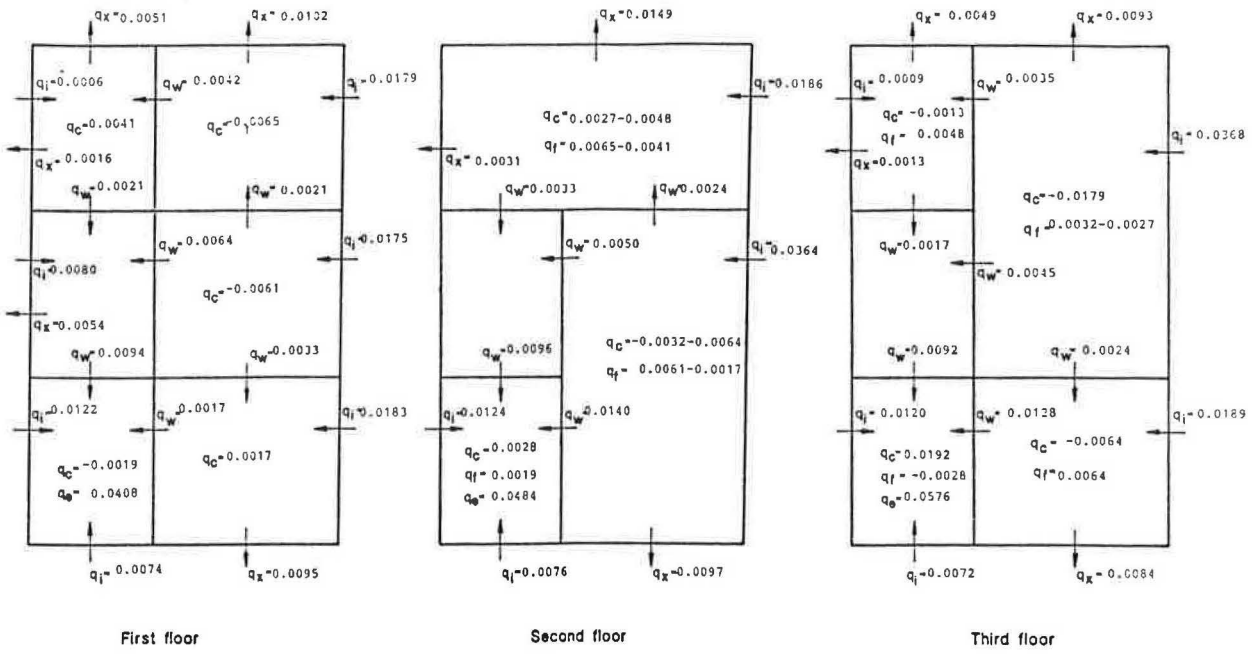


Fig. 15. Infiltration rates, exfiltration rates, ventilation flow rates, and indoor air flow rates, θ_{in} = 20°C, θ = 0°, U = 4 m/s, θ_{out} = -10°C, k_t = 0.0002, all doors are closed.

Exhaust ventilation rate

In different conditions, the exhaust ventilation rate will change from the design data due to the changing of the indoor air pressure. Fig. 16 gives the effect of room pressure on the exhaust fan total pressure. If room pressure is above atmospheric pressure then the exhaust toal pressure will decrease, and the exahsut ventilation will increase, see Fig. 5. Conversely, if the room pressure is below atmospheric pressure, an increase in exhaust fan total pressure will result, and the exhaust ventilation rate will decrease, see Fig. 5, again. Figs. 17, 18, and 19 show the influence of k_t, status of the internal doors and outside temperature on the changing of the exhaust ventilation rate. The common point was that the exhaust ventilation rate decreased when the wind speed increased because the infdoor pressure decreased. Closing an internal door, related to the ventilated room, tightening the envelope, or a reduction in the outside temperature will cause the exhaust ventilation rate to decrease.

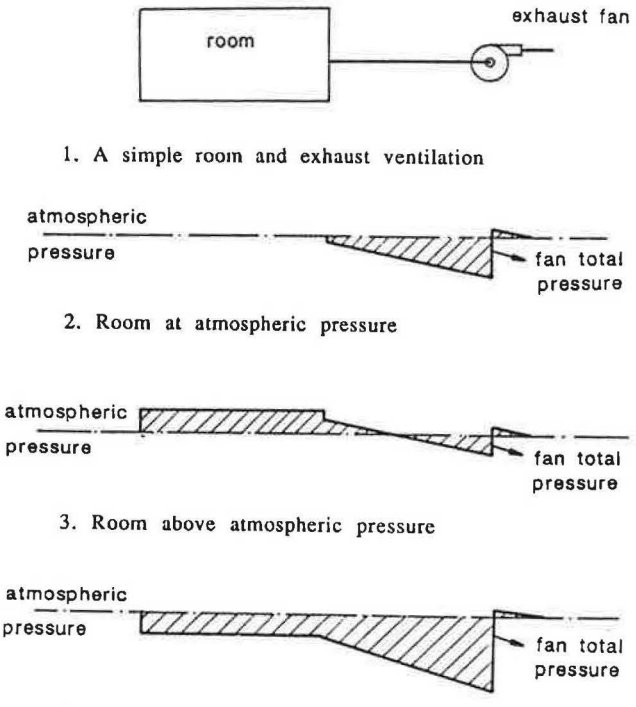


Fig. 16. Effect of room pressure on exhaust ventilation.

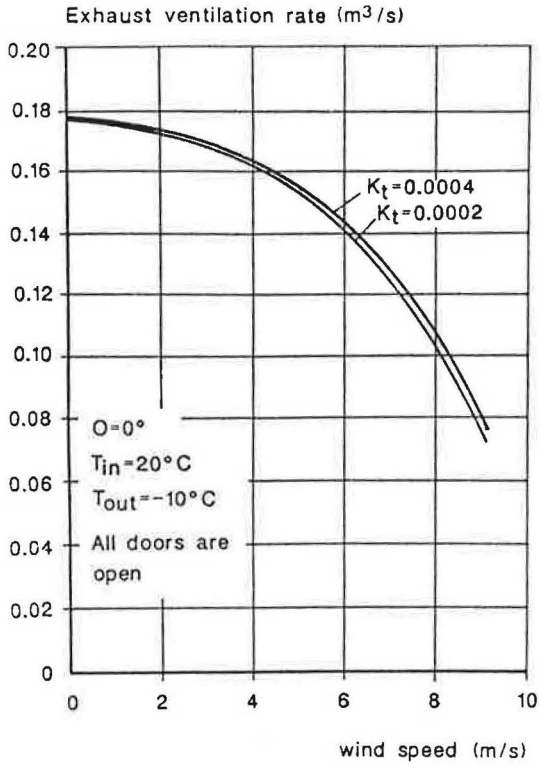


Fig. 17. The changing of the exhaust ventilation flow rates for different k_t-values.

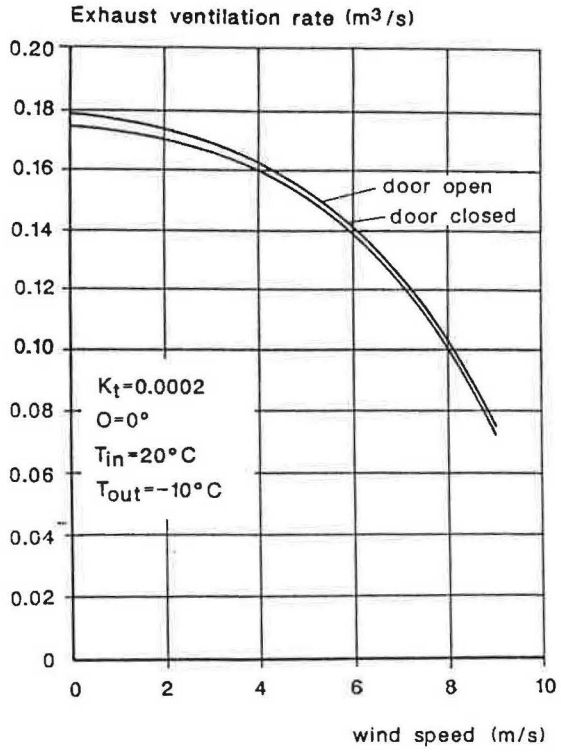


Fig. 18. The changing of the exhaust ventilation flow rates for door open and closed.

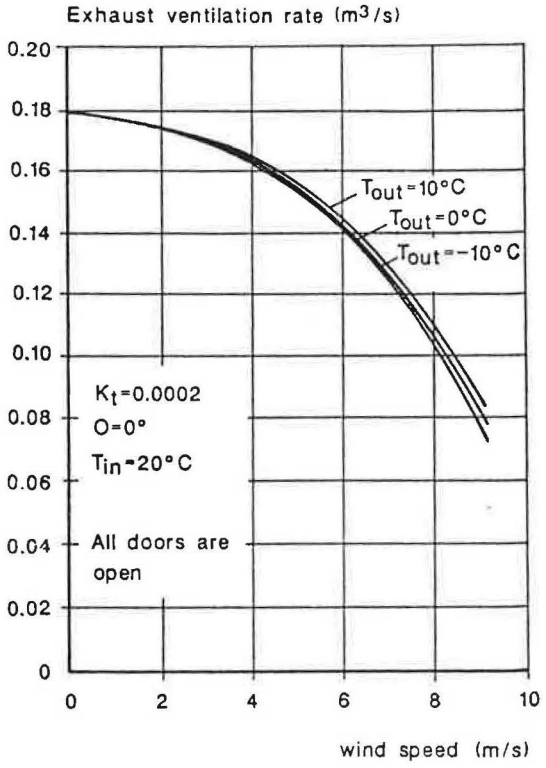


Fig. 19. The changing of exhaust ventilation flow rates for different outdoor temperature.

Wind speed

The results also show that the ventilation is dominated by the exhaust system at low wind speeds, but increasingly influenced by climate at higher wind speeds. The threshold wind speed at which climate influences the ventilation rate is about 2-4 m/s, see Fig. 20. Fig. 21 shows the detailed infiltration, exfiltration, and exhaust ventilation rate with different wind speeds. There are different results for different faces of the envelope. Take, for example, the building roof, when the wind speed is less than 5-6 m/s, it is an infiltration surface, but when the wind speed is greater than 5-6 m/s, it becomes an exfiltration surface, see Fig. 21.

Summary

The present multi-room indoor air flow model MIX can be used to calculate the infiltration, exfiltration, detailed indoor air flow, indoor pressure distribution, and interaction with the ventilation system. The single version SIX has been checked against the measurement data and has shown a good

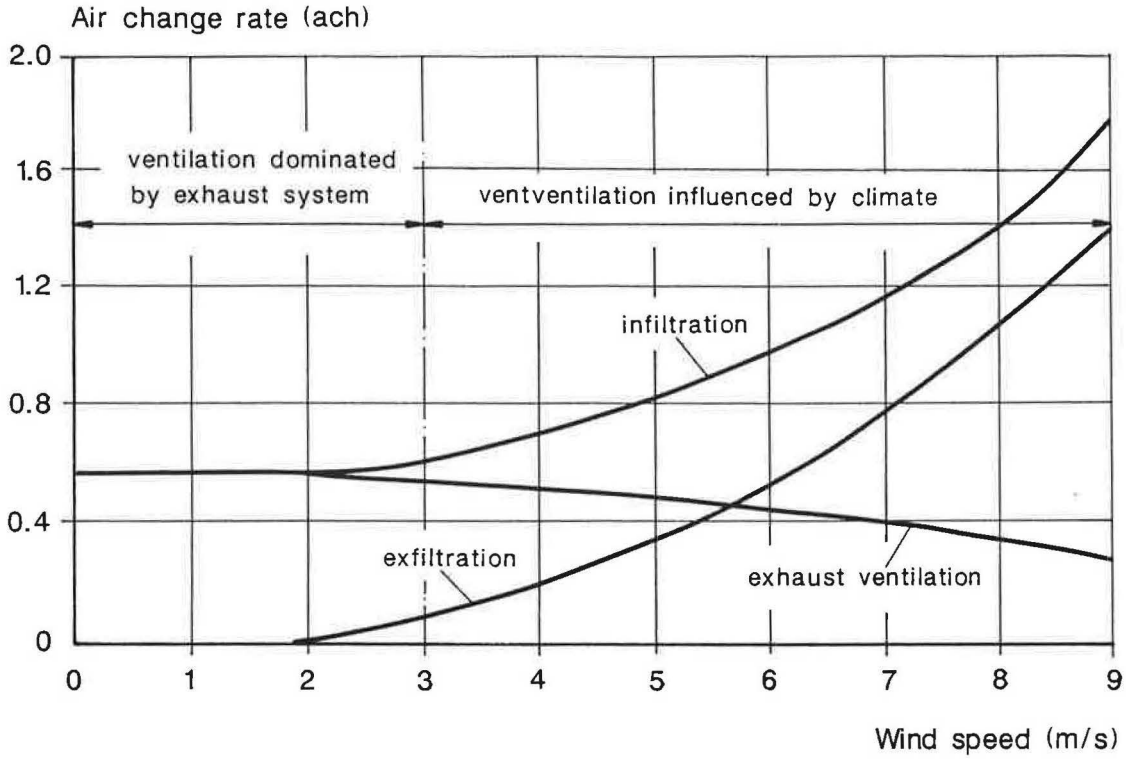


Fig. 20. Infiltration rates, exfiltration rates, exhaust ventilation flow rates for different wind speeds.

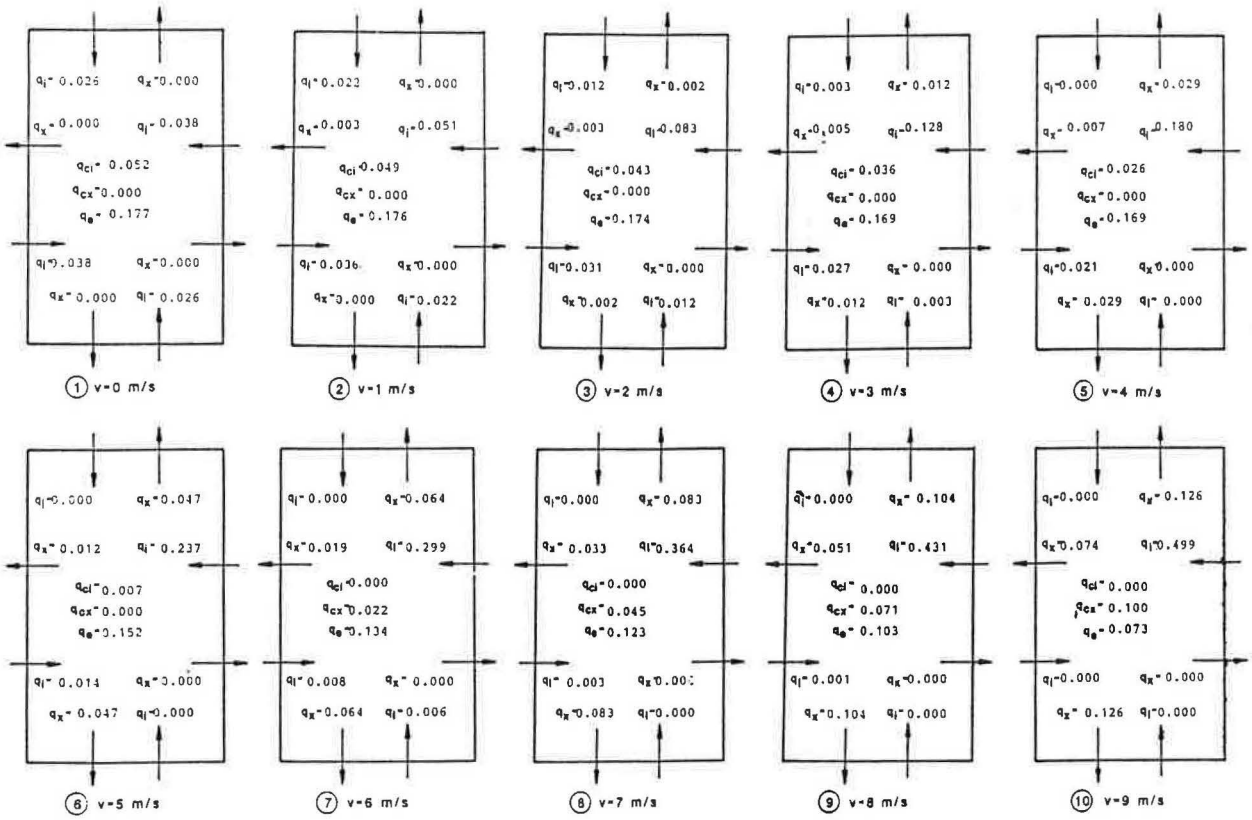


Fig. 21. Infiltration rates, exfiltration rates, and exhaust ventilation flow rates for every faces for different wind speeds.

agreement, see Li (1990). MIX will be a useful tool for further study of indoor air quality problems.

The results in the example indicate that if the openings of a multi-room building are much larger than the envelope openings there is a uniform internal pressure. In this case the single-cell model can be used. However, if the inter-room openings are smaller, then the single-cell model becomes redundant and the multi-room model should be employed.

The exhaust ventilation rate will change from the design data due to the changing of the indoor air pressure. If room pressure is above atmospheric pressure, the exhaust ventilation rate will increase, and if the indoor pressure is below the atmospheric pressure, then the exhaust ventilation rate will decrease. It has been shown that when the exhaust fan is stronger, the influence of infiltration on the exhaust ventilation system should be less.

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