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The Influence of Turbulent Wind on Air Change Rates—a Modelling Approach

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Turbulence in wind-induced pressures on a building envelope causes fluctuating air infiltration. The resultant airflows are influenced both by building characteristics, the resistance of the openings to flow, the inertia of the air mass in the openings and the compressibility of room air, and by frequency characteristics of wind pressures, their power spectra and the correlation among them. A new approach using the spectrum analysis technique is proposed to model the pulsating flows through openings of a building. The governing equations for fluctuating airflow behaviour are obtained from the pressure/force balance between the turbulence pressure differences across openings and the forces required to overcome the flow resistance and inertia of air. The proposed approach is applied to a single-zone enclosure with a single opening and with two openings, and can be easily extended to multi-zone buildings.

NOMENCLATURE

- Q, $\bar{Q}$, q airflow rate, mean airflow rate, turbulent components of airflow rate, m^3/s
- $Q(\omega)$ Fourier transform of fluctuating airflow rate q
- ΔP, $\Delta \bar{P}$, Δp pressure difference across an opening, its mean value, its turbulent component, Pascal
- C_d discharge coefficient
- A, L opening size and depth, m^2, m
- V, Δv air volume in a room, volume change, m^3
- K fluctuating airflow parameter
- B $\gamma P_a/V$
- P_a standard atmospheric pressure, Pascal
- P_w turbulent component of wind-induced pressure, Pascal
- f, ω frequency, $f=\omega/2\pi$, Hz, rad
- σ root-mean-square (RMS) value
- j $j=\sqrt{-1}$
- $S_{pw}(f)$, $S_q(f)$, $S_{pi}(f)$ power spectra of wind-induced pressure, airflow rate, and internal pressure
- $S_u^*(f)$ normalized wind velocity spectrum proposed by Davenport [23]

INTRODUCTION

FLUCTUATING air infiltration through buildings is a complex process, and has yet to be studied. Turbulence in driving forces, especially the wind-induced pressure on building external envelopes, has been recognized ever since the start of infiltration studies. However, due to the complexity of the problem, assumptions were made to consider only mean values. Only a few studies have attempted to model the fluctuating air infiltration caused by turbulent wind driving forces.

The phenomenon of air infiltration or air exchange due to turbulent wind pressure has been observed by many researchers. The causes can be due to several factors: pulsating flow, penetration of eddies and static or molecular diffusion [1]. The effect of fluctuating infiltration is especially significant when the mean pressure differences across openings are low while their turbulent components are large. This happens in several types of situations, such as single-opening enclosures, two parallel openings [2], when wind effect and stack effect are neutralizing each other [3], and flow reversal [4], etc. A few field experiments [2, 5–8] have been conducted to investigate the effects of wind turbulence on the infiltration, and found some discrepancy between measured air exchange values and those calculated by steady-state prediction procedures or models.

Fluctuating infiltration can be divided into two categories: eddy flow and pulsation flow (Fig. 1). Eddy flow

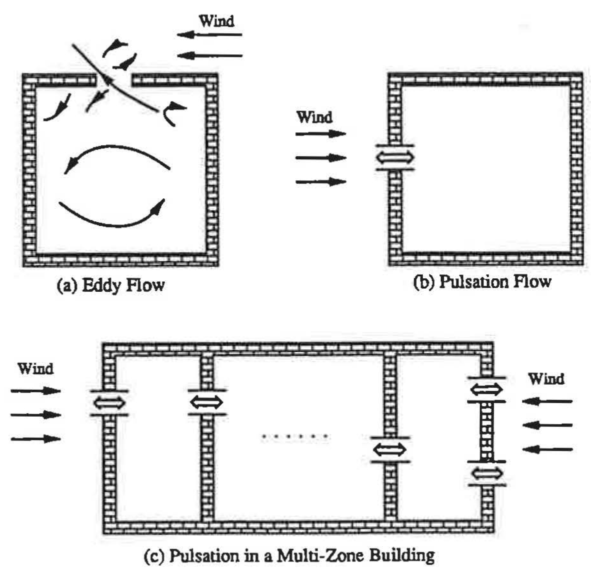

Fig. 1. Eddy flow and pulsation flow.

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represents the additional air exchange across an opening due to the penetration of eddies. Pulsation flow is the result of bulk fluctuating flow due to turbulence in the pressure difference across the opening.

Turbulence in wind-induced pressures on the surfaces of a building is due to gustiness of the wind and the interactions between the wind and the building envelope, and displays statistical regularity [9]. Although frequency characteristics of wind and its induced pressures have been studied in wind engineering [9, 10] and been used for wind load calculations, their applications in the field of air infiltration studies have yet to be examined. Nevertheless, the importance of the frequency characteristics of driving forces and the resultant airflow has been noticed. Haghighat *et al.* [11] developed a stochastic method to investigate the stochastic nature of wind on the air infiltration rate. This method provides a rational and convenient way of handling uncertainty in the analysis and design of ventilation systems, simplifies the data handling and provides the mean and standard deviation of the infiltration airflow rate. Lay and Bragg [12] attempted to employ statistical tools in representing data from experiments. Hill and Kusuda [5] calculated power spectra of wind-induced pressures. Crommelin and Vrins [13] measured power spectra of pressures on a scale model in a wind tunnel study and discussed factors that cause turbulence. Gusten [7] stressed the necessity of spectrum considerations in the calculation of fluctuating infiltration. Until now, a general theory or model has not been proposed to calculate the resultant fluctuating infiltration. As has been indicated by some research: "none of the general laws hitherto described enables us to quantify the effect of unsteady wind or of a large scale turbulence on the air exchange rate through an opening" [14].

The existing modelling approaches can be divided into several categories (Table 1). Cockroft and Robertson [15] studied the single-opening pulsation flow of an enclosure with a single opening subjected to a turbulent impinging air stream and derived a simple theoretical pulsation model to assist in understanding the physical phenomena causing airflow through the opening. The resulting theoretical solutions were compared to limited experimental data. Narasaki *et al.* [16] used this pulsation model of a single-opening enclosure and designed more comprehensive (tracer gas) testing to evaluate the effect of the wind incidence angle on the amount of turbulent airflow. Sasaki *et al.* [17] employed a dynamic analysis method to account for the fluctuating infiltration caused by wind turbulence. A building was modeled by its corresponding mechanical system consisting of mass, resistance and elastic forces. The resultant second order differential equations were solved to obtain responses excited by single-sine wind pressure. The model can be applied to multiple room buildings and to other problems such as the buffering effect of double sashed windows. A specially designed testing method has been used to provide experimental data for validation. Holmes [18] used second-order non-linear differential equations (similar to the equations in Ref. [17], but with non-linear resistance components) to model the internal pressure fluctuations due to turbulent winds. The equations were solved numerically with a lumped-impulse method. Results showed good agreements in the frequency domain with the wind tunnel scale-model testing data. Etheridge and his colleagues [4, 19] developed a multi-zone model which accounts for the pressure fluctuations due to the wind turbulence. The turbulent infiltration is assumed to be proportional to the RMS (root-mean-square) value of fluctuating wind pressure. The coefficients depend on the opening characteristics and empirical constants. Phaff and De Gidds [20] proposed a simple empirical correlation which describes the infiltration rate through an open window as a function of the temperature, wind velocity and fluctuating terms that account for the additional turbulent infiltration when the influence of wind and temperature is zero. Kazic and Novak [21] used a numerical (time-domain) simulation procedure, based on fluid mechanical principles (continuity equation, energy equation, Bernoulli's equation and the ideal gas equation) to solve the dynamic fluctuating infiltration.

Table 1. Approaches to fluctuating infiltration modelling

Model	Main features	Authors
Pulsation flow	Pulsation flow of single open enclosure is caused by longitudinal fluctuations of wind and the compressibility of air within the enclosure. Limited experiments.	Cockroft and Robertson [15]
	Effects of wind incident angles.	Narasaki *et al.* [16]
Mechanical system	Airflows through openings are modulated as mechanical system. Infiltration can be calculated for pressures of single frequency. Incorporation into building mechanical system. Specially designed tests.	Sasaki *et al.* [17]
	Fluctuating internal pressures are obtained numerically from second-order non-linear differential equations. Compared with wind tunnel test data in the frequency domain.	Holmes [18]
Parametric coefficient	Fluctuating infiltration is assumed to be proportional to RMS value of wind pressures, the coefficients depend on opening characteristics and empirical constants. Used in multi-zone air infiltration model.	Etheridge and Alexander [19]
	Empirical formula.	Phaff and De Gidds [20]
Numerical fluid mechanical	Governing equations deriving from fundamentals are solved numerically.	Kazic and Novak [21]

The emphasis of this paper is to present a new approach to model the pulsation flows induced by the turbulent wind pressures acting upon the building envelope. The fluctuating flow rates depend upon the characteristics of openings, the interconnection of openings and the rooms, the characteristics of the turbulent wind-induced pressures on individual openings, and the correlation among these turbulent pressures.

APPROACH

The mathematical study of random processes reveals that the turbulent pressure, as a random variable, can be considered to be composed of an infinite series of simple sine waves of different frequencies. Studies in wind engineering indicate that the contribution of the sine wave of a particular frequency to the total turbulent pressure is invariant with time for a given type of wind. This contribution by each frequency to the total pressure can be more precisely represented by the concept of the power spectrum function. A turbulent pressure can be fully described and represented by its power spectrum function, and the calculation of the infiltration can be based on the spectral information of the wind-induced pressures.

In the light of the power spectrum concept, the complex problem of calculating the resultant fluctuating infiltration due to the turbulent pressures can be decomposed into an infinite series of simple problems, each one calculates the corresponding portion of infiltration caused by the simple sine wave pressure on a single frequency. The total fluctuating infiltration can then be obtained by summing up all these infinitesimal portions of air infiltration at single frequencies.

The mathematical equivalent of the above description of the spectral analysis for fluctuating infiltration is the system theory. The relationship between the output (fluctuating air infiltration) and the input (turbulent wind pressures) of the linear system (building air infiltration system) is determinant and depends on the system characteristic (impulse response function). For the given driving forces, the frequency characteristics of the fluctuating infiltration can be calculated.

COMPONENT MODELLING

Modelling of fluctuating air infiltration involves two steps: component modelling and system modelling. Component modelling produces the mathematical descriptions of the individual components: the openings and rooms. In system modelling, governing equations which indicate the behaviour of the fluctuating air infiltration system are derived and solved.

Flow equation

The application of the spectral analysis approach is based on the linear assumption of fluctuating flow. At the present stage, only the orifice type of opening is considered. Let the total airflow rate through (Q) and the total pressure difference across (ΔP) an opening be composed of the mean ($\bar{Q}$ and $\bar{P}$) and the turbulent (q and p) components:

$$Q = \bar{Q} + q \tag{1}$$

$$\Delta P = \Delta\bar{P} + \Delta p. \tag{2}$$

The steady-state (mean) flow equation is expressed as:

$$\bar{Q} = C_d A \sqrt{\frac{2 \times \Delta\bar{P}}{\rho}} \tag{3}$$

where C_d is the discharge coefficient and A is the effective opening size.

It is assumed that the fluctuating airflow through the opening is governed by a linear relation as:

$$q = \frac{d\bar{Q}}{d(\Delta\bar{P})} \times \Delta p. \tag{4}$$

Carrying out the differentiation, it yields:

$$q = \frac{C_d A}{\sqrt{\rho \Delta\bar{P}}} \times \Delta p. \tag{5}$$

Let

$$K = \frac{\sqrt{2\rho\Delta\bar{P}}}{C_d}, \tag{6}$$

then:

$$q = \frac{A}{K} \times \Delta p \tag{7}$$

or

$$\Delta p = \frac{K}{A} q. \tag{8}$$

The turbulent fluctuating flow equations (7) and (8) can be graphically illustrated in Fig. 2a. The resistance to the fluctuating component of the airflow, when the mean pressure difference is $\Delta\bar{P}$, follows a linear line tangential to the steady-state flow line.

In cases where the mean pressure difference $\Delta\bar{P}$ is very small or approaches zero, this fluctuating flow relation given by equations (7) and (8) is no longer valid. These equations can be generalized to cover this situation by arbitrarily choosing the K value in a way such that the fluctuating flow is represented by a linear line as in Fig. 2b. For the case $\Delta\bar{P} = 0$, the value for K is chosen as:

$$K = 0.527 \frac{\sqrt{2\rho\sigma_{\Delta p}}}{C_d} \tag{9}$$

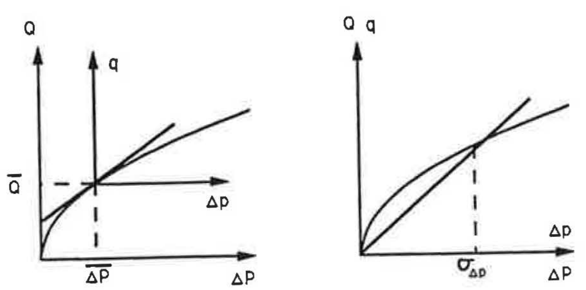

Fig. 2. Interpretation of fluctuating flow equation.

where $\sigma_{\Delta p}$ is the root-mean-square (RMS) value of the turbulent pressure difference Δp.

Compressibility of room air

The air in a room is assumed to be an ideal gas and is compressible. Initially, the room air has a volume of V, at a pressure P_0. When a pressure P ($P-P_0 = \Delta P$) is applied to the room air, the volume of the original air in the room is decreased to a smaller volume, $V-\Delta v$. The volume decrement, Δv, will be equal to the amount of outside air that is pressurized into the room, and therefore an air exchange is caused in the process.

Assume an adiabatic process for the room air and a small pressure change ($\Delta P/P_0 \ll 1$), then:

$$\Delta P = \frac{\gamma P_0}{V}\times\Delta v \doteq \frac{\gamma P_a}{V}\times\Delta v = B\cdot\Delta v \quad (10)$$

where P_a is the standard atmospheric pressure, and

$$B = \frac{\gamma P_a}{V}. \quad (11)$$

Since

$$\Delta v = \int_0^t \sum q_i \, dt \quad (12)$$

and the pressure change is equal to the turbulent component of internal pressure, $\Delta P = p^i$, therefore the turbulent internal pressure, p^i, can be written as:

$$p^i = B\int_0^t \sum q_i \, dt. \quad (13)$$

Inertial force

When the air flowing through an opening is fluctuating, the air is either accelerated or decelerated. According to Newton's second law of motion, a force is required to cause this fluctuating change in airflow rate:

$$F_i = ma = (\rho AL)\frac{d(q/A)}{dt}. \quad (14)$$

Therefore the pressure difference (across the opening) required to balance this inertial force can be expressed as:

$$f_i = \frac{F_i}{A} = \frac{\rho L}{A}\frac{dq}{dt}. \quad (15)$$

SYSTEM MODELLING—SINGLE OPENING CASE

The case of the fluctuating infiltration of a single opening enclosure is special. The mean airflow rate through the opening is zero. In reality, air exchange does occur between the inside and outside. Several studies have been conducted to examine this case with similar findings [5, 15].

An example of this case is shown is Fig. 3, a building with volume 1000 m^3 has an opening with flow area A and depth L located on the windward facade, with the wind blowing perpendicular to it (Fig. 3). Let p^w, p^i, Δp and q be the fluctuating components of wind-induced pressure, internal pressure inside the enclosure, pressure

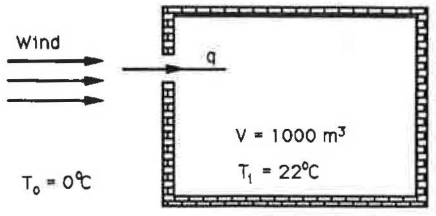

Fig. 3. A single-opening enclosure.

difference across the opening, and the flow rate through the opening, respectively.

The system of governing equations is derived from the pressure/force balance across the opening. The pressure difference across the opening, which equals the difference between external wind pressure and the internal pressure ($p^w - p^i$), is balanced by the force required to overcome the flow resistance and the inertia of air in the opening. Utilizing equations (8), (13) and (15), this balance relation can be expressed as:

$$\frac{K}{A}q + \frac{\rho L}{A}\dot{q} = p^w - B\int_0^t q\,dt. \quad (16)$$

Applying Fourier transform to both sides of equation (16), the system equation can be written in the frequency domain as:

$$\frac{K}{A}Q(\omega) + j\frac{\rho L}{A}\omega Q(\omega) = p^w - B\frac{1}{j\omega}Q(\omega), \quad (17)$$

or by simplification:

$$[(BA-\rho L\omega^2)+jK\omega]\cdot Q(\omega) = jA\omega P^w(\omega). \quad (18)$$

For a given wind-induced pressure $P^w(\omega)$, the resultant airflow through the opening can be expressed by:

$$Q(\omega) = \frac{jA\omega}{[(BA-\rho L\omega^2)+jK\omega]}\times P^w(\omega) = H(\omega)\cdot P^w(\omega), \quad (19)$$

where $H(\omega)$ is called the transfer function.

This equation indicates that, in the frequency domain, the airflow through the single opening can be expressed as a linear function of the wind pressure. According to the properties of linear system [22], when the spectrum of the wind pressure, $S_{P^w}(\omega)$, is known, the airflow spectrum can be calculated as:

$$S_q(\omega) = \|H(\omega)\|^2\cdot S_{P^w}(\omega). \quad (20)$$

The root-mean-square (RMS) value of the fluctuating infiltration can be calculated by integrations of the spectra over the frequency.

$$\sigma_q = 2\pi\int_0^\infty S_q(\omega)\,d\omega$$

$$= 2\pi\int_0^\infty \left\|\frac{jA\omega}{(BA-\rho L\omega^2)+jK\omega}\right\|^2 S_{P^w}(\omega)\,d\omega. \quad (21)$$

The wind pressure spectrum as input can be obtained either experimentally or by means of empirical formulae. To simplify the calculation procedure, it is assumed that the turbulence in wind pressure is caused only by the gustiness in wind velocity, and the empirical formula for wind velocity spectrum of Davenport [23] is employed. Therefore, the power spectrum of the wind pressure can be expressed in

$$S_{p^w}(f) = \sigma_{p^w}\times S_u^*(f) = 2I\bar{P}^w\times S_u^*(f) \quad (22)$$

$$S_u^*(f) = \frac{1}{\sigma_u^2}\times\frac{2/3\sigma_u^2}{f}\times\frac{x^2}{(1+x^2)x^{4/3}} \quad (23)$$

where $S_u^*(f)$ is the normalized wind velocity spectrum, the frequency $f = \omega/2\pi$ Hz, σ_{p^w} and σ_u are the RMS values of wind pressure and wind velocity, I is the turbulence intensity of wind velocity $I = \sigma_u/v_z$, and $x = 1200f/V_{10}$ and V_{10} is the wind velocity at 10 m height.

According to equation (13), the internal pressure of this single-opening enclosure, in frequency domain, can be expressed as:

$$S_{p^i}(\omega) = \left\|B\frac{1}{j\omega}\right\|^2 S_q(\omega) = \left\|\frac{BA}{(BA-\rho L\omega^2)+jK\omega}\right\|^2 S_{p^w}(\omega). \quad (24)$$

The RMS value of internal pressure can be calculated in a similar fashion as in equation (21), i.e. by integration of the spectrum function.

For the given values listed in Table 2, the following results are obtained. The RMS value of fluctuating infiltration rate is $\sigma_q = 0.06057$ m^3 s^{-1} = 218 m^3 h^{-1}. For an opening size of $A = 0.05$ m^2, this value is quite large. The internal pressure has a RMS value almost as large as that of the external wind pressure acting on the opening, $\sigma_{p^i}/\rho_{p^w} = 96\%$. The RMS values of pressure difference across the opening ($\Delta p = p^w - p^i$), $\sigma_{\Delta p} = 4.22$ P$_a$.

The related spectra reveal more detailed information. Figure 4 shows the spectra and transfer function involved. The wind pressure spectrum ranges from 10^{-4} to 10^{-1}, which means the pressure fluctuation is more intense for the cycles between 10,000 and 10 s. The peak occurs at about $10^{-2.1}$ Hz, which indicates the wind fluc-

Table 2. Parameters used for single-opening case

Building parameters	
Volume $V = 1000$ m^3, $\gamma = 1.4$, $P_a = 101{,}325$ Pa	
Temperature $T_{in} = 22$°C, $T_{out} = 0$°C	
Roof height $H = 6$ m	
Opening characteristics	
Type orifice	
Crack area $A = 0.05$ m^2, depth: $L = 0.2$ m	
Discharge coefficient	$C_d = 0.6$
Opening position	$Z = 5$
Wind data	
Velocity at weather station	$V_{10} = 10$ m/s
Terrain type at building site	urban
Pressure coefficient	$C_p = 0.8$
Turbulence intensity	$I = 0.16$
Derived values	
Wind speed at roof height	$V_H = 8.76$ m/s
Mean wind pressure	$\bar{P}^w = 39.4$ Pascals
RMS of wind pressure	$\sigma_{p^w} = 12.52$ Pa
Results	
$\sigma_q = 0.06057$ m^3/s	$\sigma_q/(C_d A\sqrt{\rho\sigma_{\Delta p}/2}) = 0.57$
$\sigma_{p^i} = 12.02$ Pa	$\sigma_{p^i}/\sigma_{p^w} = 0.96$
$\sigma_{\Delta p} = 4.22$ Pa	$\sigma_{\Delta p}\sigma_{p^w} = 0.34$

tuation has an approximate return period of 120 s (Fig. 4a). The transfer function $H(\omega)$, shown in Fig. 4b, indicates the relation between p^w and q. It represents the characteristics of the opening combined with the space of the enclosure. $\|H(\omega)\|^2$ has a peak at a frequency about $10^{0.9}$ Hz, which is much higher compared to the spectrum of wind pressure. The "band width" of $\|H(\omega)\|^2$ ranges from 0.2 to 3 Hz. The spectrum of q is determined by the interaction between $S_{p^w}(\omega)$ and $\|H(\omega)\|^2$. Since the spectra of p^w and $\|H(\omega)\|^2$ do not occupy the same frequency range, and only nearly overlay in a narrow band width, the resultant $S_q(\omega)$ is small (Fig. 5). The $S_q(\omega)$ sits between peaks of $S_{p^w}(\omega)$ and $\|H(\omega)\|^2$. Compared to wind turbulence, the frequency of fluctuation in q is quite high. The turbulent pressure difference across the opening is about one-third of that of p^w, although σ_{p^i} is approximately equal to σ_{p^w}. This is because the wind pressure and internal pressure do not change synchronously. There is a phase difference between p^w and p^i (Fig. 6). This creates a pressure difference and thus the infiltration.

The RMS value of fluctuating infiltration calculated by equation (21) is only a theoretical flow rate. In practice, the effective infiltration, which represents the actual air exchange rate between room air and outside air, is used. In case of the single-opening enclosure (where $\Delta\bar{P} = 0$), there are two factors that influence the conversion from σ_q to effective infiltration.

The first factor is due to the opening depth. The air in an opening must be displaced by at least the depth of the opening to enable the outside air to enter the building, or the inside air to leave the building. When the air flow rate is fluctuating, the actual amount of air that has been exchanged between inside and outside is less than σ_q (it is actually an amount of, say, σ_q'). The relationship between σ_q' and σ_q is complex, especially when the air displacement is considered to be a random process. This relationship is a function of frequency. At low frequency, the direction change of airflow through the opening is less frequent, and σ_q' is close to σ_q. At high frequency, more air exchange is lost in the openihg due to the high rate of direction change of airflow in the opening.

The second factor that affects the actual infiltration is the mixing process. If any air entering the building is mixed with the room air *immediately*, then σ_q' would indicate the actual infiltration. However, the mixing of the outside air with the room does take time (and is affected by many factors, such as the air movement field in the room, temperature difference between inside and outside, etc.). Some of the incoming air from outside might be removed through the opening to outside before mixing, due to the airflow direction change.

A sensitivity study is performed next to investigate the influence of the involved parameters, including the opening area, the opening depth, the volume of the enclosure, the wind velocity and the turbulence intensity.

1. *The influence of opening size*

Figure 7 shows the variation of the RMS value of the pulsation flow rate with respect to the change of the opening size. All other parameters related to the building, opening and climate remain unaltered as listed in Table 2. The plot indicates that the pulsation flow rate increases

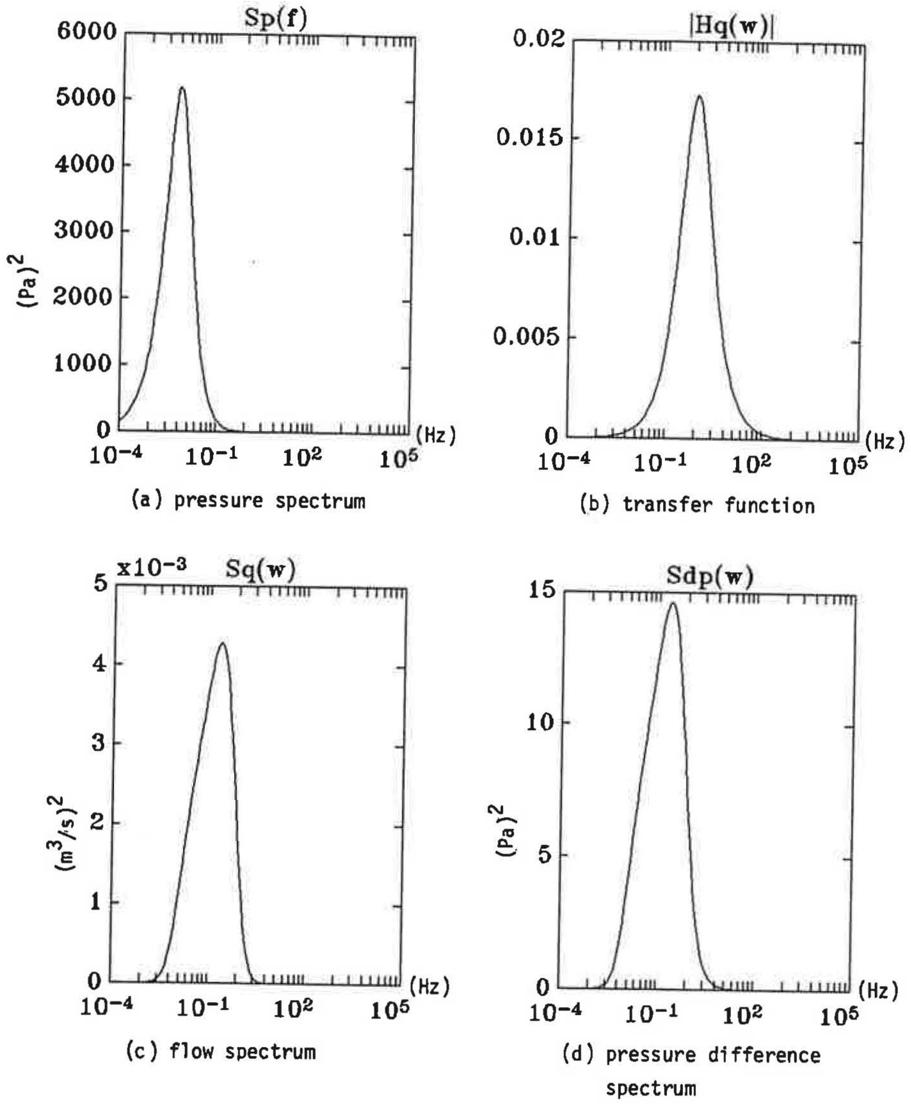

Fig. 4. Related spectra and transfer function of one-opening enclosure.

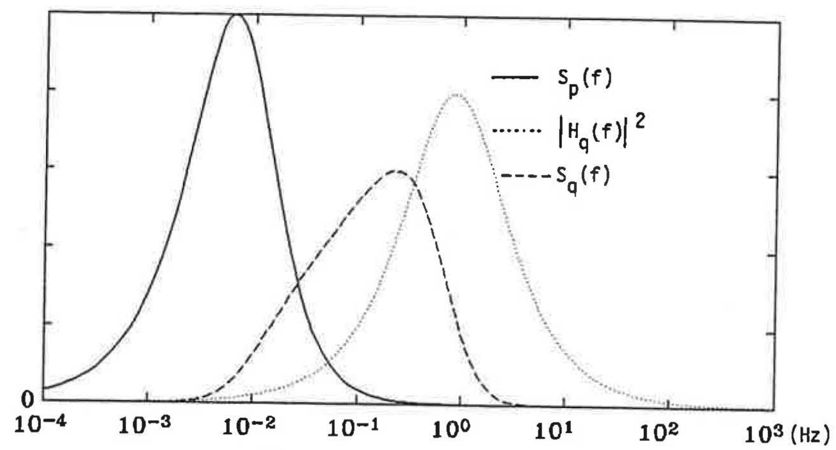

Fig. 5. Positions of related spectra.

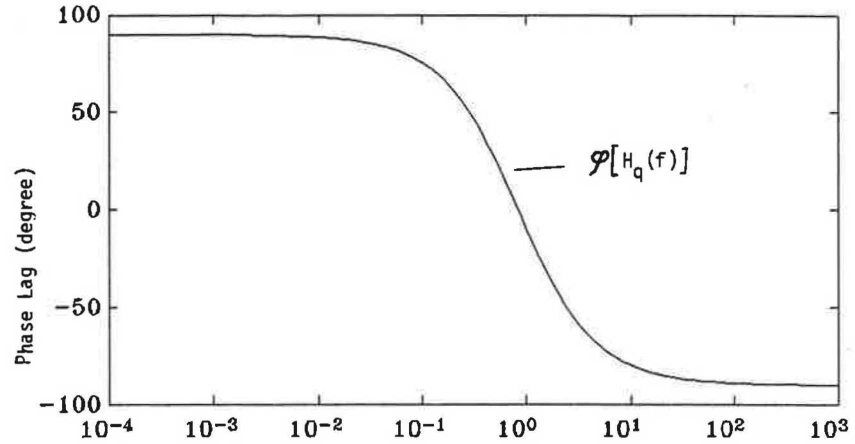

Fig. 6. Phase leg of flow spectrum to pressure.

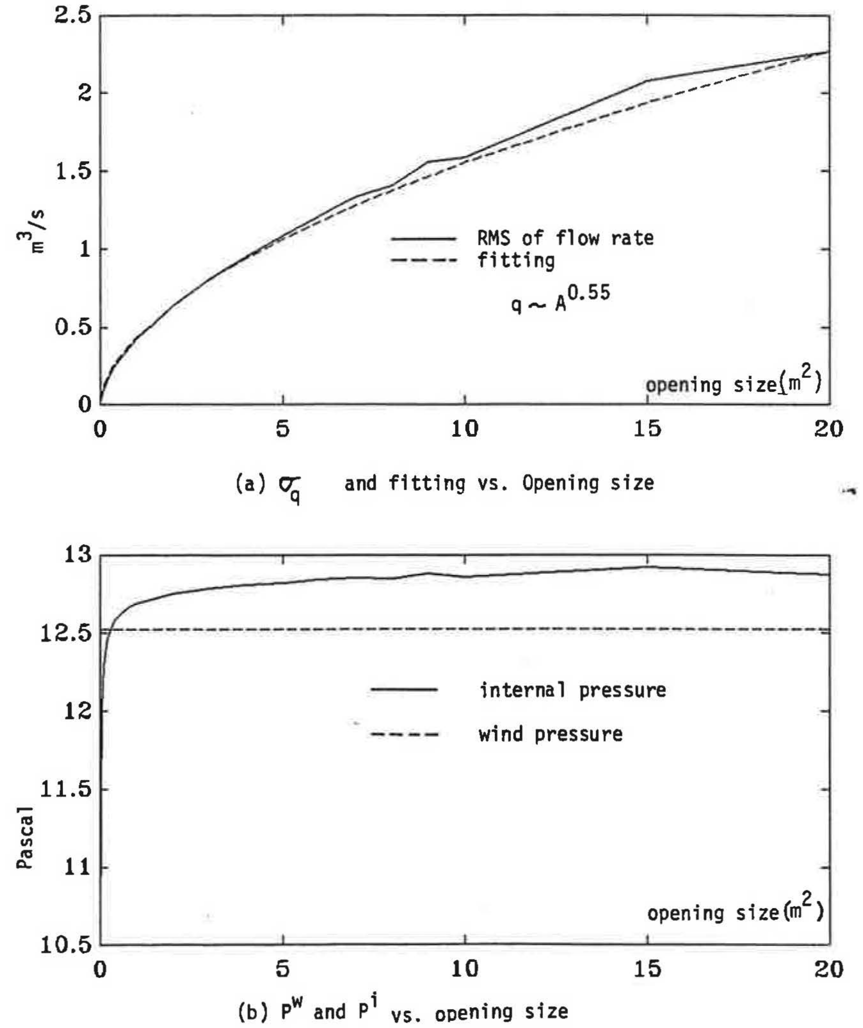

Fig. 7. Effect of opening size.

while the opening is enlarged. However, the rate of increase of σ_q is less than the linear proportion to the opening size. Regression analysis of the results shows that σ_q is proportional to an exponent of the size, i.e.:

$$\sigma_q(A) \propto A^{0.55}. \quad (25)$$

This is because there is only one opening, the pulsation flow is limited by the volume within the enclosure and thus the amount of increase or decrease of space which can be obtained by pressurization or depressurization.

Equation (25) is different from the empirical correlation of Crommelin and Vrins [13] obtained from tracer gas measurements on a scale model in a wind tunnel. Their correlation indicates a high increase in σ_q when window size increases, and their exponent is equal to 0.92. This discrepancy is reasonable, since in this study only the pulsation flow is considered. For Crommelin and Vrins's correlation, the experiments did not limit the fluctuating infiltration to pulsation flow alone, and the eddy flow may have induced more air exchange. As the opening (window) size increases, the influence of eddy flow becomes more significant. Therefore, the rate of increase in σ_q is higher, compared to the results obtained solely from pulsation flow in equation (21).

The internal pressure fluctuation should increase with the opening size, but be limited to the wind pressure. Figure 7 shows the internal pressure change over the opening size. The plot shows that the RMS value of internal pressure rises higher than the wind pressure on the opening. This is contradictory to reality. The cause behind this is the assumption that the air in the opening can be considered as a rigid mass when the inertial effect of air is considered.

2. *The influence of opening depth*

The increase of opening depth will enlarge the volume (and thus the mass) of air in the opening, and thus increases the force that is required to overcome the inertia in the fluctuation. Therefore, it is expected that σ_q will decrease as the depth increases. The variation of σ_q with respect to the opening depth is plotted in Fig. 8a. The plot shows a steady decrease in the RMS value of q as the opening depth increases. The internal pressure increases only slightly along with the opening depth (Fig. 8b), while the RMS of pressure difference across the opening increases as the opening is deepened.

3. *The influence of enclosure volume*

The pulsation flow through the only opening of an enclosure is due to the compressibility of air within the

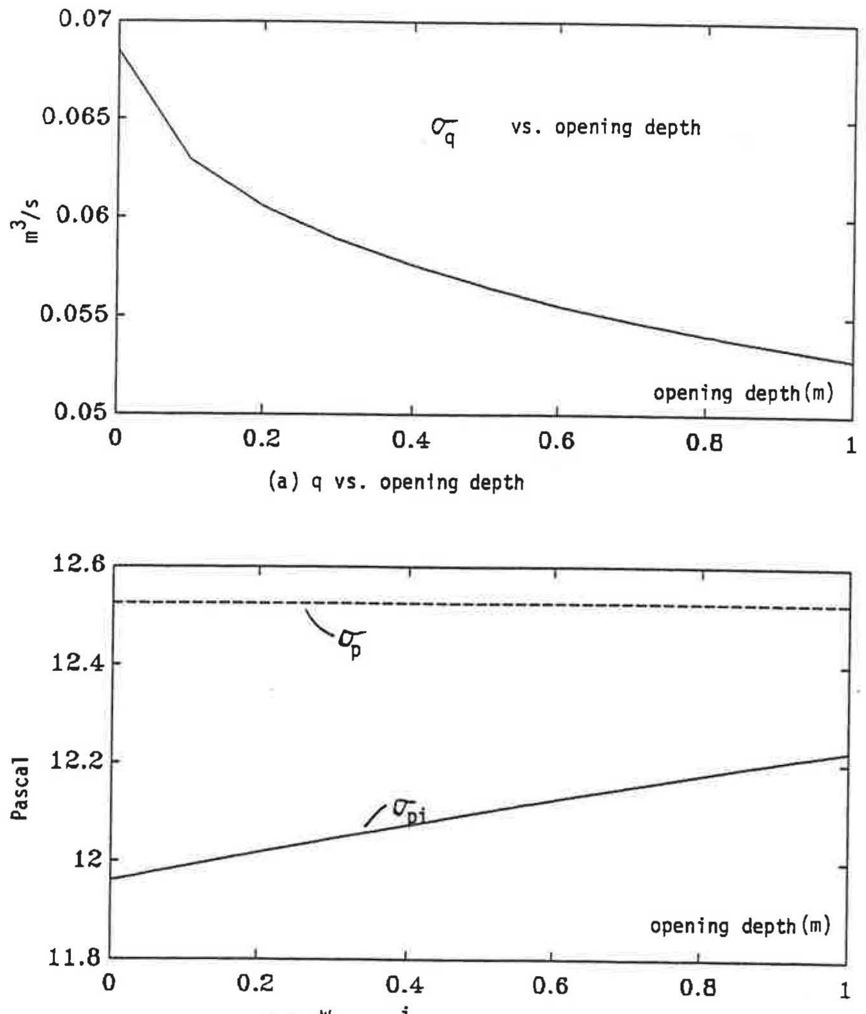

Fig. 8. Effect of opening depth variation.

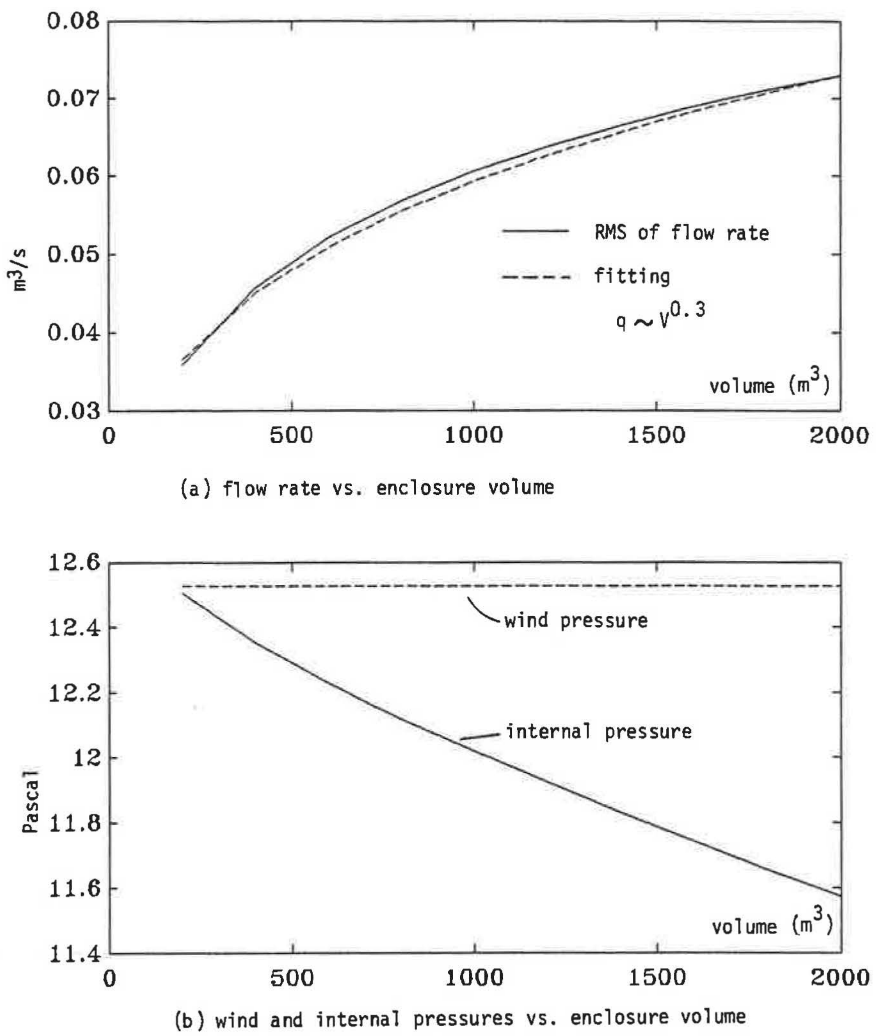

Fig. 9. Effect of room volume variation.

enclosure. The amount of air that can enter (or leave) the enclosure for a given pressure change is proportional to the enclosure volume. When the volume increases, there is more potential for the pressurization or depressurization. Therefore, the σ_q is expected to increase while other parameters remain unchanged. The variation of σ_q with respect to volume change is plotted in Fig. 9a. It can be seen that when volume is small, a change in the volume causes large change in σ_q. As the volume increases, the required force of pressurization and depressurization is less, the resistance force plays a dominant role, and thus further change in the volume causes less change in σ_q. The relation between σ_q and the volume is approximately:

$$\sigma_q \propto V^{0.3}. \quad (26)$$

With the increase in the volume, the RMS of internal pressure decreases slightly, the RMS of pressure difference across the opening increases (Fig. 9b).

4. *The influence of wind velocity and turbulence intensity*

The increase in both wind velocity and turbulence intensity increases the turbulent wind pressure on the external opening, which, in turn, augments the fluctuating infiltration.

Figure 10 shows the variation of σ_q with respect to the change in wind (weather station) velocity (V_{10}).

SYSTEM EQUATION—TWO OPENING CASE

A building with dimensions of $8 \times 8 \times 8$ m^3 has two openings which are located on the windward facade (Fig. 11).

The set of equations that govern the pulsation flows through the two openings is obtained from the force balance equations:

$$\frac{K_1}{A_1} q_1 + \frac{\rho L_1}{A_1} \dot{q}_1 = p_1^w - B\left[\int_0^t (q_1 + q_2)\,dt\right] \quad (27)$$

$$\frac{K_2}{A_2} q_2 + \frac{\rho L_2}{A_2} \dot{q}_2 = p_2^w - B\left[\int_0^t (q_1 + q_2)\,dt\right] \quad (28)$$

where p_1^w and p_2^w are the turbulent pressures at the two openings.

Equations (27) and (28) can then be transformed into the frequency domain (by Fourier transform) and be

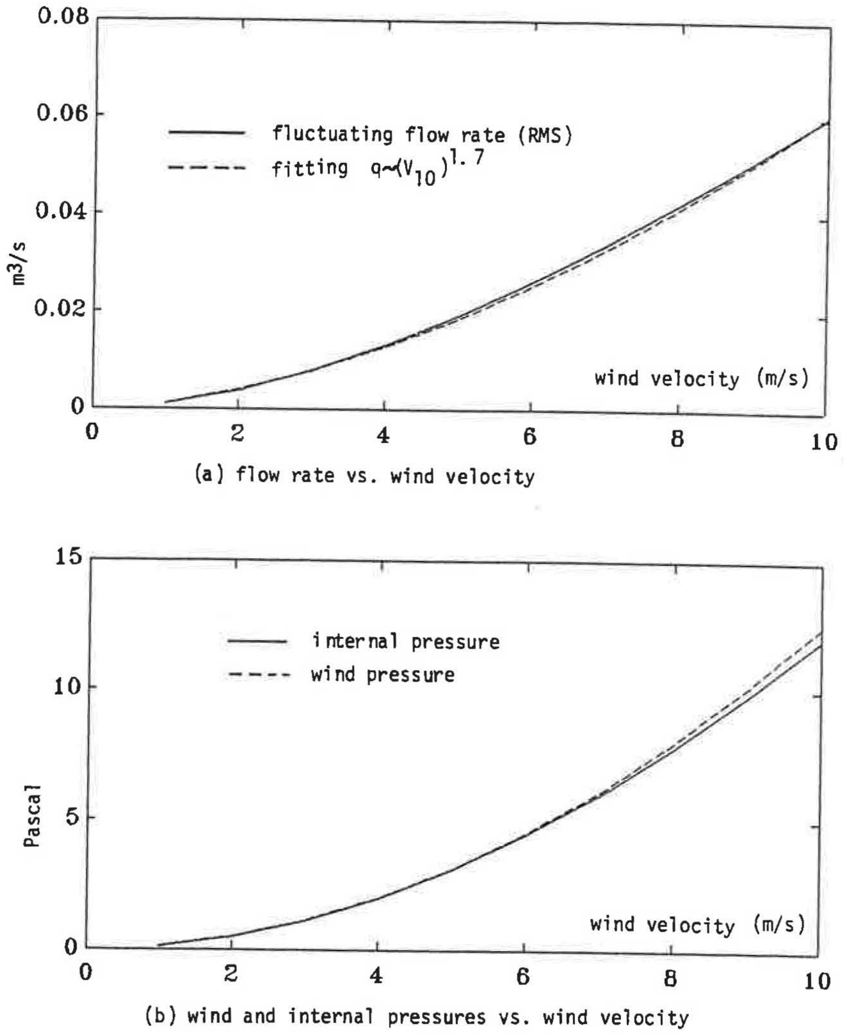

Fig. 10. Effect of wind velocity (V_{10}) change.

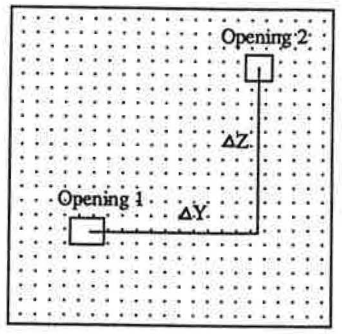

Fig. 11. The windward facade of a two-opening enclosure.

arranged into matrix form as:

$$\begin{bmatrix} BA_1-\rho L_1\omega^2+jK_1\omega & BA_2 \\ BA_1 & BA_2-\rho L_2\omega^2+jK_2\omega \end{bmatrix} \times \begin{bmatrix} Q_1(\omega) \\ Q_2(\omega) \end{bmatrix} = \begin{bmatrix} jA_1\omega & 0 \\ 0 & jA_2\omega \end{bmatrix} \begin{bmatrix} P_1^w(\omega) \\ P_2^w(\omega) \end{bmatrix}. \quad (29)$$

This set of equations can be solved as:

$$\begin{bmatrix} Q_1(\omega) \\ Q_2(\omega) \end{bmatrix} = \begin{bmatrix} BA_1-\rho L_1\omega^2+jK_1\omega & BA_2 \\ BA_1 & BA_2-\rho L_2\omega^2+jK_2\omega \end{bmatrix}^{-1} \times \begin{bmatrix} jA_1\omega & 0 \\ 0 & jA_2\omega \end{bmatrix} \begin{bmatrix} P_1^w(\omega) \\ P_2^w(\omega) \end{bmatrix}. \quad (30)$$

$$\begin{bmatrix} Q_1(\omega) \\ Q_2(\omega) \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} P_1^w(\omega) \\ P_2^w(\omega) \end{bmatrix}. \quad (31)$$

Therefore, when the spectra of wind-induced pressures and the correlation between them are known, the power spectra for the fluctuating airflows through the two openings can be expressed in the form of:

$$S_{q_i}(\omega) = \|H_{i1}\|^2 \cdot S_{p_1^w}(\omega) + \|H_{i2}\|^2 \cdot S_{p_2^w}(\omega) - 2\|H_{i1}\| \cdot \|H_{i2}\| \cdot S^{(c)}_{p_1^w p_2^w}(\omega) \quad i = 1, 2. \quad (32)$$

To simplify the data input procedure, the power spectra of (longitudinal) wind velocities are assumed to comply with Davenport's [23] formula and the turbulence in

wind pressures are caused by gustiness in wind only. Then

$$S_{p_i^w}(f) = \sigma_{p_i^w} \times S_u^*(f) = 2I\bar{P}_i^w \times S_u^*(f), \quad i = 1, 2 \quad (33)$$

where $S_u^*(f)$ is given by equation 23. The co-spectrum is affected by the correlation between the two wind pressures and is assumed to comply with the relation proposed by Vickery [24]:

$$S^{(c)}_{p_1^w p_2^w}(f) = \sqrt{S_{p_1^w}(f) \times S_{p_2^w}(f)} \times \mathrm{coh}(f) \quad (34)$$

where the coherence function coh (f) is given as:

$$\mathrm{coh}(f) = e^{-\hat{f}} \quad \hat{f} = \frac{f[C_z^2 \Delta Z^2 + C_y^2 \Delta Y^2)^2]^{1/2}}{\frac{1}{2}[U(Z_1)+U(Z_2)]}$$

where ΔZ and ΔY are the vertical and horizontal distances between the two openings, $V(Z_1)$ and $V(Z_2)$ are the wind velocities at the height of two openings, C_Z and C_Y are coefficients determined by experiments, here $C_z = 10$, $C_y = 16$.

Equation (32) indicates that the fluctuating airflow through either opening is affected by the two turbulent pressures and the correlation between them. The exact relation depends on the transfer function matrix $\mathbf{H}(\omega)$.

Figure 12 shows the spectra and co-spectrum of wind pressures and the spectra for the fluctuating airflow. Table 3 lists the parameters used in the calculation, and the steady-state solutions to the airflow of the enclosure are calculated by Walton's [25] method. The two spectra for airflow through two openings are almost the same. The maximum of the spectra $S_{q1}(\omega)$ and $S_{q2}(\omega)$ occurs at a frequency of 0.007 Hz, almost the same as those of the pressure spectra.

Equation (32) indicated that the power spectrum of the fluctuating flow through each of the two openings is composed of or affected by three terms: the effects of the two pressure spectra and the effects of co-spectrum. In Fig. 13 the three terms of the spectrum $S_{q1}(f)$ are plotted. The first two terms contribute positively to the spectrum $S_{q1}(f)$; the third term, related to the co-spectrum, contributes negatively. The spectrum $S_{q1}(f)$ would be equal to the sum of the first two terms if the two wind pressures were totally un-correlated. However, since the two pressures on the same windward surface are correlated positively only to a certain extent, the actual spectrum $S_{q1}(f)$ should be smaller than the sum of the first two terms by an amount given by the third term. In other words, the third term in equation (32) denotes the effects of correlation between the two pressures on the overall (spectrum of) fluctuating airflow.

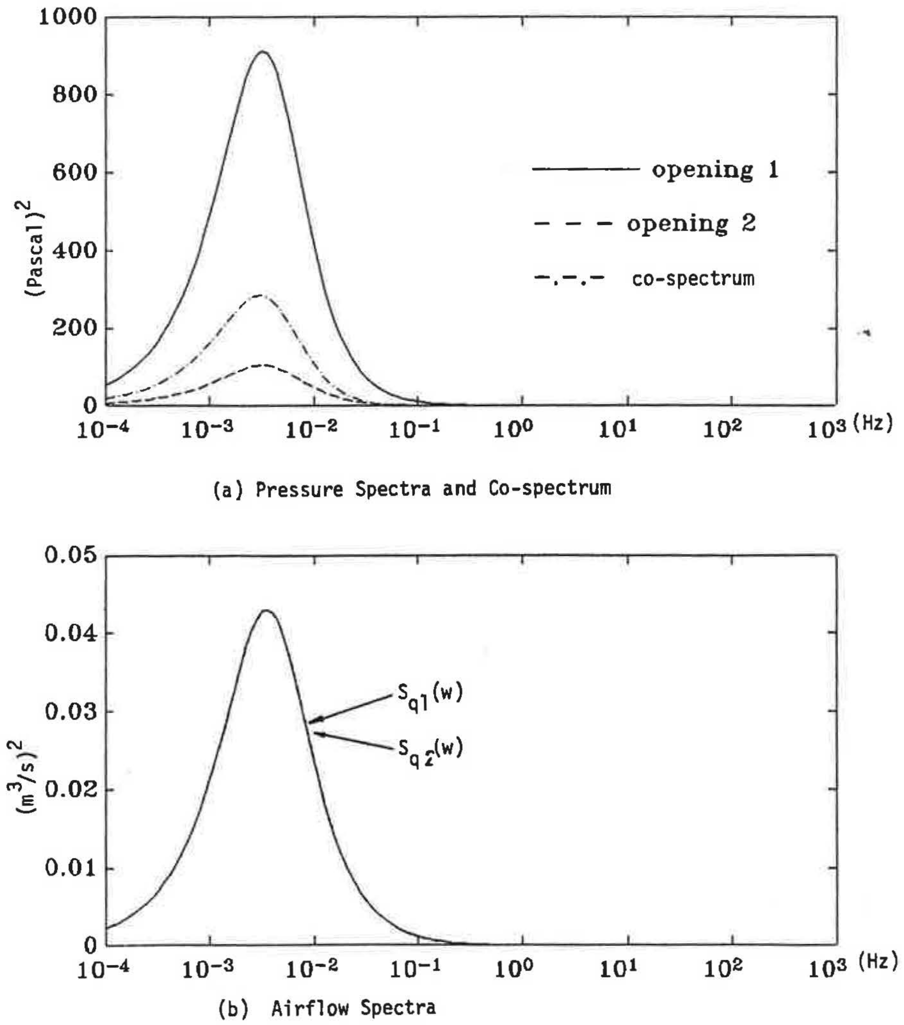

Fig. 12. Spectra in two-opening case.

Table 3. Parameters and results of two opening case study

Building parameters		
Building dimension	$8 \times 8 \times 8$ m^3	Volume = 500 m^3
Temperatures	$T_{in} = 23°C$	$T_{out} = 0°C$
Roof height	$H = 8$ m	
Opening characteristics		
Type: orifice openings		
Opening areas	$A_1 = 0.05$ m^2	$A_2 = 0.04$ m^2
Opening depth	$L_1 = 0.2$ m	$L_2 = 0.2$ m
Discharge coefficient	$C_d = 0.6$	$C_d = 0.6$
Opening positions	$Z_1 = 2$ m	$Z_2 = 6.5$ m
Distance between two openings	$\Delta Z = 4$ m	$\Delta Y = 6$ m
Wind data		
Velocity at weather station	$V_{10} = 5$ m/s	
Pressure coefficients	$C_{p1} = 0.5$	$C_{p2} = 0.9$
Turbulence intensity	$I = 0.16$	
Derived values		
Wind speed at roof height	$V_H = 4.72$ m/s	
Mean wind pressures	$\bar{P}_1^w = 7.18$ Pa	$\bar{P}_2^w = 12.92$ Pa
Stack effect	$P_s = 3.93$ Pa	
Mean airflow rates	$\bar{Q}_1 = \bar{Q}_2 = 0.031$ m^3/s	ach = 0.23
RMS of wind pressure	$\sigma_{pw1} = 1.25$ Pa	$\sigma_{pw2} = 3.71$ Pa
K values	$K_1 = 2.252$	$K_2 = 2.816$
Turbulent results		
$\sigma_{q1} = 0.026$ m^3/s	$\sigma_{q2} = 0.026$ m^3/s	
$\sigma_{q1}/Q_1 = 84.2\%$	$\sigma_{q2}/Q_2 = 84.3\%$	
$\sigma_{pi} = 1.14$ Pa	$\sigma_{pi}/\bar{P}_H = 7.9\%$	
$\sigma_{pi}/\sigma_{ph} = 27\%$		

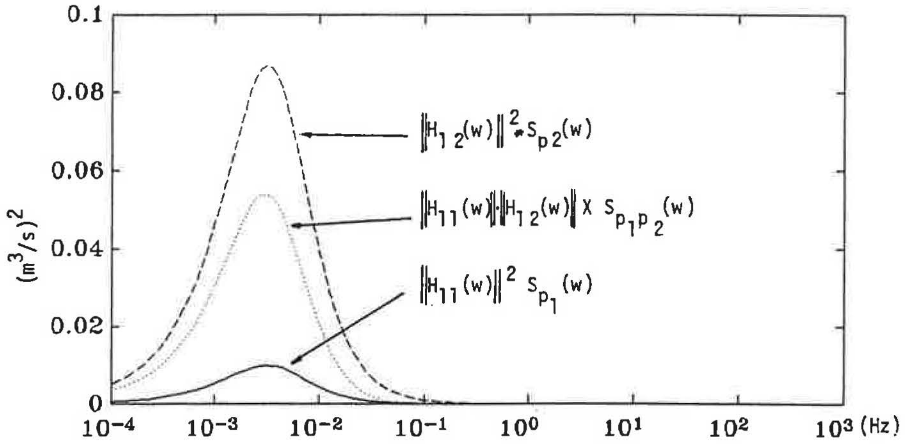

Fig. 13. Decomposition of $S_{q_1}(w)$.

Figure 14 shows that the two spectra of airflows through the two openings are not equal. The difference is due to the compressibility of the room air. When the airflow through one opening increases (or decreases) the airflow through the other opening does not necessarily need to be decreased (or increased), the room serves as a buffer for the differences between the two airflows. The curve in Fig. 14 shows that the spectrum $S_{q2}(f)$ is greater than the spectrum $S_{q1}(f)$, at a frequency above 1.26 Hz.

Employing the same principle of equation (21), the root-mean-square (RMS) values for fluctuating airflows can be obtained by integration of the corresponding spectra. The results in Table 3 show that the RMS values of airflow rates are about 84% of the mean airflow rates (for the given conditions). This is to say that on average, the turbulent components of airflow are about 84% of the mean values.

The actual effect of fluctuating airflows on the total airflow rate for an opening, when mean airflow rates are not zero, can fall into two cases, as shown in Fig. 15. In the case of Fig. 15a, the turbulent component is small, and the total airflow rate is in one direction. The time average of the turbulent component is zero. Therefore, the total infiltration is equal to the mean value. In the case of Fig. 15b, the turbulent component is large, and the total airflow rate changes direction. Flow reverse does occur. Then the reversed airflow, represented by the shaded area in Fig. 15b, presents an additional air exchange through the opening.

Figure 16 shows a sample of the total airflow through

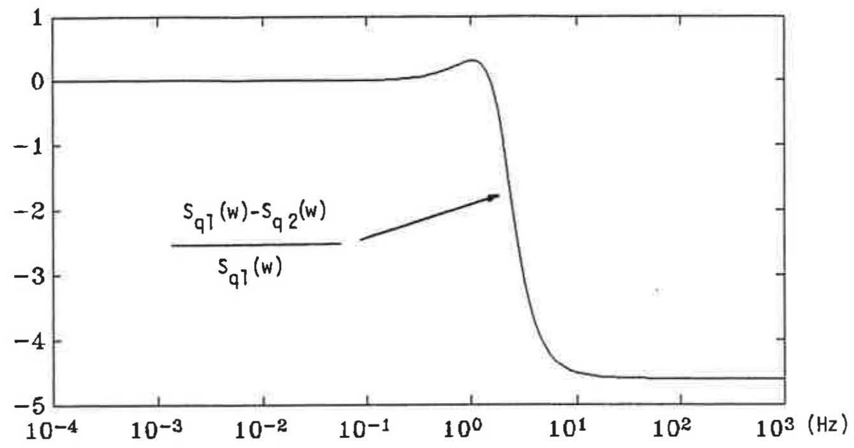

Fig. 14. Difference between two flow spectra.

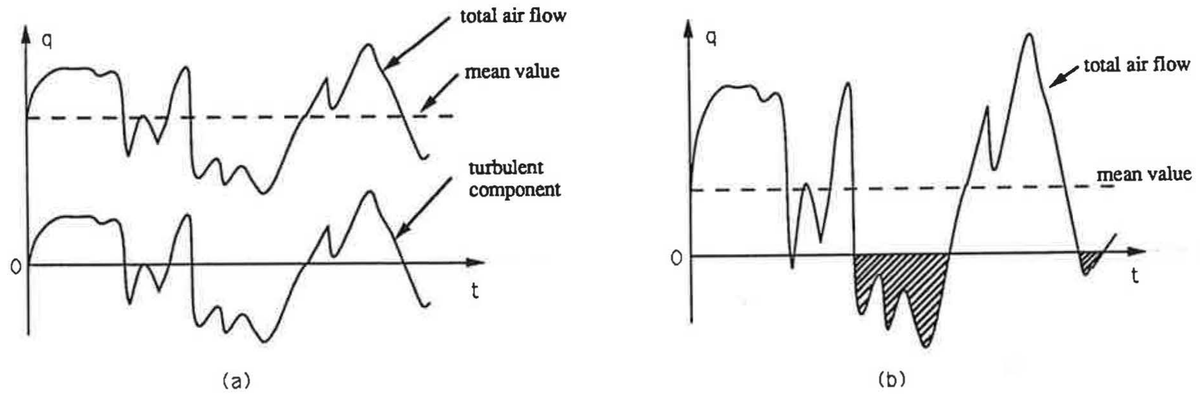

Fig. 15. Effects of fluctuating flow to total flow.

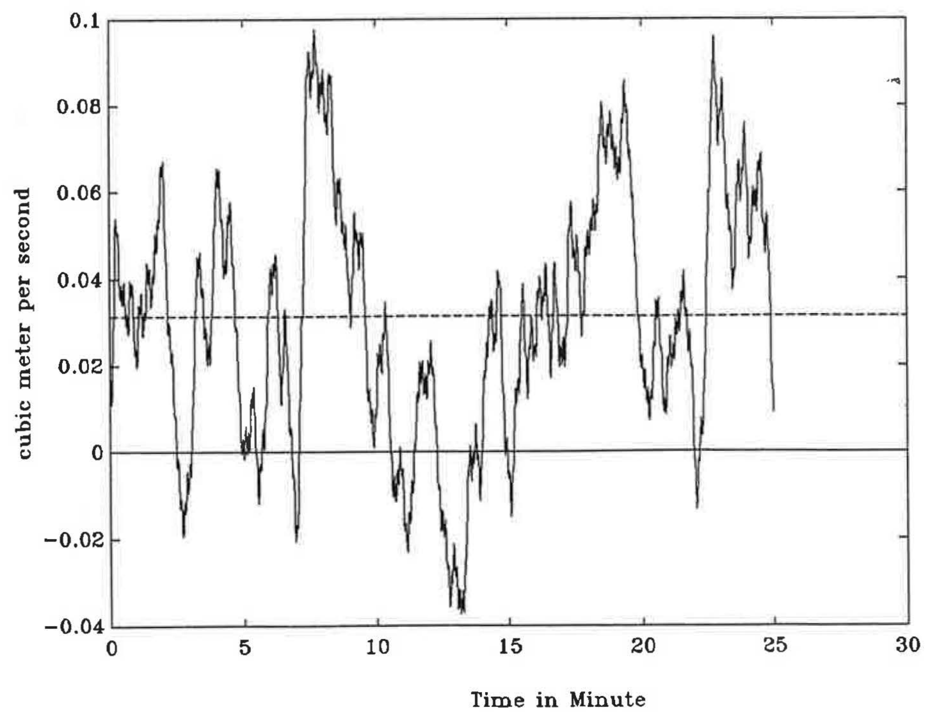

Fig. 16. A sample of air flow rate as a function of time.

opening 1, generated from the calculated spectrum function in equation (32). It is apparent that flow reversal does occur in this case. Calculations show that the reverse flow is 12% of the mean flow rate.

The turbulent internal pressure can be calculated according to equation (13), in the time domain, as:

$$p^i = B\int_0^t (q_1+q_2)\,\mathrm{d}t \tag{37}$$

or in the frequency domain as:

$$\begin{aligned} p^i(\omega) &= \frac{B}{j\omega}[Q_1(\omega)+Q_2(\omega)] \\ &= \frac{B}{j\omega}[H_{11}(\omega)+H_{21}(\omega)]P_1^w(\omega) \\ &\quad + \frac{B}{j\omega}[H_{12}(\omega)+H_{22}(\omega)]P_2^w(\omega). \end{aligned} \tag{38}$$

Therefore, the power spectrum and RMS value of the p^i can be obtained by the same principle used in calculations for q, and (for the given conditions) $\sigma_{p^i} = 1.14$ P_a, which is 27% of the turbulent wind pressure at roof height (Table 3).

CONCLUSIONS AND DISCUSSION

A power spectrum analysis approach to model the pulsating air flows due to turbulent wind-induced pressures is presented and applied to the single-opening and two-opening cases. The total air flow is divided into mean and fluctuating components. The equations that govern the latter components are linearized in order to benefit from the spectral analysis methods. It is demonstrated that the wind pressure spectra and the co-spectra, which completely describe the characteristics of turbulence in wind-induced pressures on building surfaces, are utilized directly as input to the proposed fluctuating air infiltration model.

The results of case studies show that, in the case of a single-opening, the turbulence in the airflow rate is concentrated in the higher frequency range (around 0.1 Hz), while for the two-opening case the fluctuating airflow rates are mainly caused by the instantaneous pressure differences and the frequency ranges of their turbulence are lower (around the same frequency range of the wind pressure, at about 0.008 Hz).

The predicted RMS values of fluctuating airflow rates are theoretical, and a conversion is required to obtain the effects on the total air exchange rates. Three factors influence the conversion: the "residue" (or "hidden") air in the openings, the mixing process inside rooms, and the possibility of flow reversal. In the single-opening case, only the first two factors have effects. For the two-opening case, the third factor has a major influence while the other two factors have only negligible effects. Further theoretical and experimental efforts are required to better understand and quantify these phenomena.

This approach also enables the calculation of turbulent internal pressures inside buildings, a problem which has drawn much attention in wind engineering [10, 26].

Further research is needed to overcome the linear assumption which has been made in this study in order to apply the spectral analysis method. More research is also needed to collect more information on wind-pressure spectra and co-spectra, and to derive fluctuating flow equations more accurately and for more types of openings. The present data for wind pressure on building surfaces are mainly concentrated on the mean values. The few existing studies on frequency characteristics are not consistent or systematic and thus are of little practical use.

Although the proposed approach has been applied to only two simple hypothetical cases, it can be readily extended to multi-zone buildings. The extended model can include the airflow fluctuations due to turbulence other than wind pressures, such as thermal buoyancy and unsteady operation of mechanical ventilation.

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