#4517

Technical Note

Summary   The applicability of psychrometric charts is normally limited by their dependence on particular air pressures. This Note demonstrates a method of allowing for pressure variation when using a standard psychrometric chart. Despite the approximations involved, good accuracy is obtained for pressures from 70 to 110 kPa.

Psychrometry: Use of standard charts for non-standard air pressures

I J G Berry BSc(Eng) CEng MIMechE
Middlesex Polytechnic, Faculty of Engineering, Science and Mathematics, Bounds Green Road, London N11 2NQ, UK

List of symbols

A	Constant	
c	Specific heat capacity	kJ kg K^{-1}
h	Specific enthalpy	kJ kg^{-1}
m	Mass	kg
p	Pressure	Pa
r	Pressure ratio (p/p_m)	
R	Specific gas constant	J kg^{-1} K^{-1}
t	Temperature	°C
T	Absolute temperature	K
v	Specific volume	m^3 kg^{-1}

Greek symbols

ω	Moisture content (specific humidity, m_v/m_a)	
ψ	Saturation (%)	

Suffixes

a	Dry air
g	Saturated vapour
MP	Mixture point
p	Constant pressure
s	Standard atmospheric pressure
sat	Saturation
sl	Sling
v	Water vapour
VP	Vapour point

1 Introduction

The standard *CIBSE Psychrometric Chart*[1] is based on an atmospheric pressure of 101.325 kPa, which corresponds to the pressure at sea level of the International Standard Atmosphere. Since much design is based on this pressure, the chart is widely used for this purpose. However, when practical psychrometric measurements are carried out on site or in the laboratory, or when design conditions differ, non-standard atmospheric pressures may arise which can significantly affect the results obtained. Although special psychrometric charts are available for such circumstances[2], it would clearly be advantageous if the effects of pressure variation could be taken into account when using a standard chart, without recourse to special correction tables such as are provided in the *CIBSE Guide*[3]. The purpose of this

paper is to introduce a relatively simple method of achieving this.

2 Approach

According to the Gibbs–Dalton Law for ideal gas mixtures[4], each constituent behaves as if it occupied the total volume of the mixture individually. This can be assumed to apply not only to the dry air component but also to the moisture component of humid air for the range of conditions covered by the standard *CIBSE Psychrometric Chart* (i.e. for saturation temperatures not exceeding 32°C). It follows that the vapour properties incorporated in the chart are independent of the total pressure. In this paper, the point on the standard chart at which the vapour properties (e.g. temperature and partial pressure) correspond to those in the actual situation is termed the *vapour point*. This point is first located on the chart, followed by the *mixture point*, at which the mixture temperature and moisture content (or specific humidity) correspond to the true values. When the actual and chart pressures are equal, both the vapour and the mixture properties are, of course, represented by a single point.

Following the Gibbs–Dalton Law, the *vapour point* gives the actual relative humidity directly, since this is purely a vapour property. (In fact the *CIBSE Chart* uses percentage saturation in place of relative humidity, but in practice the two are almost identical[3].) Similarly, the *mixture point* gives the specific enthalpy, which is effectively a function only of temperature and moisture content. This point also enables the specific volume to be obtained by dividing the chart volume by the ratio of actual to standard pressures, $p/p_s = r$. Details of the procedures are given in section 3 and illustrated in Figure 1.

3 Detailed procedures

3.1 Vapour point

Locating the *vapour point* depends on the information available. If the ambient (or dry-bulb) temperature and the relative humidity/percentage saturation are known, then the intersection of the corresponding lines on the chart immediately gives the required point. Similarly, the combination of ambient temperature and dew point gives the
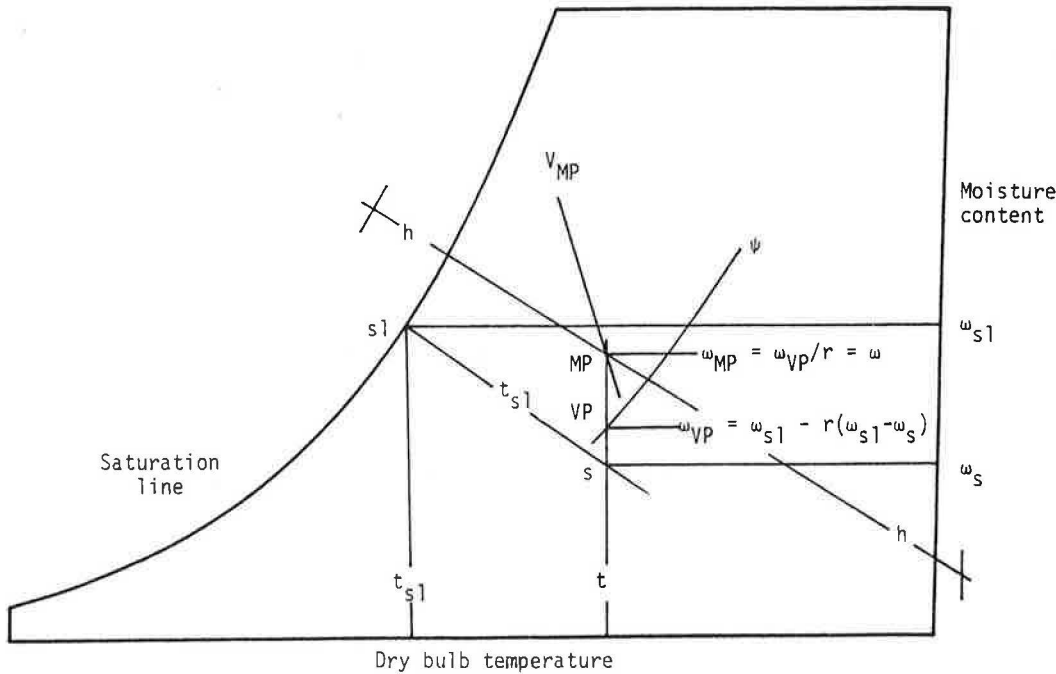
point directly, since the former again represents the required abscissa while the dew point corresponds to the ordinate. Note that we are now taking advantage of the fact that a constant moisture content at a given ambient pressure also corresponds to a constant vapour pressure, as shown below.

$$\omega = \frac{m_v}{m_a} = \frac{p_v V/(R_v T)}{p_a V/(R_a T)} = \frac{R_a p_v}{R_v(p - p_v)} \approx 0.622 \, p_v/p \quad (1)$$

Note also from equation 1 that the moisture content corresponding to a given vapour pressure varies approximately inversely with the ambient pressure (or, conversely, the vapour pressure for a given moisture content is proportional to the ambient pressure).

A further useful pair of properties for which the psychrometric chart is designed consists of the wet- and dry-bulb temperatures. In practice the normal wet-bulb temperature assumed is the sling temperature (i.e. that given by a sling psychrometer). It is related to the vapour pressure as follows[2]:

$$p_v = p_{sl} - Ap(t - t_{sl}) \quad (2)$$

where $A = 666 \times 10^{-6} \text{ K}^{-1}$ for temperatures above 0°C.

Thus the difference in vapour pressure associated with a given wet-bulb depression (i.e. wet- to dry-bulb temperature difference) is directly proportional to the ambient pressure. Accordingly, the actual reduction in vapour pressure—or in standard chart moisture content—relative to the sling saturation point is r times that given by the standard chart. Using this result again enables the correct *vapour point* to be located and consequently the relative humidity/percentage saturation to be obtained.

3.2 Mixture point

The *mixture point* on the standard chart is the point which has the same temperature and moisture content as the actual humid air. Since, from equation 1, the moisture content is inversely proportional to the ambient pressure for a given vapour pressure, it follows that the actual moisture content

is related to the apparent value (at the *vapour point*) by:

$$\omega = \omega_{VP}/r \quad (3)$$

Thus, once the *vapour point* has been located, the *mixture point* is easily found using equation 3, enabling the specific enthalpy and specific volume to be obtained as indicated above. Note that, for sub-standard pressures, the *mixture point* lies above the *vapour point* on the standard chart, and possibly also above the saturation line; nevertheless it can still be plotted by extending the appropriate dry-bulb temperature and moisture content lines and hence the required properties determined. The procedures are illustrated in Figure 1 and in the worked example of section 4.

4 Worked example

The worked example is intended both to illustrate the procedures described and to indicate the magnitude of error that can be incurred if no correction for pressure is made. Since the combination of wet- and dry-bulb temperatures represents the least simple case, this is assumed in the example, with actual temperatures of 15 and 25°C respectively. A non-standard pressure of 95 kPa is also assumed, corresponding to an altitude of approximately 540 m in the International Standard Atmosphere. As a check on the results, the example is repeated using data obtained from steam tables[5].

4.1 Chart method

The following data at standard pressure were obtained from the standard *CIBSE Psychrometric Chart*[1] for the given wet- and dry-bulb temperatures:

$$\omega_s = 0.00645 \text{ kg}_v/\text{kg}_a$$

$$\psi_s = 32\%$$

$$h_s = 41.5 \text{ kJ}/\text{kg}_a$$

$$v_s = 0.853 \text{ m}^3/\text{kg}_a$$

$$\omega_{sl} = 0.01065 \text{ kg}_v/\text{kg}_a$$

At reduced pressure:

$$r = 95/101.325 = 0.9376$$

So

$$\omega_{VP} = 0.01065 - 0.9376\,(0.01065 - 0.00645)$$
$$= 0.00671 \text{ kg}_v/\text{kg}_a$$

Hence, from the chart:

$$\psi = \psi_{VP} = \textbf{33.5\%}$$

Also

$$\omega = 0.00671/0.9376 = \textbf{0.00716 kg}_v/\textbf{kg}_a$$

Again, from the chart

$$h = h_{MP} = \textbf{43.4 kJ/kg}_a$$

Also

$$v_{MP} = 0.854 \text{ m}^3/\text{kg}_a$$

Giving

$$v = 0.854/0.9376 = \textbf{0.911 m}^3/\textbf{kg}_a$$

Comparing the results for the reduced pressure with those for the standard pressure, the derived properties are all increased by 4–7%—significant differences, even allowing for the lack of precision inherent in using a chart.

4.2 Steam table method

From tables for 15°C:

$$p_{sl} = 1704 \text{ Pa}$$

Hence at standard pressure

$$p_v = 1704 - 666 \times 10^{-6} \times 101\,325(25 - 15) = 1029 \text{ Pa}$$

So

$$\omega_s = 0.622 \times 1029/(101\,325 - 1029) = \textbf{0.00638 kg}_v/\textbf{kg}_a$$

From tables at 25°C:

$$p_{sat} = 3166 \text{ Pa}$$

So

$$\omega_g = 0.622 \times 3166/(101\,325 - 3166) = 0.02006 \text{ kg}_v/\text{kg}_a$$

Hence

$$\psi_s = 0.00638 \times 100/0.02006 = \textbf{31.8\%}$$

Also from tables at 25°C:

$$h_g = 2547 \text{ kJ/kg}\ (\simeq h_v)$$

For air

$$c_p = 1.005 \text{ kJ/kg K}$$

Hence

$$h_s = 1.005 \times 25 + 0.00638 \times 2547 = \textbf{41.4 kJ/kg}_a$$

Also for air

$$R_a = 287 \text{ J/kg K}$$

Giving

$$v_s = 287 \times 298.1/(101\,325 - 1029) = \textbf{0.853 m}^3/\textbf{kg}_a$$

At reduced pressure, proceeding as before:

$$p_v = 1704 - 666 \times 10^{-6} \times 95\,000\,(25 - 15) = 1081 \text{ Pa}$$

So

$$\omega = 0.622 \times 1081/(95\,000 - 1081) = \textbf{0.00716 kg}_v/\textbf{kg}_a$$

Also

$$\omega_g = 0.622 \times 3166/(95\,000 - 3166) = 0.02144 \text{ kg}_v/\text{kg}_a$$

Giving

$$\psi = 0.00716 \times 100/0.02144 = \textbf{33.4\%}$$

Again

$$h = 1.005 \times 25 + 0.00716 \times 2547 = \textbf{43.4 kJ/kg}_a$$

and

$$v = 287 \times 298.1/(95\,000 - 1081) = \textbf{0.911 m}^3/\textbf{kg}_a$$

The minor discrepancies between these and the previous results are well within the possible calculational errors, thus vindicating the method used. Additionally, comparisons made between the results of the proposed method and figures obtained from special charts[2] agreed well over the pressure range 70 to 110 kPa, as shown in Table 1.

Table 1 Comparison of standard chart method with special chart results (based on wet- and dry-bulb temperatures of 15°C and 30°C)

Property	Ambient pressure (kPa)			
	70		110	
	Standard chart	Special chart	Standard chart	Special chart
ψ (%)	38.5	38.0	30.0	30.0
ω (kg$_v$/kg$_a$)	0.01122	0.01126	0.00561	0.00561
h (kJ/kg$_a$)	53.8	53.9	39.5	39.5
v (m^3/kg$_a$)	1.244	1.244	0.785	0.785

5 Concluding remarks

There are essentially three means available for determining psychrometric properties: calculation, data tables and charts. With the availability of computers and calculators, pure calculation is frequently the most suitable approach. Nevertheless psychrometric charts offer simplicity and convenience, despite being restricted to specific ambient pressures. The procedures described offer to overcome this drawback at the cost of some loss of simplicity but little of accuracy. It is, perhaps, worth noting that while the pressure difference from standard assumed in the worked example represents a considerable variation in relation to sea level conditions, it amounts to little more than the pressure difference existing between the top and bottom of a major skyscraper!

References

1 *CIBSE Psychrometric Chart (−10 to +60°C)* (London: Chartered Institution of Building Services Engineers) (1987)
2 Martin P L and Curtis D M *M–C Psychrometric Charts for a Range of Barometric Pressures* (London: Troup Publications) (1972)
3 *CIBSE Guide* pages C1–2, 3 (London: Chartered Institution of Building Services Engineers) (1986)
4 Eastop T D and McConkey A *Applied Thermodynamics for Engineering Technologists* Chapters 13–14 (London: Longman) (1986)
5 Rogers G F C and Mayhew Y R *Thermodynamic and Transport Properties of Fluids* (Oxford: Blackwell) (1980)