Tracer-Gas Techniques For Measuring Ventilation in a Single Zone

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Tracer-gas techniques have become widely used to measure the ventilation rates in buildings. A tracer gas is an idealized substance used to tag volumes of air so as to be able to infer their bulk movement; the properties of a perfect tracer are discussed. The basic principle involved is that of conservation of mass (of both air and tracer gas) as expressed in the continuity equation; by monitoring the injection and concentration of the tracer, one can infer the exchange of air. This report will summarize the techniques in use. Although there is only one continuity equation, there are many different experimental injection strategies and analytical approaches. These different techniques may result in different estimates of ventilation due to uncertainties and biases of the procedures. This report will analyze the spectrum of techniques, including some of the relevant error analyses.

INTRODUCTION

THE PURPOSE of this report is to review the approaches to the use of tracer gases in the determination of ventilation (i.e., total outside air exchange) from all sources (mechanical ventilation, natural ventilation, infiltration, etc.). In a building, air exchange is the exchange of air between the outside and the test space. Such multizone techniques, however, will not be covered herein.

The vast majority of the ventilation measurements made to date have involved a single-tracer gas deployed in a single zone. This technique has proven very useful for buildings which may be treated as a single zone (e.g., houses) and for more complex buildings in which there are isolatable sub-sections. However, as the need to understand more complex buildings has grown, tracer techniques that recognize that not only does air flow between the outside and the test space. Such multizone techniques, however, will not be covered herein.

Tracer gas

Tracer gases are used for a wide range of diagnostic techniques including leak detection [5, 6] and atmospheric tracing [7]. The application of interest here is the use of tracer gases to measure ventilation (i.e., air flow) in buildings [8]. Ventilation is an important process in buildings due to its impact on both energy requirements and indoor air quality—both of which are topics of concern to society. Measurement of the tracer gas combined with conservation laws allows a quantitative determination of the tracer transport mechanism (i.e., a measurement of the air flow).

For determining ventilation rates, an ideal tracer gas must have certain properties:

- Safety: the presence of the tracer should not pose a hazard to people, materials, or activities in and around the test area. The tracer, therefore, should be non-flammable, non-toxic, non-allergenic, etc.
- Non-reactivity: because conservation of tracer will be used to infer airflow, the tracer gas should not react chemically or physically with any part of the system under study.
- Insensibility: the presence of the tracer should in no way affect the processes that are being studied. Thus, an ideal tracer gas should not affect the air flow or air density of the system.
- Uniqueness: an ideal tracer should be able to be re-

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cognized from all other constituents of air. In general it should not be a normal constituent of air in which it is being placed, but a tracer with non-zero background may be used provided that the background is stable and additional tracer concentration is significantly larger than the (steady) background.

- **Measurability**: the (true) concentration and all injected tracer gas must be quantifiable through some sort of instrumentation.

Although these criteria define an ideal tracer gas from a theoretical perspective, practical considerations—such as the cost of the tracer and necessary instrumentation—will factor strongly into the selection.

No tracer gas meets all these requirements; thus, many researchers have compared tracer gases to find those that most closely approach the ideal. Early researchers have compared many different gases in the course of their work. Warner [9] compared coal gas (a mixture of H₂ and CO obtained by passing steam over hot carbon), detected using a katharometer with CO₂ measured by Haldane gas analysis. Collins and Smith [10] used the radioactive argon isotope ⁴¹Ar as a tracer, measuring concentration with a Geiger counter and a ratemeter. They made a direct comparison of the infiltration rate using H₂ as detected by a katharometer and ⁴¹Ar; agreement within ±8% was seen in two trials. Howland et al. [11] compared radioactive isotope ⁸⁵Kr, measured using a Geiger counter and a ratemeter with CO₂. The carbon dioxide concentration was determined by drawing samples of air periodically and using chemical analysis (the Haldane apparatus), to find the amount of tracer remaining in the test space. Three separate tests showed agreement to within ±9%.

Lidwell [12] compared nitrous oxide, N₂O, with acetone, C₃H₆O. Infrared absorption was used to measure the concentration of nitrous oxide; acetone concentration was determined by measuring the change in pH which occurred when air containing acetone is absorbed into solutions of hydroxylamine hydrochloride. A single measurement (judged to be accurate to within ±10%) produced ±3% agreement.

Howard [13] compared N₂O with both H₂ and O₂. N₂O concentrations were determined with an infrared analyser; H₂ concentrations with a katharometer, and O₂ by absorption in aqueous chromous chloride. Using as their test space a four-bedroom townhouse constructed within an environmental test chamber, Hunt and Burch [14] compared air change rates using He and SF₆ to examine the influence of molecular diffusion on the infiltration process. Six trials were made, and the ratio of the air change rate measured with SF₆ to that measured with He was 1.17 with a standard deviation of 0.14.

A study of tracer gases was made [15] to compare SF₆, N₂O, C₃H₆ and CH₄ (sulphur hexafluoride, nitrous oxide, ethane and methane). SF₆ was measured by means of gas chromatography with electron capture, and the light gases were measured by infrared absorbency techniques. When combined with the earlier studies, the conclusions from this paper are that the very light gases (e.g. H₂, He) and the very heavy gases (e.g. SF₆) have anomalous behavior when compared with other gases, but there are a wide variety of acceptable gases for tracer studies.

More recently the chlorinated fluorocarbons (CFC) and perfluorocarbons compounds have been used as tracers. Although some technique intercomparisons with SF₆ have been made [16] there have been no systematic intercomparison studies. Currently the fluorine-containing compounds are the most widely used class of tracer gases.

**THE ZONAL ASSUMPTION: MIXING AND VENTILATION EFFICIENCY**

In making ventilation measurements with a tracer gas the concept of a zone is utilized. The tracer techniques are then used to determine the flow from a test zone to/from the outside (but not within the zone). For the purpose of ventilation measurement a perfect zone has the following properties:

- **Homogeneity**: the fluid properties (i.e. density, and tracer concentration) are assumed to be the same at every point within the zone.

- **Isolated**: the zone only communicates with the “outside”, an area whose concentration of tracer gas is unaffected by the zone. Thus, there is neither reentrainment of tracer (from exfiltrating air to infiltrating air) nor is there any buffer zone.

- **Perfectly mixed**: in a perfect zone any outside air or tracer injected becomes instantaneously (and homogeneously) dispersed within the zone. In practice there will be a finite time required for the air to become well mixed. As long as characteristic times of the analysis are significantly longer than this mixing time, the perfectly mixed assumption may still be valid. As will be discussed in following sections, techniques in which the concentration changes quickly compared to the mixing time, may be subject to significant errors.

The zones defined in this way are appropriate for tracer gas studies of ventilation. Other definitions of “zone” such as for thermal modeling or occupant behavior may be quite different. For example, a set of rooms connected by a continuous ventilation system may be assumed to be a single zone for ventilation purposes, but not for acoustic purposes.

The zonal assumption is necessary when there is no knowledge about the size and location of the inlet and exhaust flows to each zone. In cases (e.g. mechanically ventilated rooms) in which these quantities are known, tracer gases can be used to measure the ventilation efficiency within the zone. **Age of air** concepts are often used to describe spatial variation of ventilation. Sandberg [17] summarizes the definitions and some of the tracer techniques for determining the efficiency (by seeding inlet streams or monitoring exhaust streams). Because they are usually concerned with the highly controlled environment of mechanically ventilated buildings, those looking at age of air concepts do not normally consider the time variation of the ventilation. Conversely, the simple zonal approach assumed herein, ignores spatial variations within nominal zones. There is some preliminary modeling work [18] on combining these two approaches using more complex transfer functions for the zones. Further
discussion of intrazonal air flows is beyond the scope of this report.

In this language, the zonal assumption can be stated as "the local age of air is everywhere equal to the nominal ventilation time". This statement, however, does not usually allow one to decide whether the zone is well mixed. Methods for testing the validity of the zonal assumption and techniques for taking into account imperfect mixing are currently lacking. Perforce, good mixing will be assumed for the following analyses.

**CONTINUITY EQUATION**

Invocation of the zonal assumption removes the requirement for treating the spatial variation of concentration within the zone—reducing it to a single-node network problem that can be solved by using conservation of air and tracer gas to infer the ventilation. Since there is no source or sink of air, the change in the amount of air inside the zone is related to the air flow into and out of the zone:

\[ \frac{d}{dt}(\rho_a V) + q_{in} - q_{out} = 0. \]  

(1)

(See the Nomenclature table for the definitions of the terms.) All of the quantities in this expression may be functions of time.

The conservation of tracer gas is an analogous expression, except that there may be a source term due to the injection of tracer gas and all air flows are weighted by the concentration of tracer:

\[ \frac{d}{dt}(\rho_a V c) + C_{out} q_{in} - C_{in} q_{out} = q_T. \]  

(2)

We can combine these two expressions to get the continuity equation:

\[ \rho_a V \frac{dC}{dt} + q_{in} (C_{out} - C_{in}) = q_T. \]  

(3)

As noted earlier the continuity equation is a mass balance equation and does not directly involve volumetric flows. If we make the following identifications:

\[ Q \equiv \frac{q_{in}}{\rho_a} \]  

(4.1)

(i.e. the volumetric flow is the infiltration at inside density)

\[ Q_T \equiv \frac{q_T}{\rho_a} \]  

(4.2)

(i.e. the volumetric injection is of tracer at inside density) and:

\[ C \equiv C_{in} - C_{out} \]  

(4.3)

(invoking an ideal tracer property; i.e. any outside tracer concentration can be treated as a negligible offset)

\[ \dot{C} \equiv \frac{dC}{dt}, \]  

(4.4)

we can recover the continuity equation in its conventional (i.e. volumetric) form:

\[ V \dot{C} + QC = Q_T. \]  

(5)

Using the above definitions, this expression is correct even if the volume, ventilation rate and air density are changing in time. Although all of the quantities in the continuity equation are functions of time, the explicit time dependence will be suppressed unless required for clarity.

All tracer gas methods rely on the solution of the continuity equation to infer the ventilation from measurements of the concentrations and injected tracer flow rate. The continuity equation can be solved direction for the ventilation as follows:

\[ Q = \frac{Q_T - V \dot{C}}{C}. \]  

(6)

Equivalently, we can solve the continuity equation to predict the concentration as a function of time given all of the other quantities:

\[ C(t) = \int_{-\infty}^{t} \frac{Q_T(r)}{V(r)} e^{\frac{\dot{C} (r-t')}{V}} \, dr'. \]  

(7)

The common definition of air change rate, \( \lambda \), is used above:

\[ \lambda \equiv \frac{Q}{V} = \left( \frac{q_{in}}{\rho_a V} \right). \]  

(8)

If it were possible to solve equation (5) on an instantaneous basis to find the ventilation, most of the rest of this report would not be needed. However, because of measurement problems, including mixing issues, the determination of an instantaneous ventilation value is practically impossible. Therefore it becomes necessary to use time-series data to reduce the uncertainties to make the analysis possible. The rest of this report details some of the approaches being used to overcome these measurement limitations.

The exact analytical technique for inverting the measured data to find the ventilation depends on both the experimental technique, the assumption made about the system, and the quantity of interest. The following sections develop the analysis for techniques currently in use. Virtually all of the widely used techniques assume that the ventilation is constant over the measurement period, although this assumption is often violated. These techniques will be discussed in detail below.

**GENERAL SOLUTION**

Equations (5)–(7) suggest three different approaches for the determination of the ventilation from the data. The term general here is used to indicate that there are no particular assumptions made about the injections or concentration other than the continuity equation itself.

In following sections specific solutions which assume a particular control strategy (e.g. constant injection) will be considered.

**Regression techniques**

Regression techniques use equation (7) and assume that the ventilation (as well as the volume) is constant over the regression period \( 0 \leq t \leq T \) and then find the
The best set of parameters that fit the concentration and injection data to the following equation:

$$C(t) = C_0 e^{-\lambda t} + \int_0^t \frac{Q_r(t')}{V_r} e^{-\lambda (t-t')} dt'.$$

(9)

The parameters $C_0$ and $\lambda$ are always treated as unknowns in the regression. The effective volume, $V_\text{eff}$, may be treated as either a known or an unknown.

The regression will result in the best single estimate of ventilation to match the data. If the air change is constant, the regression estimate will result in an unbiased estimate of the true ventilation. If the ventilation is not constant, however, the regression will result in a biased estimate of the average air change.

The precision of the estimate of ventilation can usually be extracted from the details of the regression. As there are many ways of doing a regression, no uncertainty analysis will be presented herein for the regression techniques.

Integral techniques

Integral techniques make use of the fact that it is often easier to obtain integrals (or, equivalently, averages) of measured data (i.e. injection and concentration) than to get a complete time history of both. In integral techniques the flows and concentrations are integrated in the initial time (i.e. $t = 0$) to the final time (i.e. $T$). Thus, these techniques integrate equation (5), assuming the ventilation is constant over the integral period:

$$Q(t) = \int_0^T Q(t') dt + V(C_\text{initial} - C_\text{final}),$$

(10.1)

or, equivalently, in terms of averages:

$$Q_t = \frac{Q'_t - V\bar{C}}{C},$$

(10.2)

where the average concentration is given by

$$\bar{C} = \frac{1}{T} \int_0^T C(t) dt,$$

(11.1)

the average accumulation is given by:

$$\bar{C} = \frac{C_\text{final} - C_\text{initial}}{T},$$

(11.2)

and the average injection is the total volume of injected tracer divided by the length of the measurement period:

$$Q_T = \frac{1}{T} \int_0^T Q(t) dt.$$  

(11.3)

The uncertainty in this estimate of the ventilation is as follows:

$$\delta Q^2 = \frac{\delta Q'_t}{C^2} + \frac{\delta C}{C^2} + \frac{\delta V^2}{C^2} + \frac{\delta Q'_t \delta C}{C^2},$$

(12.1)

$$\delta T = \frac{\delta Q'_t}{C^2} \frac{\delta C}{C^2} + \frac{\delta Q'_t \delta Q'_t}{C^2} + \frac{\delta T}{C^2},$$

(12.2)

where:

$$\delta T = \frac{\delta C(0)^2 + \delta C(T)^2}{T^2}.$$  

(13)

In the absence of measurement error one can show, as Axley [19] has done for the pulse technique, that the ventilation value calculated with an integral technique must have occurred sometime during the measurement period. Such an existence proof may be sufficient if it is exogenously determined that the ventilation moves within a narrow band. For most other purposes one wishes to know the average ventilation over the measurement period. If the measurement period, $T$, is known to be smaller than the character time over which the ventilation changes, then the integral approach will give a good estimate of the average ventilation; otherwise, averaging techniques may be preferable.

Averaging techniques

The techniques discussed so far all have assumed that the ventilation does not vary over the measurement period. The validity of this assumption will depend, of course, upon the physical situation being measured and the technique being used. If the ventilation is not actually constant over the measurement period it is not always clear what the previous techniques are actually calculating. This kind of error is qualitatively different from the types of error associated with uncertainties in the measured concentrations and flows. Average techniques are those which yield unbiased estimates of the average ventilation over the measurement period.

Although equation (6) cannot practically be used to determine the instantaneous ventilation, it can be averaged over a measurement period in order to get the average ventilation:

$$\bar{Q} = \frac{Q_T}{T} - \frac{V}{T} \ln \left( \frac{C_\text{final}}{C_\text{initial}} \right).$$  

(14)

This general expression can now be used to determine the average ventilation and its uncertainty for the different experimental approaches previously discussed.

The uncertainty in the estimate of the average ventilation can be estimated from the following equations:

$$\delta Q^2 = \left( \frac{1}{C} \right)^2 \delta Q'_t + \left( \frac{Q'_t}{C} \right)^2 \delta C^2 + \ln \left( \frac{C_\text{final}}{C_\text{initial}} \right) \frac{\delta V}{T} + \frac{V^2}{T^2} \left( \frac{\delta C^2}{C_\text{final} - C_\text{initial}} \right).$$  

(15.1)

$$\delta T = \left( \frac{1}{C} \right)^2 \delta T^2 + \left( \frac{Q'_t}{C} \right)^2 \delta C^2 + \frac{\delta T}{C_\text{final} - C_\text{initial}} + \frac{\delta C^2}{T^2} + \frac{\delta C}{T^2} \left( \frac{\delta C}{C_\text{final} - C_\text{initial}} \right).$$  

(15.2)

**SPECIFIC TECHNIQUES**

The general techniques cited above can be simplified by controlling the injection of tracer gas in certain well-behaved ways. Different groups have used different techniques depending on practical limitations. A rather complete listing of the groups using various techniques has...
Table 1. Summary of measurement methods

<table>
<thead>
<tr>
<th>Name</th>
<th>Measurement type</th>
<th>Flow control</th>
<th>Biased average?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient methods (measure $\lambda$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Simple) decay</td>
<td>Regression</td>
<td>Decay</td>
<td>Yes</td>
</tr>
<tr>
<td>Two-point decay</td>
<td>Average</td>
<td>Decay</td>
<td>No</td>
</tr>
<tr>
<td>Integral decay</td>
<td>Integral</td>
<td>Decay</td>
<td>Yes</td>
</tr>
<tr>
<td>Charge up</td>
<td>Regression</td>
<td>Const. inj./pulse</td>
<td>*</td>
</tr>
<tr>
<td>Steady-state methods (measure $Q$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulse</td>
<td>Integral</td>
<td>Pulse</td>
<td>Yes</td>
</tr>
<tr>
<td>Constant injection</td>
<td>Regression</td>
<td>Constant injection</td>
<td>No</td>
</tr>
<tr>
<td>Long-term integral</td>
<td>Integral</td>
<td>Any</td>
<td>Yes</td>
</tr>
<tr>
<td>Constant concentration</td>
<td>Average</td>
<td>Constant concentration</td>
<td>No</td>
</tr>
</tbody>
</table>

* Usually used with other methods to ascertain effective volume.

been compiled by the Air Infiltration and Ventilation Centre [20] and will not be repeated herein.

Steady-state techniques

Specific techniques tend to fall into two categories, transient and steady-state. Steady-state techniques, measure $Q$ and tend to minimize the effect of the accumulation of tracer in the test volume. In all steady-state techniques the experiment is so arranged that:

$$\frac{\delta Q}{Q^2} + \frac{\delta V}{V^2}$$

(16)

and the accumulation can be ignored in all the steady-state techniques. Since the steady-state techniques measure $Q$, an estimate of $\lambda$ will have an increased error:

$$\frac{\delta \lambda^2}{\lambda^2} = \frac{\delta Q^2}{Q^2} + \frac{\delta V^2}{V^2}.$$  

(17)

Transient techniques

Transient techniques tend to emphasize the change in tracer-gas concentration in the room and measure $\lambda$; inequality (16) is never true. Although $Q$ and $\lambda$ are simply related by the volume, the building volume that participates in air exchange is usually not known very precisely and the precision of the result may depend on which of the two types of approaches is used. Because the decay techniques are transient techniques and measure $\lambda$, the error in $Q$ is increased by the uncertainty in the volume:

$$\frac{\delta Q^2}{Q^2} = \frac{\delta \lambda^2}{\lambda^2} + \frac{\delta V^2}{V^2}.$$  

(18)

Table 1 summarizes the various techniques currently in use. The sections that follow will describe the techniques in more detail.

TRACER DECAY

The most widely used type of tracer measurement is that of a tracer decay. In a tracer decay test the test space is initially charged up to a concentration of tracer gas appropriate to the instrumentation and then shut-off and allowed to decay. The analysis in all decay techniques is simplified because the terms in both the ventilation and uncertainty equations, involving the injection rate vanish.

The concentration is monitored during the period of zero injection and then used to infer the ventilation rate, $\lambda$. There are three different approaches to analysing a tracer decay, all of which are transient approaches.

Decay regression

If there is no tracer injection and the concentration is allowed to decay from some initial value, the integral for equation (10) vanishes and we are left with the decay equation:

$$C(t) = C_R e^{-\lambda t}.$$  

(19)

This equation can be fit to the measured data using regression methods. The parameters of the fit are $C_R$ and $\lambda_R$. An estimate of the precision can be made from the regression.

The normal method of analysis uses a linear regression on the log of the concentration. This technique is extremely sensitive to zero drift and tends to overweight the contribution of lowest concentration values to the fit. A detailed error analysis of this kind of regression is beyond the scope of this report, but the uncertainty analysis from the other decay techniques can be used to give a reasonable estimate of the precision of this technique.

Integral decay

If over the measurement period there is no injection of tracer, equation (10.2) simplifies to the following:

$$Q_i = -\frac{V}{C},$$  

(20.1)

or, equivalently:

$$\lambda_i = \frac{C_{\text{initial}} - C_{\text{final}}}{TC}.$$  

(20.2)

The uncertainty in these quantities can be calculated from the uncertainties in the concentrations:
Two-point \((\text{average})\) decay

For a decay test the term involving the flow and concentration becomes identically zero and the average ventilation can be determined from the initial and final concentrations:

\[
\dot{Q} = \frac{V}{T} \ln \left( \frac{C_{\text{initial}}}{C_{\text{final}}} \right).
\]

This analysis technique has been used in what is known as the \textit{container method} [21]. If the air change rate varies over the measurement period, this technique assures an unbiased estimate of the average; it is a less precise technique than the others.

The uncertainty in the ventilation can be calculated from this expression:

\[
\delta \dot{Q} = \frac{1}{T^2} \left( \frac{\delta C_{\text{initial}}}{C_{\text{initial}}} + \frac{\delta C_{\text{final}}}{C_{\text{final}}} \right).
\]

Assuming the initial and final concentration can be measured equally well:

\[
\delta \dot{Q} \approx \frac{\sqrt{1 + e^{2\lambda T}}}{T} \frac{\delta C}{C_{\text{initial}}},
\]

the uncertainty is minimized by making the measurement about the same at the decay time, \(T_{\text{opt}} \approx 1/\lambda\). Thus:

\[
\frac{\delta \dot{Q}}{\dot{Q}} \leq 3 \frac{\delta C}{C_{\text{initial}}}.
\]

Note that this limit is inferior to the precision possible with the integral decay; such is the price of having an unbiased measurement.

\textbf{PULSE TECHNIQUE}

The pulse technique is a steady-state variant on the integral decay technique [22]. Unlike the decay techniques the measurement period begins before the tracer is injected and then waiting until the concentration decays. Also unlike decay techniques, the total volume of tracer as injected into the space must be known. When the concentration has decayed sufficiently that the inequality in equation (16) has been satisfied, the measurement can be terminated and the ventilation can be estimated as follows:

\[
\dot{Q}_p = \frac{Q_T}{C_T}.
\]

The uncertainty in this estimate is as follows:

\[
\frac{\delta Q^2}{Q^2} = \frac{\delta C^2}{C^2} + \frac{(\delta C + \delta C \cdot \lambda T)^2}{Q^2} + \frac{\delta V^2}{V^2} + \frac{\delta Q^2}{Q^2}.
\]

Since it takes infinitely long to decay to zero the accumulation term and its uncertainty must be considered, even though it does not contribute to the estimate of the ventilation. Because the injection is usually known quite precisely and the accumulation term is made small, the last two terms can usually be neglected.

Care must be taken not to wait too long or the concentration errors could increase because of the low average value.* The concentration-term error will increase as the square root of the measurement time and the accumulation-term error will decrease exponentially as does the instantaneous concentration. The optimum time will occur when these two terms are approximately equal:

\[
e^{-\lambda T} \approx 1.
\]

Since the total amount of tracer injected can be known quite well, the uncertainty only depends on the ability to measure the average concentration and the measurement time:

\[
\frac{\delta Q^2}{Q^2} \approx \frac{\delta C^2}{C^2} + e^{-\lambda T}.
\]

\textbf{Charge up}

If the pulse is injected quickly and the mixing in the test space is quite good. The rise in concentration due to the pulse of tracer gas can be used to determine the effective volume of the test space. However, care must be taken to ensure that good mixing is present and that the air change rate is not too high, before this volume measurement technique should be considered.

\textbf{CONSTANT INJECTION}

If the tracer injection is non-zero, but constant the integral in equation (7) can be simply evaluated:

\[
C(t) = \frac{Q_T}{Q_k} + \left( C_k - \frac{Q_T}{Q_k} \right) e^{-\lambda t}.
\]

Although some researchers [3, 23] have fit this curve directly using non-linear methods to find the ventilation and the effective volume, most use of this method has been to wait until \textit{steady-state} has been reached and then the data is fit to the steady state equation:

\[
Q_k = \frac{Q_T}{C(t)} \quad \text{for} \quad e^{-\lambda T} \ll 1.
\]

Regression techniques in this instance are equivalent to an average. Multiple independent measurements of the ventilation can be used to increase the precision accordingly. Again, it should be noted that if the ventilation is not constant during the measurement period the result may be biased. The result will not be biased, however, if the following condition is met:

\[
C_{\text{final}} - C_{\text{initial}} \ll \lambda T.
\]

Often, however, an average concentration is used in the denominator; such an approach is actually an integral approach (see the long-term average technique):

\[
\dot{Q}_l = \frac{Q_T}{C_l}.
\]

* A possible improvement on this technique would be to use a non-zero starting and stopping value by pre-injecting a small amount of tracer before beginning the experiment; thus allowing the accumulation term to be made smaller without needing such a long integration time. Such a technique approaches a modified \textit{constant concentration} method, which is discussed in a later section.
This approach will be biased, but not in the same way as above. As long the accumulation term remains small:

\[ Q_i \approx Q_h \approx Q. \]  

(34)

Thus, \( Q_h \) is preferable to \( Q_i \).

Charge up

As noted above constant injection is usually done after steady-state has been achieved. However, if the mixing is good, it may be possible to use the period of tracer accumulation (i.e. \( \lambda T \approx 1 \)) to determine \( \lambda \) (or, alternatively, the effective volume). Although the mixing constraints are not as severe as in the pulse charge-up technique, similar care must still be taken.

**LONG-TERM INTEGRAL**

The long-term integral method—sometimes called the long-term (concentration) average method—is a steady-state approach in which the accumulation term is minimized by using a continuous injection and averaging long enough for the \( T \) in the denominator of the accumulation term to make the term arbitrarily small. Equation (10.2) then becomes the following:

\[ Q_i = \frac{\dot{Q}_T}{C} \quad \text{for} \quad \lambda T \gg 1. \]  

(35)

Note that although this technique is usually used with constant injection, any injection pattern could be used. (When done with constant injection, this technique is sometimes referred to as the passive ventilation measurement technique most typically used with perfluorocarbon tracer [24].)

If we assume that the accumulation term is sufficiently small so that errors attributable to the accumulation term are negligible, the uncertainty in this steady-state result is simply:

\[ \frac{\delta Q^2}{Q^2} = \frac{\delta C_i^2}{C_i^2} + \frac{\delta \dot{Q}_T^2}{\dot{Q}_T^2}. \]  

(36)

One potential problem with this technique is that it requires a measurement time very long compared to the decay time, but it also assumes that ventilation is constant over that time. It has been shown [25] that even if the injection is constant, this technique always leads to a negative bias in the estimation of the ventilation unless the ventilation is quite constant. This technique is not recommended for use in determining the average ventilation in buildings.

The average ventilation is not always the quantity of interest in making tracer gas measurements. It has been shown [26] that the quantity of interest for determining the amount of pollutant dilution provided by ventilation is not the average ventilation, but a quantity called the effective ventilation. The passive ventilation technique directly measures the effective ventilation and so may be the technique of choice for determining pollutant dilution.

**CONSTANT CONCENTRATION**

In the constant concentration technique a steady-state technique in which an active control system is used to change the amount of injected tracer to maintain the concentration of tracer at some target level, \( C_T \). With that assumption the accumulation term is identically zero and equation (14) can be used to calculate the average ventilation from the average injection:

\[ \dot{Q} = \frac{\dot{Q}_T}{C_T}. \]  

(37)

Although it does not enter the estimate of the ventilation the accumulation term can effect the uncertainty, if the control is not perfect:

\[ \frac{\delta Q^2}{Q^2} = \frac{\delta Q_T^2}{Q_T^2} + \frac{\delta C_T^2}{C_T^2} + \frac{\nu^2 \delta C_T^2}{Q_T^2}. \]  

(38)

The last term is due to the uncertainty in the accumulation term. The error in the second term comes from the difference between the average concentration during the period and the target.

**EXAMPLE ERROR ANALYSES**

In this section a few examples of how various techniques might be used in specific situations and what kind of uncertainties could be expected. For the purposes of these examples, it will be assumed that a single concentration measurement can be made with a 10% uncertainty scaled to the initial concentration. Although almost all analysers have better accuracy than 10%, the vagaries of sampling and mixing make 10% a reasonable number. In any case, the calculated uncertainties will scale with this number. It will also be assumed that it is possible to make 10 measurements per hour.

The volume of the example zone will be assumed to be 100 m³. For the decay methods (see Table 2) only \( \lambda \) will be calculated and for the steady-state methods (see Table 3) only \( \dot{Q} \) will be calculated so that the uncertainty in the volume will not enter. It should be kept in mind, however, that zone volumes are rarely known to better than 20% so that the additional error could be significant.

Average decay

The last row of Table 2a should never be used in a real experiment, but it serves to demonstrate what can happen to the precision if the decay goes on too long.

Integral decay

As can be seen (Table 2b) from the average-decay uncertainty analysis, the uncertainties begin to increase for long measurement times and higher air exchange rates. This increase is due to the fact that the final concentration begins to get quite low and, hence, quite uncertain. The alternative decay procedures do not suffer from this and can give better precision in this range—provided the air change rate can be assumed to be constant.

The last row of this table suggests that good precision can be achieved for longer measurement periods with an integral decay. However, an 8 h measurement period may only be justified if the ventilation were known to be constant. It should be noted that this technique is quite
Table 2a. Uncertainties for an average decay

<table>
<thead>
<tr>
<th>$\delta t$</th>
<th>$T$</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 min</td>
<td>0.433 (177%)</td>
<td>0.464 (93%)</td>
<td>0.515 (52%)</td>
<td>0.657 (33%)</td>
<td></td>
</tr>
<tr>
<td>1 h</td>
<td>0.163 (65%)</td>
<td>0.193 (39%)</td>
<td>0.290 (29%)</td>
<td>0.746 (37%)</td>
<td></td>
</tr>
<tr>
<td>3 h</td>
<td>0.078 (31%)</td>
<td>0.153 (31%)</td>
<td>0.670 (67%)</td>
<td>13.4 (67%)</td>
<td></td>
</tr>
<tr>
<td>8 h</td>
<td>0.093 (37%)</td>
<td>0.583 (13%)</td>
<td>37.0 (37%)</td>
<td>$\infty$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2b. Uncertainties for an integral decay

<table>
<thead>
<tr>
<th>$\delta t$</th>
<th>$T$</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 min</td>
<td>0.442 (177%)</td>
<td>0.462 (92%)</td>
<td>0.503 (50%)</td>
<td>0.607 (30%)</td>
<td></td>
</tr>
<tr>
<td>1 h</td>
<td>0.160 (64%)</td>
<td>0.181 (36%)</td>
<td>0.229 (23%)</td>
<td>0.356 (18%)</td>
<td></td>
</tr>
<tr>
<td>3 h</td>
<td>0.067 (27%)</td>
<td>0.093 (19%)</td>
<td>0.160 (16%)</td>
<td>0.359 (18%)</td>
<td></td>
</tr>
<tr>
<td>8 h</td>
<td>0.041 (17%)</td>
<td>0.076 (15%)</td>
<td>0.167 (17%)</td>
<td>0.456 (23%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3a. Uncertainties for the pulse method

<table>
<thead>
<tr>
<th>$\delta Q$</th>
<th>$T$</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>42 (85%)</td>
<td>72 (72%)</td>
<td>103 (51%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>27 (61%)</td>
<td>37 (37%)</td>
<td>30 (15%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>11 (23%)</td>
<td>8 (8%)</td>
<td>22 (11%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>2 (5%)</td>
<td>9 (9%)</td>
<td>36 (18%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Errors caused by offsets in the concentration at. If there is no zero offset (and if the measurement is still random at low values), the method has the potential for greater precision in the integral decay method; the cost for this is potentially the ventilation is not constant.

Weighting is done correctly, the regression decay potential for the same level of error as the integral decay, however, if the standard method is used, the 3h decay will give unreliable results when the ventilation is allowed to decay too far.

Method

The pulse method (see Table 3a) it is assumed that the amount of injected tracer is known to better than 1% an be neglected in the error analysis; this assumption may not be valid if crude injection methods are used.

In the decay methods, an eight hour measurement may only be justified if the ventilation was known to be constant. Also like the decay methods, this technique is sensitive to zero offsets.

Because of the trade-off between additional measurement points, accumulation term, and low average concentration, the pulse technique's precision behavior is complex. However, there is an optimal measurement time for any given experimental set of conditions, which tends to be longer than for the decays.

Constant concentration

The constant concentration method (Table 3b) will be used as an example of the general steady-state method. It is assumed that the control is unbiased (i.e. that there is no difference between the mean concentration and the target). It is also assumed that the average flow can be determined to 1% precision; not all constant concentration systems can achieve this, but many claim to. With these assumptions and for the example values chosen, the error is dominated by the uncertainty in the accumulation term.

The absolute error in the table does not depend much on the ventilation, but on the measurement time. (This result is a consequence of the high measurement rate and the small injection error.) If the measurement time were extended, this trend would change.

DISCUSSION

The use of tracer gases to determine total ventilation is a powerful tool. All of the techniques, however, make use of certain assumptions, and the validity of these assumptions must be checked to insure confidence in the results.

The most universally important assumption is the zonal assumption, which encompasses many of the processes lumped under the rubric of mixing. The most common mistakes made in tracer-gas analyses usually involve the analysis of measurements made under poorly mixed conditions.

Even if the zonal assumption has been verified and the tracer gas meets all of the criteria, it is not necessarily clear which of the approaches is best suited for a particular use. Depending on the quantity desired, the stability of the air flows, and the experimental limitations, various choices can be made. The choice of method to use in a given situation will depend on the practical details of the experiment as well as the reason for measuring the air change in the first place.

Decay and pulse methods require the least time and usually the least preparation. With the exception of the two-point method, all of these give biased estimates...
of the average air change rate. In most circumstances, however, the biases will be negligible as long as the measurement period is limited to times on the order of the air change time. Although unbiased, the two-point method may yield unacceptably large uncertainties in the air change rate, unless the precision in the measurement of those two concentrations can be made quite high.

Like the pulse technique, the long-term integral technique may yield a biased estimate of the average ventilation. Because of much longer measurement period, however, the bias may not be negligible. Although the long-term technique may prove unacceptable for determining average ventilation, it can nevertheless provide an unbiased estimate of the effective air change rate.

The constant concentration technique can be both accurate and precise, but it requires the most equipment as well as sophisticated control systems and real-time data acquisition. The constant injecting technique (without charge-up) can be considered a somewhat simpler version of the constant concentration technique, in that no active control of the injection rate is needed. Rather segments of data which meet equation (32) are used to determine the average ventilation. If this condition is not met equation (14) must be used to analyze the data.

The majority of recent research in the field has involved more complex experimental situations than the single-zone ones considered herein. A detailed discussion of such techniques is beyond the scope of this report, but there are several references of interest which the reader may wish to consult.* Some of the multi-zone research has focused on the precision and accuracy of measurement techniques [27–29] but can also be applied to single-zone techniques.

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