EXPERIMENTAL VALIDATION OF A SINGLE GAS TRACER TECHNIQUE FOR ANALYZING AIRFLOWS AND EFFECTIVE VOLUMES IN MULTIZONE SYSTEMS

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This paper presents and experimentally validates a method, based upon tracer gas techniques, for determining interzonal airflows and effective volumes in a multizone enclosure. Presently used tracer gas techniques have a number of drawbacks including the need for multiple tracers when analyzing a multizone structure. Also, traditional techniques cannot be used to independently determine volumetric flow rates and effective volumes in the multizone case. The method described in this paper eliminates some of the problems introduced by multiple tracers and allows the independent determination of both volumetric flow rates and effective volumes.

The proposed method uses a single tracer gas to disturb the zones. A state-space formulation is used to model the multizone system. The concentration data are used in combination with a least-squares identification algorithm to determine all of the interzonal airflows and effective volumes. A three-zone experimental facility is used to validate the method. The experimental results show that this technique may be an effective alternative to presently used multiple tracer methods.

INTRODUCTION

In this paper, a method is proposed for determining interzonal airflows and effective volumes in a multizone enclosure. The method is based upon tracer gas techniques and uses inputs of a single tracer to disturb each of the zones. A state-space formulation is used to model the multizone system and the concentration data are used in combination with a least-squares identification algorithm to determine all of the interzonal airflows and effective volumes. The method also shows promise for identifying multizone model orders and for use in systems with slowly varying parameters and transport delays.

MULTIZONE MODEL

The following is a model formulation which follows directly from (1). When examining a general multizone system, the number of unknown parameters which must be identified becomes quite large. For example, in a three-zone system, there are a total of 15 unknown system parameters. These include 12 interzonal airflows–including exchange with the outdoors. There are also 3 unknown effective volumes. This type of modeling makes two very important assumptions. The first assumption is that the number of well mixed zones is known. The second is that the locations of each individual zone are known. While there may be cases where physical barriers make zone locations obvious, it may prove difficult in many systems to determine the actual locations of the zones. The assumption is also made that the air in each zone is uniformly mixed.

For such a multizone system, conservation of mass for the tracer gas in a single zone, i, can be written as

$$V_i(t)\,\dot{c}'_i(t) = \sum_{j=0}^{n}(1-\delta_{ij})F_{ji}(t)c'_j(t) - c'_i(t)\sum_{j=0}^{n}(1-\delta_{ij})F_{ij}(t) + g_i(t) \qquad (1)$$

where

- $g_i(t)$ = tracer input into zone i (mass/time)
- $V_i(t)$ = effective volume of zone i
- $c'_i(t)$ = tracer concentration in zone i (mass/volume)
- $\dot{c}'_i(t)$ = time derivative of tracer concentration in zone i (mass/volume-time)
- $F_{ij}(t)$ = flow from zone i to j (volume/time)
- δ_{ij} = Dirac delta function ($\delta_{ij} = 0$ for $i \neq j$; $\delta_{ij} = 1$ for $i=j$)
- n = total number of zones

The subscript "0" represents outdoor air. If the concentration of tracer in the outdoors is considered constant or relatively slowly varying, a change of variables can be made. If this approximation is incorporated, the outdoor concentration, c'_0, can be eliminated from Equation (1) by defining the other concentration terms to be the difference between the actual zone concentration and the outdoor value

$$c_i \equiv c'_i - c'_0 \tag{2}$$

To be completely general, Equation (1) also allows for the possibility that the interzonal airflows and effective volumes may vary during the duration of the tracer gas test.

Equation (1) represents n first-order simultaneous differential equations for the multizone system. They can be written compactly by introducing state-space notation. This form has become the standard for presenting the mass conservation equations. For the three-zone case, Equation (1) can be rewritten in state-space form as

$$\begin{bmatrix} V_1(t) & 0 & 0 \\ 0 & V_2(t) & 0 \\ 0 & 0 & V_3(t) \end{bmatrix} \begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \\ \dot{C}_3(t) \end{bmatrix} = \begin{bmatrix} -Q_1(t) & F_{21}(t) & F_{31}(t) \\ F_{12}(t) & -Q_2(t) & F_{32}(t) \\ F_{13}(t) & F_{23}(t) & -Q_3(t) \end{bmatrix} \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \end{bmatrix} + \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} \tag{3}$$

where

$$Q_i(t) = \sum_{j=0}^{n}(1-\delta_{ij})F_{ij}(t) = \sum_{j=0}^{n}(1-\delta_{ij})F_{ji}(t) \tag{4}$$

The first summation in Equation (4) represents the net outflow from zone i to all the other zones including the outdoors. The second summation represents the net inflow to zone i from all the other zones including the outdoors. In steady-state, net inflow equals net outflow.

Equation (3) can be represented more compactly in matrix form as

$$\mathbf{V}(t)\,\dot{\mathbf{c}}(t) = \mathbf{F}(t)\,\mathbf{c}(t) + \mathbf{g}(t) \tag{5}$$

or multiplying through by $\mathbf{V}^{-1}(t)$,

$$\dot{\mathbf{c}}(t) = \mathbf{V}^{-1}(t)\mathbf{F}(t)\,\mathbf{c}(t) + \mathbf{V}^{-1}(t)\mathbf{g}(t) \tag{6}$$

Equation (6) is known as the time varying state-space representation of the system of linear differential equations described by Equation (1). It is this equation, along with the accumulated tracer gas data, which is then often used to estimate the parameters **V**(t) and **F**(t) for the unknown multizone system.

LEAST-SQUARES IDENTIFICATION PROCEDURE

Since most data is collected at a finite number of points during the course of a test, a number of discrete-time methods have been developed for analyzing the data to extract the necessary information. However, before one of these methods can be described (the least-squares algorithm) it is necessary to transform the continuous-time system model to its discrete-time equivalent. Details on discretization can be obtained from (2). The discrete-time form of Equation (6) is

$$\mathbf{c}[(k+1)T] = \mathbf{A}\,\mathbf{c}(kT) + \mathbf{B}\,\mathbf{g}(kT) \tag{7}$$

where **A** and **B** are defined as

$$\mathbf{A} = \exp[(\mathbf{V}^{-1}\mathbf{F})T] = \mathbf{I} + (\mathbf{V}^{-1}\mathbf{F})T + \frac{[(\mathbf{V}^{-1}\mathbf{F})T]^2}{2!} + \frac{[(\mathbf{V}^{-1}\mathbf{F})T]^3}{3!} + \cdots \tag{8a}$$

$$B = \int_0^T \exp[(V^{-1}F)t]V^{-1}dt = \left[IT + \frac{(V^{-1}F)T^2}{2!} + \frac{(V^{-1}F)^2T^3}{3!} + \cdots\right]V^{-1} \quad (8b)$$

Equation (7) is valid if the flow matrix, **F**(t), volume matrix, **V**(t), and input vector, **g**(t), are constant during the sampling interval, T. The matrices V^{-1} and **F** in Equation (8) are defined as the values of $V^{-1}(t)$ and **F**(t) on the interval (kT, [k+1]T).

To formulate the least-squares estimate of the system parameters it is useful to first transform the discrete-time state-space equation into a multiple-input/multiple-output (MIMO) form. To do this, Equation (7) is rewritten in a slightly different form

$$c[k] = \theta^T\phi(k-1) + v(k-1) \quad (9)$$

where the output, **c**[k], is a vector containing the measured tracer concentrations in each zone at time step k. The symbol, θ, is used to denote the parameter matrix,

$$\theta = [\,A \;\; B]^T \quad (10)$$

and contains the unknown parameters of interest. The variable, ϕ(k-1), is the regression vector whose components are comprised of past observations of the inputs and outputs of the system (regression variables)

$$\phi(k-1) = [c_1(k-1) \;\; c_2(k-1) \ldots c_n(k-1) \;\; g_1(k-1) \;\; g_2(k-1) \ldots g_n(k-1)]^T \quad (11)$$

The vector, **v**(k-1), contains unknown and unmeasurable disturbances to the system (eg. measurement noise).

The method of least-squares is described by the criterion function

$$S(\theta) = \sum_{k=1}^{N} \beta(k)\left\{[c^T[k]-\phi^T(k-1)\theta]\,[c[k]-\theta^T\phi(k-1)]\right\} \quad (12)$$

where β(k) is a sequence which can be used to give varying weight to the data. Equation (12) can be readily solved for the optimal value of θ which minimizes the squared error between the predicted and actual tracer gas concentrations. A recursive solution is presented below (5). Using this procedure, the new estimate of the parameter matrix, $\hat{\theta}(k)$, is equal to the old estimate, $\hat{\theta}(k-1)$, plus a gain matrix, **L**(k), times the error between the predicted and actual values of the output(s). The algorithm is thus,

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k)[y^T[k]-\phi^T(k-1)\hat{\theta}(k-1)] \quad (13a)$$

where

$$L(k) = \frac{P(k-1)\phi(k-1)}{1/\beta(k-1) + \phi^T(k-1)P(k-1)\phi(k-1)} \quad (13b)$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k-1)\phi^T(k-1)P(k-1)}{1/\beta(k-1) + \phi^T(k-1)P(k-1)\phi(k-1)} \quad (13c)$$

EXPERIMENTAL PROCEDURE

The single gas tracer technique proposed above is meant to be an experimental tool which can be used to evaluate the internal dynamics (flows and effective volumes) of a multizone enclosure. To validate the identification technique, an experimental facility was constructed. Figure 1 shows a schematic of the three-zone test facility developed at the University of Illinois. The internal physical volumes of Zones 1, 2, and 3 are 25.5, 12.5, and 12.5 m^3 respectively.

Figure 2 shows the experimental data for a typical three-zone test. The total length of the test was arbitrarily chosen to be 3 hours (10800 seconds). The tracer input to each zone was a single pulse injection of 0.0593, 0.0502, and 0.0528 kg applied to Zones 1, 2, and 3 respectively. The injected amounts for Zones 2 and 3 were lower than that for Zone 1 because of their smaller physical volumes. The duration of each of the pulse inputs was approximately 9 seconds. The concentrations were sampled at 120 second intervals.

The figure shows that the concentration data is fairly smooth and corresponds very well (as will be shown) to a third-order system model. Since the pulse inputs result in sharp increases in tracer concentration, the tracer gas pulses mix within a single sample period. Thus, each of the three physical volumes corresponds well to an individual effective volume. Figure 2 also shows that the outdoor tracer concentration remained relatively constant for the duration of the test.

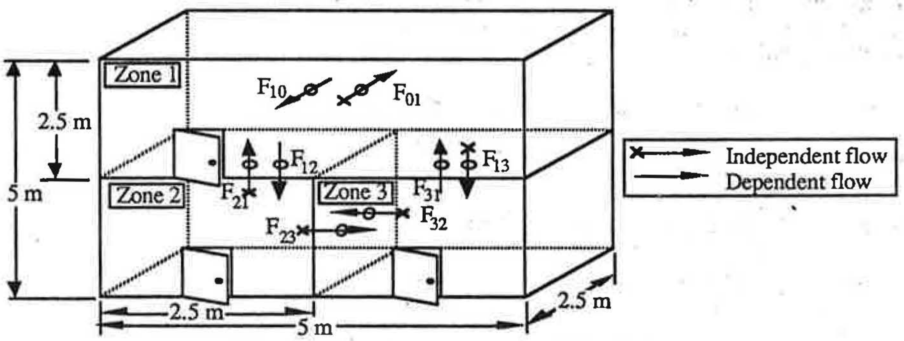

Figure 1. Three-Zone Experimental Test Facility

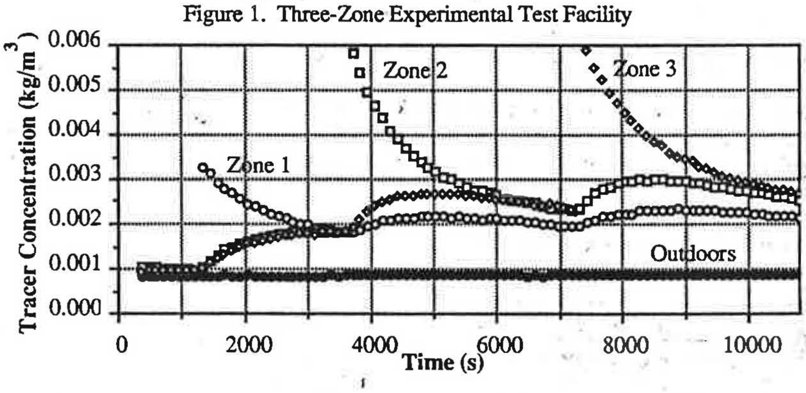

Figure 2. Experimental Tracer Concentration Data for Three-Zone

EXPERIMENTAL RESULTS

The interzonal airflows and effective volumes of this three-zone system were identified using the method of recursive least-squares. The identified parameters were compared to the actual values by introducing the dimensionless parameter ratio, Ω. The parameter ratio is defined as the ratio between the predicted value of a parameter to its actual measured value

$$\Omega_{ij} = \frac{F_{ij\ predicted}}{F_{ij\ actual}} \tag{14}$$

for airflow rates F_{ij} ($i \neq j$) and

$$\Omega_i = \frac{V_{i\ predicted}}{V_{i\ actual}} \tag{15}$$

for effective volumes. The actual values of the flows were measured experimentally. The actual values of the effective volumes were assumed to be equal to the physical volumes of the zones (since well mixed).

Figures 3 through 5 show the effective volume parameter ratios Ω_1, Ω_2, and Ω_3 as a function of time for the data of Figure 2. The figures indicate that all of the effective volumes are identified to within 3% of their measured value. The figures also show that the effective volume of zone i is identified within a few samples following the input of tracer to that zone. The identified effective volume is also relatively insensitive to inputs applied to other zones.

Figures 6 through 10 shows the airflow parameter ratios Ω_{01}, Ω_{21}, Ω_{32}, Ω_{13}, and Ω_{23} as a function of time for the data of Figure 2. The figures indicate that all of the interzonal airflows are identified to within 15% of their measured value. In fact, all but one are identified to within 10%. As each successive pulse input is applied to the zones, more parameters are identified. However, complete identification does not occur until after the third pulse input has been applied to Zone 3. Thus, to estimate the values of all the unknown parameters, a pulse input must be applied to each of the zones.

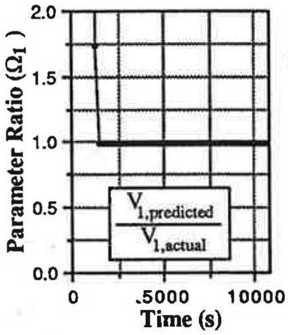

Figure 3. Ω_1 versus Time

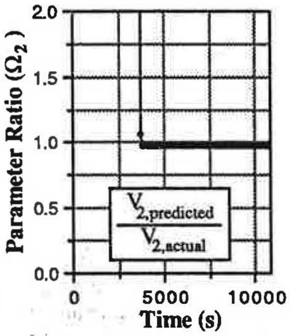

Figure 4. Ω_2 versus Time

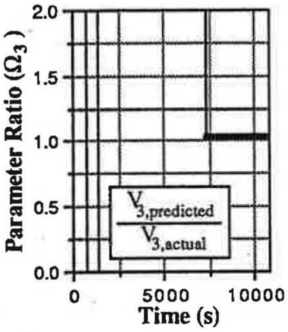

Figure 5. Ω_3 versus Time

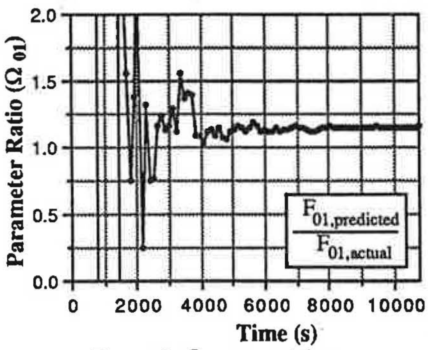

Figure 6. Ω_{01} versus Time

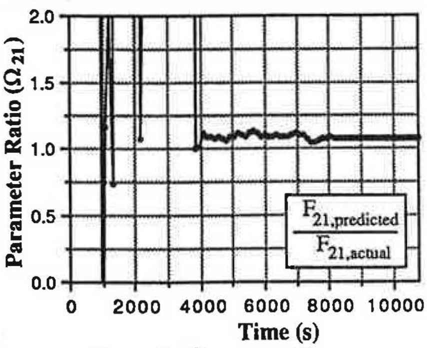

Figure 7. Ω_{21} versus Time

CONCLUSIONS

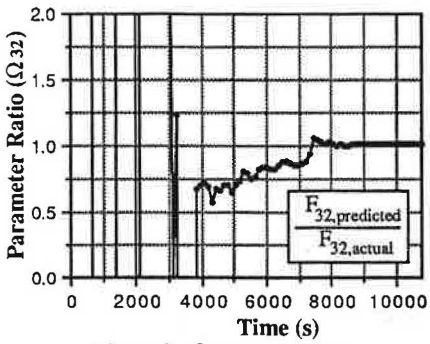

Figure 8. Ω_{32} versus Time

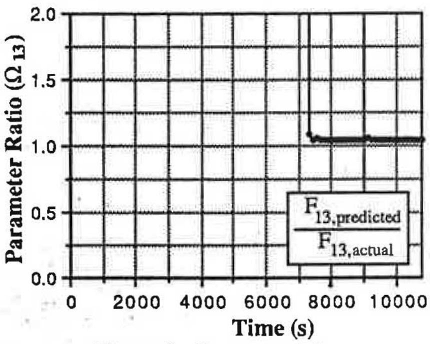

Figure 9. Ω_{13} versus Time

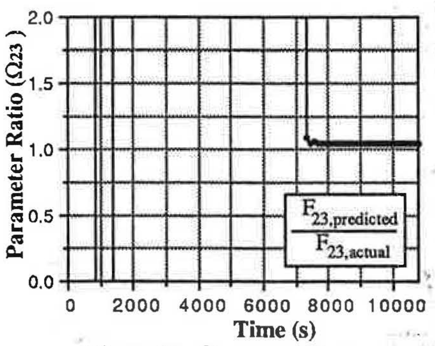

Figure 10. Ω_{23} versus Time

The experimental results show that the recursive least-squares technique is able to correctly estimate all of the unknown independently controlled flows to within 15%. It is also able to estimate the effective volumes of the zones to within 3%. The recursive technique has the advantage of allowing an on-line examination of the identification procedure. This enables the investigator to select appropriate times to apply inputs and terminate the test. Since, for the test shown, the identified parameters vary little following the third pulse input, the recursive procedure indicates that the test could have been terminated earlier. Future work is needed to determine optimal test durations and sampling intervals, develop methods to estimate system order, and examine cases in which non-uniform mixing occurs.

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