Using the T-Method for duct system design

The T-Method offers significant advantages over other methods for designing optimal HVAC air duct systems

By Robert J. Tsal, Ph.D., and Herman F. Behls, P.E.
Member ASHRAE *Member ASHRAE*

Studies show that HVAC air duct systems are one of the major energy consumers in industrial and commercial buildings. Inefficient design of a duct system means that either energy is being wasted and/or excessive ductwork material is being installed.

Duct system optimization offers the opportunity to realize significant owning and energy savings. Although some advances in optimization procedures have been made in the literature (ASHRAE 1985), their application to HVAC systems design remains the exception, rather than the rule, in practice.

Four methods of duct design are presented in the ASHRAE 1985 *Handbook of Fundamentals*. These methods were developed as expedient procedures and do not address optimization nor simulation. Even available duct design computer programs are simply automated versions of these procedures. To this day, duct design is more an art than a science. The same air distribution system calculated by different engineers results in different duct sizes and costs.

Another problem is duct simulation. The pressure losses at nodes of a duct system will always balance naturally. If the design does not provide this balancing, the flow rates will balance themselves and the total flow and pressure loss will be in accordance with the fan performance curve. The purpose of simulation is to find the actual flow in a duct system.

There is no duct simulation method in the ASHRAE publications. However, the need to calculate flow distribution in a duct system occurs any time an engineer is studying the effect of system performance or control.

About the authors

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Following are questions that designers face when designing or retrofitting a duct system.

- How does the change in damper blade angle influence airflow at existing terminal outlets?
- What is the operating point on the fan performance curve when changes in duct size or damper position are applied?
- What happens when a fan in a multiple-fan system is shut down?
- Is it necessary when retrofitting an air distribution system to replace the motor and/or fan?
- What is the flow distribution in a variable air volume (VAV) system when terminal box flows approach a minimum?
- How should additional terminal outlets be connected to an existing system?
- What is the possibility of damper self-noise?
- What happens in a system in case of a fire when some dampers are closed and others open?

A simulation paper will be presented at ASHRAE's 1990 Annual Meeting in St. Louis (Tsal *et al.* 1990). The simulation method can promote energy saving during the period of partial building occupation by predicting flow distribution and assisting in selecting and locating dampers.

Duct system optimization

A comprehensive analysis of the four traditional duct design methods (equal friction, static regain, velocity reduction and constant velocity) is presented in Tsal and Behls (1986). Three requirements for optimizing a duct system are: the fan must operate at optimum system pressure; the ratio between the velocities in all sections of the duct system must be optimal; and pressure balancing must be obtained by changing duct sizes, not dampers or other devices. None of the traditional methods satisfies all of these requirements.

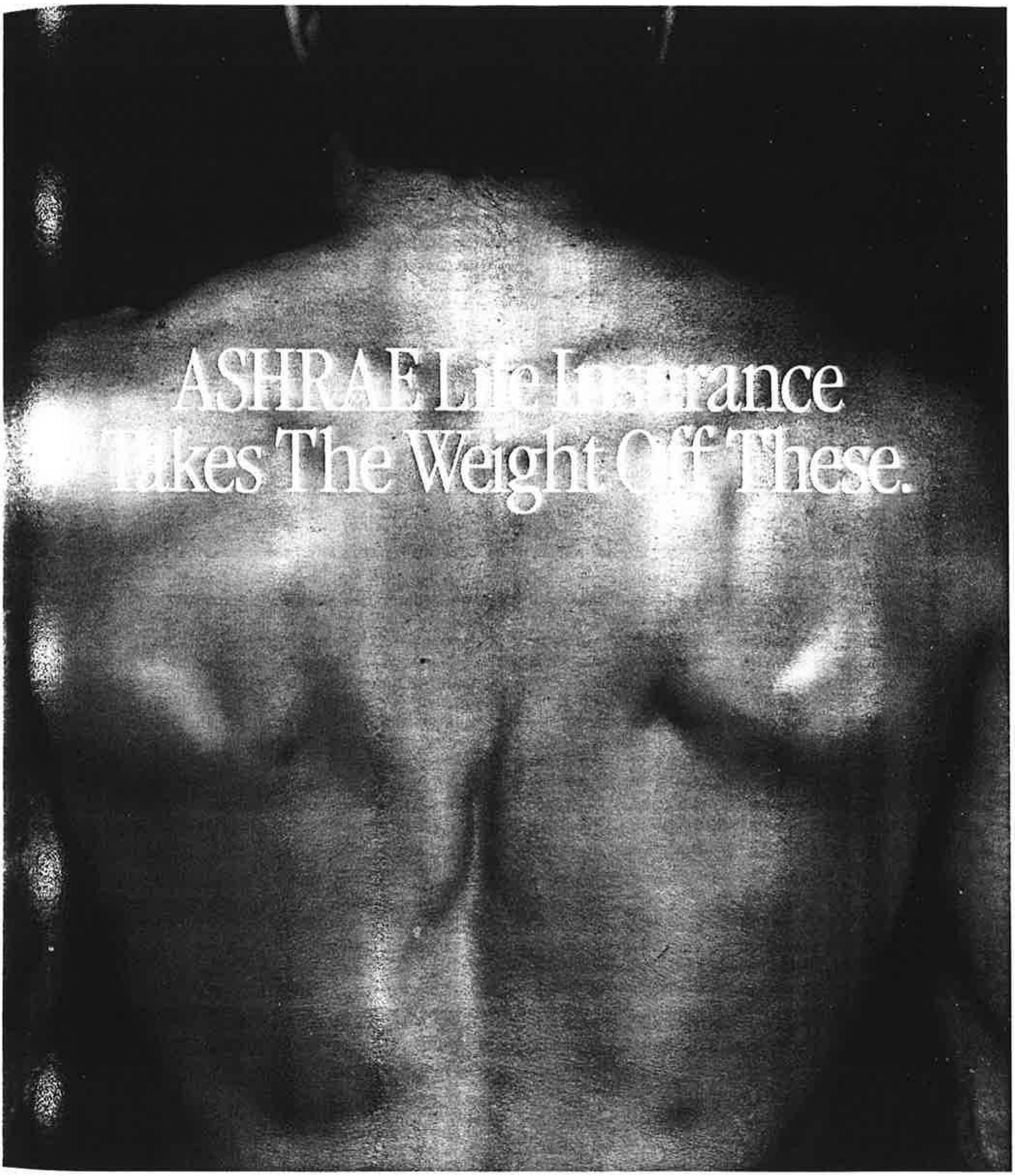
The first optimization method was developed by Grasshoff in 1875 for a single pipeline. Since then, many analytical and numerical methods for pipe and duct optimization have been developed. For a comprehensive survey of the existing numerical duct optimization methods, see Tsal and Adler (1987).

Several of the calculation procedures attempt to minimize total cost by establishing optimum velocities or friction rates. These procedures are based on the classical calculus minimization technique of setting the first derivative to zero to find the diameter of the pipe or duct.

The classical method of optimization for a multi-path district heating system was first applied by Shifrinson (1937) and for multi-path duct systems by Lobaev (1959). Even though these were before the computer era, their techniques are impractical for manual calculation. According to Tsal and Adler (1987), analytical approaches may be effectively used only to identify trends in system behavior. A comprehensive analysis of such a duct system was published by Bouwman (1982).

The computer-aided numerical optimization methods are divided into two categories: the discrete methods which are coordinate descent and dynamic programming (Tsal and Chechik 1968); and the continuous methods which are penalty function, Lagrange multipliers (Zanfirov 1933, Kovaric 1971, Stoecker *et al.* 1971); gradient method (Arklin and Shitzer 1979); and quadratic search (Leah *et al.* 1987).

Many of these methods can find the minimum of an unconstrained concave problem, but most fail to yield a successful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solution that can be used in practice. There is no analytical or numerical method that can easily find the global minimum and satisfy all duct system constraints.

System cost

The goal of duct optimization is to determine duct sizes and select a fan that minimizes system life-cycle cost. The owning cost includes initial cost, property taxes, insurance and salvage value. The operating cost includes the costs of energy, maintenance, operation labor, income tax and cost escalation.

The purpose of optimization is to compare system cost for different fan total pressures. Therefore, many of the above factors are constant and can be excluded from the objective function. Only initial cost, energy cost, time period, escalation rate and interest rate are used for optimization. Life-cycle cost is given by,

$$E = Ep\ (PWEF) + Es \tag{1}$$

Electric energy cost is determined by,

$$Ep = (Qfan)\,\frac{(Ec)\ Y + Ed}{10^5\ g_f\ g_e}\,(Pfan) \tag{2}$$

(For definition of variables, see Nomenclature.)

Electric energy cost depends on residential, commercial and industrial retail prices of electricity as well as on the differences in demand and consumption costs. It must be considered that the electricity demand cost (Ed) must be paid, not only for HVAC systems, but for the entire building; and the difference between the fan pressures (Pfan) for optimized and non-optimized systems is a part of this demand. To simplify the calculation procedure, assume that Ed is a constant.

The present worth escalation factor is,

$$PWEF = \frac{[(1 + (AER)) / (1 + (AIR)]^a - 1}{1 - [(1 + (AIR)) / (1 + (AER))]} \tag{3}$$

Heating and cooling loads depend on many probability factors. Therefore, there is no need for over-accurate economic data for duct design. If the interest rate (AIR) is unknown, the recommended interest rate is 6 percent. If the amortization period (a) is unknown, 10 years is recommended.

The initial cost includes the cost of ducts and HVAC equipment. The duct cost is presented as a function of the cost per unit area of duct surface, adjusted for straight duct and fittings. For round ducts, the cost is,

$$Es = Sd\ \pi\ D\ L \tag{4a}$$

For rectangular ducts, the cost is,

$$Es = 2\ Sd\ (H + W)\ L \tag{4b}$$

An important factor of duct optimization is the cost of space required by ducts and equipment. This additional cost reduces the size of ducts and thereby increases energy consumption. If saved space is available for use as rentable area, it must be included into the objective function.

Constraints

The following list of constraints is necessary for duct optimization. A detailed explanation of each constraint can be found in Tsal and Adler (1987).

- Kirchoff's first law. For each node, the flow in is equal to the flow out.
- Pressure balancing. The total pressure loss in each path must be equal to the fan total pressure. This restriction, called Kirchoff's second law, is analogous to electric networks.
- Nominal duct sizes. Each diameter of a round duct or height and width of a rectangular duct is rounded to the nearest lower or upper nominal size. Nominal duct size normally depends on the manufacturer's standard increments. Such increments may be 25 mm (1 in.) for sizes up to 500 mm (20 in.), then 50-mm (2-in.) increments. Standard sizes can differ by country.
- Air velocity restriction. This is an acoustic (ductwork regenerated noise) or particle conveyance limitation.
- Preselected sizes. Duct diameters, heights and/or widths can be preselected.
- Construction restrictions. Architectural space limitations may restrict duct sizes.
- Equipment. Central air-handling units and duct-mounted equipment are selected from the set produced by industry.

T-Method theory and procedures

The T-Method is a new optimization method (Tsal *et al.* 1986) that minimizes the objective function. This method is based on the same tee-staging idea as dynamic programming (Bellman 1957, Tsal and Chechik 1968).

However, the necessity for phase level solutions is eliminated by introducing local optimization at each stage. This modification drastically reduces the number of calculations, but requires iterations.

The T-Method incorporates the following major procedures:

- System condensing. Condensing a branched tree system into a single imaginary duct section with identical hydraulic characteristics and the same initial cost as the entire system.
- Air-handling unit selection. Selecting an optimal fan and establishing the optimal system pressure loss.
- System expansion. Expanding the condensed imaginary duct section into the original system with optimal distribution of pressure loss.

By substituting Equation 2 and Equation 4 into Equation 1, the objective function for a single duct becomes:
For round duct:

$$E = z_1\ (Pfan) + Sd\ \pi\ D\ L \tag{5a}$$

For rectangular duct:

$$E = z_1\ (Pfan) + Sd\ 2\ (H + W)\ L \tag{5b}$$

where the intermediate coefficient z_1 is,

$$z_1 = (Qfan)\,\frac{(Ec)\ Y + Ed}{10^5\ g_f\ g_e}\,(PWEF) \tag{6}$$

The Darcy-Weisbach equation for round and rectangular ducts is:

$$\Delta P = \left(\frac{f\ L}{D} + \Sigma C\right)\frac{V^2\ \rho}{2\ g_c} \tag{7}$$

Introduce the coefficient r,

$$r = f\ L + \Sigma C\ D \tag{8}$$

and then substitute r into the Darcy-Weisbach equation to yield,

$$\Delta P = 0.811\ g_c^{-1}\ r\ \rho\ Q^2\ D^{-5} \tag{9}$$

To express diameter in terms of a pressure loss using coefficient r yields,

$$D = 0.959\ (r\ \rho)^{0.2}\ Q^{0.4}\ (g_c\ \Delta P)^{-0.2} \tag{10}$$

Substituting D from Equation 10 into Equation 4 yields the initial cost as follows,

$$Es = z_2\ K\ (\Delta P)^{-0.2} \tag{11}$$

where the intermediate cofficient z_2 is,

$$z_2 = 0.959\ \pi \left(\frac{\rho}{g_c}\right)^{0.2} Sd \tag{12}$$

and section characteristic,

$$K = r^{0.2}\ Q^{0.4}\ L \tag{13}$$

Finally, the objective function is derived by substituting Equation 11 into Equation 5 for Es (see Equation 1),

$$E = z_1\ (Pfan) + z_2\ K\ (\Delta P)^{-0.2} \tag{14}$$

The objective function for condensing two duct sections in series is,

$$\begin{aligned} E &= E_1 + E_2 \\ &= z_1\ (\Delta P_1 + \Delta P_2) + z_2\ [K_1\ (\Delta P)_1^{-0.2} + K_2\ (\Delta P)_2^{-0.2}] \end{aligned} \tag{15}$$

where,

$$Pfan = \Delta P_1 + \Delta P_2$$

The optimum pressure losses are obtained by taking the partial derivatives of Equation 15 with respect to ΔP, setting equal to zero, and solving for pressure loss. The optimum pressure ratio is therefore,

$$\frac{\Delta P_1}{\Delta P_2} = \left(\frac{K_1}{K_2}\right)^{0.833} \tag{16}$$

Introduce an imaginary duct section (1-2) called condensed. This section must satisfy the following conditions:

- Flow of the condensed section 1-2 must be the same as the flow of the original sections,

$$Q_{1-2} = Q_1 = Q_2 \tag{17}$$

- Pressure loss of the condensed section 1-2 must be equal to the sum of the pressure losses for the original sections,

$$\Delta P_{1-2} = \Delta P_1 + \Delta P_2 \tag{18}$$

- The initial cost of the condensed section must be equal to the sum of the initial cost of the original sections,

$$Es_{1-2} = Es_1 + Es_2 \tag{19}$$

The cumulative initial cost for the condensed section is,

$$\begin{aligned} Es_1 + Es_2 &= z_2\ [\ K_1\ (\Delta P)_1^{-0.2} + K_2\ (\Delta P)_2^{-0.2}] \\ &= z_2\ (\ K_1^{0.833} + K_2^{0.833})^{1.2}\ \Delta P_{1-2}^{-0.2} \end{aligned} \tag{20}$$

According to Equation 11, the initial cost for the imaginary section (1-2) is,

$$Es_{1-2} = z_2\ K_{1-2}\ (\Delta P)_{1-2}^{-0.2} \tag{21}$$

After substituting Equations 20 and 21 into Equation 19, the relationship between the characteristics of the imaginary duct section and the original ducts in series is,

$$K_{1-2} = (K_1^{0.833} + K_2^{0.833})^{1.2} \tag{22}$$

When condensing two sections, 1 and 2, in parallel into an imaginary section (1-2), the condensed section must satisfy the following conditions:

- Flow: $Q_{1-2} = Q_1 + Q_2$ (23)
- Pressure: $\Delta P_{1-2} = \Delta P_1 = \Delta P_2$ (24)
- Initial cost: $Es_{1-2} = Es_1 + Es_2$ (25)

By substituting Equation 24 into Equation 11 for each section and adding, the cumulative initial cost for the condensed section is,

$$\begin{aligned} Es_1 + Es_2 &= z_2\ [\ K_1\ (\Delta P)_1^{-0.2} + K_2\ (\Delta P)_2^{-0.2}] \\ &= z_2\ (\ K_1 + K_2)\ (\Delta P)_{1-2}^{-0.2} \end{aligned} \tag{26}$$

According to Equation 11, the initial cost for the condensed section (1-2) is,

$$Es_{1-2} = z_2\ K_{1-2}\ (\Delta P)_{1-2}^{-0.2} \tag{27}$$

After substituting Equations 26 and 27 into Equation 25, the relationship between the characteristics of the imaginary duct section and the original ducts in parallel is,

$$K_{1-2} = K_1 + K_2 \tag{28}$$

Let us condense the tee, containing one node, two children (sections 1 and 2) in parallel, and one parent (section 3) in series. First, condense the parallel sections 1 and 2 using Equation 28. Then, condense the tee using Equation 22 and assume only section (1-2) and section 3 are connected in series. Thus,

$$\begin{aligned} K_{1-3} &= (K_{1-2}^{0.833} + K_3^{0.833})^{1.2} \\ &= [(K_1 + K_2)^{0.833} + K_3^{0.833}]^{1.2} \end{aligned} \tag{29}$$

The purpose of condensing a system is to decrease the tree depth from the maximum to one by a series of repetitive calculations for all tees. Therefore, the entire subsystem is condensed into one section only.

The selection of an optimal fan for one section then becomes easy since Pfan = ΔP_{root} of the one-section system. The following four situations are applicable.

- Case 1. Optimum fan pressure needs to be calculated. For this case, the following equation defines the total pressure requirement.

$$\Delta P^{(opt)} = 0.26 \left(\frac{z_2}{z_1} K\right)^{0.833} + \Delta Px \tag{30}$$

where K is the characteristic of the condensed root section. The constant ΔPx is an additional pressure loss that does not influence optimization and is not a part of the derivative process. This additional loss is that of equipment contained within the central air-handling unit (preheat coil, cooling coil, filter). Fan and motor costs are not included in Equation 30.

- Case 2. The cost of a central air-handling unit or fan and motor can be represented by the curve fit function,

$$Sfan = a_{fan}\ (Pfan)^{-0.2} + b_{fan}\ (Pfan) + c_{fan} \tag{31}$$

where a_{fan}, b_{fan}, and c_{fan} are constants. The optimum fan pressure is,

$$\Delta P^{(opt)} = Pfan^{(opt)} = 0.26 \left(\frac{z_2\ K + a_{fan}}{z_1 + b_{fan}}\right)^{0.833} + \Delta Px \tag{32}$$

- Case 3. A number of central air-handling units or fans with motors (j = 1,2,... ,ν) may be considered. The cost of each fan and motor, $Sfan_j$, and the total pressure, $Pfan_j$, and coefficients of efficiency, g_f and g_e, are known. A comparison is made between life-cycle costs of the system equipped with each fan using the following equation,

$$E = z_1\ (Pfan)_j + z_2\ K_{root}\ (Pfan)_j^{-0.2} + Sfan_j \tag{33}$$

where j = 1,2,...ν. The optimum fan is the one that minimizes the objective function E. To select the optimum fan, the variables Pfan, Sfan, g_f and g_e have to be substituted into Equations 6 and 33 for each fan candidate.

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• Case 4. Fan and motor are preselected. The fan pressure is considered optimum, and the best pressure distribution is accomplished.

The expansion procedure distributes the available fan pressure throughout the system sections. Unlike the condensing procedure, the expansion procedure starts at the root section and continues in the direction of the terminals. Optimum pressure loss for r-section is,

$$\Delta P_r = P_r\, T_r \tag{34}$$

where,

$$T_r = \left(\frac{K_r}{K_{1-r}}\right)^{0.833} \tag{35}$$

For root section P_r is the fan pressure Pfan.

Kirchoff's first and second laws are always satisfied by the T-Method. If velocity or construction constraints are violated during iteration, a permissible boundary duct size is calculated and the duct section is considered as preselected for this iteration. The pressure loss is then calculated for this duct size and is considered an additional loss. Equipment located within a section is also considered an additional pressure loss.

Let us analyze a duct system with a fan and two sections connected in series. Each section has an additional total pressure loss of ΔPz_1 and ΔPz_2. Since these losses are constants, they do not influence pressure distribution. These losses are subtracted from the fan pressure.

The additional pressure loss for a condensed section of ducts in series is the sum of the additional total losses for each duct. For pre-sized ducts, the sectional pressure loss (ΔPz) is considered an additional loss.

For condensing two duct sections in parallel with constraints, three situations need to be analyzed:

• Section 1 is presized and its total pressure loss is ΔPz_1. Because of pressure balancing, section 2 must be the same (i.e. $\Delta P_2 = \Delta Pz_1$). No optimization occurs. The pressure loss for the condensed section (1-2) is therefore the same as ΔPz_1.

• Both sections 1 and 2 have additional pressure losses, ΔPz_1 and ΔPz_2. It is assumed that an additional pressure loss for the condensed section (1-2) is,

$$\Delta Pz_{1-2} = \max\,(\Delta Pz_1, \Delta Pz_2) \tag{36}$$

• Only section 1 has an additional pressure loss, ΔPz_1. The same additional pressure loss is considered for the condensed section (1-2). This is a partial case of the previous assumption (see Equation 36) when $\Delta Pz_2 = 0$.

When condensing a tee with constraints, assume that each section of the tee has the additional pressure loss ΔPz_1, ΔPz_2 and ΔPz_3. Using the previous assumptions, the additional pressure loss for the condensed section (1-3) is,

$$\Delta Pz_{1-3} = \max\,(\Delta Pz_1, \Delta Pz_2) + \Delta Pz_3 \tag{37}$$

When condensing a system with constraints, the maximum additional pressure is calculated at each tee for each condensed section and kept separately from the condensing coefficients K. The existence of additional pressure losses does not affect the condensing process. However, the expansion procedure is different.

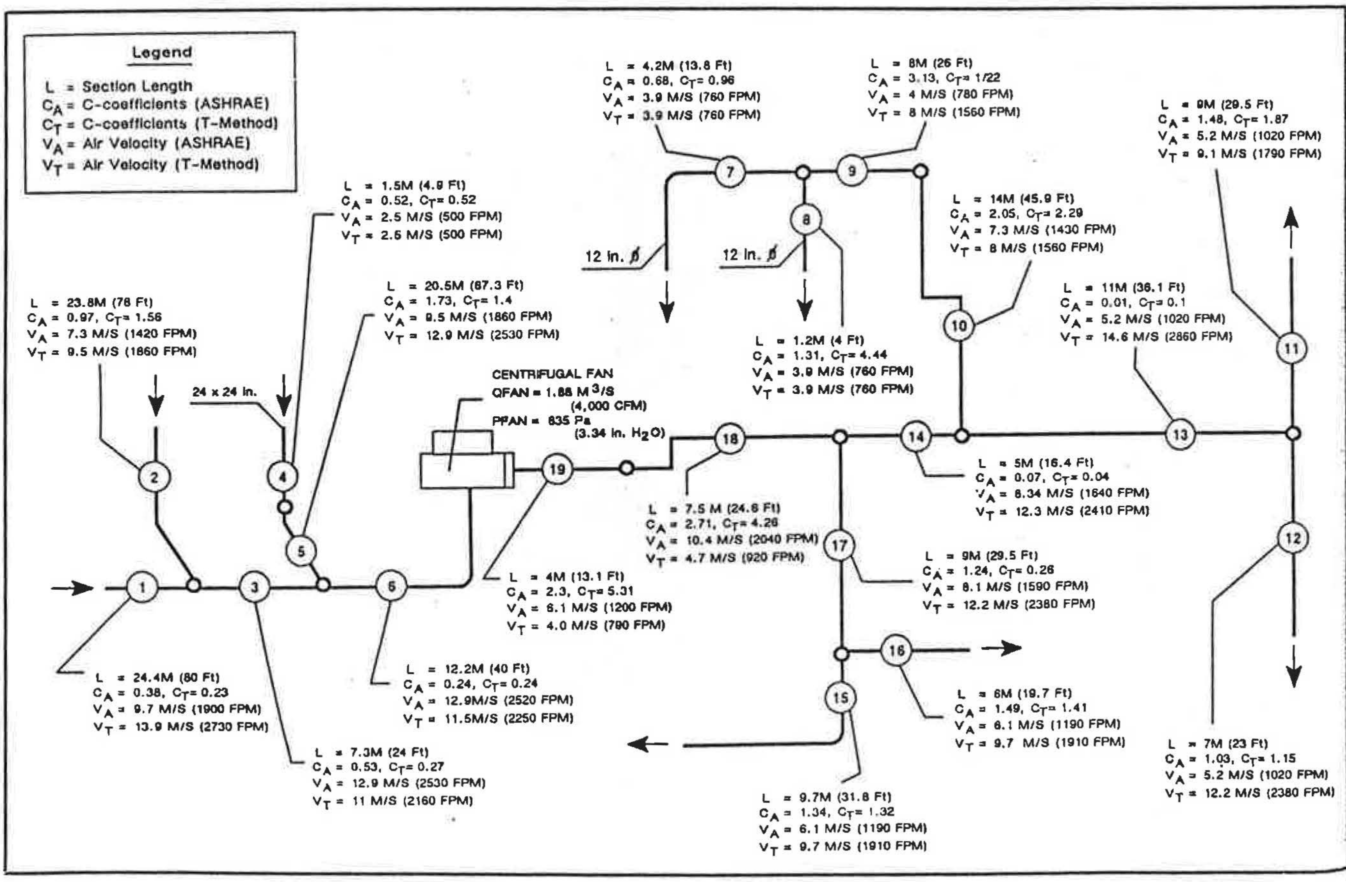

Figure 1. Comparison of ASHRAE and T-Method results.

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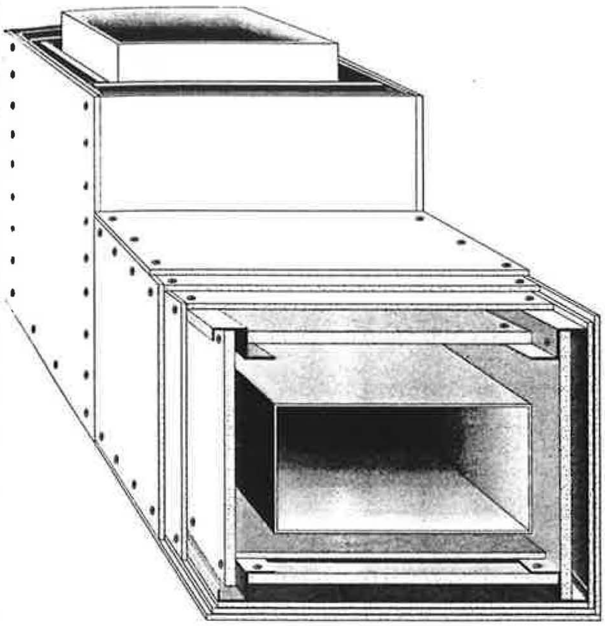

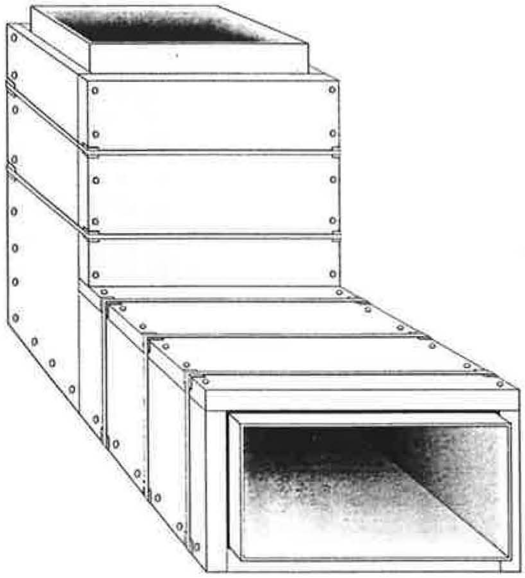

1. Allow 3" - 6" minimum dead air space
2. Install C/Z channel supports and core stud framing (not shown) as necessary
3. Cut and install 1" core board all 4 sides and screw to C/Z frame
4. Cut and install 1st application of 1/2" X-board; allow for 1-1/2" dead air space as required
5. Fasten with screws 16" o.c. (on center)
6. Cut and install 2nd application of 1/2" X-board
7. Fasten with screws 16" o.c. top and bottom
8. Fasten with screws 12" o.c. both sides
9. Cut and install 3rd application of 1/2" X-board top and bottom; insure staggered joint
10. Fasten with screws 12" o.c.
11. Install 1/2" X-board on top of C/Z channel, under grease duct
12. Finish all outside joints with fire tape

1. Field measure and precut Super Firetemp™
2. Install Super Firetemp™ full wrap, section-by-section using Super Calstik™ adhesive on all joints
3. Fasten with pneumatic nailer 8" o.c.
4. Apply bands 24" o.c.

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in. wc) compared to 835 Pa (3.34 in. wc) for the ASHRAE example. For the ASHRAE design, the life-cycle cost is $39,800, the owning cost is $7,100 and the operating cost is $32,700. For the same system designed by the T-Method, life-cycle cost is $18,500, owning cost is $10,500 and operating cost $8,000. For this example, the economic effect on life-cycle cost is 53.4 percent. This result is obtained from a 62.1 percent energy saving at the expense of 8.7 percent higher owning cost.

If the same system was built in Seattle, with industrial electrical rates and stainless steel ductwork (case 11), the T-Method design saves 12.2 percent of the life-cycle cost, consisting of a 16.5 percent reduction of owning cost and a 4.3 percent increase in energy cost. Seattle is a city where energy cost is the cheapest in the nation.

Conclusion

Economic analysis of the 1985 Handbook example showed that significant owning and/or operating costs are obtainable. In addition, for the optimized designs, the three requirements in Tsal and Behls (1986) are satisfied (optimum fan selection, pressure balancing and optimum sectional velocity ratios).

T-Method advantages, compared to other optimization methods, are that it: applies to any duct shape, material and air density; applies for supply, return, exhaust and combination supply/return systems; acknowledges constraints (space limitations and preselected sizes); includes pressure balancing; recognizes variable fitting loss coefficients, including fan system effect factors; rounds to nominal duct sizes; selects the optimum fan-motor or central air-handling unit; optimizes the duct system for a preselected fan; and provides an efficient converging process.

Using the T-Method, duct design becomes a science rather than an art. Computer programs for duct optimization and simulation with duct leakage incorporated will be available in November 1990.

Acknowledgment

T-Method duct optimization and simulation reported in this article are the results of cooperative research between ASHRAE and Fluor Daniel Corporation.

Terminology

Children and parent—Duct sections connected at the same node. The parent section is the one that collects or distributes the total flow. The rest are children sections. Terminal sections have no children, and root sections have no parents.

Node level—The maximum number of nodes (including terminals) in any path within a tree or subtree.

Path—A set of descendants connected in series.

Root—The oldest parent section in a subsystem, usually the ducts connected to the fan.

Subsystem—A part of the duct system that includes all descendant duct sections.

Tee—sections linked at the same node.

Tree—A system of duct sections connected at nodes (junctions).

Tree depth—The maximum node level for the whole subtree.

Nomenclature

AER = annual escalation rate, decimal
AIR = annual interest rate, decimal
C = local loss coefficient, dimensionless
D = duct diameter, m (in.)
E = present worth owning and operating cost, dollars
Ec = unit energy cost, dollars/kWh
Ed = energy demand cost, dollars/kW
Ep = first year energy cost, dollars
Es = initial cost, dollars
H = duct height, m (in.)

Table 2. Comparison of Six American Cities

Electricity Cost (Cents/kWh)	Duct Cost ($/m², $/ft²)	Life-Cycle Costs T-Method ($)	Life-Cycle Costs ASHRAE ($)	Comparison Life Cycle	Comparison Owning Cost	Comparison Operating Cost
1. New York, Residential, Spiral ducts						
16.34	33.36 (3.10)	18,500	39,800	53.4%	−8.7%	62.1%
2. New York, Commercial, Spiral ducts						
15.85	33.36 (3.10)	18,300	38,800	52.8%	−8.6%	61.4%
3. San Diego, Industrial, Spiral ducts						
11.88	33.36 (3.10)	16,200	30,900	47.6%	−9.0%	56.6%
4. Atlanta, Commercial, Low pressure galvanized steel ducts						
8.52	41.01 (3.81)	16,300	25,700	36.7%	−8.8%	45.5%
5. Detroit, Residential, Galvanized steel insulated ducts						
7.26	55.43 (5.15)	19,100	26,300	27.4%	−7.1%	34.5%
6. Denver, Industrial, Aluminum ducts						
4.83	43.27 (4.02)	14,200	18,800	24.5%	−6.2%	30.7%
7. San Diego, Industrial, Stainless steel ducts						
11.88	127.98 (11.89)	40,000	50,900	21.3%	−4.9%	26.2%
8. Seattle, Commercial, Spiral ducts						
2.4	33.36 (3.10)	9,800	11,900	17.7%	−2.6%	20.3%
9. Seattle, Industrial, Spiral ducts						
2.03	33.36 (3.10)	9,400	11,100	15.7%	−0.9%	16.6%
10. Seattle, Residential, Spiral ducts						
1.89	33.36 (3.10)	9,200	10,800	15.0%	−0.1%	15.1%
11. Seattle, Industrial, Stainless steel ducts						
2.03	127.98 (11.89)	27,300	31,100	12.2%	16.5%	−4.3%

L = duct length, m (ft)
P_{fan} = fan total pressure, Pa (in. wc)
PWEF = present worth escalation factor, dimensionless
Q = duct airflow, m^3/s (cfm)
Q_{fan} = fan airflow rate, m^3/s (cfm)
Sd = unit ductwork cost, including material and labor, dollars/m^2 (dollars/sq ft)
S_{fan} = central air-handling unit cost, dollars
V = mean air velocity, m/s (fpm)
W = duct width, m (in.)
Y = system operating time, h/yr
a = amortization period, years
f = friction factor, dimensionless
g_c = dimensional constant, 1.0 (kg-m)/(N-s^2): 32.2 (lb_m-ft)/(lb_f-s^2)
ΔP = total pressure loss, Pa (in. wc)
ΔPz = additional pressure loss in a section, Pa (in. wc)
ΔPx = additional pressure loss in a system, Pa (in. wc)
η_e = motor-drive efficiency, decimal
η_f = fan total efficiency, decimal
ρ = air density, kg/m^3 (lb_m/cu ft)

Equations

The equations in this article are SI metric, not inch-pounds.

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