AN ANALYSIS OF STOCHASTIC PROPERTIES OF ROOM AIR TEMPERATURE AND THE HEATING LOAD DURING "INTERMEDIATE PERIOD"

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[1] ABSTRACT

External climatic conditions such as outdoor air temperature and solar radiation fluctuate randomly with time, and cause the random fluctuation of room air tempereture and heating and/or cooling load. To reduce operating costs or energy consumption, therefore, it is important to have a knowledge of maximum value of the heating load and the probability distribution of the heating load. In spring and autumn in Japan, there is a term when room air temperature can be kept within the target temperature range without air-conditioning. If the length of this period can be extended by means of ventilation without heating/cooling and of building structure, energy consumption can be reduced. From those points of view, in this paper, we present a method of analysis and analyze the stochastic properties of room air temperature in this period, and the effects of various means expanding this period are studied. It is shown that this period is extended about half a month by the means above mentioned under the climate of middle Japan.

[2] INTRODUCTION

External climatic conditions such as outdoor air temperature and solar radiation fluctuate randomly with time, and cause the random fluctuation of room air temperature and heating and/or cooling load. So far, the authors have presented a paper which analyzed the stochastic properties of room air temperature and heating load during summer and winter, where only cooling in summer and only heating in winter were needed[1]. However, during spring and autumn in Japan, there is a period when room air temperature can be kept within the target temperature range without air-conditioning (in this paper this period is called the "intermediate period"). If the length of this period can be extended by means of ventilation without heating/cooling and of building structure, energy consumption can be reduced . In this paper, the stochastic properties of room air temperature in the "intermediate period" , and the effects of various means expanding the "intermediate period" are analyzed. The gradient of (deterministic) annual cyclical variation of external climatic conditions is larger in spring and autumn in comparison with that in summer and winter. Therefore, cyclical variation must be taken into consideration in analyzing the "intermediate period" . Although the analysis of room air temperature is the main object of this study, we have also investigated the stochastic properties of heating and/or cooling load to maintain the target temperature, since knowledge concerning simultaneous occurence of heating and cooling during the day is important in air-conditioning practice.

[3] MATHEMATICAL MODEL

3-1. Basic equations

(1) Input outdoor temperature $\theta_o(t)$, solar radiation J(t) Data of the "intermediate period" has not been examined throughly. However, judging from the results of summer and winter, outdoor temperature and solar radiation can be assumed to be expressed by the sum of deterministic and random components as follows[2];

$$J(t) = \sigma_J(t)J'(t)+J_P(t) \qquad \theta_o(t) = \theta'_o(t)+\theta_P(t) \qquad (1)(2)$$

Discrete random components of outdoor temperature $\theta'_o(t)$ and solar radiation $J'(t)$ may be expressed by ARMA and ARMAX model as follows;

$$Z_{j+1} = a_1 Z_j+a_2 Z_{j-1}+a_3 Z_{j-2}+e_j+b_1 e_{j-1}+b_2 e_{j-2}+b_3 e_{j-3} \qquad (3)$$
$$y_{j+1} = c_1 y_j+c_2 y_{j-1}+d_1 \sigma_{J,j+1} Z_{j+1}+d_2 \sigma_{J,j} Z_j+d_3 \sigma_{J,j-1} Z_{j-1}+e'_j+g_1 e'_{j-1}+g_2 e'_{j-2} \qquad (4)$$

(2) Thermal model For simplicity, the heat flow equation of a single wall is dealt with in the following formulation. Equations of the wall temperature θ and the room air temperature θ_R are as follows;

$$\dot{\theta} = a_w(\partial^2\theta/\partial x^2) \quad (\cdot = d/dt) \tag{5}$$
$$A_s J + \alpha_0(\theta_0 - \theta) = -\lambda_w(\partial\theta/\partial x) \qquad \alpha_i(\theta - \theta_R) = -\lambda_w(\partial\theta/\partial x) \quad (6)(7)$$
$$c\gamma V\dot{\theta}_R = S_w\alpha_i(\theta - \theta_R) + (S_g K_g + c\gamma Vn)(\theta_0 - \theta_R) + S_g\tau_g J + Q \tag{8}$$

(3) Heat input (control) model   The stochastic properties of heating and/or cooling load to maintain the target temperature is also investigated, since simultaneous occurrence of heating and cooling in a day is important in air-conditioning practice. Intermittent heating load Q is approximately expressed as follows[1];

$$\dot{Q} = A_1 G_1(t)(\theta_S - \theta_R) - A_2 G_2(t) Q \tag{9}$$

3-2. State equation

Eqs.(5)-(9) are discretized by the finite difference method, and the system is expressed by state equations. Making use of the vector notation, T denotes transpose.

$$X^j = [Z_j, Z_{j-1}, Z_{j-2}, e_{j-1}, e_{j-2}, e_{j-3}, y_j, y_{j-1}, e'_{j-1}, e'_{j-2}, \Theta^{jT}] = [X^j]^T \tag{10}$$

where, Θ : state variance vector such as wall temperature, room air temperature, heat load and so on

Eqs.(5)-(8) are rewritten as follows;

$$X^{j+1} = [A]^j X^j + [B]^j f_P{}^j + [C]^j e_j \tag{11}$$

From eq.(11), mean and variance equations are obtained as follows[3];

$$E[X^{j+1}] = [A]^j E[X^j] + [B]^j f_{Pj} \qquad D^{j+1} = [A]^j D^j [A]^{jT} + [C]^j V^j [C]^{jT} \quad (12)(13)$$

where, $E[\cdot]$: average operator
D^j : variance matrix, V^j : variance matrix of Gaussian white noise

By solving these two equations, all stochastic properties of the room air temperature and heat load can be easily obtained.

3-3. Stochastic properties of heat load

For the analysis of the "intermediate period", not only the room air temperature but also the probability distribution of total heat load can give us useful informations.

(1) Probability distribution of cumulative heat load   The probability $F(Q_0)$ that heat load Q is above the level Q_0 during the period [0,T] is expressed as follows[4];

$$F(Q_0) = \frac{1}{T}\int_0^T \int_{Q_0}^{+\infty} f(Q|t)\,dQ\,dt \tag{14}$$

From eqs.(12)(13), the mean and the variance of heat load Q at every time can be obtained. Since the system studied is linear with Gaussian noise as an input, probability density function $f(Q|t)$ is also Gaussian. Thus, since the Gaussian probability density can be determined by its mean and variance, probability density of heat load Q is calculated.

(2) Probability of switching between heating and cooling   During the "intermediate period", the maintenance of room air temperature at a set point usually requires an alternation between heating and cooling. From air-conditioning point of view, it is important to know how often this switching occurs in a day. Although the analysis in §3-3 cannot give us information about the switching probability, the frequency of simultaneous occurrence in a day of heating and cooling may be judged by the probability that heat load changes from -(+) to +(-). This is expressed as follows[4];

$$n_{Q_0}{}^+ = \int_{t1}^{t2}\int_0^{+\infty} f(Q_0,\dot{Q})\dot{Q}\,d\dot{Q}\,dt \qquad n_{Q_0}{}^- = -\int_{t1}^{t2}\int_{-\infty}^{0} f(Q_0,\dot{Q})\dot{Q}\,d\dot{Q}\,dt \quad (15)(16)$$

The probability that heat load changes from -(+) to +(-) is calculated by setting Q_0 to 0. The joint probability density function of Q_0 and $\dot{Q}$ can be obtained by substituting eq.(9) into $f(Q, \theta_R)$ calculated in §3-2.

[4] EXTERNAL CLIMATIC CONDITION IN THE "INTERMEDIATE PERIOD"

4-1. Period for analysis

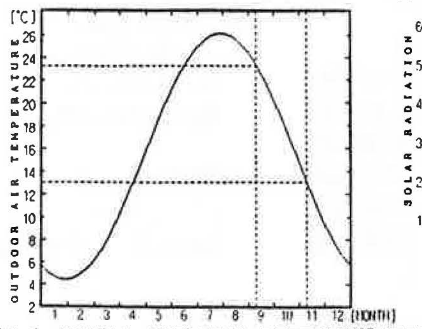

FIG.1 ANNUAL CYCLICAL DETERMINISTIC COMPONENT OF OUTDOOR TEMPERATURE

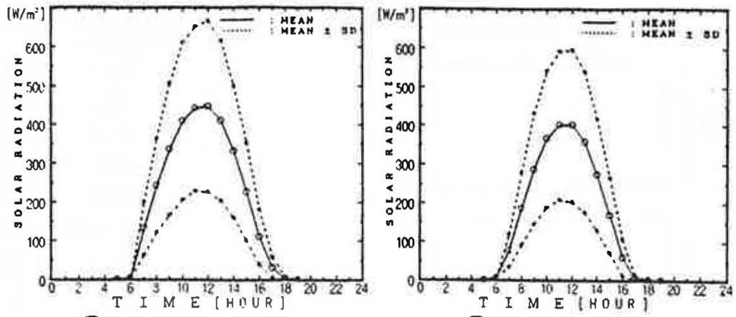

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FIG.3 HOURLY SOLAR RADIATION -HORIZONTAL SURFACE-

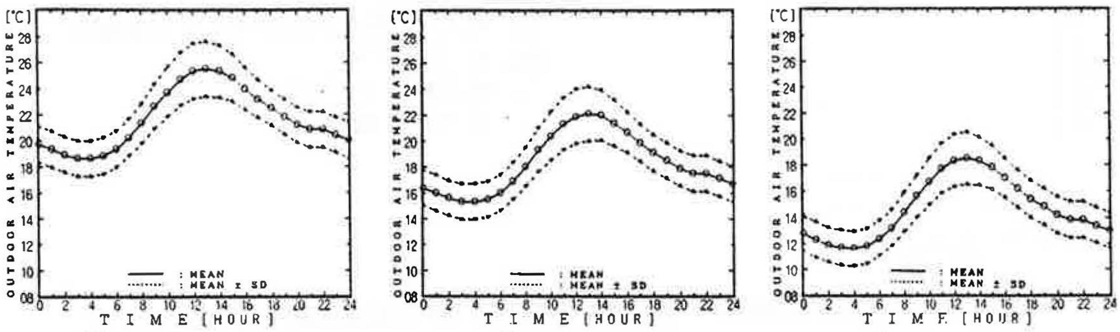

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FIG.2 HOURLY OUTDOOR TEMPERATURE

The analysis was executed during 60 days in autumn from Sep.11th to Nov.9th. The period studied is subdivided into 3 different periods to reflect the about 10°C decrease of the daily mean temperature that spaned over the 60 days. The first period is from Sep.11th to Sep.30th; the second period is from Oct.1st to Oct.20th; and the third period is from Oct. 21st to Nov.9th.

4-2. Outdoor temperature

The deterministic annual cyclical variation of outdoor temperature in Tokyo[5)] is shown in Fig.1. The deterministic daily cyclical variation and the random component are assumed to be the same as that in summer period[2)] (fig.2).

4-3. Solar radiation

(1) Global solar radiation on the horizontal surface Fig.3 shows daily means and standard deviations of global solar radiation in autumn, used in this paper. Those values are determined by the interpolation between those in summer and winter in ref.6).

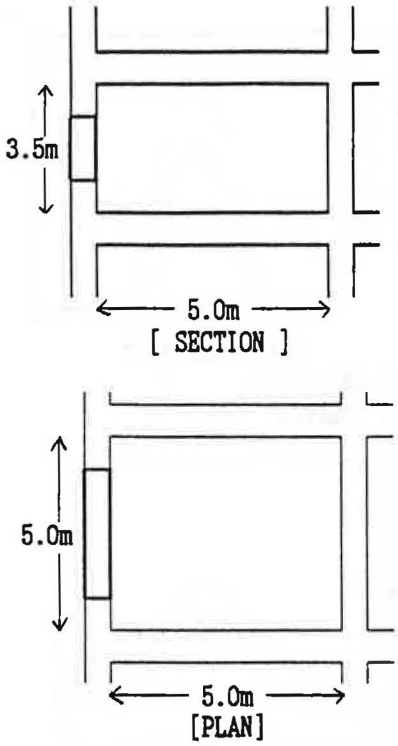
(2) Separation of solar radiation into direct and diffuse components The methods of separating solar radiation into direct and diffuse component which is suitable for stochastic analysis are developed, here. Although several methods of separation have been proposed[7)], it is assumed that direct component can be given by Bouguer equation and that diffuse component can be given by Berlage equation. That is,

$$J_H = J_0 P^{\mathrm{cosec}\,h} \sin h \qquad J_{SH} = 0.5 J_0 \sin h \times (1-P^{\mathrm{cosec}\,h})/(1-1.4\log_e P) \qquad (17)(18)$$

In an analysis of stochastic properties, it is desirable that the system is linear. Thus, P is approximated as a linear equation of J(t) which fits well in the range of h(t) between 0°~60° and in the range of P between 0.3~0.7. By making use of this expression of P, J_H and J_{SH} are approximated as a liner equations of J(t) as follows;

$$P = \left(\frac{J(t)}{J_0} - \frac{92 \times h(t)}{40000} \right) \times \frac{20000}{301 \times h(t)} + 0.1 \qquad (19)$$

$$J_H = \left\{ \frac{275 \times h(t) - 1200}{20000} \times (P-0.1) + \frac{18 \times h(t)}{40000} \right\} \times J_0 \qquad (20)$$



☆NUMERICAL VALUES

[COMMON]

$As = 0.8$

$\alpha o = 23.3 (W/m^2K)$

$\alpha i = 9.3 (W/m^2K)$

$\lambda w = 1.62 (W/mK)$

$Cw = 0.88 (J/kg)$

$\gamma w = 2200.0 (kg/m^3)$

$Kg = 4.06 (W/m^2K)$

$\tau g = 0.8$

[EXTERNAL WALL]

$l_1 = 0.15 (m)$

$S_{w1} = 5.0 \times 3.5 (m^2)$

$Sg_1 = 3.0 \times 1.75 (m^2)$

[PARTION WALL]

$l_3 = 0.1 (m)$

$Sw_3 = 5.0 \times 3.5 \times 2 (m^2)$

[CELLING AND FLOOR]

$l_2 = 0.12 (m)$

$l_P = 0.006 (m)$

$\alpha i' = 1.0/(1.0/\alpha i + Rair + l_P/\lambda_P) (W/m^2K)$

$Rair = 0.17 (m^2K/W)$

$S_{w2} = 5.0 \times 5.0 \times 2 (m^2)$

$\lambda_P = 0.464 (W/mK)$

FIG.4 SCHEMATIC OF ROOM EMPLOYED IN SIMULATION

$$J_{SH} = \left\{ \frac{26 \times h(t) + 1200}{20000} \times (P - 0.1) + \frac{74 \times h(t)}{40000} \right\} \times J_a \qquad (21)$$

Since the coefficients of these equations include h(t) only, which is the function of time only, these equations are linear.

[5] NUMERICAL RESULTS : STOCHASTIC PROPERTIES OF ROOM AIR TEMPERATURE AND THE HEATING LOAD

5-1. Thermal model and numerical values used (Fig.4)

A room in the middle floor of an office building is considered. The room is enclosed on three sides by partition walls and on one side by an external wall with 30% glazing area. The air temperature in the neighbouring rooms varies in the same manner as that of the room being considered. The wall opposite the external wall is well insulated. The air condition period of 10 hours from 8:00 to 18:00 with a target temperature of 26.0°C is assumed.

5-2. Results

(1) Room air temperature (without heating or cooling) Fig.5 shows the room air temperature. The exchange rate n is 1/3600. The external wall faces south and is not insulated. In the second and third period during the "intermediate period", the room air temperature is kept in a range between 22 °C (target temperature in winter) and 26 °C (target temperature in summer) in the daytime. Furthermore the maximum standard deviation is about

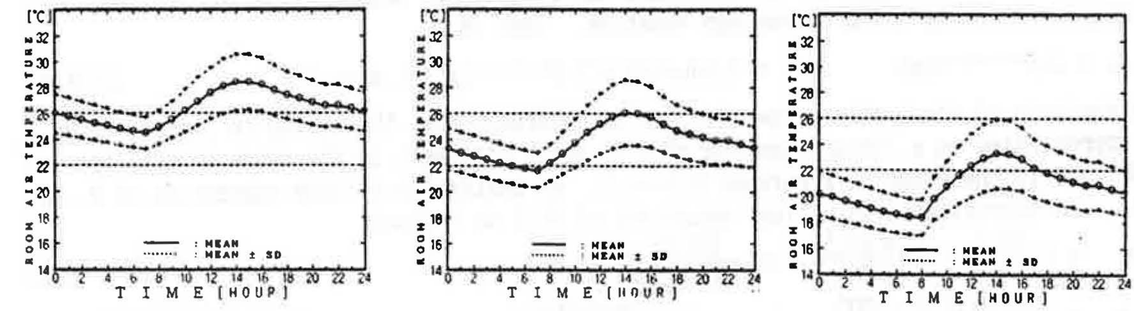


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FIG.5 HOURLY ROOM AIR TEMPERATURE

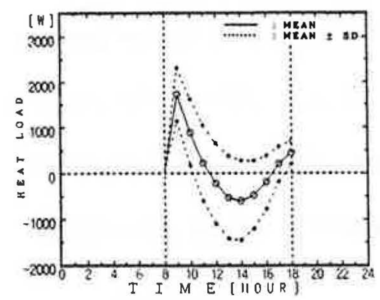

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FIG.6 HOURLY HEAT LOAD

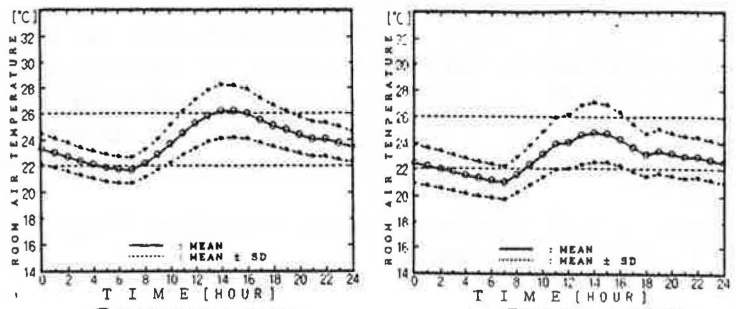

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FIG.7 EFFECT OF VENTILATION ON ROOM AIR TEMPERATURE

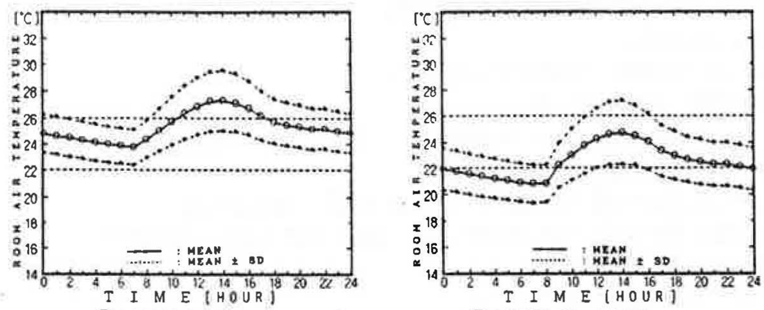

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FIG.8 EFFECT OF INSULATION ON ROOM AIR TEMPERATURE

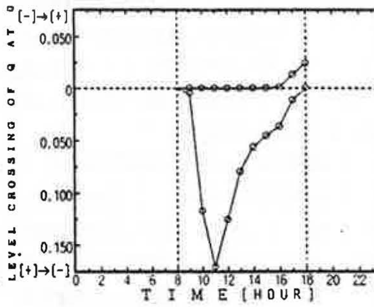

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FIG.9 EXPECTED NUMBER OF CROSSING OF Q AT 0

2.5 °C. On the contrary, room air temperature in the first period exceed the maximum value of target temperature range as shown in fig.5-①.
In fig.6, the heat load distribution is shown, under the condition that air temperature is kept at 26.0 °C.

(2) Effect of ventilation (case ①②) To study the effect of the exchange rate on the room air temperature, the following two cases are analyzed.

①exchange rate n=5/3600 during 1st period

②exchange rate n=5/3600 (11:00-18:00),1/3600 (0:00-11:00,18:00-24:00) during 2nd period

Fig.7 shows that by adjusting the exchange rate n, the room air temperature can be brought into the target temperature range without heating or cooling during autumn (from first period to third period). Therefore, ventilation is very effective in enlarging the "intermediate period" . Standard deviation becomes smaller as n becomes greater.

(3) Effect of thermal insulation (case ③) To study the effect of insulation, we analyse the following case.

③exchange rate n=1/3600, with insulation

By insulating exterior wall, the room air temperature becomes warmer by about 1.5~2.0 °C in all periods, as shown in fig.8. The effect of insulation is noticeable especially in the third period.
The results of (1)(2) and (3) suggest that the "intermediate period" can be extended with a suitable combinations of ventilation and insulation.

5-3. Probability of switching between heating and cooling

Using the method of §3-4, the probability of switching between heating and cooling in a day is calculated. Fig.9 shows the expected number of crossing of Q at 0. From these results, the probability of switching from cooling to heating is very small, but the probability of switching from heating to cooling is about 18% which occurs prior to or at noon. Thus, these results can be used to evaluate whether air-conditioning is needed.

[6] CONCLUSIONS

In this paper, the stochastic properties of room air temperature in the "intermediate period" are analyzed, and the effects of various means expanding the "intermediate period" are studied. The results are summarized as follows;

1. The method of analysis is proposed which gives the stochastic properties of the heating load and room air temperature in the "intermediate period".
2. The method of separation of global solar radiation into direct and diffuse components, which is suitable to the stochastic analysis, is proposed.
3. Proper combinations of ventilation and thermal insulation can extend the "intermediate period".
4. The method of calculation of the probability of switching between heating and cooling in a day is investigated.

NOMENCLATURE

$J_P(t)$: deterministic component of solar radiation (W/m²)
$\sigma_J(t)$: hourly standard deviation of solar radiation (W/m²)
$J'(t)$: random component of solar radiation
$\theta_P(t)$: deterministic component of outdoor temperature (°C)
$\theta'_0(t)$: random component of outdoor temperature
Δt : time increment e_j, e'_j : Gaussian white noises x : coordinate
a_w : thermal diffusivity of the wall ($=\lambda_w/c_w\gamma_w$) (m²/s)
A_S : absorptivity of the exterior surface of the wall to the solar radiation
α_0, α_i : heat transfer coefficient through the outer and inner air layer (W/m²°C)
λ_w : thermal conductivity of the wall (W/m°C)
$c\gamma$: volumetric heat capacity of the air (J/m³°C)
S_w : area of the wall (m²) S_g : area of the window (m²)
$c_w\gamma_w$: volumetric heat capacity of the wall (J/m³°C)
τ_g : transmittance of the glazing to the solar radiation
K_g : overall heat transfer coefficient of the window (W/m²°C)
n : exchange rate (1/s) Q : heat input by the air-conditioning system (W)
A_1, A_2 : constants θ_S : target temperature (°C)
$f(Q|t)$: probability density function of Q at time t
n_{Qa}^+, n_{Qa}^-: the expected number of crossing of Q at Q_a with positive and negative slopes within the interval $[t_1, t_2]$, respectively
$f(Q_a, \dot{Q})$: joint probability density function of Q_a and $\dot{Q}$
J_H : direct radiation (W/m²) J_{SH} : diffuse radiation (W/m²)
J_0 : solar constant (W/m²) h : solar altitude at time t
P : atmospheric transmittance
$Z_j=J'(t_j)$, $y_j=\theta'_0(t_j)$, $t_j=j\Delta t$
$G_1(t)$ = 1.0 (air-conditioned) or 0.0 (not air-conditioned) , $G_2(t)=1.0-G_1(t)$

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