#4316

AN ANALYSIS OF STOCHASTIC PROPERTIES OF INDOOR AIR TEMPERATURE, HUMIDITY AND HEATING LOAD OF BUILDING INTERMITTENTLY AIR-CONDITIONED

S. HOKOI and M. MATSUMOTO
Kobe University, Kobe, 657, Japan
K. NIWA
Nikken Sekkei Co.,Ltd., Tokyo, 112, Japan

[1] ABSTRACT

In this paper, a method is presented that gives the probability distribution of the indoor air temperature, humidity and heating load of the room intermittently air-conditioned, taking account of the stochastic nature of the external climate. The influence of the hygroscopic nature of the building envelope on indoor air quality is also analyzed. Furthermore, the method of analysis of the probability distribution of indoor air relative humidity is given since relative humidity is a crucial factor for the evaluation of indoor air quality. Our formulation of this stochastic problem is based on the impulse response functions. By making use of discrete Fourier Transform, this stochastic problem can be analyzed easily.

[2] INTRODUCTION

For air quality control and efficient use of energy, an analysis of not only air temperature and sensible heating load but also of air humidity and latent heating load is very important. In the analysis of air humidity, the hygroscopic nature of the building envelope must be taken into account. Thus, the combined system of heat and moisture transfer should be dealt with for prediction of air quality and heating load. Furthermore, since external climatic conditions fluctuate randomly, it is necessary to take the randomness of the external variables into consideration in determining the indoor air temperature, humidity and heating load.

In this paper, a method is presented that gives the probability distribution of the indoor air temperature, humidity and heating load of the room intermittently air-conditioned, taking account of the stochastic nature of the external climate. The influence of the hygroscopic nature of the building envelope on indoor air quality is also analysed.

Our formulation of this stochastic problem is based on the impulse response functions. No particular modeling of the external climate is necessary, since only the correlation functions of the external climates, which can be obtained directly from the original weather data without difficulty, are needed in this formulation. Furthermore, this method of calculation can be easily applied to multi-room problems. On the other hand, the calculation of the impulse response functions is very difficult and time-consuming since these functions are time variant when the room is air-conditioned intermittently. By making use of discrete Fourier transformation, this difficulty is avoided and we can analyze the stochastic problems with ease.

[3] GOVERNING EQUATIONS

3-1 Heat and Mass Transfer Equations in a Hygroscopic Material Wall

Suppose that the inner condensation doesn't occur in a wall studied. Then, the heat and mass transfer in the wall can be described by the simultaneous transfer equations of heat and moisture in a hygroscopic region. These linear equations are used here[1].

$$\frac{\partial X}{\partial t}=a'\frac{\partial^2 X}{\partial x^2}+w\frac{\partial T}{\partial t},\quad \frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial x^2}+b\frac{\partial X}{\partial t} \qquad [1][2]$$

where,

$$a'=\frac{\lambda'}{c'\gamma'+\kappa}\qquad w=\frac{\nu}{c'\gamma'+\kappa}\qquad a=\frac{\lambda}{c\gamma+r\nu}\qquad b=\frac{r\kappa}{c\gamma+r\nu} \qquad [3]$$

and

- X : humidity ratio [Kg/Kg'], T : temperature [K], t : time [s]
- a' : moisture diffusivity [m^2/s], a : thermal diffusivity [m^2/s]
- λ : thermal conductivity [W/mK], λ' : water vapour conductivity [Kg/ms/(Kg/Kg')]
- c' : porosity [m^3/m^3], κ : $=\rho_w \partial\psi/\partial X$ [$Kg/m^3/(Kg/Kg')$]
- γ' : density of the air [Kg/m^3], ν : $=-\rho_w \partial\psi/\partial T$ [Kg/m^3K]
- c : specific heat [KJ/KgK], r : latent heat of vaporization [KJ/Kg]
- γ : specific weight of the material [Kg/m^3], ρ_w : density of the water [Kg/m^3]
- x : coordinate [m], ψ : water content [-]

3-2 Heat and Mass Balance of a Room

a) Heat balance equation

$$\begin{aligned}
&Q_r\frac{dT_r(t)}{dt}+c_a\gamma_a VnT_r(t)+\Sigma F_g k_g T_r(t)\\
&\quad+\Sigma F_o\int_0^\infty \phi_{s.t}(s)T_r(t-s)ds+\Sigma F_o\int_0^\infty \phi_{s.x}(s)X_r(t-s)ds\\
&=c_a\gamma_a VnT_o(t)+\Sigma F_g k_g T_o(t)\\
&\quad+\Sigma F_o\int_0^\infty \phi_{T.t}(s)T_o(t-s)ds+\Sigma F_o\int_0^\infty \phi_{T.x}(s)X_o(t-s)ds+q_{ac}(t)+q_{gen}(t) \qquad [4]
\end{aligned}$$

where,

- $\phi_{s.t}$: impulse response of heat absorption caused by temperature input
- $\phi_{s.x}$: impulse response of heat absorption caused by humidity ratio input
- $\phi_{T.t}$: impulse response of heat transmission caused by temperature input
- $\phi_{T.x}$: impulse response of heat transmission caused by humidity ratio input
- Q_r : heat capacity of the room [KJ/K], F_g : glazing area [m^2]
- T_r : room air temperature [K], k_g : transmittance of the window [W/m^2K]
- X_r : room air humidity ratio [Kg/Kg'], n : air exchange rate [1/s]
- $c_a\gamma_a$: volumetric heat capacity of the air [KJ/m^3K], V : room volume [m^3]
- F_o : area of the external wall [m^2], q_{ac} : heat input [W]
- q_{gen} : internal heat source generated by human body, illumination etc. [W]
- X_o, T_o : outdoor humidity ratio and temperature [Kg/Kg'],[K], respectively

b) Moisture balance equation

$$\begin{aligned}
&Q_r'\frac{dX_r(t)}{dt}+c_a VnX_r(t)+\Sigma F_o\int_0^\infty \psi_{s.x}(s)X_r(t-s)ds+\Sigma F_o\int_0^\infty \phi_{s.t}(s)T_r(t-s)ds\\
&=c_a VnX_o(t)+\Sigma F_o\int_0^\infty \psi_{T.x}(s)X_o(t-s)ds+\Sigma F_o\int_0^\infty \psi_{T.t}(s)T_o(t-s)ds\\
&\quad+w_{ac}(t)+w_{gen}(t) \qquad [5]
\end{aligned}$$

where,

- $\psi_{s.t}$: impulse response of moisture absorption caused by temperature input
- $\psi_{s.x}$: impulse response of moisture absorption caused by humidity ratio input
- $\psi_{T.t}$: impulse response of moisture transmission caused by temperature input
- $\psi_{T.x}$: impulse response of moisture transmission caused by humidity ratio input
- Q_r' : moisture capacity of the room [Kg], w_{ac} : moisture supply [Kg/s]
- w_{gen} : moisture generated by human body, cooking, etc. [Kg/s]

3-3 Heat Input Equation

The heat input to the room is controlled by the following equation [2].

$$\frac{dq_{ac}(t)}{dt}=A_1 g_1(t)[T_s-T_r(t)]-A_2 g_2(t) q_{ac}(t) \quad [6]$$

where,

$g_1(t), g_2(t)$: step functions with the value 1 when air-conditioned and non-air-conditioned, respectively; otherwise they equal to 0

A_1, A_2 : constants (in this paper, $A_1=3000$, $A_2=1$), T_s : target temperature

The moisture supply w_{ac} is assumed to be proportional to heat input, that is,

$$w_{ac}(t)=K_{wq}\cdot q_{ac}(t) \quad [7]$$

[4] FUNDAMENTAL EQUATIONS WHEN THE EXTERNAL CLIMATES FLUCTUATE RANDOMLY

In linear systems with input of normal probability distribution, the probability density function of the output can be calculated only by its mean and variance[3]. Thus, only the means and variances are calculated in the followings.

4-1 Expression by Impulse Response Function (Continuous Time)

In this paper, the mean and the variance are expressed by the impulse response functions. The expression for room air temperature $T_r(t)$ in continuous time is given as

$$T_r(t)=\int_0^\infty g_T(t;\tau)X_o(t;\tau)d\tau+\int_0^\infty h_T(t;\tau)T_o(t;\tau)d\tau \quad [8]$$

where, g_T and h_T are impulse responses of room air temperature (at time t) to the outdoor humidity ratio and temperature input, respectively (past by time τ).

The mean of room air temperature, $E[T_r(t)]$, is given by[3]

$$E[T_r(t)]=\int_0^\infty g_T(t;\tau)E[X_o(t;\tau)]d\tau+\int_0^\infty h_T(t;\tau)E[T_o(t;\tau)]d\tau \quad [9]$$

where, $E[\cdot]$ denotes ensemble average.

The variance of the room air temperature, $\sigma_{Tr}(t)^2$, is given as [3]

$$\begin{aligned}\sigma_{Tr}(t)^2=&\int_0^\infty\int_0^\infty g_T(t;\tau)g_T(t;s)\sigma_{Xo}(t)^2 R_{Xo}(|s-\tau|)d\tau ds\\&+\int_0^\infty\int_0^\infty h_T(t;\tau)h_T(t;s)\sigma_{To}(t)^2 R_{To}(|s-\tau|)d\tau ds\\&+2\int_0^\infty\int_0^\infty g_T(t;\tau)h_T(t;s)\sigma_{Xo}(t)\sigma_{To}(t)R_{XoTo}(s-\tau)d\tau ds\end{aligned} \quad [10]$$

where,

R_{Xo}, R_{To}: auto-correlation functions of X_o and T_o, respectively

R_{XoTo} : cross-correlation function between X_o and T_o

$\sigma(t)$: diurnal standard deviation of the external climate

The quite similar relations are obtained as to the room air humidity ratio.

4-2 Probability Distribution of Relative Humidity

Although the room air temperature and humidity ratio are normally distributed, relative humidity ψ_r is not Gaussian because it is not a linear function of temperature and humidity ratio. In this case, the probability distribution of the relative humidity, $P'(\psi_r, T_r)$, can be expressed in terms of the joint probability density function of room air temperature and humidity ratio, $P(X_r, T_r)$.

Relative humidity ψ_r can be expressed as follows,

$$\psi_r=f(X_r, T_r) \quad [11]$$

If we set as,

$$z_1=\psi_r,\ z_2=T_r \text{ and } y_1=X_r,\ y_2=T_r \quad [12]$$

then, $P'(z_1, z_2)$ can be obtained as follows [3],

$$P'(\psi_r, T_r)=P'(z_1, z_2)=|J|\cdot P(y_1, y_2)=|J|\cdot P(f^{-1}(z_1, z_2), z_2) \quad [13]$$

where, J is a Jacobian.

[5] EXPRESSION BY DISCRETE FOURIER TRANSFORM [4)5)]

Eqns.[8]~[10] and the impulse responses are calculated in discrete times. The values of impulse response at discrete times [4)], have the dimension of time-integral of the impulse response. In discrete time systems, the integral $\int$ in the above relations must be replaced by sum Σ. As to the details, please refer to reference 5.

[6] RESULTS AND DISCUSSIONS

6-1 Numerical Values Used in Simulation

A 5×8 (m^2) residential room with 2.5 m height is simulated. 30% of the external walls are glazed. The floor is assumed to be insulated. The air exchange rate is set at 1(1/h). The walls are composed of 12cm thick concrete and the 1cm thick fibre board. The fibre board is hygroscopic. The case where the 5cm insulation is inserted between them is also examined. Physical properties of these materials are listed in Table 1. Moisture generation rate by unit heat input is assumed as $K_{wq}=4.34\times10^{-5}$ [Kg/KJ]. The external conditions used in this simulation are shown in Figures 1~5. These are obtained from the data in winter 1962 (Tokyo, Japan). Solar radiation is not included. In the calculation of the impulse response functions by discrete Fourier Transform, calculation was carried out for 3 days and the time increment was set 15 minutes in order to make the computational error small [5)].

TABLE 1. Characteristics of materials.

	Concrete	Glass Wool	Fibre Board
λ	1.4	0.049	0.105
c	0.80	0.84	1.26
γ	2220	24	330
λ'	0.97E-6	2.72E-5	0.47E-5
c'	0.05	0.99	0.773
γ'	1.2	1.2	1.2
κ	2920	16.7	4710
ν	1.08	8.0E-3	2.56

$\alpha_i=9.3$, $\alpha_i'=4.44E-3$, $\alpha_o=23.3$
$\alpha_o'=1.78E-2$, $r=2470$

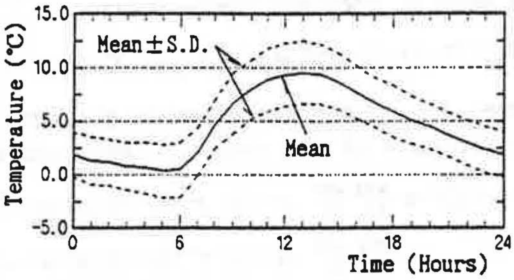

FIGURE 1. Outdoor temperature.

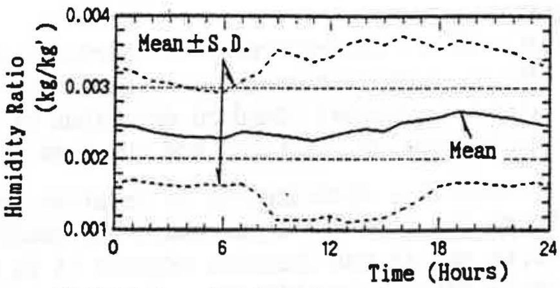

FIGURE 2. Outdoor humidity ratio.

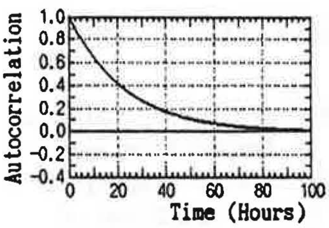

FIGURE 3. Autocorrelation function of outdoor temperature.

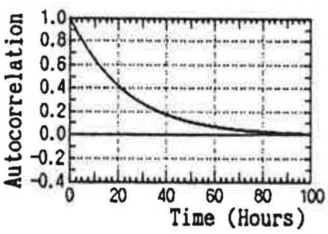

FIGURE 4. Autocorrelation function of outdoor humidity ratio.

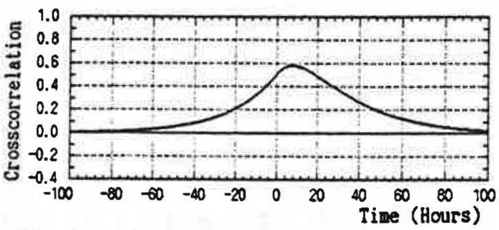

FIGURE 5. Crosscorrelation between outdoor temperature and humidity ratio.

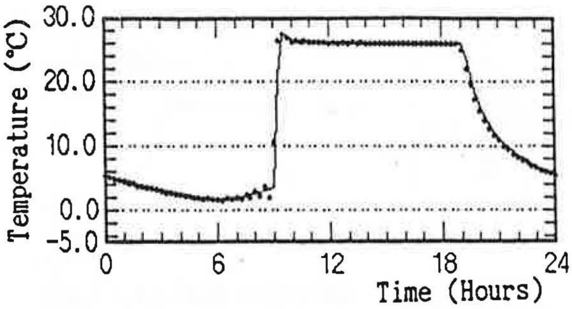

FIGURE 6. Mean of room air temperature.

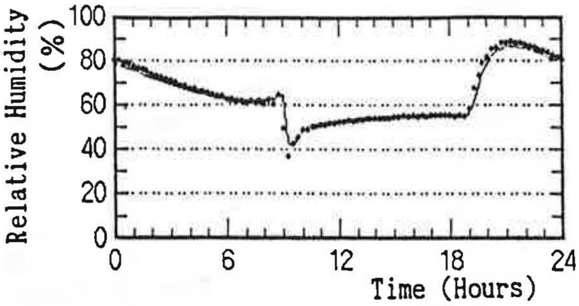

FIGURE 7. Mean of relative humidity.

6-2 Stochastic Characteristics of Room Air Temperature, Humidity and Heat Input

a) Means Figures 6,7 and 12,13 show the means of the room air temperature, humidity and heat input. Circles represent the results obtained by the discrete Fourier Transform, and the solid lines are those by a finite difference method which can be regarded as exact here. The fact that both compare favourably well confirms the method of discrete Fourier Transform to be useful in this time variant system with some numerical cautions. The mean of room air temperature is maintained nearly at set point temperature 26°C during an occupancy period, which means that the integral control works well. Humidity ratio of the room air increases during an occupancy period because of the moisture generation accompanying heating while the relative humidity rather decreases with the rise of the air temperature.

b) Standard deviations Figures from 8 to 10 show the standard deviations of the room air temperature, humidity and heat input. Although the discrepancies of the results by discrete Fourier Transform (solid circles) from the exact solutions (solid lines) are larger compared with that of the means, these discrepancies may be regarded as insignificant. The standard deviation of the heat input is about 10% of the mean heat input and it changes in a similar manner as the mean. In an occupancy period, fluctuation of the humidity ratio is small since the standard deviation of the room air temperature is small.

6-3 Probability Distribution of Relative Humidity

The probability distribution of the relative humidity obtained by using the method in 4-2 is shown in Figure 11. This distribution compares well with the normal distribution with the same mean and standard deviation. Thus, in ordinary indoor conditions, the probability distribution could be regarded as normal.

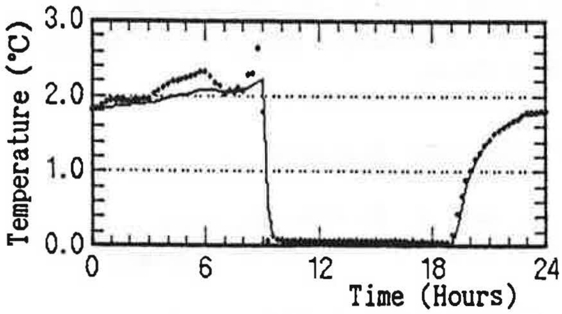

FIGURE 8. S.D. of room air temperature.

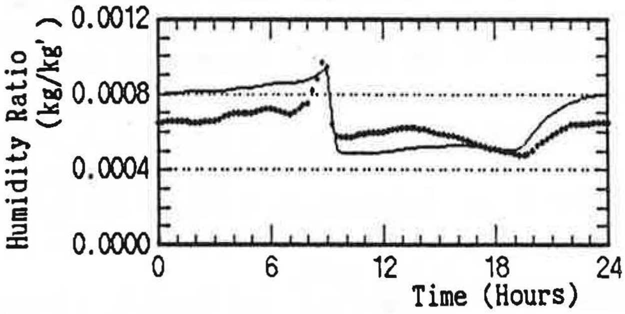

FIGURE 9. S.D. of room air humidity ratio.

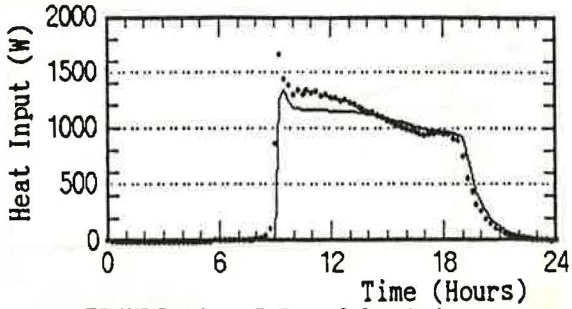

FIGURE 10. S.D. of heat input.

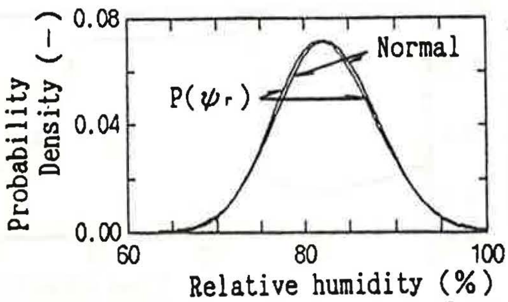

FIGURE 11. Probability density of RH.

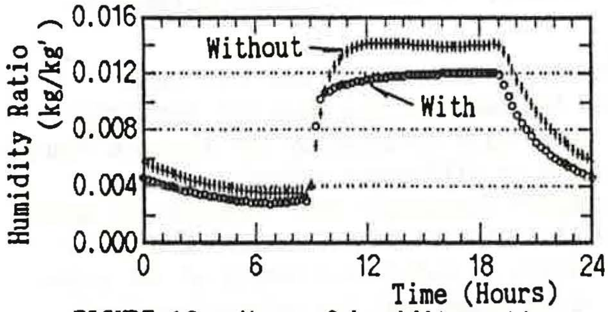

FIGURE 12. Mean of humidity ratio.

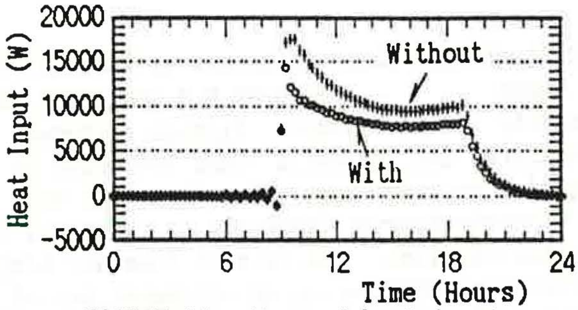

FIGURE 13. Mean of heat input.

6-4 Effect of Hygroscopic Material

Figures 12 and 13 show the means of room air humidity ratio and heat input, respectively. When the hygroscopic materials are added to the wall (○), the increase of humidity ratio caused by moisture generated from the gas furnace is much smaller than that without hygroscopic materials (+). Furthermore, the heat input with hygroscopic materials decreases by about 20% compared with that without hygroscopic materials (Fig. 13). This decrease of heat input is caused by the heat generation followed by the moisture absorption by the walls, although it should be noted that these results include the influence of the difference of thermal properties.

[7] CONCLUSIONS

An analysis of the intermittent heating is dealt with, which takes into account the hygroscopic nature of the wall. The deterministic governing equations of heat and moisture transfer are extended to include the randomness of external climates. The solutions are expressed in terms of the impulse responce functions which simplify the modeling of the external climatic conditions. With the proper period and time increment, the discrete Fourier Transform method can be used to calculate the stochastic properties. In addition, the probability distribution of relative humidity is obtained in terms of the room air temperature and humidity ratio.

REFERENCES

(1)Matsumoto,M., Fujihana,R. : Proc. of CIB/RILEM Symposium, Rotterdam, 2.1.1, PP.1-10, 1974.
(2)Hokoi,S. and Matsumoto,M. : Energy and Buildings, Vol.11, PP.259-266, 1988.
(3)T.T.Soong : Random Differential Equations in Science and Engineering, Academic Press, 1973.
(4)Matsusita,T., Yoshida,H. and Terai,T. : Trans. Environ. Eng. Architect., Architect. Inst. of Japan, No.4, PP.119-124, (in Japanese), 1982.
(5)Matsumoto,M., Hokoi,S. and Niwa,K. : Proc. of Kinki Branch, Architect. Inst. of Japan, No.28, PP.141-144, (in Japanese), 1988.