



# Calculation of vertical temperature gradients in heated rooms

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SUMMARY

This paper deals with a simple model which predicts the vertical temperature gradient in a heated room. The gradient is calculated from a dimensionless temperature profile which is determined by two room air temperatures only, the mean temperature in the occupied zone and the mean temperature in the zone above the occupied zone. A model to calculate the two air temperatures has been developed and implemented in SUNCODE-PC, a thermal analysis programme for residential and small commercial buildings. The dimensionless temperature profile based on measurements in a laboratory test room is presented, and the improvement of the programme is validated by comparing laboratory measurements with simulated results.

LIST OF SYMBOLS

- A area ( $m^2$ )  
a, b empirical coefficients in correlating equation (8)  
 $c_p$  fluid specific heat at constant pressure ( $J/kg^\circ C$ )  
e emissivity factor  
 $F_{i-F}$  overall radiation interchange factor between surface i and F  
G generated air flow in the boundary layer in the distance h ( $kg/s\ m$ )  
 $G_{con}$  amount of air in the plume at the distance y above the radiator ( $kg/s$ )  
H room height  
h height (m)  
Gr Grashof number  
g acceleration of gravity ( $m/s^2$ )  
 $h_c$  surface-average convective heat transfer coefficient ( $W/m^2\ ^\circ C$ )  
L characteristic length of heat transfer surface (m). For horizontal surfaces  $L = A/P$ , where A is the surface area and P is its perimeter. This is analogous to the hydraulic diameter  
 $L_{con}$  length of heat source (m)  
k, m, p exponents in correlating equation (8)  
n number of surfaces  
q convective power that leaves or enters a zone ( $W/m$ )  
 $q_{con}$  convective power delivered from the heat source (W)

Ra	Ralaigh number
R	radiant heat flux ( $W/m^2$ )
$r_{rad}$	radiant power from radiator (W)
$r_{sun}$	radiant power from the sun which has been transmitted into the room (W)
$s_i$	fraction of solar radiation absorbed on surface i
T	absolute temperature (K)
t	temperature ( $^{\circ}C$ )
$\Delta t$	mean temperature difference ( $^{\circ}C$ )
$t_h$	measured air temperature in the height h
$t^*$	dimensionless temperature i the height h/H
y	distance between heat source and the fictive line that separates zones (m)
$y_o$	distance between the inlet and the virtual origin. This can in practice be set equal to the width of the heat source (m)
$\sigma$	Stefan - Boltzmann constant ( $W/m^2 K^4$ )
$\beta$	coefficient of cubic expansion ( $K^{-1}$ )
$\nu$	fluid kinematic viscosity ( $m^2/s$ )
$\rho$	fluid density ( $kg/m^3$ )

## INDICES

a	actual used value
F	related to fictive surface
g	related to the generating zone
H	related to horizontal surfaces
o	related to the occupied zone
R	related to the room
r	related to the receiving zone
s	related to the surface
u	related to the upper zone
V	related to vertical surfaces
z	realted to the zone

double index mean from index one to index two,  
 example:  $G_{uo}$  = generated air flow in boundary layer that  
 is passing from upper zone to occupied zone.

## INTRODUCTION

An accurate modelling of the dynamic thermal behaviour of a room requires the solution of a thermal network representing all convective and radiant heat transfers within the room. The method involves the solution of a set of equations: one for each heat transfer surface and one for the room air. This way the room air is assumed to have one homogeneous temperature which never is the case in a normal occupied room. To calculate the room temperature gradients it requires not one but several equations for the room air: one for each air node, and it also requires a calcula-

tion of the convective energy transport between every air node in the room. The consumption of CPU time is increased enormously to make an accurate calculation which involves modelling of the air flow patterns for determination of the convective air transport in the room.

In this paper it will be shown that it is possible to calculate a vertical average gradient in a room heated by a radiator and without forced air movement in a quite simple way. The model is validated by comparing simulated results with measured temperatures in a laboratory test room.

The model described in this paper is implemented in the simulation programme Suncode which is a PC-version of the mainframe programme SERIE-RES /1/. It has been necessary to make a few improvements in Suncode to make the new room model fit to the calculation structure of the simulation programme. The original programme does not calculate an air temperature but a weighted air and surface temperature, therefore, it has been necessary to implement a new model for distribution of radiant energy. The new programme is also provided with an algorithm to calculate the actual convective heat transfer coefficient. Except for these changes the main structure of the original programme is unchanged and therefore Suncode itself is not described further.

#### DESCRIPTION OF THE ROOM MODEL

The main aspect in this paper is to calculate the vertical average temperature gradient in a heated room. The description of the vertical temperature gradient is based on two air temperatures: The mean temperature in the occupied zone and the mean temperature in the zone above the occupied zone called the upper zone and a dimensionless temperature gradient. This way the model requires only two air nodes and the description of the air movement in the room is possible in a relatively simple way.

##### **Model for convective air movement between zones.**

The model that describes the convective air movement between the occupied and the upper zone is shown in figure 1. The height of the occupied zone is 1.2 meter and the upper zone is the zone from 1.2 meter above the floor and up to the ceiling. This way the mean temperature in the lower zone is approximately the same as the temperature 0.6 meter above the floor which is the height for measuring the comfort temperature for a seated person.

The air movement in the model is only caused by temperature differences between the air and the surfaces in the zone. To determine this air flow the following equation is used /2/

$$G_z = 0.0033 \Delta t_z^{\frac{1}{4}} h_z^{\frac{3}{4}} \quad (1)$$

This expression is based on graphical integration of a velocity profile measured in the convection boundary layer at a non-isothermal room sur-

face and with a normal temperature stratification in the surrounding air. In ref. /2/ there is not described any limits in use of the equation but the laboratory experiments are carried out with the Grashofs number in

the interval  $1.2 \times 10^7 < Gr < 455.0 \times 10^7$ . For that reason the connection between the height of the zone and the temperature difference ought to be expressed by equation (2).

$$1.2 \times 10^7 < \frac{g \beta \Delta t_z h_z^3}{\nu^2} < 455.0 \times 10^7 \quad (2)$$

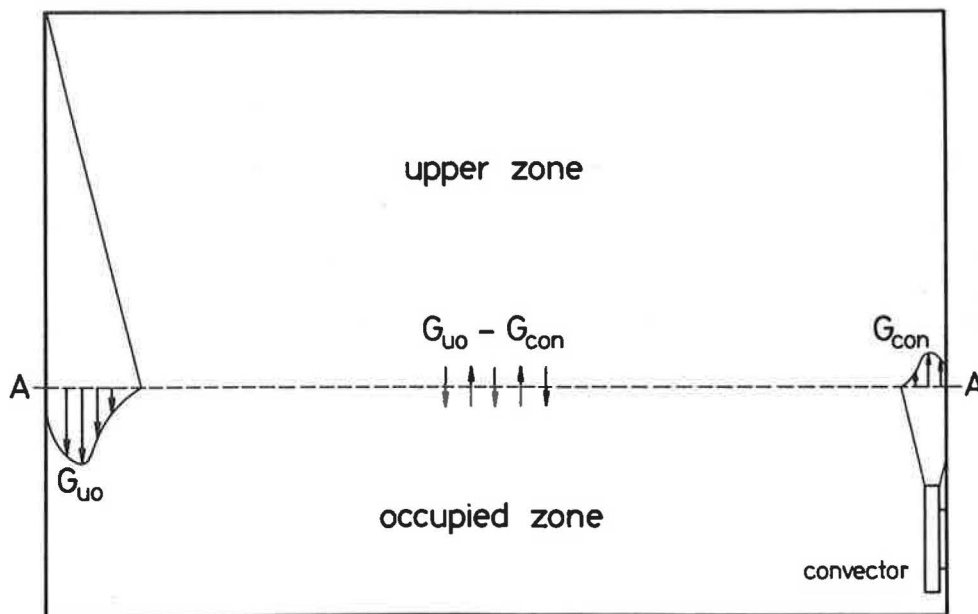


Figure 1. Air movement model.

If in the upper zone, there is generated cold down draught and there is generated an upstream plume along the same surface in the occupied zone then the actual air change between the zones along that surface shall be calculated by equations (3) and (4).

if  $G_{ou} > G_{uo}$  :

$$G_{ou,a} = 1.5(G_{ou} - G_{uo}) \quad (3)$$

$$G_{uo,a} = 0.5(G_{ou} - G_{uo}) \quad (4)$$

and vice versa if  $G_{uo} > G_{ou}$

This relation has been found satisfactory for the simulating procedure.

The plume generated by the heat source which in the laboratory test room is an electrical radiator is calculated from the equation for a line source /3/.

$$G_{con} = 0.014 \left( \frac{q_{con}}{L_{con}} \right)^{\frac{1}{3}} (y - y_0) L_{con} \rho \quad (5)$$

Based on equations (1) to (5) the final air movements between the upper and the occupied zone can be calculated. If there is a difference in the upstream and the downstream air volumes then it is accepted that this difference is exchanged directly between the zones so that the total sum of air passing the fictive line that separates the zones is equal to zero.

#### Model for convective energy exchange between zones.

The model for energy exchange (fig. 2) is built upon the main concept that the plume entrains air from the zone in which it is generated, and that the plume in the generating zone thermally separates the wall from the zone air. That means that there is not any mixing between the air and the plume in the generating zone and therefore the calculation of the energy that leaves the generating zone and the energy that passes through the fictive line that separates the zones can be based on equations (6) and (7).

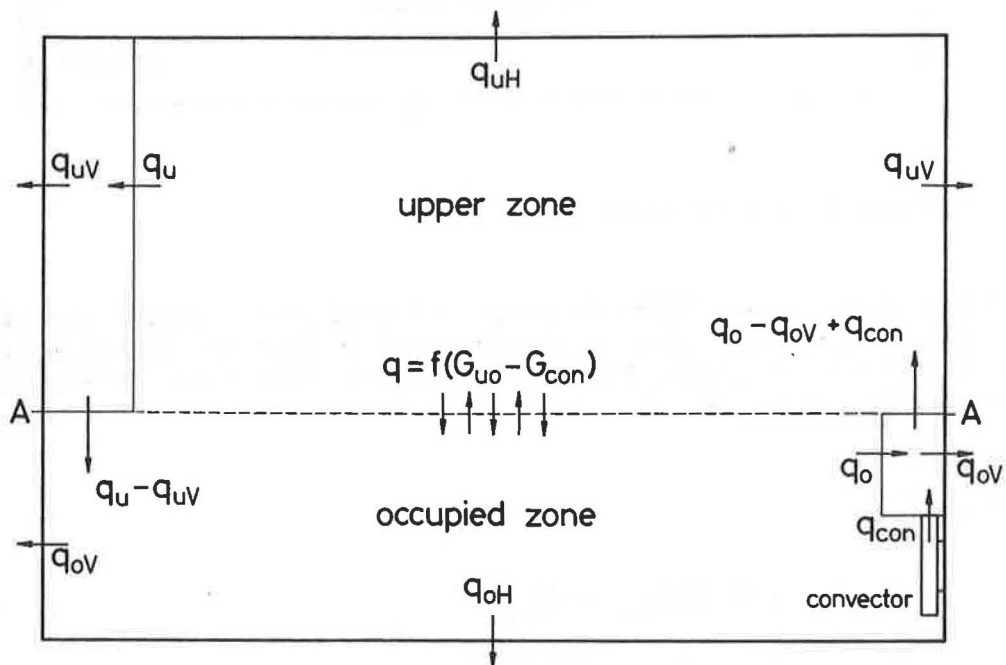


Figure 2. Model for convective energy exchange between zones and heat loss from the room.

$$q_g = G_g c_p t_z \quad (6)$$

$$q_r = q_g - q_v \quad (7)$$

When the plume enters the receiving zone it is presumed that the plume will be totally mixed with the zone air.

#### Model for calculation of convective heat transfer coefficient

To improve the original structure of Suncode the programme is provided with an algorithm to calculate the convective heat transfer coefficient. The chosen algorithm is a convenient set of equations which are developed to incorporate into computer programmes for thermal simulations /4/.

The heat transfer coefficients are calculated by equations (8) or (9)

$$h_c = \left[ \left( a \left( \frac{\Delta t}{L} \right)^p \right)^m + \left( b (\Delta t)^k \right)^m \right]^{\frac{1}{m}} \quad (8)$$

$$h_c = 0.60 \left( \frac{\Delta t}{L^2} \right)^{\frac{1}{5}} \quad (9)$$

The empirical constants to be used in equation (8) are given in table 1.

Flow and surface orientation	a	b	p	k	m
Buoyancy-driven convection over vertical surfaces	1.50	1.23	0.25	0.33	6
Buoyancy-driven flow on horizontal surfaces	1.40	1.63	0.25	0.33	6

Table 1. Constants to be used in equation (8) for calculation of convective heat transfer coefficients.

For calculation of  $h_c$  at a horizontal surface it should be noticed that equation (8) is only usable if a surface that faces down is colder than the air, or if it faces up is warmer than the air. If the opposite is the case then the coefficient is calculated by equation (9).

In the simulating model there are calculated two convective heat transfer coefficients for each surface in the room, one for the upper part of the wall and one for the part in the occupied zone.

If the plume is flowing down the calculation in the upper zone is based on the air and surface temperatures plus the height of the zone. In the occupied zone the used convective heat transfer coefficient is an average between the coefficient for the upper part of the wall and a calculation based on the total height of the wall and the actual temperature difference in the occupied zone.

Equation (8) is valid over an extensive range (  $10^4 < Ra < 10^{12}$  ) which encompasses all practical building environments.

#### Model for radiant interchange

The model for calculation of the radiant interchange in the room is built upon the idea that each surface radiates to a fictitious surface which has an area, emissivity, and temperature which causes the same heat transfer from the surface as the real multi-surface case /5/.

The area of the fictitious surface is the sum of all other surfaces in the room:

$$A_F = \sum_{j \neq i}^n A_j \quad (10)$$

The emissivity is an area weighted average of all other surface emissivities:

$$e_F = \sum_{j \neq i}^n A_j e_j / \sum_{j \neq i}^n A_j \quad (11)$$

The temperature of the fictitious surface is an area multiplied by emissivity weighted temperature:

$$T_F = \sum_{j \neq i}^n A_j e_j T_j / \sum_{j \neq i}^n A_j e_j \quad (12)$$

$T_F$  is the mean radiant temperature (MRT) seen by surface  $i$ . The radiation interchange factor is given by equation (13). Equation (13) is the expression for the radiant conductance between surface  $i$  and the fictitious surface  $F$ . Two conditions must be observed; no part of surface  $i$  can view itself and  $F$  shall completely enclose  $i$  (or  $i$  and  $F$  shall form a complete enclosure).



$$F_{i-F} = \frac{1.0}{\frac{(1 - e_i)}{e_i} + 1 + \frac{A_i(1 - e_F)}{A_F e_F}} \quad (13)$$

Equation (14) of the radiant interchange gives a slight imbalance in the total radiant energy because individual view factors have been ignored.

$$R_i = \sigma F_{i-F} (T_F^4 - T_i^4) \quad (14)$$

That imbalance is handled by redistributing it equally on all surfaces to give conservation of energy. The radiation balance is given by equation (15)

$$r_{bal} = \sum_{i=1}^n \sigma F_{i-f} (T_F^4 - T_i^4) \frac{A_i}{\sum_{i=1}^n A_i} \quad (15)$$

The radiant balance for surface  $i$  can therefore be calculated according to equation (16)

$$R_i = \sigma F_{i-F} (T_F^4 - T_i^4) + r_{bal} \quad (16)$$

In the model it is assumed that the radiant energy from the radiator is distributed equally on all surfaces. The energy from the sun is distributed by a self-elected fraction connected to the separate surface so that the user can decide how the solar radiation shall be spread in the room. Equation (17) is the final equation for radiant distribution in the room.

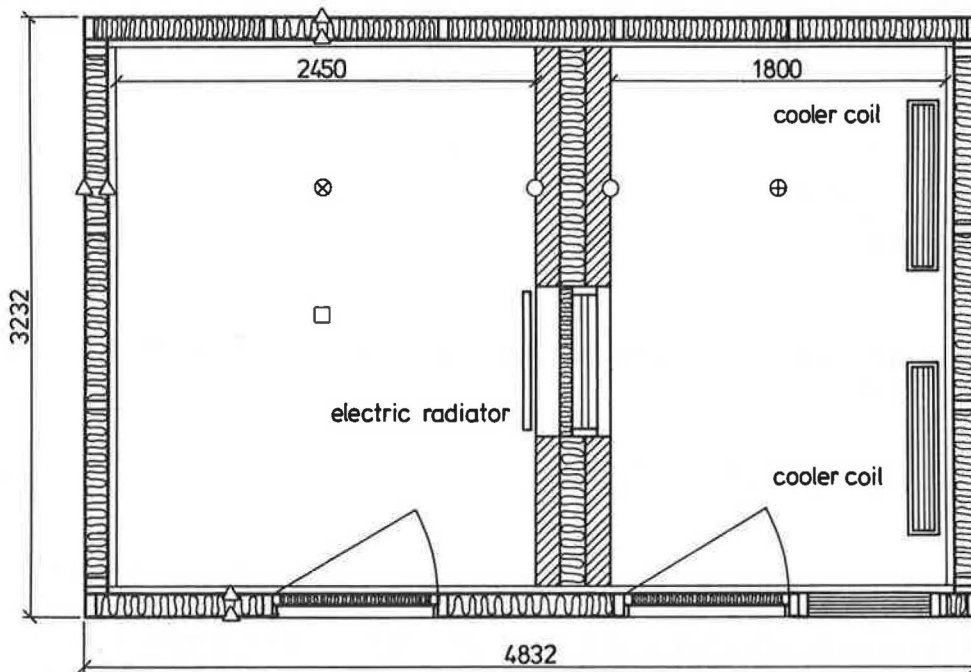
$$R_i = \sigma F_{i-F} (T_F^4 - T_i^4) + r_{bal} + \frac{r_{rad}}{\sum_{i=1}^n A_i} + s_i \frac{r_{sun}}{A_i} \quad (17)$$

Suncode approximates the wall constructions with a thermal network, and this network is solved using the explicit finite differences or Euler's method. The algorithm for radiation is therefore solved only once in every time step, and the radiant energy is kept constant through this step.

The algorithm for radiant distribution is appropriate for typical geometries and temperatures that normally occur in buildings, but the model is less correct if there in specific cases are large temperature differences between the surfaces.

DESCRIPTION OF THE TEST ROOM AND THE EXPERIMENT

The enclosure of the test room consists of a wooden frame structure. The room is divided in two parts by a heavy brick wall, in the brick wall there is a double-glazed window with an area of  $1.08 \text{ m}^2$ . Both the heavy wall and the light frame construction are insulated with  $100 \text{ mm}$  mineral wool. The smallest room can be cooled down to approximately  $-10 \text{ }^\circ\text{C}$  by two cooler coils. The other part of the enclosure has the internal dimension of  $2450 \times 2960 \times 3309 \text{ mm}$ . This room is regarded as an indoor room with a heavy outside wall and three light inside walls, and it is heated by an electrical radiator. The test room is located in an open space in the laboratory. A horizontal plan of the test room is shown in fig. 3.



Figur 3. Horizontal plan of the laboratory test room, and positions of thermocouples.

- △ : Surface temperatures in the height  $0.6 \text{ m}$  and  $2.25 \text{ m}$  above the floor.
- : Surface temperatures in the height  $0.4 \text{ m}$ ,  $1.13 \text{ m}$ ,  $1.8 \text{ m}$  and  $2.6 \text{ m}$  above the floor.
- : Surface temperatures under the floor and over the ceiling.
- ⊕ : Air temperature  $1.2 \text{ m}$  above the floor.
- ⊗ : Air temperature in the height  $0.05 \text{ m}$ ,  $0.4 \text{ m}$ ,  $0.6 \text{ m}$ ,  $1.13 \text{ m}$ ,  $1.86 \text{ m}$ ,  $2.25 \text{ m}$  and the surface temperature on the floor and the ceiling. The temperature is also measured on the surface of the window.

The experiment which is the main subject of this paper starts from a thermal steady state condition, where the temperature in the "outdoor" room is  $-10 \text{ }^\circ\text{C}$  and the indoor steady state situation is reached when the energy transport from the laboratory through the light walls is equal to the energy that is lost from the room through the brick wall. This steady

state temperature for the warm room is approximately 13°C. After stationary condition has been reached the room is supplied with 400 watt (50% of this energy supply is convective) for 29 hours, after which the heat supply has stopped again and the temperature is falling towards a new stationary condition over a period of 115 hours.

The experiment in the test room shows that in the steady state situation there is nearly a linear vertical gradient with approximately 1°C air temperature difference between floor and ceiling. After 1.5 hours of heating (400 watt) the same gradient is approximately 2°C and the gradient curve has been bent upwards. The shape of the gradient is now the same throughout the heating period, and the air temperature difference between floor and ceiling is 2.6 °C after 29 hours of heating (fig. 8).

To make the gradient curves dimensionless the measured air temperatures are expressed according to equation (18), and the height is made dimensionless by dividing with the total room height.

$$t^* = \frac{t_h - t_o}{t_u - t_o} \quad (18)$$

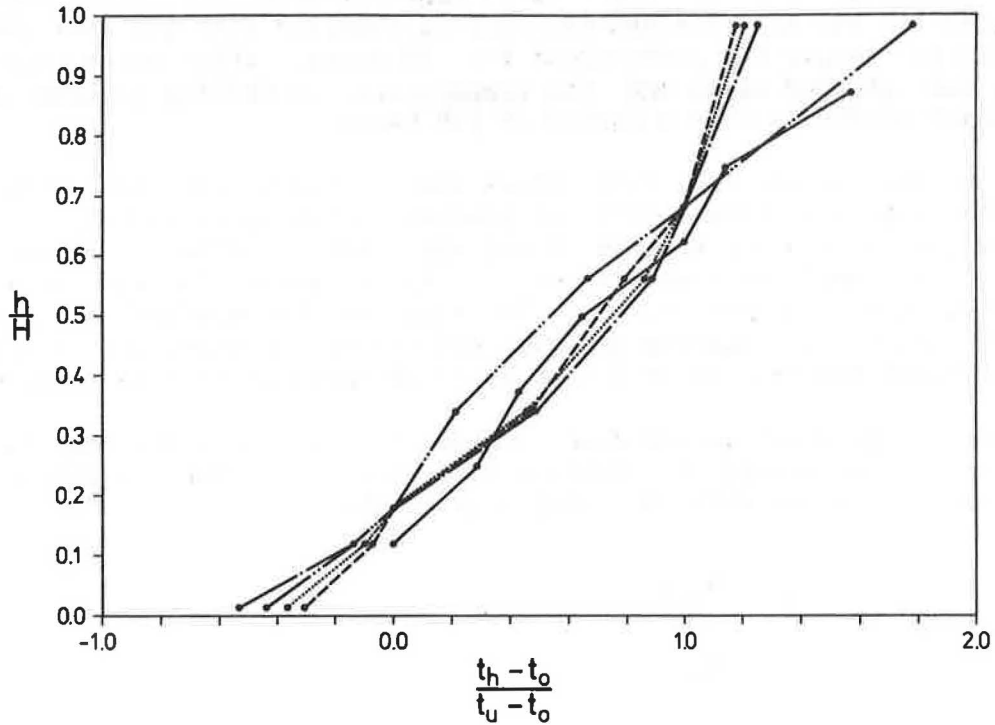
The four dimensionless temperature gradients which are shown in figure 4 are nearly all identical except for the gradient in the steady state situation which is slightly different. This identity of the dimensionless temperature gradients are found in the same test room with different kinds of heat sources (small and big electrical radiators) and different fractions of the radiant part of the total energy supply.

The fifth curve shows that the influence of room geometry has little significance. This gradient is measured by Parczewski and Renzi /6/ in a cubic test room with the side length 2.44 m. This independence of the height is caused by the convective heat transfer coefficient which is mostly in the turbulent domain. This is because the Grashof's number in a free turbulent flow is raised to 1/3 power for calculation of Nusselt's number, after which the length dimension disappears.

Based on these dimensionless gradients it is possible to determine the full shape of the vertical temperature gradient in the room in every case just by using the described model and known (calculated) values of the temperature in the occupied and the upper zone as demonstrated in equation (19).

$$t_h = t^* (t_u - t_o) + t_o \quad (19)$$

The constants  $t^*$  that are used in the later calculations of the vertical temperature gradients are given in tabel 2. These constants are taken as a mean value of the dimensionless gradient measured after 1.5, 8 and 29 hours of heating.



Figur 4. Dimensionless temperature gradients.

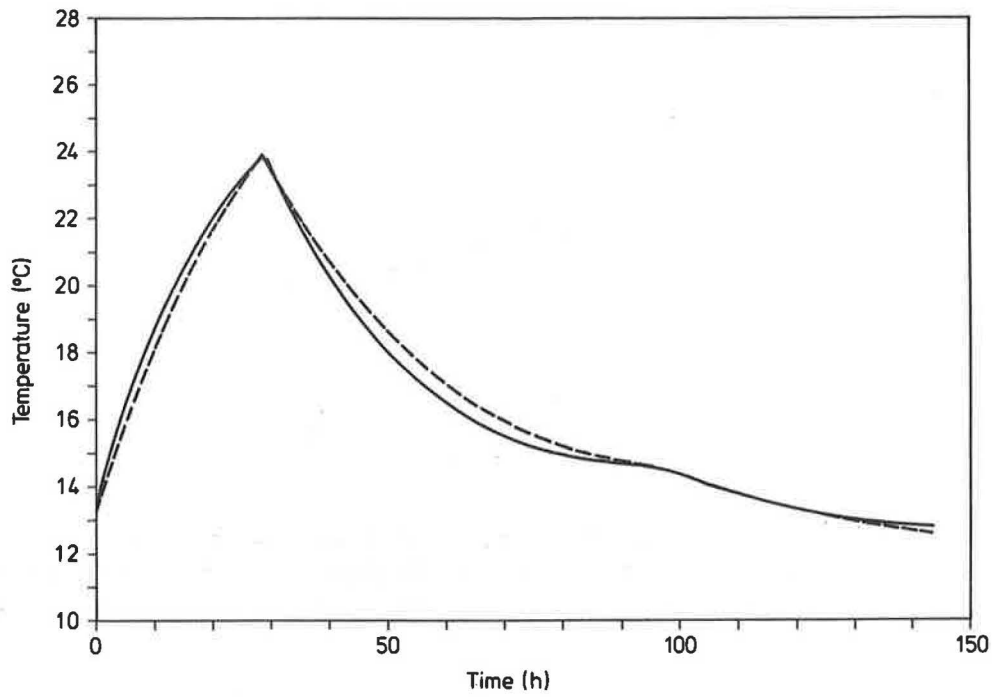
- in thermal steady state situation
- - - after 1.5 hours of heating (400 W)
- ..... after 8 hours of heating (400 W)
- · - after 29 hours of heating (400 W)
- gradient measured in a cubic test room by Parczewski and Renzi /6/.

Dimensionless height $h/H$	Dimensionless temperature
0.014	- 0.37
0.120	- 0.10
0.180	0.00
0.340	0.46
0.560	0.83
0.680	1.00
0.980	1.28

Tabel 2. Dimensionless temperatures  $t^*$  in 7 selected heights

COMPARISONS BETWEEN MEASURED AND CALCULATED TEMPERATURES

The previously described laboratory experiment has been simulated with the improved version of SUNCODE-PC. In the following section selected groups of temperature comparisons between laboratory measurements and simulated results are grafically presented.



Figur 5. Temperature on the brick wall in the occupied zone.  
———— measured ————— simulated

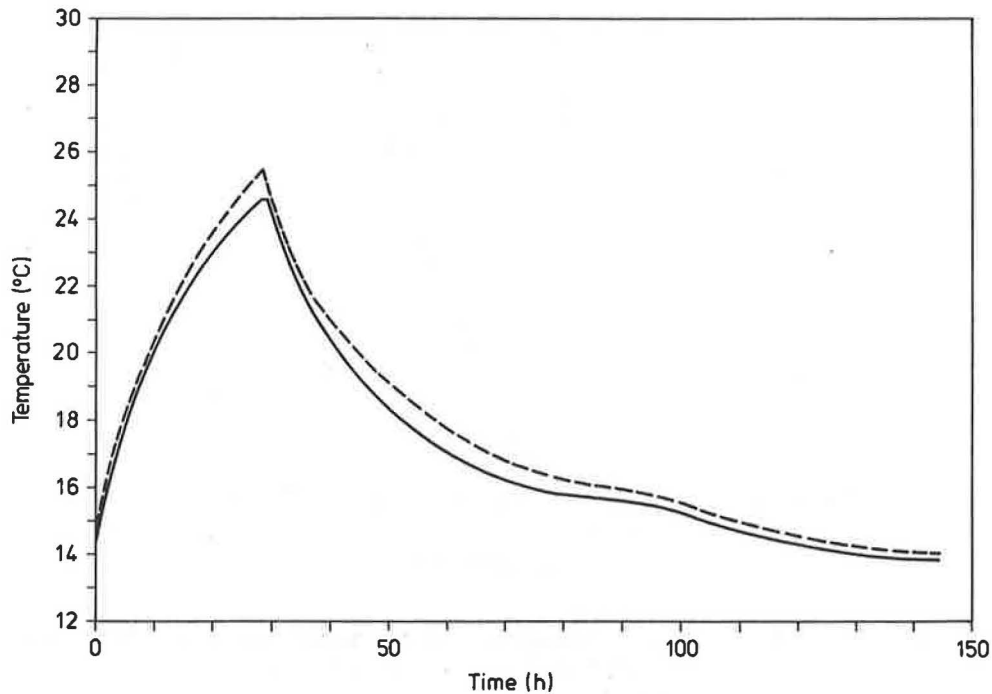


Figure 6. Temperature on the floor  
———— measured ————— simulated

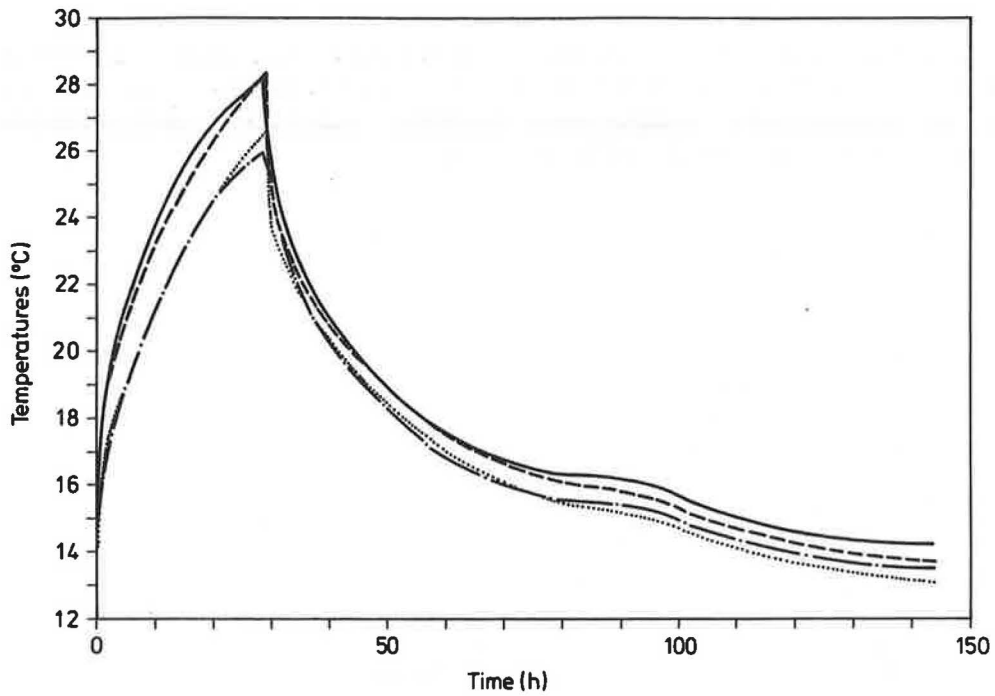


Figure 7. Air temperature in the occupied and the upper zone  
 occupied zone: ——— measured ..... simulated  
 upper zone : ——— measured - - - - - simulated

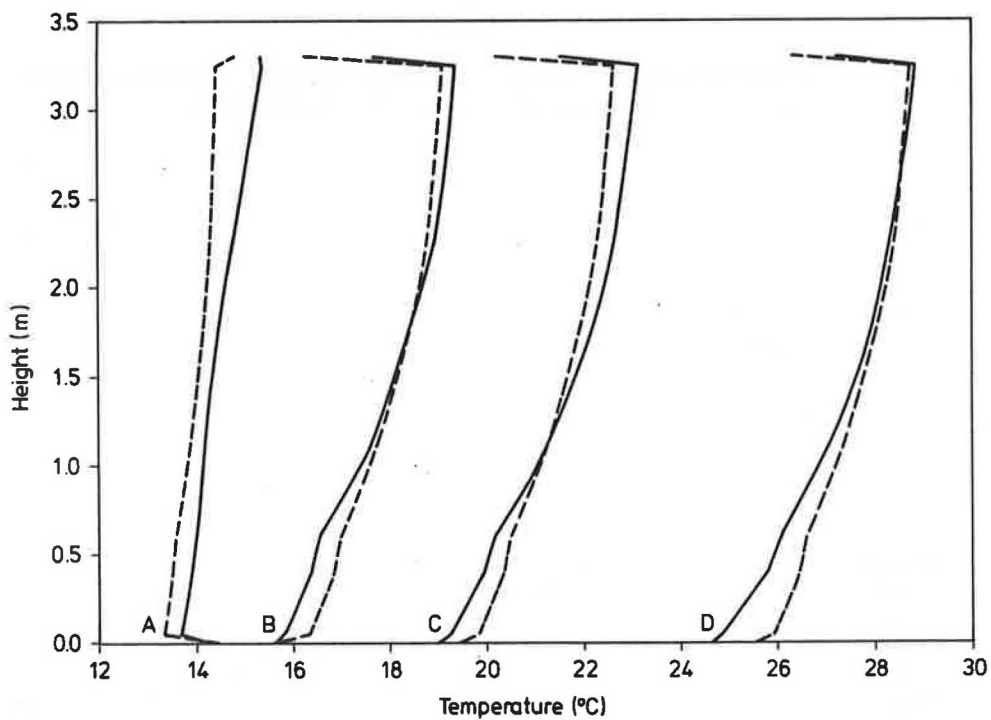


Figure 8. Vertical air temperature gradient in the test room  
 ——— measured - - - - - simulated  
 curve A: Gradient in steady state situation  
 curve B, C and D: Gradients after 1.5, 8 and 29 hours  
 heating (400 W)

### CONCLUSION

In the present paper it has been shown that with a simple model it is possible to calculate the vertical main temperature gradient in a heated room without forced air movement. In the model two temperatures are calculated, the mean temperature in the occupied zone and the mean temperature in the upper zone of the room. To make a total gradient description it demands also a knowledge of a dimensionless temperature gradient for the room, but this dimensionless gradient is possibly independent of the room dimension because of the turbulence intensity in the boundary layer. The model is built upon the assumption that it is mainly the convective air movements along the room surfaces and the radiator that generates the vertical temperature gradient, and therefore it is likely that a global dimensionless temperature profile can be used with success in general room loads calculations.

The validation of the programmes against measured temperatures in a laboratory test room shows that the calculated surface and air temperatures are closely following the real conditions; even the floor temperature that normally in single node models (air node) always is calculated too high, is determined with satisfaction in this model.

A good estimate of the vertical temperature gradient in the test room is obtained, even though the calculated gradients have a tendency to be more steep than the real ones.

The described hybrid model shows an improvement which could be included in commercial room loads programmes.

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