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Laboratory and matematical moduls of natural ventilation

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SUMMARY

Heat sources within a building lead to temperature differences between the building and the outside air, and this can be used to drive ventilation flows using the "stack effect". It is these buoyancy-driven flows that are investigated in this paper. A description of the use of laboratory models to investigate spaces with internal heat gains is given, together with the scaling laws necessary to apply the results to a full-scale building. For this technique salt solutions of different concentrations are used in the model to represent air at different temperatures, and thus different densities, in the real building. An outline of a mathematical model is also given, and the general features of natural ventilation are discussed. It is found that the flows can be divided into two broad categories: mixing and displacement modes. The modelling techniques are applied to proposed building projects. The use of modelling studies in the design process is discussed.

1. Introduction

Natural ventilation is the use of naturally occurring pressure differences to drive a ventilation flow. These pressure differences may be caused by the action of the wind or by temperature differences between the internal and external air. It is the latter type of natural ventilation, using the "stack effect", that will be discussed in this paper. In recent years there has been increased interest in the use of natural, rather than mechanical, ventilation, especially for the ventilation of large spaces with high heat gains such as atria (1). Clearly the use of novel ventilation techniques requires extensive testing and modelling at the design stage.

The ventilation flows found in naturally ventilated, heated spaces can be divided into two broad categories called *mixing ventilation* and *displacement ventilation*. In a mixing system the fresh, cooler air is introduced near the top of the space, and descends, mixing throughout the space, giving a nearly uniform temperature distribution. In displacement ventilation the cooler air is introduced near the floor of the space and warm air is allowed to leave through openings near the top of the space, giving a stably stratified temperature distribution. These cases are sketched in figure 1. In reality many systems are intermediate between these two, but we will show that most cases can be discussed in terms of one or other of these categories.

In section 2 of this paper we describe techniques for the laboratory modelling of ventilation flows driven by temperature differences, and in section 3 we give an outline of a mathematical model of such flows (greater detail of this model is given in (2)). The application of these techniques to particular building projects is given in section 4, and in the final section, section 5, we conclude with a discussion of the general features of natural ventilation and the use of modelling techniques.

2. Laboratory Modelling of Ventilation Flows

2.1 Scaling Laws

For the ventilation flows considered here the effects of viscosity and heat diffusion are small compared with advection, as is evident from the values of the Reynolds and Péclet numbers (10^3 and greater). These parameters are given by UL/ν and UL/κ respectively, where U is a typical velocity, L is a typical length, ν is the viscosity and κ is the diffusivity of heat. At such large values the flow will be independent of these parameters. To simulate these flows in the laboratory it is necessary to ensure that the values of the Reynolds and Péclet numbers in the experiment are sufficiently high. This can be achieved in the laboratory in small scale models using water as the working fluid and adding salt to produce density differences. Thus different salt concentrations represent different temperatures (3,4,5). In the laboratory heat is modelled by salt and so it is the diffusivity of salt in water that gives the relevant Péclet number.

The driving force in these flows is the buoyancy force caused by density differences between different parts of the fluid. A parcel of fluid of density $\rho+\Delta\rho$ surrounded by fluid of density ρ will experience an acceleration of approximately $g\Delta\rho/\rho$, if $\Delta\rho/\rho$ is small (where g is the acceleration due to gravity). This acceleration is known as the *reduced gravity*, and it will be denoted by g'. For air, it is useful to note that $\Delta\rho/\rho$ is approximately equal to $-\Delta T/T$, where T is the temperature measured in degrees Kelvin. It is not necessary to use the same density difference in the model as in the real situation, rather it is chosen to give high enough Reynolds numbers. The relation between the experimental results and the real situation is found by considering the appropriate scalings. The subscripts M and F will be used to denote the scales in the model and real (full-scale) cases. Length-scales will be denoted by L, velocities by U, times by t and buoyancy fluxes by B. (The buoyancy flux is the flux of g'.) These scales can all be constructed from the length-scale and g' as shown in table 1.

		model	full-scale
scale:	times	$(L_M/g'_M)^{1/2}$	$(L_F/g'_F)^{1/2}$
	velocities	$(L_M g'_M)^{1/2}$	$(L_F g'_F)^{1/2}$
	buoyancy fluxes	$L_M^{5/2}\, g'^{3/2}_M$	$L_F^{5/2}\, g'^{3/2}_F$

Table 1. Scales for other variables in terms of the length-scale and the reduced gravity ($g' \equiv g\Delta\rho/\rho$).

Thus, for example, the ratio if velocities in an experiment to those at full scale will be $(L_M g'_M)^{1/2}:(L_F g'_F)^{1/2}$. It is useful to note that the buoyancy flux due to a heat source of strength W (in kilowatts) in air at room temperature is $B=0.0281W$, where B is measured in m^4s^{-3}. Further discussion of scalings will be given below with reference to the study of particular buildings.

2.2 Experimental Techniques

The laboratory models are constructed from clear perspex, which allows visualisation of the flows. The concentration of salt solutions may be measured either by taking samples or by measurement *in situ* using conductivity probes. The latter technique makes use of the change in conductivity of salt solutions with changing concentration. Note that since salt water is denser than fresh water higher salt concentrations would represent lower temperatures. In many cases, including the case described below, it is convenient to turn the model upside down so that higher salt concentrations represent higher temperatures and heat sources can be represented by sources of salty water.

The flows are visualised by shining light through the apparatus onto a translucent screen forming a *shadowgraph*. Since different concentrations of salt solution have different refractive indices, changes in salt concentration are shown by this technique. Dyes are also used to mark different parts of the fluid as they are advected by the flow. The flow is recorded using video cameras and by taking still photos, and can be examined in detail using image processing techniques.

3. Mathematical Model of Natural Ventilation

In this section the main results of mathematical models of ventilation flows will be given. For a full description of modelling of ventilation flows see (2,5), for flow through openings see (6,7,8,9), for turbulent plumes see (10), and for plumes in a closed space see (11,12).

3.1 Mixing Ventilation

Mixing ventilation occurs where a heated space is connected to cooler ambient air by openings near the top of the space. Warm air leaves through these openings and cool air enters, often mixing throughout the space as it descends giving a fairly uniform temperature throughout the space. A steady situation will develop, with the heat lost through the openings being balanced by the heat sources within the space. If there is a single opening in a vertical wall, of height d and area A, connecting a space at temperature $T+\Delta T$ with ambient air at temperature T, then the air flow through the opening will be

$$F = kA\,(gd\ (\Delta T/T))^{1/2}, \tag{1}$$

where k is a constant equal to 0.25. For an opening in a ceiling, such as a skylight, the flux can again be found from equation (1) with $k=0.05$ and d the longest dimension of the opening. If the space contains a heat source of W kW, and lengths are measured in metres, the steady temperature is given by $T+\Delta T_M$, with

$$\Delta T_M = 2.8(W/kAd^{\,1/2})^{2/3}. \tag{2}$$

3.2 Displacement Ventilation

If a heated space has openings near the floor and openings near the ceiling connecting the space to cooler ambient air, then warm air will leave through the upper openings and cooler air will enter through the lower openings. The air in the space will be highly stratified with cool air near the floor and warm air near the ceiling. We consider here the simple case of a single, point heat source on the floor of a space with an opening near the ceiling of area a_1 and an opening near the floor of area a_2. A turbulent plume will rise above the heat source, entraining ambient air and becoming cooler as it rises. A layer of warm air will build up near the ceiling, driving a flow through the openings due to the stack effect. Eventually a steady state will develop, with a stationary interface between warm air in the upper layer and cool air in the lower layer, as shown in figure 2. The only flow from the lower to upper layer will be within the rising plume, so the volume flux in the plume at the interface level must match the volume flux out through the upper opening, which in turn must be equal to the inflow through the lower opening. In addition, in the steady state the heat flux into the upper layer, due to the plume, must equal the heat flux out through the upper opening, thus the temperature in the rising plume at the interface level must equal the temperature of the air leaving the space through the upper opening. Therefore the temperature of the upper layer must be uniform throughout the layer, equal to the temperature in the plume at the interface level.

To calculate the interface level it is necessary to match the flow driven by the layer of warm air to the flow in the plume at the interface level, and then use the fact that the temperature in the upper layer is equal to the temperature in the plume at the interface level. The flow driven by the upper layer can be found by considering the position of the neutral level (where the pressure is equal inside and outside the space) and by using Bernoulli's theorem. Writing H for the height of the space and h for the height of the interface, the flow driven by the warm layer is

$$F = A\,(g\,(H-h)\Delta T/T)^{1/2}, \tag{3}$$

where A is the *effective area*, given by

$$A = (a_1 a_2)/(1/2((a_1^2/c) + a_2^2))^{1/2}. \tag{4}$$

In equation (4) c is a constant lying between one half, for a sharp expansion at the inlet, to unity, for a perfectly smooth expansion. The effective area is dominated by the smaller of the total area of the upper and lower openings. The flow rate in the turbulent plume increases with height, whereas its temperature decreases, see (10) for a full analysis. The steady state balances described above lead to a relation between the area of the openings and the interface height

$$A/H^2 = 0.04\,(h/H)^{5/2}(1-h/H)^{-1/2}. \tag{5}$$

Notice that the strength of the heat source does not appear in equation (5): the interface height is a geometric quantity determined by the geometric relation between the area of the openings and the height of the space. The temperature difference between the upper and lower layer, which will be denoted by ΔT_D, does depend on the heat source, being given by

$$\Delta T_D = 24 W^{2/3} h^{-5/3}, \tag{6}$$

where h is measured in metres, W in kilowatts and ΔT_D in degrees Celsius or Kelvin.

If, instead of a point source, the heat source is a line source spreading the full length of the space, the interface height is given by

$$A_L/H = 0.2\,((h/H)^3/(1-h/H))^{1/2}, \tag{7}$$

where A_L is the effective area per unit length. The temperature difference between the upper and lower layer is given by

$$\Delta T_D = 8 W_L^{2/3} h^{-1}, \tag{8}$$

where W_L is the heat strength per unit length. Note that all these results for displacement flows are for a source on the floor of the space: if the source is elevated then h and H should be measured from the height of the source. Note also that the height of the lower opening is immaterial, so far as the position of the interface is concerned, provided it is below the level of the interface.

The results for both mixing and displacement flows are shown in graphical form on figures 3, 4 and 5, together with results from laboratory experiments conducted in a simple rectangular box. The results given in this section apply to a single point or line source, but the analysis can be extended to multiple sources and to area sources as shown in (2).

4. Modelling Studies

The modelling techniques described above have been applied to various building designs, and we give brief case studies of three such projects. In all cases a scale model of the significant part of the building was constructed from clear perspex, of overall size about 0.5m in all dimensions. The model was suspended upside down in a large tank of water, of depth 0.7m, width 0.6m and length 13m. Heat sources were represented by pumping in relatively dense salt water and, for the last study, a body of cool air was represented by injecting a relatively light alcohol solution. The results of such studies are best appreciated by viewing videos of the experiments and we give here some digitised images from the videos.

4.1 Courtroom

Modelling

The first study we describe is of a proposed design of courtroom, investigated for the PSA. The courtroom has a cruciform design, with a raised roof over the central crossing, as shown in figure 6. The main upper openings were four skylights, of area 0.6m^2 each, and windows in the walls of the raised roof, of total area 14m^2. There were many small vents in the lower part of the walls and the floor. A model was constructed of the courtroom to 1:30 scale. The main source of heat was from occupants, with a typical value being 8kW. This heat source was represented in the model by pumping in salt water, 10% denser than fresh water (so, at the source, $g'=0.981\text{ms}^{-2}$), at a rate of $30\times10^{-6}\text{m}^3\text{s}^{-1}$, giving a buoyancy flux of $29.6\times10^{-6}\text{m}^4\text{s}^{-3}$, through three distributed sources. In the real situation the buoyancy flux is $0.225\text{m}^4\text{s}^{-3}$. Given this ratio between the buoyancy fluxes (29.6×10^{-6}:0.225) and the ratio in the lengths (1:30) this implies, from table 1, a ratio in the scales for g' of 0.75:1 (model:full-scale). Cancelling g from the expressions for g' this gives $(\Delta\rho/\rho)_M/(\Delta T/T)_F=0.75$, where T is measured in Kelvin. Thus a temperature difference of 1°C at full-scale is represented by a density difference of 0.25% in the model (since T is approximately 300K). This also fixes the time-scaling ratio to 0.2:1, again from table 1, so that one second at full-scale is represented by 0.2 seconds in the model.

Results

Digitised images of the flow are shown in figure 7, for various configurations of openings. Note the sharp interface level shown in figure 7a, with a displacement mode of ventilation established. When the large upper windows were opened a mixing type of ventilation generally resulted, since the area of the windows was much larger than the lower vents. This results in the mixing down of warm air, and in increased temperatures in the occupied zone compared with displacement ventilation (see figure 7b). The modelling suggests that a displacement type of ventilation could be achieved with the skylights and the lower level vents open, giving adequate flow rates for comfort.

4.2 Atrium

Modelling

The new Westminster and Chelsea hospital will have a large central atrium, as shown in figure 8. To model this a 1:100 scale model was constructed of one third of the atrium. Obviously this limits the model to representing flows which do not have a large component along the length of the atrium, but we have found that for buoyancy-driven flows such horizontal variations are weak due to the effects of stratification. The main source of heat in this case was sunlight falling on the north wall of the atrium. This was modelled by injecting salt solution through a series of small, upwards directed holes in a tube placed at the bottom of the sunpatch. This position was altered, and heat sources also placed on the west or east facing walls, to represent the heated patch at different times of day and different times of year. The atrium has openings in the upper part of the side walls, where the atrium extends above the level of the building. There are openings from the rooms adjacent to the atrium, and other ducted vents to the atrium: these openings were also represented in the model.

Results

Digitised images of the displacement flows are shown in figure 9. It was found that in mixing mode, with upper level openings only, the temperature in the atrium would build up to intolerable levels. It was also found that, especially in the summer months, displacement mode ventilation (with openings at upper and lower levels) did not raise the interface between warm and cool air sufficiently high. It was recommended that more openings be provided at lower levels (the total area of the lower openings was much less than that of the upper openings in the original design), and that an increased atrium height (more “dead space”) would be useful.

4.3 Atrium and connected rooms

Modelling

A proposed building for the Department of Humanities at Seville University on the Expo '92 site has a large atrium with lecture rooms to the north and offices to the south, see figure 10a. The design of the atrium roof is such that sunlight will fall on the upper part of the northern wall, with shading devices preventing sunlight reaching the lower part of the atrium directly. It is intended that a displacement flow will be set up in the atrium, with warm air leaving through openings in the roof and cool air being drawn through the surrounding room, thus ventilating them. A perspex 1:100 scale model of the central third of the building was constructed, as shown in figure 10b. The lecture rooms and library space on the northern side of the building were included, as was the central atrium, but the offices on the southern side were not included. The lecture rooms and library had openings to the atrium and to the ambient fluid. In addition there was a large opening along the roof ridge of the atrium, some doorway openings at ground level in the atrium and openings on the southern wall of the atrium representing connections with the (unmodelled) offices. It is intended to allow the building to cool during the night, and the stability of a pool of cool air at the base of the atrium was investigated by representing the cool air by alcohol solution. Note that the Richardson number (the relevant parameter in considering the mixing of stratified fluids) is preserved in the laboratory scaling, so that mixing will be accurately modelled in the laboratory.

Results

Digitised images of the flow for two configurations of heat sources and openings are shown in figure 11. The warm air generated by sunlight falling on the interior wall of the atrium is seen to drive a flow through the building, with warm air flowing out through the opening in the roof being replaced by ambient air flowing in through the lecture rooms and other openings. An adequate ventilation flow can be driven in the lecture rooms by the heated atrium. The interface between warm and cool air in the atrium can be controlled between acceptable limits by adjusting the area of the openings.

In the lecture rooms the heat generated by occupants moves into the upper part of the lecture room, and thence out of the room into the atrium. The cool pool of air at the bottom of the atrium is only slightly disturbed by the ventilation flows, the most disruption occurring when unheated side rooms are ventilated. This is because if the flow is from heated rooms it rises on entering the atrium, whilst if it comes from an unheated room it mixes throughout the atrium. Cool air can be prevented from escaping through ground level openings by the ventilation flow drawing in air through these openings at a fast enough rate to prevent outflow.

5. Conclusions

5.1 Natural Ventilation

We have outlined a mathematical model of ventilation flows and shown how it gives reliable estimates of the observed flows. We have shown how ventilation flows may be divided into two categories, mixing and displacement flows, and described the steady states of both these flows using mathematical models. An important result is that the *distribution* of temperature may depend on the distribution of heat sources, but not their strength, whereas the *magnitude* of the temperature does depend on the heat source strengths with ΔT varying like $W^{2/3}$. For displacement flows we have found that a two-layer stratification will result, with the height of the interface between warm and cool air dependent on the area of the openings and the height of the space. Increasing the area of the openings increases the height of the interface, but it is important to note that it requires large increases in the opening area for small increases in interface height once the interface is above about three-quarters the height of the space. It is clearly useful to have a certain amount of "dead space" at the top of a room that is to be ventilated by displacement ventilation. It is also important to note that the effective area of the openings is dominated by the smaller of the upper and lower openings. It may often be convenient to have a large area of openings at low levels, to ensure comfortably low inflow velocities, and to control the ventilation by adjusting the area of the upper openings.

In this paper we have not discussed transient effects, such as the flushing of a space initially filled with buoyant gas (as may happen after a gas leak) or the effects of altering the strength of the heat sources; these topics are covered in (2). Here we note that the flushing time for mixing flows is $(2SH/kA)(gd\,(\Delta T/T))^{-1/2}$, and that for displacement flows is $(2SH/A)(gH\,(\Delta T/T))^{-1/2}$ where S is the floor area of the space. Thus, for example, oscillations in the heat source strength over periods much shorter than these will have little effect on the overall flow.

In general we have found that displacement ventilation gives the greater ventilation flow rates, and appears to be the most appropriate means of ventilating many large spaces. It also provides the possibility of driving a ventilation flow in smaller spaces adjacent to a large atrium as shown in the last case study.

5.2 Modelling and Modelling Techniques

The basic features of natural ventilation flows can be modelled by the mathematical models described above, these are particularly useful in the early stages of the design process. We have also shown how ventilation flows may be modelled in the laboratory. It is possible to model flows due to a variety of distributed heat sources in a complex geometry which are, at present, beyond the scope of numerical techniques. This type of modelling is suited to flows where the advection of heat is the most important means of heat transport, but is not suitable where the diffusion or radiation of heat are more important. The scaling rules necessary to interpret the implications of the results of laboratory experiments for full-scale flows have also given. Laboratory experiments can give detailed, quantitative information, but the visual results are also accessible to a wide audience with little technical expertise. Laboratory modelling is best suited to the later stages of the design process when it is necessary to establish the details of the flow and adjust the design accordingly. There are some areas which require more detailed investigation, such as the representation of the heat generated by a group of people.

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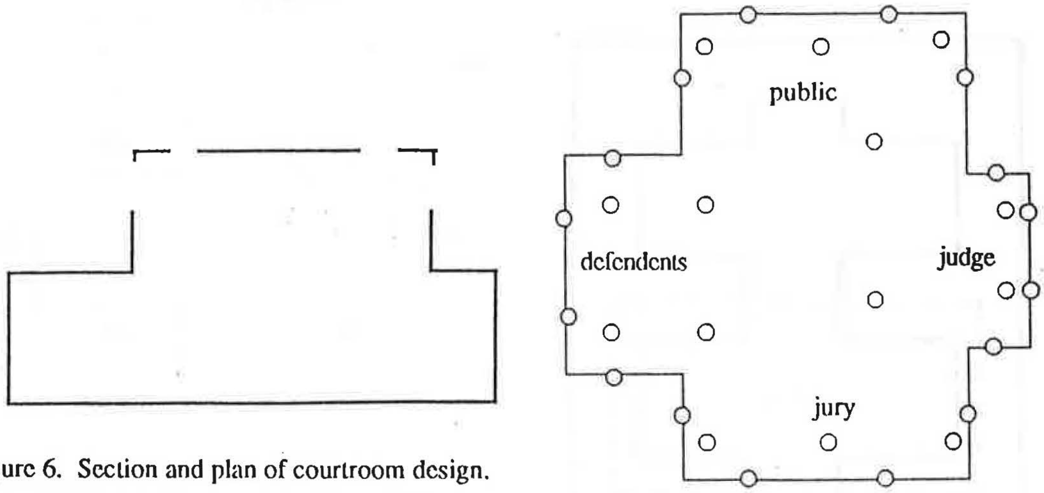
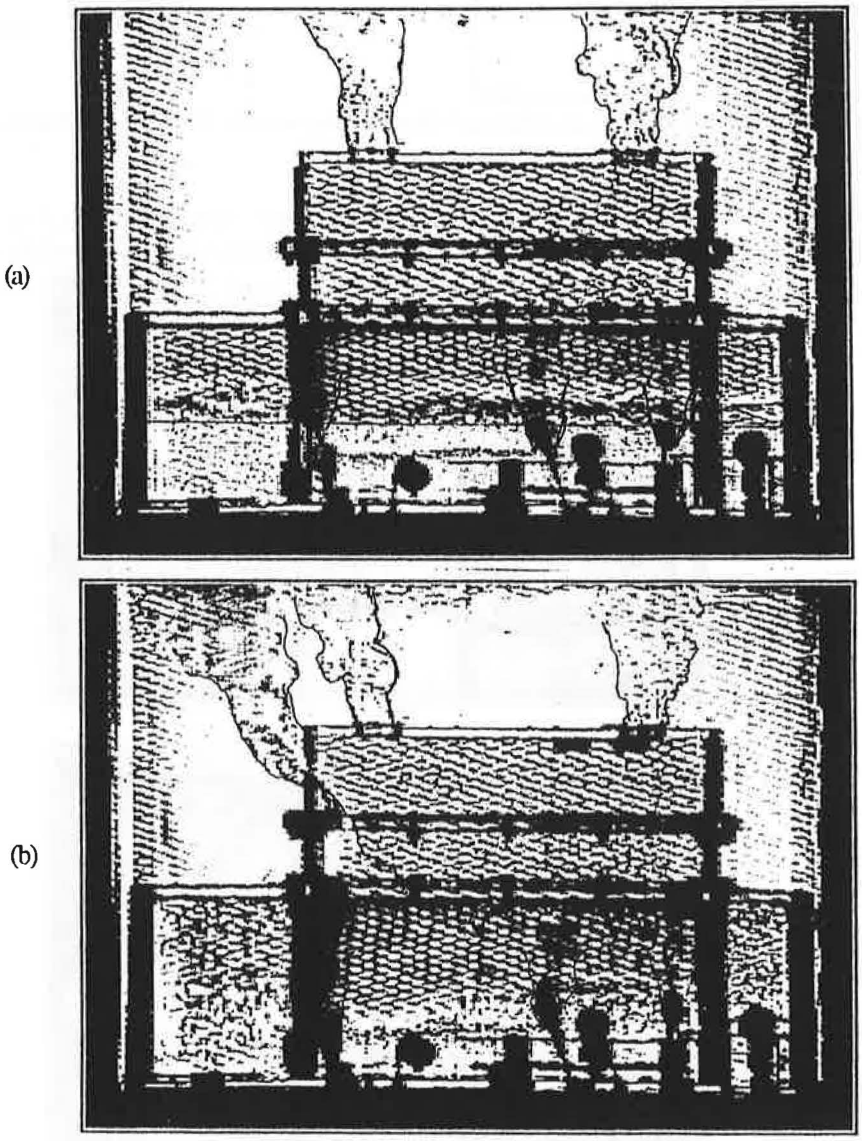


Figure 6. Section and plan of courtroom design.

Figure 7. Digitised images from videos of experiments using the courtroom model with internal sources of buoyancy (dyed). (a) Displacement flow with fluid entering through vents in the lower wall and leaving through skylights and a sharp interface between the buoyant and clear fluid. (b) Mixing flow (predominantly) with, in addition to the openings in (a), a large upper window open. Fluid enters through this window and descends into the space mixing down the buoyant fluid.



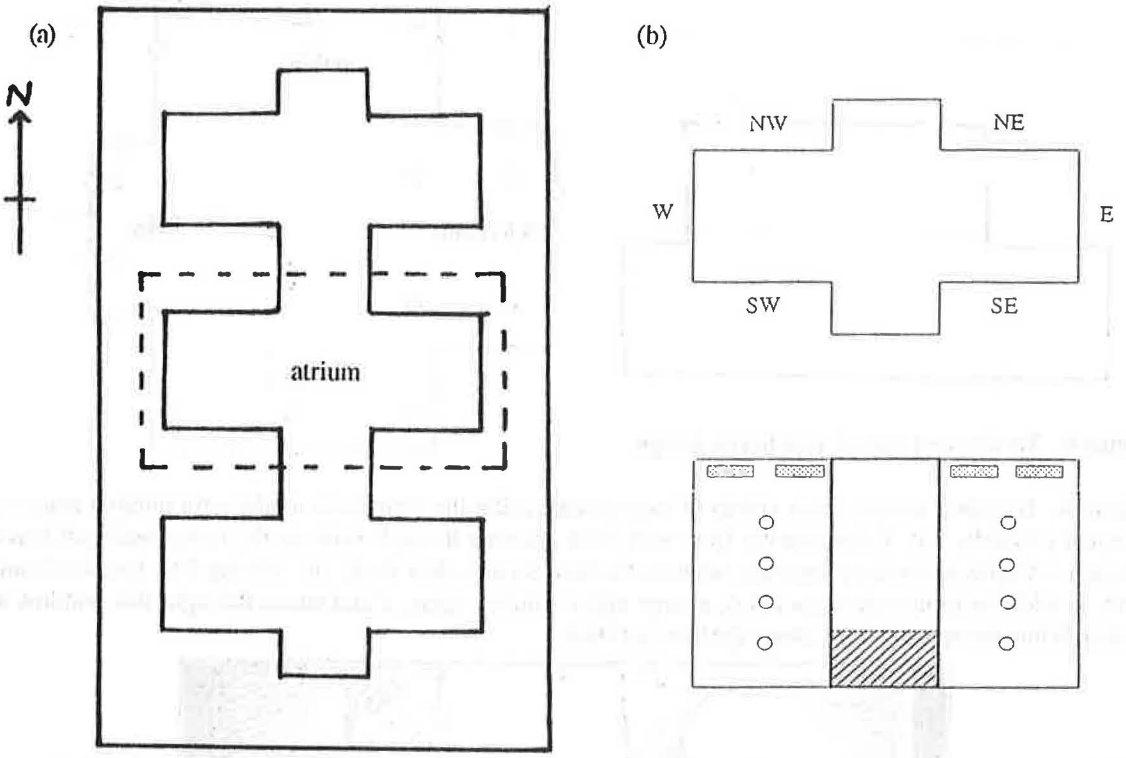
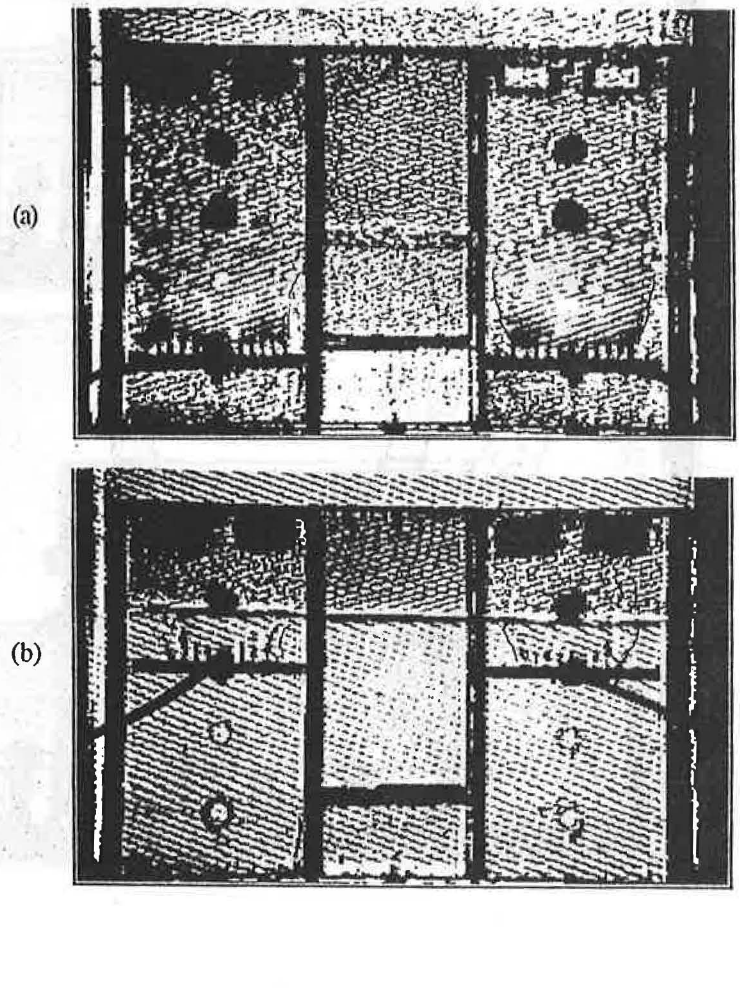


Figure 8. (a) Plan of Westminster and Chelsea hospital showing the section of the atrium that was modelled. (b) Plan and elevation of laboratory model.

Figure 9. Displacement ventilation in the atrium model driven by sources representing heating by sunlight, (a) with the sun high in the sky (and thus the sunpatch extending down to near the floor) and (b) with the sun low.



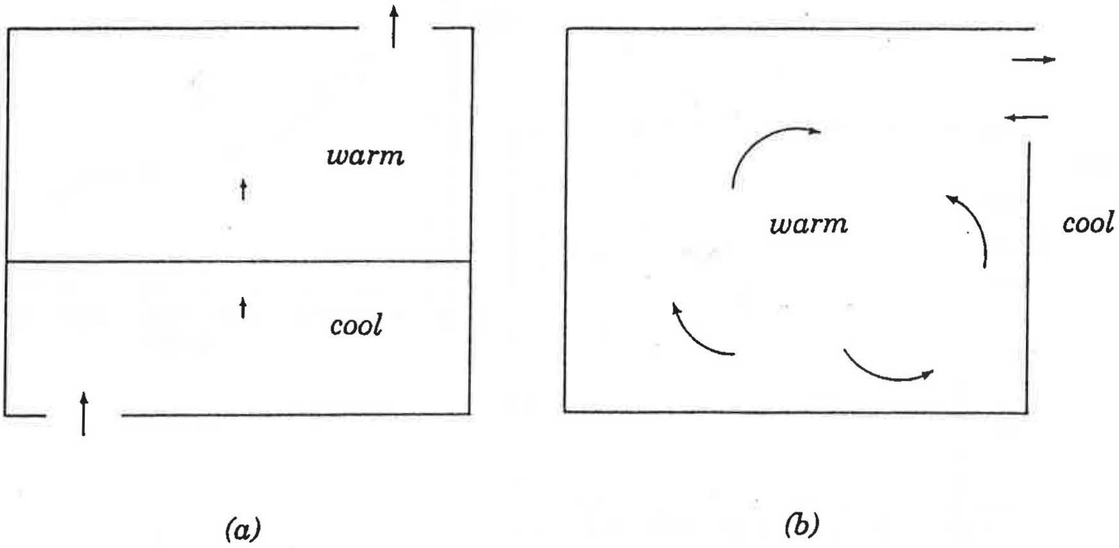


Figure 1. (a) Displacement and (b) mixing ventilation.

Figure 2. Steady displacement flow in a space containing a source of buoyancy.

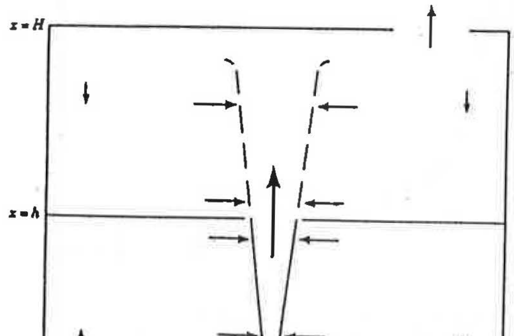
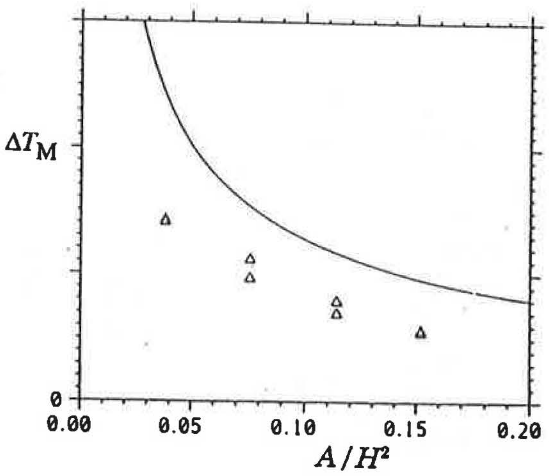


Figure 3. Theoretical and experimental results for steady mixing ventilation, showing ΔT_M as a function of opening area.



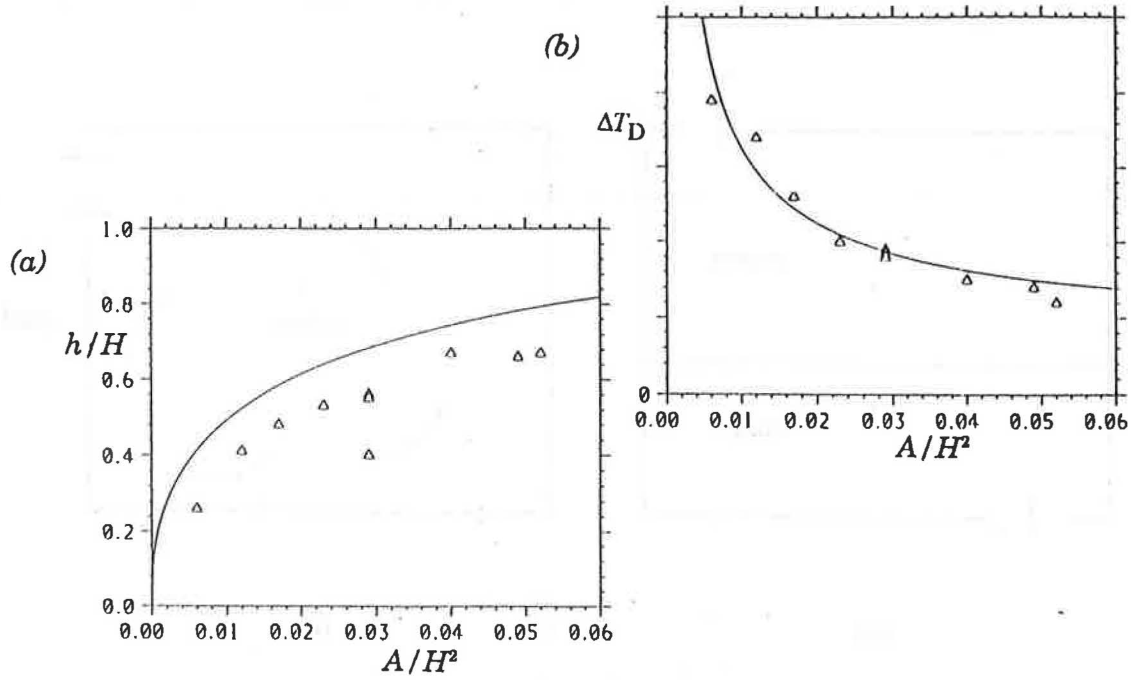
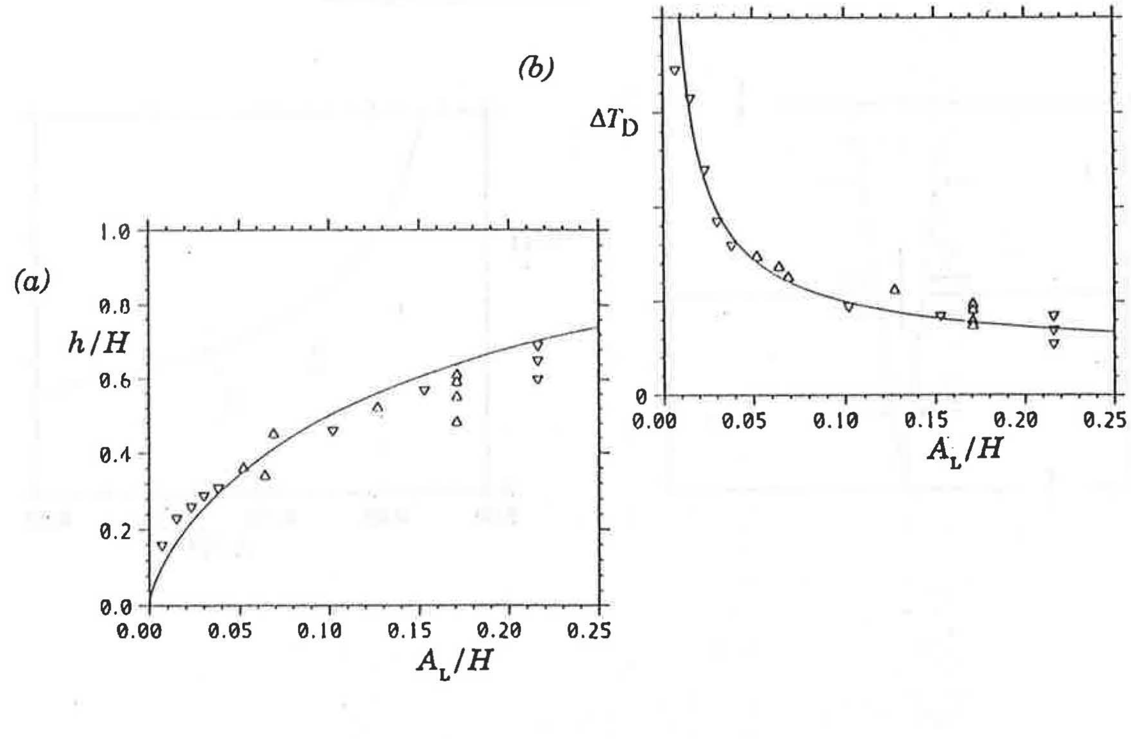


Figure 4. Theoretical and experimental results for steady displacement flow in a space containing a point source of buoyancy, showing (a) interface height and (b) ΔT_D as functions of opening area.

Figure 5. As figure 4, but for a line source.



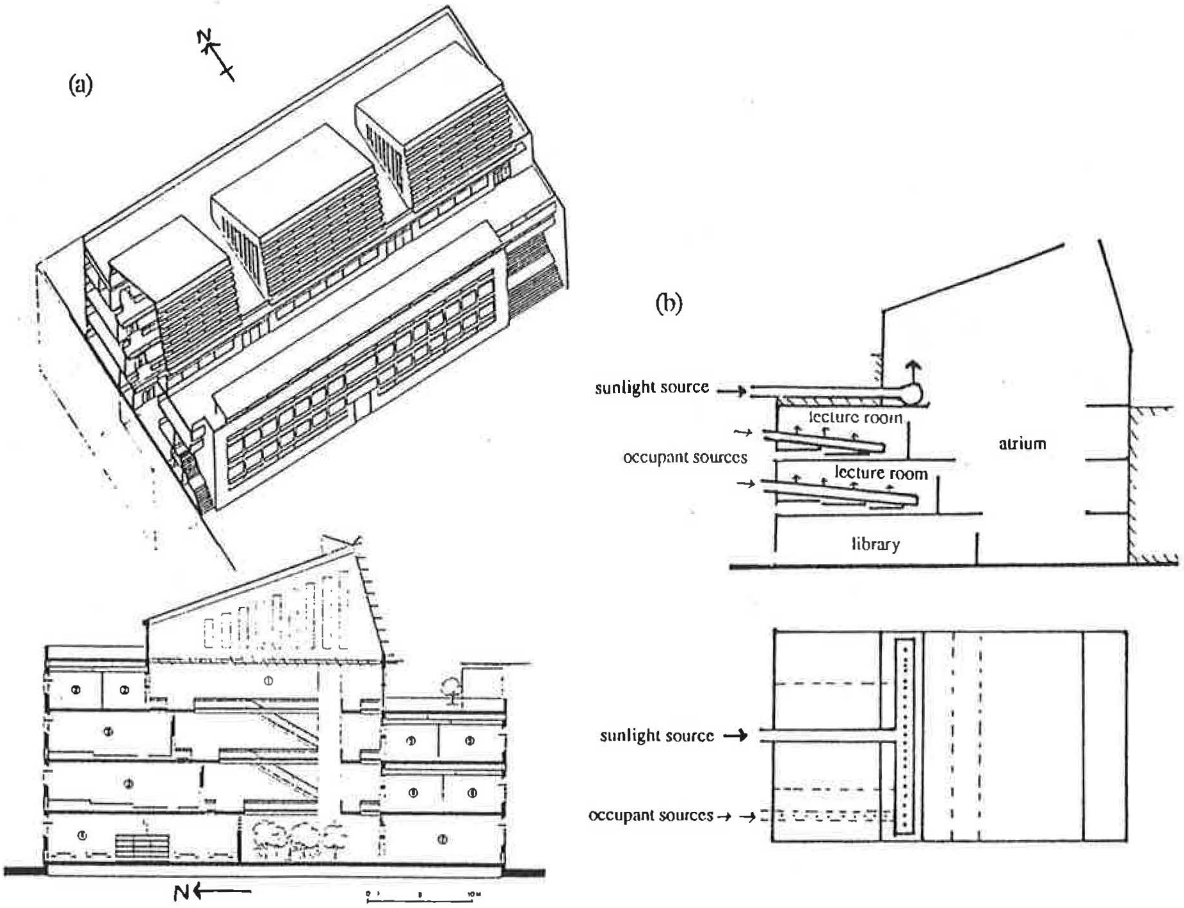
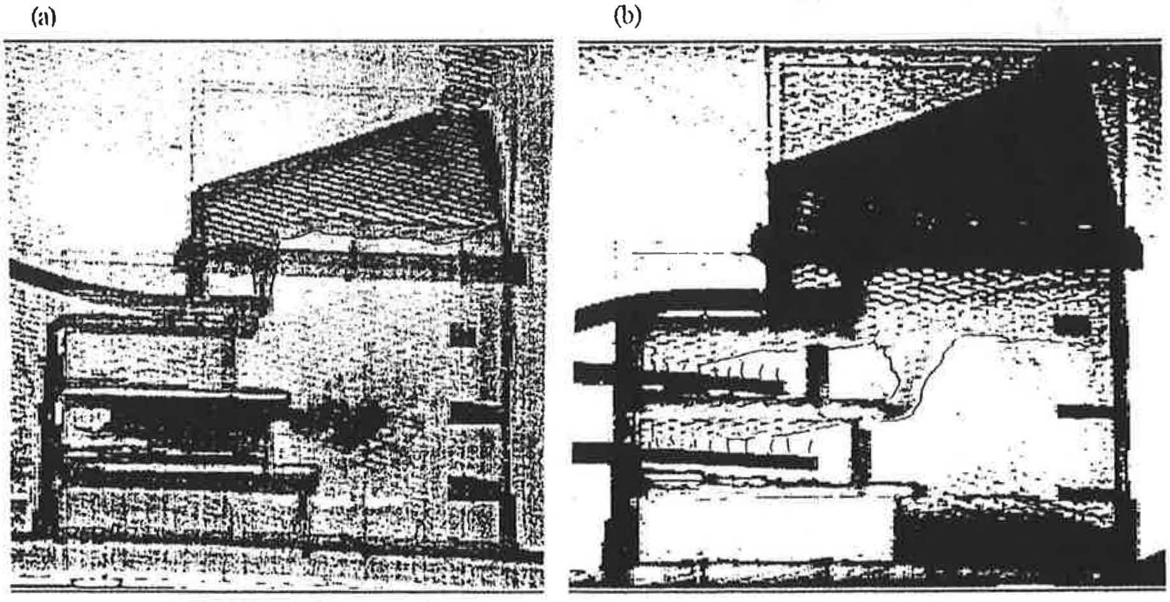


Figure 10. (a) Perspective view and section of the university building. (b) Section and plan of the laboratory model showing sources of salty water (representing heat sources).

Figure 11. Displacement flow in the laboratory model. (a) Sunlight source only: dye has been injected into one of the lecture rooms showing the flow from the lecture room into the atrium. (b) Sunlight source, occupant sources and cool pool: note that fluid flowing from the lecture rooms rises on entering the atrium, rather than mixing throughout the atrium as in (a).


(a)
(b)