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Coupled airflow and thermal analysis for building system simulation by element assembly techniques

James Axley
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Richard Grot
National Institute of Standards and Technology
Gaithersburg, Maryland, USA

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James Axley
Massachusetts Institute of Technology
Cambridge, Massachusetts, U.S.A.

Richard Grot
National Institute of Standards and Technology
Gaithersburg, Maryland, U.S.A.

Summary

Element assembly techniques have been applied to developed methods to model a) airflows driven by wind pressures, buoyant forces, and HVAC systems and b) heat transfer due to conduction, convection, advection, and radiation in complex multi-zone building systems. These methods, the Airflow Network Analysis Method and the Discrete Thermal Analysis Method, allow an integrated consideration of the building envelope, construction, and HVAC system interaction in each area of analysis, are computationally non-demanding, and can be employed to model building systems of arbitrary complexity. This paper will consider the integration of these methods to solve the coupled airflow and thermal analysis problem in macroscopic (i.e., whole-building, multi-zone) building system simulation. The theoretical bases of these related methods will be reviewed and a framework for integrating these methods to solve the coupled airflow and thermal analysis problem in building system simulation will be presented. A general approach for solving the resulting coupled system of equations, based on Newton-Raphson techniques, and special cases derived from this approach will be outlined.

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James Axley
Massachusetts Institute of Technology
Cambridge, Massachusetts, U.S.A.

Richard Grot
National Institute of Standards and Technology
Gaithersburg, Maryland, U.S.A.

Introduction

Building energy performance and indoor air quality of buildings are both intimately linked to infiltration, exfiltration, and interzonal airflows in building systems. There is, therefore a clear need for mathematical models to predict these airflows for the design of new buildings, for redesign modifications and diagnosis of existing buildings, for research in building thermal and air quality behavior, and to improve our understanding of airflow in buildings in general. Presently, mathematical models exist, that may be used for whole-building, steady-state airflow analysis, that account for airflow driven by wind pressures, building mechanical systems, and buoyant forces. These *macroscopic* models (i.e., as distinguished from *microscopic* models used to study the details of airflows in rooms (10)) have been applied to modeling quasi-steady state changes in airflows due to changes in these driving forces (19), but modeling unsteady conditions, especially those due to the coupled interaction between heat transfer and airflow, remains a challenge.

A large variety of macroscopic models for multi-zone, unsteady heat transfer in buildings have been developed, yet few researchers have attempted to integrate these models with existing macroscopic airflow models to directly address the coupled airflow/thermal problem. Two exceptions, however, deserve special note. Walton (22) integrated a simple network airflow analysis technique with the conduction transfer function approach to multi-zone building thermal analysis to solve the coupled airflow/thermal problem and later Clarke (11) described a similar airflow analysis technique and briefly outlined a computational solution strategy for the coupled problem that has been implemented as part of the ESP building thermal simulation program (1). The strategies employed by these authors are similar to two special cases of the approach that will be presented in this paper and will be discussed subsequently. Suffice it to say, these approaches do not explicitly account for the nonlinear dependency of the flow and thermal problems and, as a result, must be considered somewhat less general than the approach presented herein.

Element assembly techniques have been applied to develop methods to model airflows and heat transfer in multi-zone building systems. These methods, the Airflow Network Analysis Method and the Discrete Thermal Analysis Method, allow an integrated consideration of the building envelope, construction, and HVAC system interaction in each area of analysis, are computationally non-demanding, and can be employed to model building systems of arbitrary complexity. This paper will present an approach to modeling the coupled airflow/thermal problem, based on the integration of these methods, that explicitly accounts for the full nonlinearity of coupled problem Although an emphasis will be placed on the macroscopic modeling of whole building systems the approach may also be applied to the macroscopic modeling of building subsystems and provides a framework for the integration of microscopic modeling techniques, based on the finite element method, with macroscopic techniques (2). The approach presented has grown out of an informal and formal collaboration between the authors and George Walton at NIST and is the basis of a program, DTFAM (Discrete Thermal-Flow Analysis Method), presently being developed at NIST by the second author of this paper, Grot.

Spatial Discretization

A building system may be considered to be a three-dimensional continuum within which we seek to completely describe the temporal, t, and spatial, x,y,z, variation of the *state* of the system. The state of solid portions of the continuum will be defined by temperature, T, and the state of the air portions of the continuum will be defined by the temperature, pressure, P, and velocity, **v**, of infinitesimal air parcels within these portions of the building system.

The determination of the spatial and temporal variation of the temperature field will be referred to as *thermal analysis*, and the determination of spatial and temporal variation of the flow field will be referred to as *flow analysis*. An approach to the solution of both analysis problems may be based on replacing the continuously defined state variables:

$$T(x,y,z,t), \quad P(x,y,z,t), \quad \vec{v}(x,y,z,t)$$

by a finite set of *discrete state variables* that are meant to approximate, in some sense, the values of the continuous variables at discrete points or regions, identified by *nodes*, in the building system. Here, the temperature and pressure fields will be approximated by spatially discrete, but temporally continuous, sets of temperature and pressure variables (organized as vectors) while the velocity vector field will be replaced by a collection of discrete mass flow rates, w, (i.e., having units of mass per time) corresponding to mean mass flow rates through discrete flow paths connecting well-mixed zones within the building airflow system:

$$\{\mathbf{T}(t)\}, \quad \{\mathbf{P}(t)\}, \quad \text{and} \quad \{\mathbf{w}(t)\}$$

(Note: Column vector quantities will be expressed by bold-faced variables enclosed in braces, { }, and matrix quantities by bold-face variables enclosed in brackets, .)

Both the thermal and airflow analysis problems will be formulated using *element assembly* techniques, borrowed from the closely-related fields of structural and finite element analysis (8, 12), wherein equations approximating the behavior of the macroscopic system as a whole, the *system equations*, are assembled from equations that describe the behavior of discrete *elements* of the system model. An intuitively useful relationship exists between the mathematics of the assembly process and the diagrammatic conventions that support it, as suggested by Figure 1.

The *element equations* will be defined in terms of subsets of the discrete system state variables:

$$\{\mathbf{T}^e(t)\}, \{\mathbf{P}^e(t)\} \text{ and } \{\mathbf{w}^e(t)\}$$

and will be referred to as the *element* state variables; vectors of discrete temperature, pressure, and mass flow rates associated with a given element "e". Inasmuch as there exists a one-to-one correspondence between each of the element's state variables and the system state variables, we may describe this correspondence by Boolean transformations of the form:

$$\{T^e\} = [B^e]\{T\} \quad \text{and} \quad \{P^e\} = [B^e]\{P\} \quad ; \quad e = a, b, c, \ldots \tag{1}$$

where $[B^e]$ is a matrix of ones and zeros defined for each element, a, b, c, ..., in the assembly.

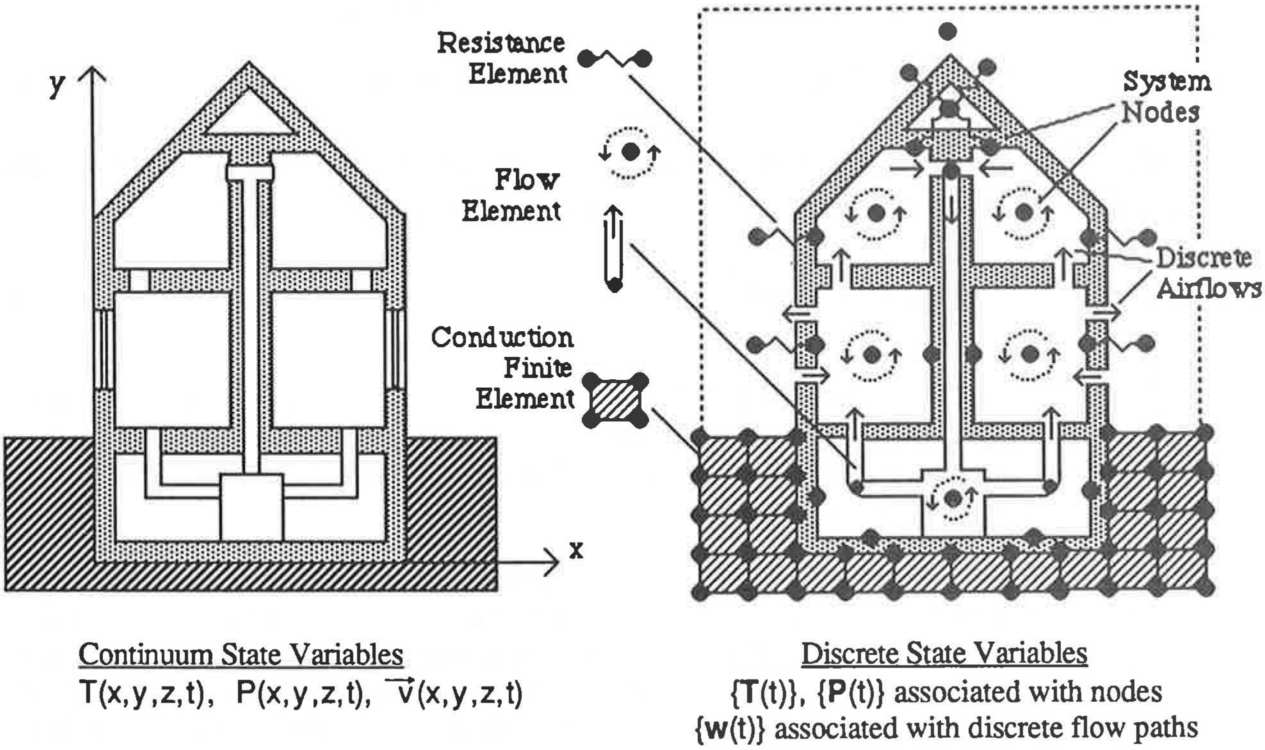

Fig. 1. Spatial discretization, element assembly, and the state variables.

Discrete Thermal Analysis

Building energy simulation has been approached using a variety of methods including methods based upon resistance-capacitance networks, LaPlace transform techniques (e.g., conduction response function techniques), Fourier transform techniques (e.g., harmonic transmission matrix methods), finite difference techniques, etc. The authors have favored an element assembly approach, because it is felt that such an approach may serve to unify the various and diverse simulation methods presently used within a single theoretical framework and, importantly, because it allows the inclusion of the powerful finite element method, and the numerical techniques associated with it, within the repertoire of techniques that may be applied to building energy simulation.

In its application to building thermal analysis, the element assembly approach is based on the assertion that: *building thermal systems may be idealized by assemblages of discrete thermal elements chosen to model specific instances or aspects of thermal transport that occur within the building system.* The program DTAM1, developed to provide a demonstration of the basic approach, provides five thermal elements including a simple thermal resistance element and a well-mixed zone or "lumped" capacitance element (i.e., the elements of the RC network analysis approach), a fluid flow loop element, and 1D and 2D conduction elements based on isoparametric finite element formulations. Equations describing a variety of other elements for radiant and fluid transfer have been presented but not yet implemented (3, 5). This approach is outlined below.

We distinguish flow elements from nonflow elements and describe the behavior of these subclasses of elements by equations of the general forms given below:

$$\{q^e_{net}\} = \mathcal{L}^e(\{T^e\}) - \{q^e\} \qquad \text{; for } \textit{nonflow elements} \tag{2}$$

$$\{h^e_{net}\} = \mathcal{L}^e(\{T^e\}) - \{h^e\} \qquad \text{; for } \textit{flow elements} \tag{3}$$

where $\{q^e_{net}\}$, $\{h^e_{net}\}$ are vectors of element net-heat and net-enthalpy flow rates, respectively, $\{q^e\}$, $\{h^e\}$ are vectors of element-derived heat and enthalpy generation rates and:

$$\mathcal{L}^e(\{T^e\}) \equiv [k^e]\{T^e\} + [c^e]\frac{d\{T^e\}}{dt} \tag{4}$$

$\mathcal{L}^e(\{T^e\})$ is a transformation of $\{T^e\}$ that has the form of a linear transformation, specific to a given element type, where $[k^e]$ and $[c^e]$ – the element *conductance* and *capacitance* matrices respectively – are square transformation matrices that may, in general, vary with time (i.e., be nonsteady) or temperature (i.e., be nonlinear).

The meaning of the element variables employed in these general element expressions may be clarified by the diagrammatic representations of hypothetical nonflow and flow elements shown in Figure 2. An element (equation) defines the nature of heat transfer between specific nodes in the system corresponding to a specific heat transfer process being modeled. Nodal temperature and either nodal heat flow rates or enthalpy flow rates are associated with each node with the convention assumed that flow into the element is positive.

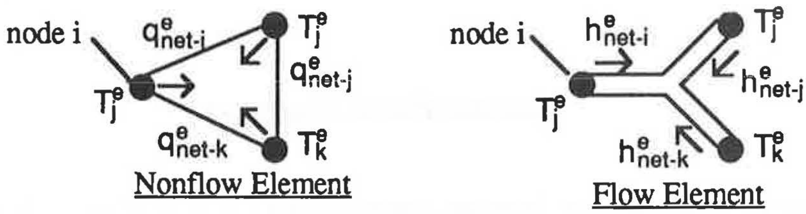

Fig. 2 Thermal element variables.

The three simplest element equations follow directly from fundamental considerations:

The 1-Node Well-Mixed Zone or Simple Capacitance Element: A single-node element, say element e associated with node i, that models the (ideal) capacitance of a well-mixed zone enclosing a mass of air m^e having a specific heat capacity of C^e_p.

$$\{h^e_{net\text{-}i}\} = m^e C^e_p[1]\frac{d\{T^e_i\}}{dt} \quad ; \text{ or } \quad [c^e] = m^e C^e_p[1] \tag{5}$$

The 2-Node Simple Resistance Element: A two-node element, say element e with nodes i and j, that models one-dimensional heat transfer through a material having a resistance of R^e and an area available for heat transfer of A^e:

$$\begin{Bmatrix} q^e_{net-i} \\ q^e_{net-j} \end{Bmatrix} = (A^e/R^e)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T^e_i \\ T^e_j \end{Bmatrix} \quad ; \text{ or } \quad [k^e] = (A^e/R^e)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{6}$$

The 2-Node Simple Flow Element: A two-node element, say element e with nodes i and j, that models heat transfer due to a (practically) instantaneous flow of rate w^e of air of specific heat capacity C^e_p through a discrete airflow path :

$$\begin{Bmatrix} h^e_{net-i} \\ h^e_{net-j} \end{Bmatrix} = (w^e C^e_p)\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}\begin{Bmatrix} T^e_i \\ T^e_j \end{Bmatrix} \quad ; \text{ or } \quad [k^e] = (w^e C^e_p)\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} \quad ; \quad w^e \geq 0 \tag{7}$$

Although a variety of other element equations could be presented, these simple element equations will be sufficient to discuss the essential features of the coupled airflow/thermal problem. Even with these simple elements building thermal systems of considerable complexity may be modeled.

Two points should be noted at this time. First, the resistance element, being representative of those elements that may be used to model conduction in solids, is defined by a symmetric system of equations while the simple flow element, as other more complex flow elements, is defined by a nonsymmetric system of equations. Second, due to thermally induced buoyancy, the air mass flow rate, w^e, will in general be dependent upon the nodal temperatures, $w^e = w^e(\{T^e\})$, thus, the simple flow element equations will be nonlinear.

Demanding the conservation of thermal energy at each of the system nodes, the element equations may be directly *assembled* to yield the *system equations* that describe heat transfer in the building system as a whole:

$$\boxed{[K]\{T\} + [C]\frac{d\{T\}}{dt} = \{E\}} \tag{8a}$$

where:

$$[K] = \underset{e = a, b, \ldots}{\mathbf{A}} [k^e] + \underset{e = \alpha, \beta, \ldots}{\mathbf{A}} [k^e] \qquad \text{the } \textit{system conductance matrix} \tag{8b}$$

$$[C] = \underset{e = a, b, \ldots}{\mathbf{A}} [c^e] + \underset{e = \alpha, \beta, \ldots}{\mathbf{A}} [c^e] \qquad \text{the } \textit{system capacitance matrix} \tag{8c}$$

$$\{E\} = \{Q\} + \underset{e = a, b, \ldots}{\mathbf{A}} \{q^e\} + \underset{e = \alpha, \beta, \ldots}{\mathbf{A}} \{h^e\} , \qquad \text{the } \textit{system excitation vector} \tag{8d}$$

$a, b, \ldots$ = nonflow element indices
$\alpha, \beta, \ldots$ = flow element indices

A, above, is the assembly operator, a generalization of the conventional summation operator, Σ. It is defined in terms of the Boolean transformation matrices, presented above. The assembly of a class of element matrices, $[x^e]$, or a class of element vectors, $\{y^e\}$, is defined as:

$$\underset{e = a, b, \ldots}{\mathbf{A}} [x^e] \equiv \sum_{e = a, b, \ldots} [B^e]^T [x^e][B^e] \quad \text{and} \quad \underset{e = a, b, \ldots}{\mathbf{A}} \{y^e\} \equiv \sum_{e = a, b, \ldots} [B^e]^T \{y^e\} \tag{9}$$

The assembly operation, as represented formally above, is a direct result of demanding conservation at each system node and provides a mathematically rigorous definition that is useful for theoretical analysis and development. It defines, however, a computationally inefficient strategy for assembly, therefore, more direct computational algorithms are used in practice (8, 12).

To apply this system of equations to the solution of practical problems, prescribed temperature conditions and the possibility of zero capacitance system nodes must be accounted for. When this is done, one is left with a reduced set of equations of the same form as Equation 8, hence, we shall simply consider operations with this equation and not consider these details here. Consideration of temperature boundary conditions and zero capacitance nodes become, however, key issues when considering computational strategies for implementing this approach.

The system conductance matrix, $[\mathbf{K}]$, being an element assembly sum of element matrices will, in general, be nonsymmetric and nonlinear (i.e., due to flow element contributions) $[\mathbf{K}] = [\mathbf{K}(\{\mathbf{T}\})]$. It may be shown however, that $[\mathbf{K}]$ will be a nonsingular M-matrix that may be factored by LU decomposition without pivoting when one or more prescribed temperature boundary conditions have been imposed (see (6) for an analysis of a similar set of equations). This fact may be used to develop efficient computational strategies to solve these equations.

Steady Airflow Analysis

Macroscopic approaches to steady airflow analysis, based upon idealizing building airflow systems by collections of well-mixed zones linked by discrete airflow paths, have been developed by several groups (see (14, 17, 18) for a review of these models). These *multi-zone airflow network models* share a close relationship to water piping network analysis models (16). In these models, airflow is, most commonly, described by power-law *pressure-flow models*, wind pressures are modeled via pressure coefficients, and the dynamic pressure variation of the wind and buoyant affects are accounted for. The authors (15) and Walton (21) have shown that these models can be reformulated on an element assembly basis so that multiple pressure-flow models may be considered in a single system model. A variety of flow elements have been introduced that may be used to model flow within both the building construction and through the HVAC system.

The general features of the element assembly approach follow that presented above for thermal analysis; the building airflow system is idealized by assemblages of flow elements that model the pressure flow characteristics of discrete flow paths in the building/HVAC system. Flow element equations are formulated and, for each specific system idealization, element equations are assembled to form the system equations.

Although there is much work to be done to refine existing flow element models and to develop additional ones, the basic procedure to do so is in hand. These element equations are based on the Bernoulli equation for incompressible flow between an entry, subscript "1," and an exit, subscript "2," of a flow path:

$$\Delta P_{loss} = (P_1 + \rho\bar{v}_1^2/2) - (P_2 + \rho\bar{v}_2^2/2) + \rho g(z_1 - z_2) \qquad (10)$$

where ρ is the density of air in the flow path (assumed constant), $\bar{v}$ is the mean or bulk fluid velocity, g is the acceleration of gravity, and z is the vertical height from an arbitrary datum. These and the other element variables used in the expressions below are illustrated in Figure 3.

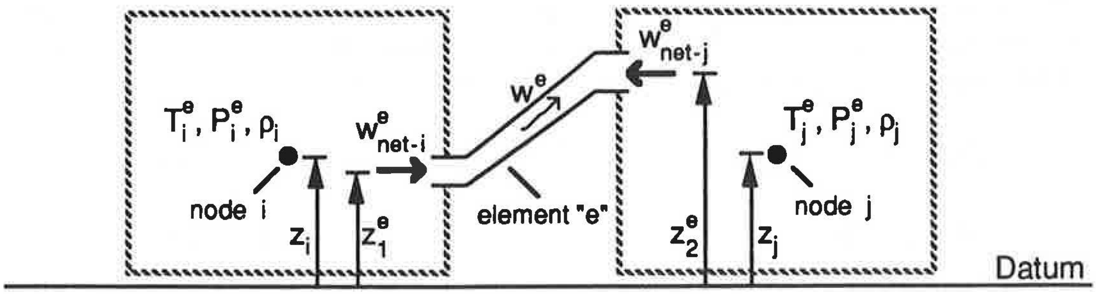

Fig. 3 Representative two-node flow element variables.

The Bernoulli equations are then complemented by one of several *pressure–flow models* that relate the air mass flow rate, $w^e = \rho A\bar{v}$, with A the cross-sectional area, to the frictional and dynamic losses, ΔP^e_{loss}, for the class of flow path being considered. Examples include:

Power-Law Correlations : used for complete constructions (e.g., walls or window units).

$$w^e = \begin{cases} \left\{C\rho^{1/2}\right\}\left\{\dfrac{1}{\left|\Delta P^e_{loss}\right|^{1-n}}\right\}\Delta P^e_{loss} & ; \ \textit{transition / turbulent flow} \\ \text{generally not available} & ; \ \textit{creeping / laminar flow} \end{cases} \tag{11}$$

where C and n are correlation constants (typically $0.5 \le n \le 1.0$).

Orifice Equations : used for openings from cracks to doorways (9).

$$w^e = \begin{cases} \left\{C_d A_o\left(\dfrac{2\rho}{1-(A_o/A)^2}\right)^{1/2}\right\}\left\{\dfrac{1}{\left|\Delta P^e_{loss}\right|^{1/2}}\right\}\Delta P^e_{loss} & ; \ \textit{transition / turbulent flow} \\ \left\{k^2\left(\dfrac{2\rho A_o D_o}{1-(A_o/A)^2}\right)\right\}\Delta P^e_{loss} & ; \ \textit{creeping / laminar flow} \end{cases} \tag{12}$$

where C_d is a coefficient correlated with the geometry of the flow path and the flow Reynolds number, A_o and A are cross-sectional areas of the orifice and the flow path, respectively, D_o is the diameter of the orifice, and k is a constant.

Duct Equations : for modeling flow in HVAC system ductwork (9).

$$w^e = \begin{cases} \left\{\left(\dfrac{2\rho A^2}{C_T}\right)^{1/2}\right\}\left\{\dfrac{1}{\left|\Delta P^e_{loss}\right|^{1/2}}\right\}\Delta P^e_{loss} & ; \ \textit{transition / turbulent flow} \\ \left(\dfrac{2\rho A D}{C_L \mu}\right)\Delta P^e_{loss} & ; \ \textit{creeping / laminar flow} \end{cases} \tag{13}$$

where C_T is a coefficient correlated with the geometry of the flow path and the flow Reynolds number, C_L is a constant, A and D are the cross-sectional area and (hydraulic) diameter of the duct flow path and μ is the viscosity of the air in the flow path.

To emphasize the nonlinearity of the transitional-to-turbulent flow expressions and the linearity of the creeping-to-laminar flow expressions, these representative correlations have been written in the form of linear relations, between mass flow rate and pressure loss as:

$$w^e = \begin{cases} \{C_{1T}\}\{C_{2T}\}\,\Delta P^e_{loss} \;;\; C_{2T} = 1/|\Delta P^e_{loss}|^{1-n} & ;\ \textit{transition / turbulent flow} \\ \{C_{1L}\}\,\Delta P^e_{loss} & ;\ \textit{creeping / laminar flow} \end{cases} \tag{14}$$

The leading coefficients, C_{1T} and C_{1L}, are more or less independent of pressure loss, depending primarily upon the specific geometry of the flow path and secondarily on flow intensity, while the coefficient C_{2T} isolates the primary source of nonlinearity ($n = 1/2$ or $1/2 \leq n < 1.0$). In addition to these relations, crack correlations (7, 13), expressions for flow through large openings (20, 21, 23), and fan pressure-flow models (4, 21) have been formulated.

Finally, the entry and exit pressures, P_1 and P_2, are related to the element state pressure variables, assuming hydrostatic conditions exist in each zone (e.g., with reference to Fig. 3: $P_1 \approx P^e_i + \rho_i g(Z_i - Z^e_1)$). The resulting element equations will have the general form:

$$\{w^e_{net}\} = [a^e]\Big(\{P^e\} + \{P^e_B\}\Big) + \{w^e_o\} \tag{15}$$

where $\{w^e_{net}\}$ is the vector of air mass flow rates (representing the mass flow rates from each of the element's nodes into the element,) $[a^e]$ is the *element pressure-flow coefficient matrix*, $\{w^e_o\}$ is a vector of zero-ΔP air mass flow rate terms (e.g., the free-delivery mass flow rate for fans), and $\{P^e_B\}$ is a vector of buoyancy-induced pressure terms dependent on air densities associated with element nodes. For a two-node flow element:

$$\{P^e_B\} = g\,[z^e]\begin{Bmatrix} \rho^e_i \\ \rho^e_j \end{Bmatrix} \;;\; [z^e] = \begin{bmatrix} (z_i - z^e_k) & 0 \\ 0 & (z_j - z^e_k) \end{bmatrix} \;;\; \begin{matrix} k = 2 \text{ for flow from i to j} \\ k = 1 \text{ for flow from j to i} \end{matrix} \tag{16}$$

For a two-node flow element, say element e connecting nodes i and j, based on the simplified pressure-flow model defined by Equation 14, the element pressure flow matrix would be:

$$[a^e] = a^e\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \;;\; a^e = \begin{cases} \{C_{1T}\}\left\{\dfrac{1}{|(P^e_i - P^e_j) + (P^e_{Bi} - P^e_{Bj})|^{1-n}}\right\} & ;\ \textit{transitional / turbulent flow} \\ \{C_{1L}\} & ;\ \textit{creeping / laminar flow} \end{cases} \tag{17}$$

For transitional-to-turbulent flow the element pressure flow matrix will, in general, be nonlinear, $[a^e] = [a^e(\{P^e\}, \{P^e_B\})]$, while for creeping-to-laminar flow it should be expected to be linear.

Demanding the conservation of mass flow at each of the system nodes, element equations corresponding to a specific system idealization may be *assembled* to form *system equations* that govern the behavior of the system as a whole:

$$\boxed{\{W\} = [A]\{P\} + \{W_B\} + \{W_o\}} \tag{18}$$

where

$$[A] = \mathop{\mathbf{A}}_{e=a,b,\ldots} [a^e] \; ; \qquad \{W_B\} = \mathop{\mathbf{A}}_{e=a,b,\ldots} [a^e]\{P_B^e\} \; ; \qquad \{W_o\} = \mathop{\mathbf{A}}_{e=a,b,\ldots} \{w_o^e\}$$

$\{W\}$ is a vector of the direct generation rates of air mass at each of the systems nodes. It is reasonably assumed to be a zero vector for the usual cases of building thermal or indoor air quality analysis. For building fire analysis, on the other hand, this vector will be non-zero.

The airflow equations, Equation 18, may be solved by a variety of methods (i.e., to determine the system pressure vector, $\{P\}$) although variants of the Newton-Raphson method appear to be most effective (21). The Newton-Raphson method is an iterative scheme based upon Taylor's expansion of Equation 18 written in residual form:

$$\{R(\{P\})\} \equiv [A]\{P\} + \{W_B\} + \{W_o\} - \{W\} = \{0\} \tag{19}$$

that leads to the following iterative algorithm:

$$\left[\frac{\partial\{R(\{P\})\}}{\partial\{P\}} \bigg|_{\{P\}^k} \right] \{\Delta P\}^{k+1} = -\{R(\{P\}^k)\} \tag{20a}$$

$$\{P\}^{k+1} = \{P\}^k + \{\Delta P\}^{k+1} \tag{20b}$$

With an initial estimate of the system pressure vector, $\{P\}^k$, one forms and solves Equation 20a to obtain $\{\Delta P\}^k$, which is then substituted into Equation 20b to obtain a better estimate of the system pressure vector, $\{P\}^{k+1}$. This process is repeated until the system pressure estimates converge. Element flow rates can then be determined from the element equations using the solution for the system pressure vector. The solution of Equation 20a will require the specification of one nodal pressure, typically the outside air node pressure, or, for those cases where the system is composed of uncoupled groups of zones, a single node pressure must be specified for each group.

The square matrix on the left-hand side of Equation 20a is known as the *system Jacobian*. It follows from Equation 18 that the Jacobian may be directly assembled from the element expressions for the partial derivatives $\partial\{w^e_{net}\}/\partial\{P^e\}$ as:

$$\left[\frac{\partial\{R(\{P\})\}}{\partial\{P\}} \bigg|_{\{P\}^k} \right] = \mathop{\mathbf{A}}_{e=a,b,\ldots} \left[\partial\{w^e_{net}\}/\partial\{P^e\} \big|_{\{P\}^k} \right] + \left[\frac{\partial\{W\}}{\partial\{P\}} \bigg|_{\{P\}^k} \right] \tag{21}$$

(The last term on the right-hand side may be ignored when $\{W\}$ is the zero vector.) For the simplified flow resistance elements presented above (i.e., Equations 14 and 17) the *element Jacobian*, $\partial\{w^e_{net}\}/\partial\{P^e\}$, has a particularly simple form:

$$\left[\frac{\partial\{w^e_{net}\}}{\partial\{P^e\}} \right] = \begin{cases} \left(\dfrac{n w^e}{((P_i^e - P_j^e) + (P_{Bi}^e - P_{Bj}^e))} \right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & ;\ \textit{transitional / turbulent flow} \\ C_{1L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & ;\ \textit{creeping / laminar flow} \end{cases} \tag{22}$$

It is important to note that the transitional-to-turbulent flow expressions lead to practically unbounded terms for near-zero flows (i.e., when $((P_i^e - P_j^e) + (P_{Bi}^g - P_{Bj}^g)) \approx 0$). These will, when assembled, lead to practically unbounded terms in the system Jacobian and, as a result, slow the convergence of the solution procedure or, in extreme cases, lead to nonconvergence. By employing the physically consistent creeping-to-laminar expressions for low-flow conditions, this type of convergence problem will be avoided (4, 21). For flow through building constructions described by power-law correlations that may not apply to creeping-to-laminar flow regimes one may reasonably assume orifice behavior at low flow conditions to achieve this end.

Coupled Airflow-Thermal Analysis

The thermal equations, Equations 8, may be used to determine the thermal response of a building system to an arbitrary thermal excitation. If, however, a given building thermal idealization includes flow elements it will be necessary to complement these equations with equations to determine the airflows in the building system as they vary with time. If it can be assumed that the airflows in the discrete flow paths considered are not changing rapidly and remain practically isothermal with density constant (i.e., the assumptions underlying the Bernoulli equations used to formulate the flow element equations remain valid) then one may reasonably use the steady airflow equations, Equation 18, for this determination. Recognizing, however, that the Bernoulli conditions may be satisfied in the discrete flow paths yet bulk density within each well-mixed zone may vary with time, it becomes necessary to account for the rate of change of mass due to these density variations by adding an accumulation term, $[\mathbf{V}]d\{\rho\}/dt$, to Equation 18 as:

$$[\mathbf{A}]\{\mathbf{P}\} + [\mathbf{V}]\frac{d\{\rho\}}{dt} = \{\mathbf{W}\} - \{\mathbf{W_B}\} - \{\mathbf{W_o}\} \tag{23}$$

where $[\mathbf{V}]$ is a diagonal matrix of zone volumes, assumed constant, and $\{\rho\}$ is a vector of zone densities. Assuming ideal gas behavior in each zone i, $\rho_i = P_i/RT_i$ (R is the gas constant and T is the absolute temperature of the air in the zone) this accumulation term may be expanded to yield:

$$[\mathbf{A}]\{\mathbf{P}\} + [\mathbf{M_P}]\frac{d\{\mathbf{P}\}}{dt} + [\mathbf{M_T}]\frac{d\{\mathbf{T}\}}{dt} = \{\widehat{\mathbf{W}}\} \tag{24}$$

where $[\mathbf{M_P}]$ is a diagonal matrix with terms $M_{P\text{-}i,i} = V_i/RT_i$, $[\mathbf{M_T}]$ is a diagonal matrix with terms $M_{T\text{-}i,i} = V_iP_i/RT_i^2$., and $\{\widehat{\mathbf{W}}\} \equiv \{\mathbf{W}\} - \{\mathbf{W_B}\} - \{\mathbf{W_o}\}$.

We may then combine these quasi-dynamic air flow equations with the dynamic thermal equations, Equations 8, to describe the coupled analysis problem:

$$\begin{bmatrix} [\mathbf{A}] & [0] \\ [0] & [\mathbf{K}] \end{bmatrix} \begin{Bmatrix} \{\mathbf{P}\} \\ \{\mathbf{T}\} \end{Bmatrix} + \begin{bmatrix} [\mathbf{M_P}] & [\mathbf{M_T}] \\ [0] & [\mathbf{C}] \end{bmatrix} \begin{Bmatrix} \dfrac{d\{\mathbf{P}\}}{dt} \\ \dfrac{d\{\mathbf{T}\}}{dt} \end{Bmatrix} = \begin{Bmatrix} \{\widehat{\mathbf{W}}\} \\ \{\mathbf{E}\} \end{Bmatrix} \tag{25}$$

Considering realistic numerical values for these and the other terms of Equation 25, however, it appears that the contribution of the pressure-related accumulation terms, $[\mathbf{M_P}]d\{\mathbf{P}\}/dt$, will be negligible and the contribution of the temperature-related accumulation terms, $[\mathbf{M_T}]d\{\mathbf{T}\}/dt$, are likely to be small in comparison to the uncertainties in the air mass flow rates terms $[\mathbf{A}]\{\mathbf{P}\}$, thus we shall ignore these contributions and describe the coupled problem with the following equations:

$$\begin{bmatrix} [A] & [0] \\ [0] & [K] \end{bmatrix}\begin{Bmatrix} \{P\} \\ \{T\} \end{Bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & [C] \end{bmatrix}\begin{Bmatrix} \frac{d\{P\}}{dt} \\ \frac{d\{T\}}{dt} \end{Bmatrix} = \begin{Bmatrix} \{\widehat{W}\} \\ \{E\} \end{Bmatrix} \tag{26}$$

It will be appropriate, however, to evaluate these contributions in future numerical experiments to better determine their importance and, thus evaluate the validity of this simplification.

Depending on the nature of the thermal excitation and the nature of the building system being studied, a variety of solution options for the coupled equations, Equation 26, may be considered. These include a) steady linear and nonlinear analysis, b) steady linear harmonic analysis, and c) dynamic linear and nonlinear analysis.

Under steady excitation the derivative terms vanish thus the steady problem is defined as:

$$\begin{bmatrix} [A] & [0] \\ [0] & [K] \end{bmatrix}\begin{Bmatrix} \{P\} \\ \{T\} \end{Bmatrix} = \begin{Bmatrix} \{\widehat{W}\} \\ \{E\} \end{Bmatrix} \tag{27}$$

As written, these equations appear to be uncoupled but, in fact, for the (usual) nonlinear case these equations are implicitly coupled through the dependency of both block diagonal matrices on both temperature and pressure through the flow element equations (i.e., $[A] = [A(\{P\}, \{T\})]$ and $[K] = [K(\{P\}, \{T\})]$). These steady coupled flow equations may be solved using the Newton-Raphson approach discussed above for steady flow analysis alone.

The full dynamic problem defined by Equation 26 (i.e., after accounting for temperature and pressure-prescribed boundary conditions) may be solved numerically using one of several finite difference schemes. A general semi-implicit method has been employed by the authors for the solution of the linear thermal problem (5) and is presently under investigation for solution of the nonlinear coupled airflow/thermal problem. This method employs the difference approximation:

$$\begin{Bmatrix} \{P\}_{n+1} \\ \{T\}_{n+1} \end{Bmatrix} \approx \begin{Bmatrix} \{P\}_{n} \\ \{T\}_{n} \end{Bmatrix} + (1-\alpha)\delta t \begin{Bmatrix} \{dP/dt\}_{n} \\ \{dT/dt\}_{n} \end{Bmatrix} + \alpha\delta t \begin{Bmatrix} \{dP/dt\}_{n+1} \\ \{dT/dt\}_{n+1} \end{Bmatrix} \tag{28}$$

where $0 \leq \alpha \leq 1$, the time domain has been divided into discrete steps, $t_{n+1} \equiv t_n + \delta t$, and an abbreviated notation has been introduced : $\{P\}_n \equiv \{P(t_n)\}$ and $\{dP/dt\}_n \equiv (d\{P\}/dt)|_{t_n}$. (Note that; $\alpha = 0$ corresponds to the Forward Difference Scheme; $\alpha = 1/2$ the Crank-Nicholson scheme; $\alpha = 2/3$ the Galerkin scheme; and $\alpha = 1$ the Backward Difference Scheme.)

Substituting Equation 28 into Equation 26 leads to the following time-stepping algorithm:

$$\begin{bmatrix} [A]_{n+1} & [0] \\ [0] & [\widehat{K}]_{n+1} \end{bmatrix}\begin{Bmatrix} \{P\}_{n+1} \\ \{T\}_{n+1} \end{Bmatrix} = \begin{Bmatrix} \{\widehat{W}\}_{n+1} \\ \{\widehat{E}\}_{n+1} \end{Bmatrix} \tag{29a}$$

where:

$$[\widehat{K}] \equiv \left[\alpha\delta t[K] + [C]\right] \equiv \textit{the dynamic conductance matrix} \tag{29b}$$

$$\{\widehat{E}\}_{n+1} \equiv \alpha\delta t\{E\}_{n+1} + (1-\alpha)\delta t\{E\}_n + [C]\{T\}_n - (1-\alpha)\delta t[K]\{T\}_n \tag{29c}$$

This algorithm is self-starting (i.e., given initial conditions the right-hand side of Equation 29a is determined) and is implicitly nonlinear due to the dependency of both $[\mathbf{A}]$ and $[\widehat{\mathbf{K}}]$ (or $[\mathbf{K}]$) on $\{\mathbf{P}\}$ and $\{\mathbf{T}\}$. In those cases when this nonlinear dependency can be ignored the algorithm will be unconditionally stable for $\alpha \geq 1/2$ (12).

With a given initial system state vector specified, $\{\ \{\mathbf{P}\}_0\ \ \{\mathbf{T}\}_0\ \}^T$, Equation 29 may be solved to determine the system state vector vector at the next time step, $\{\ \{\mathbf{P}\}_1\ \ \{\mathbf{T}\}_1\ \}^T$. Repeating this process, in a step-wise manner, provides an approximate solution for the response of the building system, $\{\ \{\mathbf{P}(t)\}\ \ \{\mathbf{T}(t)\}\ \}^T$, to an arbitrary system excitation, $\{\ \{\widehat{\mathbf{W}}(t)\}\ \ \{\widehat{\mathbf{E}}(t)\}\ \}^T$. System air flows and heat transfer quantities may, at any time step, be directly determined from this response using the appropriate element equations.

The matrix of flow coefficients, $[\mathbf{A}]_{n+1}$, will, in general, be dependent on the system pressure vector and the system temperature vector: $[\mathbf{A}]_{n+1} = \left[\mathbf{A}\left(\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T\right)\right]$. The dependency on the system pressure vector was discussed above. The dependency on the system temperature vector results from the dependency of the flow coefficients on air density which, in turn is dependent on nodal temperatures.

The *dynamic system conductance matrix*, $[\widehat{\mathbf{K}}]_{n+1}$, will also be dependent on the system pressure and temperature vectors when the thermal system includes flow elements. Thermal flow elements depend on the flow through the elements which, in turn, will be dependent on nodal pressures and temperatures, as above: $[\widehat{\mathbf{K}}]_{n+1} = \left[\widehat{\mathbf{K}}\left(\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T\right)\right]$.

One may approximate a solution to Equation 29 using the Newton-Raphson method discussed above for isothermal steady flow analysis. Expanding and rewriting Equation 29 in residual form:

$$\left\{\mathbf{R}\left(\left\{\begin{matrix}\{\mathbf{P}\}_{n+1}\\ \{\mathbf{T}\}_{n+1}\end{matrix}\right\}\right)\right\} = \left\{\begin{matrix}[\mathbf{A}]_{n+1}\{\mathbf{P}\}_{n+1} - \{\widehat{\mathbf{W}}\}_{n+1}\\ [\widehat{\mathbf{K}}]_{n+1}\{\mathbf{T}\}_{n+1} - \{\widehat{\mathbf{E}}\}_{n+1}\end{matrix}\right\} \tag{30}$$

the Newton-Raphson method may be directly represented by the following iterative algorithm:

- For time step t_{n+1}
- Step 1: Set initial estimate equal to solution from previous time step;

$$\left\{\{\mathbf{P}\}^1_{n+1} \mid \{\mathbf{T}\}^1_{n+1}\right\}^T = \left\{\{\mathbf{P}\}_n \mid \{\mathbf{T}\}_n\right\}^T$$

- Step 2: Nonlinear Iteration
  - Initialize iteration counter; $k = 0$
  - Until convergence is realized repeat Steps 2.1, 2.2, and 2.3:
  - Step 2.1: Increment iteration counter; $k = k + 1$

    Form the *coupled system Jacobian*:

$$[\mathbf{J}]^k_{n+1} \equiv \left[\left.\frac{\partial \mathbf{R}\left(\left\{\begin{matrix}\{\mathbf{P}\}_{n+1}\\ \{\mathbf{T}\}_{n+1}\end{matrix}\right\}\right)}{\partial\left\{\begin{matrix}\{\mathbf{P}\}_{n+1}\\ \{\mathbf{T}\}_{n+1}\end{matrix}\right\}}\right|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right]$$

• Step 2.2: Solve (using Gauss Elimination or variant):

$$[J]^k_{n+1} \begin{Bmatrix} \{\Delta P\}^{k+1}_{n+1} \\ \{\Delta T\}^{k+1}_{n+1} \end{Bmatrix} = -\left\{ R\left(\begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix} \right) \right\}$$

• Step 2.3: Update

$$\begin{Bmatrix} \{P\}^{k+1}_{n+1} \\ \{T\}^{k+1}_{n+1} \end{Bmatrix} = \begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix} + \begin{Bmatrix} \{\Delta P\}^{k+1}_{n+1} \\ \{\Delta T\}^{k+1}_{n+1} \end{Bmatrix}$$

• Step 3: Report solution for time step as;

$$\left(\{P\}_{n+1} \mid \{T\}_{n+1}\right)^T = \left(\{P\}^{k+1}_{n+1} \mid \{T\}^{k+1}_{n+1}\right)^T$$

• Continue to next time step.

Convergence evaluation may be based upon system pressures and temperatures, element mass flow rates, or a combination of these.

From Equation 30 it follows that the coupled system Jacobian matrix, $[J]^k_{n+1}$, consists of the following four submatrices:

$$[J]^k_{n+1} \equiv \begin{bmatrix} \left.\dfrac{\partial\left([A]_{n+1}\{P\}_{n+1} - \{\widehat{W}\}_{n+1}\right)}{\partial\{P\}_{n+1}}\right|_{\begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix}} & \left.\dfrac{\partial\left([A]_{n+1}\{P\}_{n+1} - \{\widehat{W}\}_{n+1}\right)}{\partial\{T\}_{n+1}}\right|_{\begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix}} \\ \left.\dfrac{\partial\left([\widehat{K}]_{n+1}\{T\}_{n+1} - \{\widehat{E}\}_{n+1}\right)}{\partial\{P\}_{n+1}}\right|_{\begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix}} & \left.\dfrac{\partial\left([\widehat{K}]_{n+1}\{T\}_{n+1} - \{\widehat{E}\}_{n+1}\right)}{\partial\{T\}_{n+1}}\right|_{\begin{Bmatrix} \{P\}^k_{n+1} \\ \{T\}^k_{n+1} \end{Bmatrix}} \end{bmatrix} \quad (31)$$

The upper left submatrix is seen to be identical to the steady flow system Jacobian (Equations 19 & 20a) and may be directly assembled from element contributions as before, Equation 21. The upper right submatrix <u>may</u> also be assembled from element contributions where now the element contributions may be approximated as:

$$\left[\frac{\partial\{w^e_{net}\}}{\partial\{T^e\}}\right] = \frac{\partial}{\partial\{T^e\}}\left([a^e](\{P^e\} + \{P^e_B\}) + \{w^e_B\}\right) \approx [a^e]\frac{\partial\{P^e_B\}}{\partial\{T^e\}} \quad (32)$$

Here, by analogy with the Boussinesq assumption, we consider only the temperature dependency of the buoyancy terms and ignore the dependency of the flow coefficient terms in $[a^e]$ on air density and, hence, temperature. Using the definition of $\{P^e_B\}$, Equation 16, and the ideal gas law $\rho_i = P_i/RT_i$ we obtain, for two-node flow elements:

$$\left[\frac{\partial\{w^e_{net}\}}{\partial\{T^e\}}\right] = -\left(\frac{g}{R}\right)[a^e][z^e]\begin{bmatrix} P^e_i/(T^e_i)^2 & 0 \\ 0 & P^e_j/(T^e_j)^2 \end{bmatrix} \quad (33)$$

where, again T is the absolute temperature at the respective zone nodes. In a similar manner, the other two submatrices may be assembled from the individual thermal element contributions.

In some situations the off-diagonal submatrices of the coupled system Jacobian, Equation 31, may be small relative to the diagonal submatrices. The upper-right submatrix, for example, would become the zero matrix for a flow idealization consisting of horizontal flow paths connecting zones and the lower-left submatrix may prove insignificant if, for example, heat transfer by flow is either dominated by forced airflow or insignificant relative to other modes of heat transfer. For either of these last two conditions, the lower-right submatrix could, reasonably, be approximated as:

$$\frac{\partial\{[\hat{K}]_{n+1}\{T\}_{n+1} - \{\hat{E}\}_{n+1}\}}{\partial\{T\}_{n+1}} \approx [\hat{K}]_{n+1} \tag{34}$$

with the result that the coupled system Jacobian becomes a block diagonal matrix:

$$[J]^k_{n+1} \equiv \begin{bmatrix} \left.\dfrac{\partial\{[A]_{n+1}\{P\}_{n+1} - \{\hat{W}\}_{n+1}\}}{\partial\{P\}_{n+1}}\right| \begin{Bmatrix}\{P\}^k_{n+1} \\ \{T\}^k_{n+1}\end{Bmatrix} & [0] \\ [0] & [\hat{K}]_{n+1}\Big| \begin{Bmatrix}\{P\}^k_{n+1} \\ \{T\}^k_{n+1}\end{Bmatrix} \end{bmatrix} \tag{31}$$

With this simplified Jacobian the solution strategy outlined above becomes an iterative process of solving the nonlinear flow problem practically identical to that defined by Equations 20, then solving a linearized thermal problem. This process is equivalent to the second of two solution options proposed by Walton (22). If small time step increments, δt, are employed one may, possibly, find that iteration is unnecessary and choose to avoid it altogether. Employing the simplified Jacobian in the solution strategy outlined above without iteration is equivalent to the first of the two options proposed by Walton and the approach employed by Clarke (11). By explicitly considering the nonlinear dependency of the flow and thermal problems the proposed solution strategy outlined above (i.e., including the diagonal submatrices) may be considered to provide a more general approach to the problem.

Conclusion

The theoretical bases of a steady building airflow analysis technique and a dynamic building thermal analysis technique have been reviewed and an approach to integrate these methods to solve problems of coupled airflow and thermal analysis has been outlined. These techniques and are based on an element assembly approach that allows the analyst to consider a practically unlimited variety of system idealizations of arbitrary complexity and leads to highly modular computer programs facilitating development and future changes.

The integrated approach may be applied to the solution of both steady state and dynamic problems. The steady airflow equations are adapted to formulate a system of equations that describe the dynamic airflow analysis problem. These equations are then integrated with the dynamic thermal analysis equations to form a system of equations that describe the coupled problem. It is argued that key terms of the dynamic airflow equations are likely to be insignificant and by ignoring these terms a quasi-dynamic approach is formulated to solve the dynamic coupled airflow/thermal analysis problem. Although, this simplification may be justified when airflows are not changing rapidly within the building system this assumption must be critically evaluated.

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