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Numerical simulation of room-airflow with inclined outlet

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SUMMARY

We describe the calculation methods of seven finite different schemes :(1)CDS,(2)UDS,(3)Power-Low,(4)SUDS,(5)SUWDS,(6)QUICK and (7)Pseudo-Spectral. And we calculated the transport of a nondimensional scalar value with them, assuming a uniform velocity field. We found from the calculation results that (1)when the flow is inclined to the coordinates, UDS and Power-Low have poor accuracy because of the numerical diffusion but SUDS,QUICK and Pseudo-Spectral are well coincided with the exact solution ,and (2)when the cell Pecret number is large, QUICK and Pseudo-Spectral occur the oscillation ,and (3)for the three dimensional case, SUDS,QUICK and Pseudo-Spectral are roughly coincided with the exact solution.

We calculated the actual room airflow and the temperature distribution using $k-\varepsilon$ model with Power-Low and SUDS. The calculation results indicate that SUDS restrains the numrical diffusin relatively than Power-Low. But we can't compare the calculations with the measurements as this atrium is now under the construction.

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1. Introduction

The finite difference schemes are often used to solve the complex problems including fluid flow, heat and mass transport. However it is very difficult to apply them to the room airflow with reasonable accuracy and cost.

As well known, Upstream Differencing Scheme (UDS) is very stable for the numerical calculation but poor in accurate because it generates the numerical diffusion. On the other hand, Central Differencing Scheme (CDS) is relatively accurate but not stable.

Many investigators have developed the finite difference schemes to improve the above problems. Several new schemes were proposed by S.V.Patanker, G.D.Raithby, B.P.Leonard, P.J.Roache and so on.

In this paper, we consider seven different schemes as follows

(1)CDS
(2)UDS
(3)Power-Low Scheme (Power-Low)[1]
(4)Skew Upstream Differencing Scheme (SUDS)[2]
(5)Skew Upstream Weighted Differencing Scheme (SUWDS)[2]
(6)Quadratic Upstream Interpolation for Convective Kinematic (QUICK)[3]
(7)The Pseudo-Spectral Scheme (Pseudo-Spectral)[4]

Power-Low was developed by S.V.Patanker. For the large cell Peclet number, this scheme improves the Hybrid scheme.[1]

SUDS and SUWDS for the two-dimensional problems were developed by G.D. Raithby. These schemes improve UDS for the case where the flow is inclined to the coordinates. SUDS is the method used for the case that the convection dominates more than the diffusion. SUWDS is the method used for the case that the convection as well as the diffusion dominate in the region.

QUICK was developed by B.P.Leonard.This scheme improves instability of CDS.

Pseudo-Spectral was developed by H.Wengle.This scheme uses Fourier transform for the spatial derivative and the forwards difference for the temporal term.

At first, we argue the calculation methods of seven finite difference schemes and apply these methods to a simple problem. And we describe the difference and characteristics of these schemes.

Next, we calculate the actual room airflow and the temperature distribution using $k-\varepsilon$ model with Power-Low and SUDS. Then we describe the difference and characteristics between Power-Low and SUDS.

2. The comparison of the finite difference schemes

2-1. The problem to be tested

We calculate the transport of ϕ, assuming a uniform velocity field. Here ϕ is a nondimensional scalar value. The calculation are carried out in three case shown in the figure1, 2 and 3, where each velocity has the different

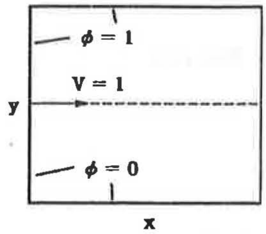


FIG1 The condition for the case(a)

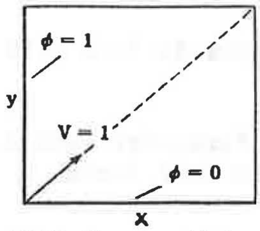


FIG2 The condition for the case(b)

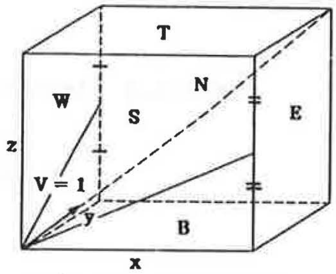


FIG3 The condition for the case(c)

angle one another.

In the case(a) shown in the figure**1**, the velocity is parallel to x axis everywhere in the region. At the center on the left boundary, $\phi = 0.5$. Avobe this center, the boundary value $\phi = 1$ and below $\phi = 0$. On the right boundary, a value of ϕ is calculated by the linear extrapolation.

In the case(b) shown in the figure**2**, the direction of the velocity makes the angle of $\pi / 4$ to x axis, where $u = v = 1/\sqrt{2}$ ($V = \sqrt{u^2 + v^2} = 1$). On the left boundary, $\phi = 1$ and on the down, $\phi = 0$. On the left down corner, $\phi = 0.5$. On the right and upper boundary, ϕ is calculated by the linear extrapolation.

In the case(a) and (b), the conservative equation for ϕ is written by

$$\frac{\partial \phi}{\partial t} = -u\frac{\partial \phi}{\partial x} - v\frac{\partial \phi}{\partial y} + \Gamma\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \tag{1}$$

where there is no source terms.

In the case(c) shown in the figure**3**, the velocity vector in the case(b) is additionally inclined α in z direction, where $\tan\alpha = 1/\sqrt{2}$, and $u = v = w = 1/\sqrt{3}$ ($V = \sqrt{u^2 + v^2 + w^2} = 1$). On the boundary B, $\phi = 0$. On the boundary W and S, where the solid lines go across, $\phi = 0$ below these lines and $\phi = 0$ above and $\phi = 0.5$ on these lines. On the boundary E, S and T, a value of ϕ is calculated by the linear extrapolation. In this case the conservative equation for ϕ is written by

$$\frac{\partial \phi}{\partial t} = -u\frac{\partial \phi}{\partial x} - v\frac{\partial \phi}{\partial y} - w\frac{\partial \phi}{\partial z} + \Gamma\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) \tag{2}$$

2-2 The calculation method for Eq(1)

①CDS, UDS and Power-Low[1]

We consider the control volume shown in the figure**4**. We integrate Eq(1) and the continuity equation over the control volume and arrange these equations. Then we can obtain the two-dimensional discretization equations as

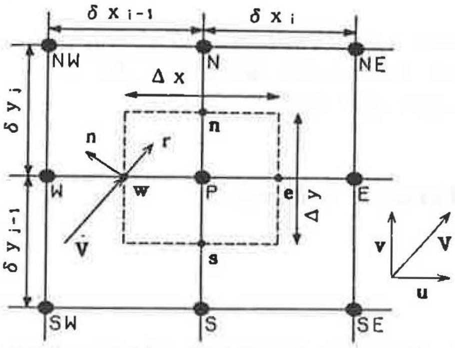


FIG4 The control volume for two dimension

$$\phi_P a_P = \phi_E a_E + \phi_W a_W + \phi_N a_N + \phi_S a_S + \frac{\Delta x \Delta y}{\Delta t}\phi_P^{\,n} \tag{3}$$

where

$$a_E = D_e A(|P_e|) + \max(-F_e, 0)$$
$$a_W = D_w A(|P_w|) + \max(F_w, 0)$$
$$a_N = D_n A(|P_n|) + \max(-F_n, 0)$$
$$a_S = D_s A(|P_s|) + \max(F_s, 0)$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta x \Delta y}{\Delta t} \tag{4}$$

$$F_w = u_w \Delta y \quad D_w = \frac{\Gamma_w \Delta y}{\delta x_i} \quad P_w = \frac{F_w}{D_w}$$

The function $A(|P|)$ is expressed for CDS, UDS and Power-Low as follows

$$A(|P|)=1-0.5|P| \quad \text{for } \textbf{CDS} \tag{5}$$
$$A(|P|)=1 \quad \text{for } \textbf{UDS} \tag{6}$$
$$A(|P|)=\max(0,\ (1-0.1|P|^5) \quad \text{for } \textbf{Power Low} \tag{7}$$

②SUDS

When the velocity V in the vicinity of the western face of the control volume is inclined to x axis shown in the figure**4**, we assume that the ϕ profile for the convective flux is expressed as[2]

$$\phi_w = C_1 + C_2 n = C_1 + C_2\left(y'\frac{u_w}{V_w} - x'\frac{v_w}{V_w}\right) \tag{8}$$

where n is the normal distance to the flow and x' and y' are measured from w. The constants C_1 and C_2 are determined from two values of the upstream grid point. The one is P or W and the other is either of four grid points NW, SW, N and S. We can select two grid points from the sign of u and v. These conditions are written by

$$\phi = \phi_{lw,j} \quad \text{at} \quad x' = -S_{uw}\delta x_i/2 \quad y' = 0$$
$$\phi = \phi_{lw,mw} \quad \text{at} \quad x' = -S_{uw}\delta x_i/2 \quad y' = -S_{uw}\delta y_{kw} \tag{9}$$

where

$$lw = i - (1+S_{uw})/2$$
$$mw = j - S_{vw}, \quad kw = j + (1-S_{vw})/2 \tag{10}$$

S_{uw} has a magnitude of unity and the sign of u; similarly S_{vw} is unity with the sign of v. We can estimate ϕ for the eastern, the southern and the northern face in the same procedure. We assume a linear distribution for the diffusive flux. Thus we can obtain the two-dimensional discretization equation as

$$\phi_P a_P = \phi_E a_E + \phi_W a_W + \phi_N a_N + \phi_S a_S + \frac{\Delta x \Delta y}{\Delta t}\phi_P{}^n$$
$$+ K_w\phi_{lw,mw} + K_s\phi_{ls,ms} - K_e\phi_{le,me} - K_n\phi_{ln,mn} \tag{11}$$

where

$$a_E = D_e - (F_e - K_e)\cdot(1-S_{ue})/2$$
$$a_W = D_w + (F_w - K_w)\cdot(1+S_{uw})/2$$
$$a_N = D_n - (F_n - K_n)\cdot(1-S_{vn})/2$$
$$a_S = D_s + (F_s - K_s)\cdot(1+S_{vs})/2$$
$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta x \Delta y}{\Delta t} + K_w + K_s - K_e - K_n \tag{12}$$
$$K_w = S_{uw}\cdot\min\left(|F_w|,\ \frac{\Delta y}{\delta y_{kw}}|v_w|\frac{\delta x_i}{2}\right)$$

③SUWDS

SUDS is used in such a case where the convection dominates more than the diffusion. On the other hand, SUWDS is used in such a case where both the convection and the diffusion dominate in the region. We assume that in the vicinity of the western face of the control volume in the figure**4** the ϕ profile for the convective and diffusive fluxes is expressed as[2]

$$\phi_w = C_1 + C_2\left(y'\frac{u_w}{V_w} - x'\frac{v_w}{V_w}\right) + C_3\exp\left(\frac{u_w}{\Gamma_w}x' + \frac{v_w}{\Gamma_w}y'\right) \tag{13}$$

The constants C_1, C_2 and C_3 are determined from three values of P, W and either one of four grid points NW, SW, S and N, which is selected from the sign of u and v. These conditions are written by

$$
\begin{aligned}
&\phi=\phi_P && \text{at} \quad x'=\delta x_i/2 && y'=0 \\
&\phi=\phi_W && \text{at} \quad x'=-\delta x_i/2 && y'=0 \\
&\phi=\phi_{lw,mw} && \text{at} \quad x'=-S_{uw}\delta x_i/2 && y'=-S_{uw}\delta y_{kw}
\end{aligned}
\tag{14}
$$

We can estimate ϕ for the eastern , the southern and the northern face in the same procedure. Then we can obtain the two-dimensional discretization equation as

$$
\begin{aligned}
\phi_P a_P = {} & \phi_E a_E + \phi_W a_W + \phi_N a_N + \phi_S a_S + \frac{\Delta x \Delta y}{\Delta t}\phi_P^n \\
& + (F_w B_w' - D_w B_w'')\phi_{lw,mw} + (F_s B_s' - D_s B_s'')\phi_{ls,ms} \\
& - (F_e B_e' - D_e B_e'')\phi_{le,me} - (F_n B_n' - D_n B_n'')\phi_{ln,mn}
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
a_E &= D_e(1 - A_e'' - B_e'') - F_e(0.5 - A_e' - B_e') \\
a_W &= D_w(1 - A_w'') + F_w(0.5 + A_w') \\
a_N &= D_n(1 - A_n'' - B_n'') - F_n(0.5 - A_n' - B_n') \\
a_S &= D_s(1 - A_s'') + F_s(0.5 + A_s') \\
a_P &= a_E + a_W + a_N + a_S + \frac{\Delta x \Delta y}{\Delta t} \\
&\quad + (F_w B_w' - D_w B_w'') + (F_s B_s' - D_s B_s'') \\
&\quad - (F_e B_e' - D_e B_e'') - (F_n B_n' - D_n B_n'')
\end{aligned}
\tag{16}
$$

Here the variables with the subscript w are defined as follows

$$
\begin{aligned}
&\alpha_w = v_w \delta x_i \qquad P_{xw} = \frac{|u_w|\delta x_i}{\Gamma_w} \qquad \beta_w = u_w \delta y_{kw} \qquad P_{yw} = \frac{|v_w|\delta y_{kw}}{\Gamma_w} \\
&A_w' = \left(\cosh\left(\frac{P_{xw}}{2}\right) - 1\right) A_w \qquad A_w'' = S_{uw}\left(2\sinh\left(\frac{P_{xw}}{2}\right) - P_{xw}\right) A_w \\
&B_w' = \left(\cosh\left(\frac{P_{xw}}{2}\right) - 1\right) B_w \qquad B_w'' = S_{uw}\left(2\sinh\left(\frac{P_{xw}}{2}\right) - P_{xw}\right) B_w \\
&A_w = \frac{\beta_w - |\alpha_w/2|(1+S_{uw})}{|\alpha_w|\exp(-P_{xw}/2)(1-\exp(-P_{yw})) + 2|\beta_w|\sinh(P_{xw}/2)} \\
&B_w = \frac{|\alpha_w|}{|\alpha_w|\exp(-P_{xw}/2)(1-\exp(-P_{yw})) + 2|\beta_w|\sinh(P_{xw}/2)}
\end{aligned}
\tag{17}
$$

We can define the valuables with the subscript e , s and n similarly.

④QUICK[3]

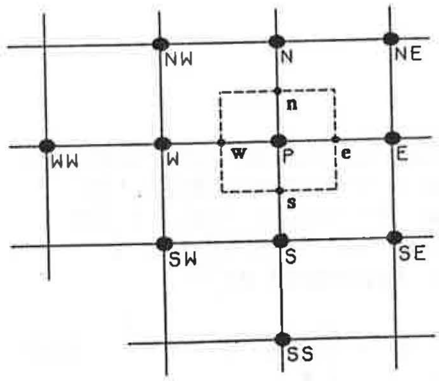


FIG5 The control volume for QUICK

We show the control volume in the figure**5** for QUICK. The differential equation of Eq(1) is written by

$$
\begin{aligned}
\phi_P^{n+1} = {} & \phi_P^n + \mathbf{FX}(I,J) - \mathbf{FX}(I+1,J) \\
& + \mathbf{FY}(I,J) - \mathbf{FY}(I,J+1)
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
&\mathbf{FX}(I,J) = \mathbf{CONX}\cdot\phi_w - \mathbf{DIFFX}\cdot(\phi_P - \phi_W) \\
&\mathbf{CONX} = u_w \frac{\Delta t}{\delta x_i} \qquad \mathbf{DIFFX} = \frac{\Gamma_w}{\Delta x}\,\frac{\Delta t}{\delta x_i} \\
&\phi_w = \frac{1}{2}(\phi_P + \phi_W) - \frac{1}{8}\left(\mathbf{CURVN} - \frac{1}{3}\mathbf{CURVT}\right)
\end{aligned}
\tag{19}
$$

Here we assume the central difference for the diffusive term and that the convective term in the western face is able to determined from five grid points WW, W, P, NW and SW if u is positive and determined from W, P, E, N and S if u is negative. The difference equation for the convective term is expressed as

at $u_W > 0$
$$(\mathbf{CURVN} - \frac{1}{3}\mathbf{CURVT}) = \phi_{WW} - 2\phi_W + \phi_P - \frac{1}{3}(\phi_{SW} - 2\phi_W + \phi_{NW})$$
$$= \mathbf{CURVX}\,(I-1, J)$$

at $u_W < 0$
$$(\mathbf{CURVN} - \frac{1}{3}\mathbf{CURVT}) = \phi_W - 2\phi_P + \phi_E - \frac{1}{3}(\phi_N - 2\phi_P + \phi_S)$$
$$= \mathbf{CURVX}\,(I, J) \tag{20}$$

Then **FX** (I, J) is written by

$$\mathbf{FX}\,(I, J) = \frac{1}{2}\mathbf{CONX}\,(\phi_P + \phi_W) - \mathbf{DIFFX}\,(\phi_P - \phi_W)$$
$$- \frac{1}{16}[(\mathbf{CONX} - |\mathbf{CONX}|)\,\mathbf{CURVX}\,(I, J) + (\mathbf{CONX} + |\mathbf{CONX}|)\,\mathbf{CURVX}\,(I-1, J)] \tag{21}$$

Similarly we can estimate ϕ in the southern face and get **FY** (I, J). Thus we can solve Eq(18).

⑤Pseudo-Spectral

The differential equation of Eq(1) is written by

$$\phi_P^{n+1} = \phi_P^n + \Delta t\,[-u\,QX\,(I, J) - v\,QY\,(I, J) + \Gamma\,(GX\,(I, J) + GY\,(I, J))] \tag{22}$$

where

$$QX\,(I, J) = \left(\frac{\partial \phi}{\partial x}\right)_P \qquad QY\,(I, J) = \left(\frac{\partial \phi}{\partial y}\right)_P \tag{23}$$

First, we consider the case where the boundary is periodic. As this condition allows us to use Fourier sine transform, $\phi\,(x, y)$ in x direction can be expressed as

$$\phi\,(x, y) = \sum_{k=0}^{N} \hat{\phi}_x\,(k)\,\sin\frac{kn\pi}{L_x} \tag{24}$$

where $L_x = N\Delta x = 1$ and N is the number of meshes. Then the first and the second derivatives of Eq(24) are expressed as follows

$$\frac{\partial \phi}{\partial x} = \sum_{k=0}^{N} \left(\frac{k\pi}{L_x}\hat{\phi}_x\,(k)\right)\cos\frac{kn\pi}{L_x} \tag{25}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \sum_{k=0}^{N} \left(-\left(\frac{k\pi}{L_x}\right)^2 \hat{\phi}_x\,(k)\right)\sin\frac{kn\pi}{L_x} \tag{26}$$

As $\phi\,(x, y)$ is given at discrete grid points,

$$x = (I-1)\,\Delta x \quad , \quad I = 1, 2, 3, \cdots\cdots, N+1$$

$\hat{\phi}\,(k)$ is obtain by inverse Fourier transform of Eq(24) as

$$\hat{\phi}_x\,(k) = \frac{2}{N}\sum \phi\,(x, y)\,\sin\frac{k\pi x}{L_x} = \frac{2}{N}\sum_{I=1}^{N+1} \phi\,(I, y)\,\sin\frac{k\pi\,(I-1)}{N} \tag{27}$$

Thus we can get $QX(I,J)$ and $GX(I,J)$ in Eq(24) as

$$QX(I,J)=\sum_{I=1}^{N+1}\left(\frac{k\pi}{L_x}\hat{\phi}_x(k)\right)\cos\frac{k\pi(I-1)}{N} \tag{28}$$

$$GX(I,J)=\sum_{I=1}^{N+1}\left(-\left(\frac{k\pi}{L_x}\right)^2\hat{\phi}_x(k)\right)\sin\frac{k\pi(I-1)}{N} \tag{29}$$

Similarly we express $\phi(x,y)$ in the y direction using Fourier sine transform and get $QY(I,J)$ and $GY(I,J)$. Then we can soleve Eq(22).

Second, we consider the case where the boundary isn't periodic. Here we use the method of 'Reduction to periodicity' proposed by Roache[5]. Following this method, the function $\phi(x,y)$ is decomposed into sum of two functions as

$$\phi(x,y)=f(x,y)+g(x) \tag{30}$$

where $\phi(x,y)$ is such a function as 0 at $x=0$ and 1, and $g(x)$ is the N'th degree polynomial. Here we take N as 3. Then $g(x)$ is expressed as

$$g(x)=a_0+a_1x+a_2x^2+a_3x^3 \tag{31}$$

Four coefficients $a_0\sim a_3$ are determined from the periodic conditions of the derivatives of $f(x,y)$ as

$$f^{(n)}(1,y)-f^{(n)}(0,y)=0 \quad (n=0,1,2):f(0,y)=0 \tag{32}$$

Eq(32) for $n=0$ gives as

$$\begin{aligned}&f(0,y)=\phi(0,y)-g(0)=\phi(0,y)-a_0=0\\&\phi(0,y)=a_0\end{aligned} \tag{33}$$

Then we can get a_0. Next, we define D_n as

$$D_n=\phi^{(n)}(1,y)-\phi^{(n)}(0,y) \tag{34}$$

Eq(34) is arrenged using Eq(32) as

$$D_n=g^{(n)}(1)-g^{(n)}(0) \tag{35}$$

For $n=0$, 1 and 2, we get as

$$\begin{array}{lllll}\text{at } n=0 & g(1) & -g(0) & =a_1+a_2+a_3 & =D_0\\ \text{at } n=1 & g^{(1)}(1) & -g^{(1)}(0) & =2a_2+3a_3 & =D_1\\ \text{at } n=2 & g^{(2)}(1) & -g^{(2)}(0) & =6a_3 & =D_2\end{array} \tag{36}$$

Then we obtain the coefficients $a_1\sim a_3$ as follows

$$a_1=D_0-a_2-a_3 \qquad a_2=\frac{1}{2}D_1-\frac{3}{2}a_3 \qquad a_3=\frac{1}{6}D_2 \tag{37}$$

The derivatives of $\phi(x,y)$ are approximated here by the second-order one sided finite difference.
As $f(x,y)$ is a periodicity function, it can be solved from the described above method. Then we can obtain ϕ, QX and GX for Eq(22) as follows

$$\begin{aligned}\phi(x,y)&=f(x,y)+a_0+a_1x+a_2x^2+a_3x^3\\QX(x,y)&=f^{(1)}(x,y)+a_1+2a_2x+3a_3x^2\\GX(x,y)&=f^{(2)}(x,y)+2a_2+6a_3x\end{aligned} \tag{38}$$

If we take the same procedure in y direction, we can obtain QY and GY. Thus we can solve Eq(23).

2-3 The calculation methods for Eq(2)

We consider the control volume shown in the figure6. Each methods of CDS, UDS, Power-Low and QUICK and Pseudo-Spectral can be extended easily for three dimensional. So we abbreviate these explanation. Here we indicate the methods of SUDS and SUWDS for Eq(2).

①SUDS

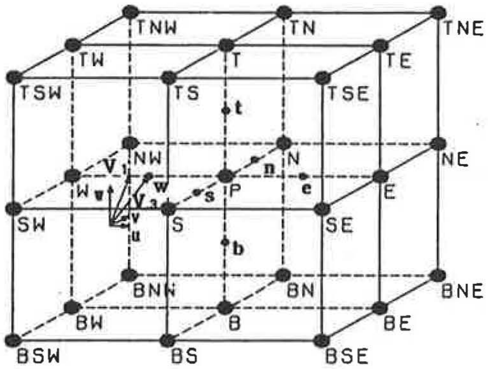


FIG6 The control volume for three dimension

We consider that for the vicinity of the western face of the control volume shown in the figure6, r axis is the direction of the flow (V_3). We rotate $x-y-z$ coordinate around w as x axis is coincident with r axis. Then the rotation y and z axis regard n_1 and n_2 axis, and we consider $r-n_1-n_2$ coordinate in the vicinity of w. We assume the ϕ profile for the convective flux is expressed as

$$\phi_w = C_1 + C_2 n_1 + C_3 n_2 \tag{39}$$

Using V_1 and V_3, Eq(39) is rewritten by

$$\phi_w = C_1 + C_2\left(y'\frac{w_w}{V_{1w}} - z'\frac{v_w}{V_{1w}}\right) + C_3\left(x'\frac{V_{1w}}{V_{3w}} - y'\frac{v_w u_w}{V_{1w}V_{3w}} - z'\frac{w_w u_w}{V_{1w}V_{3w}}\right) \tag{40}$$

where

$$V_{1w} = \sqrt{(v_w^2 + w_w^2)} \qquad V_{3w} = \sqrt{(u_w^2 + v_w^2 + w_w^2)} \tag{41}$$

and x', y' and z' are measured from w. The constants C_1, C_2 and C_3 are determined from three values of the upstream of w. The one is P or W and the other one is either of eight grid points BSW, TSW, TNW, BNW, BS, TS, TN and BN from the sign of u, v and w. The last one is either of four grid points SW, NW, N and S, or either of four TW, BW, B and T from the sign of u, v and w and a magnitude of v and w. These conditions are written by

$$\phi = \phi_{mw} \text{ at } x' = -S_{uw}\delta x_i/2 \quad y' = 0 \quad z' = 0$$
$$\phi = \phi_{qw} \text{ at } x' = -S_{uw}\delta x_i/2 \quad y' = -S_{uw}\delta y_{kw} \quad z' = -S_{ww}\delta z_{lw}$$
$$① \left|\frac{w_w}{v_w}\right| \leqq 1 \tag{42}$$
$$\phi = \phi_{rw} \text{ at } x' = -S_{uw}\delta x_i/2 \quad y' = -S_{uw}\delta y_{kw} \quad z' = 0$$
$$② \left|\frac{w_w}{v_w}\right| > 1$$
$$\phi = \phi_{rw} \text{ at } x' = -S_{uw}\delta x_i/2 \quad y' = 0 \quad z' = -S_{ww}\delta z_{lw}$$

where

$$kw = j + (1 - S_{vw})/2 \qquad lw = k + (1 - S_{ww})/2 \tag{43}$$

S_{uw} has a magnitude of unity and the sign of u; similarly S_{vw} is unity with the sign of v and S_{ww} is unity with the sign of w. We can estimate ϕ for the eastern, the southern, the northern, the bottom and the top face in the same procedure. We assume a linear distribution for the diffusive flux. Then we can obtain the three-dimensional discretization equation as

$$\phi_P a_P = \phi_E a_E + \phi_W a_W + \phi_N a_N + \phi_S a_S + \phi_T a_T + \phi_B a_B + \frac{\Delta x \Delta y \Delta z}{\Delta t}\phi_P^n$$

$$+(K_{1w}-K_{2w})\phi_{rw}+(K_{1s}-K_{2s})\phi_{rs}+(K_{1b}-K_{2b})\phi_{rb}$$
$$-(K_{1e}-K_{2e})\phi_{re}-(K_{1n}-K_{2n})\phi_{rn}-(K_{1t}-K_{2t})\phi_{rt}$$
$$+K_{2w}\phi_{qw}+K_{2s}\phi_{qs}+K_{2b}\phi_{qb}-K_{2e}\phi_{qe}-K_{2n}\phi_{qn}-K_{2t}\phi_{qt} \quad (44)$$

where

$$a_E=D_e-(F_e-K_{1e})\cdot(1-S_{ue})/2$$
$$a_W=D_w+(F_w-K_{1w})\cdot(1+S_{uw})/2$$
$$a_N=D_n-(F_n-K_{1n})\cdot(1-S_{vn})/2$$
$$a_S=D_s+(F_s-K_{1s})\cdot(1+S_{vs})/2$$
$$a_T=D_t-(F_t-K_{1t})\cdot(1-S_{wt})/2$$
$$a_B=D_b+(F_b-K_{1b})\cdot(1+S_{wb})/2$$

$$a_P=a_E+a_W+a_N+a_S+a_T+a_B+\frac{\Delta x\Delta y\Delta z}{\Delta t}$$
$$+K_{1w}+K_{1s}+K_{1b}-K_{1e}-K_{1n}-K_{1t} \quad (45)$$

The variables with the subscript w are defined as

$$F_w=u_w\Delta y\Delta z \qquad D_w=\frac{\Gamma_w\Delta y\Delta z}{\delta x_i}$$

$$① \left|\frac{w_w}{v_w}\right|\leqq 1 \qquad K_{1w}=S_{uw}\cdot\min\left(|F_w|,\frac{\Delta y\Delta z}{\delta y_{kw}}|v_w|\frac{\delta x_{i+1}}{2}\right)$$
$$K_{2w}=S_{uw}\cdot\min\left(|K_{1w}|,\frac{\Delta y\Delta z}{\delta z_{lw}}|w_w|\frac{\delta x_{i+1}}{2}\right)$$
$$② \left|\frac{w_w}{v_w}\right|>1 \qquad K_{1w}=S_{uw}\cdot\min\left(|F_w|,\frac{\Delta y\Delta z}{\delta z_{lw}}|w_w|\frac{\delta x_{i+1}}{2}\right)$$
$$K_{2w}=S_{uw}\cdot\min\left(|K_{1w}|,\frac{\Delta y\Delta z}{\delta y_{kw}}|v_w|\frac{\delta x_{i+1}}{2}\right) \quad (46)$$

We can define the variables with the other subscript by the same procedure.

②SUWDS

We assume that ϕ profile for the convective and diffusive fluxes are expressed as

$$\phi_w=C_1+C_2\left(y'\frac{w_w}{V_{1w}}-z'\frac{v_w}{V_{1w}}\right)+C_3\left(x'\frac{V_{1w}}{V_{3w}}-y'\frac{v_wu_w}{V_{1w}V_{3w}}-z'\frac{w_wu_w}{V_{1w}V_{3w}}\right)$$
$$+C_4\exp\left(\frac{u_w}{\Gamma_w}x'+\frac{v_w}{\Gamma_w}y'+\frac{w_w}{\Gamma_w}z'\right) \quad (47)$$

where the constants C_1, C_2, C_3 and C_4 are determined from P of the upstream of w and three values of the downstream of w. These conditions are written by

$$\phi=\phi_P \text{ at } x'=\delta x_i/2 \quad y'=0 \quad z'=0$$
$$\phi=\phi_W \text{ at } x'=-\delta x_i/2 \quad y'=0 \quad z'=0$$
$$\phi=\phi_{qw} \text{ at } x'=-S_{uw}\delta x_i/2 \quad y'=-S_{uw}\delta y_{kw} \quad z'=-S_{ww}\delta z_{lw}$$

$$① \left|\frac{w_w}{v_w}\right|\leqq 1$$
$$\phi=\phi_{rw} \text{ at } x'=-S_{uw}\delta x_i/2 \quad y'=-S_{uw}\delta y_{kw} \quad z'=0 \quad (48)$$

$$② \left|\frac{w_w}{v_w}\right|>1$$
$$\phi=\phi_{rw} \text{ at } x'=-S_{uw}\delta x_i/2 \quad y'=0 \quad z'=-S_{ww}\delta z_{lw}$$

After we estimate ϕ for other face similarly, we can obtain the three-dimensional discretization equation as

$$\phi_P a_P = \phi_E a_E + \phi_W a_W + \phi_N a_N + \phi_S a_S + \phi_T a_T + \phi_B a_B + \frac{\Delta x \Delta y \Delta z}{\Delta t} \phi_P{}^n$$
$$+ (F_w B_w' - D_w B_w'') \phi_{qw} + (F_s B_s' - D_s B_s'') \phi_{qs} + (F_b B_b' - D_b B_b'') \phi_{qb}$$
$$- (F_e B_e' - D_e B_e'') \phi_{qe} - (F_n B_n' - D_n B_n'') \phi_{qn} - (F_t B_t' - D_t B_t'') \phi_{qt}$$
$$+ [(F_w (C_w' - B_w') - D_w (C_w'' - B_w'')] \phi_{rw}$$
$$+ [(F_s (C_s' - B_s') - D_s (C_s'' - B_s'')] \phi_{rs}$$
$$+ [(F_b (C_b' - B_b') - D_b (C_b'' - B_b'')] \phi_{rb}$$
$$- [(F_e (C_e' - B_e') - D_e (C_e'' - B_e'')] \phi_{re}$$
$$- [(F_n (C_n' - B_n') - D_n (C_n'' - B_n'')] \phi_{rn}$$
$$- [(F_t (C_t' - B_t') - D_t (C_t'' - B_t'')] \phi_{rt} \qquad (49)$$

where

$$a_E = D_e (1 - A_e'' - C_e'') - F_e (0.5 - A_e' - C_e')$$
$$a_W = D_w (1 - A_w'') + F_w (0.5 + A_w')$$
$$a_N = D_n (1 - A_n'' - C_n'') - F_n (0.5 - A_n' - C_n')$$
$$a_S = D_s (1 - A_s'') + F_s (0.5 + A_s')$$
$$a_T = D_t (1 - A_t'' - C_t'') - F_t (0.5 - A_t' - C_t')$$
$$a_B = D_b (1 - A_b'') + F_b (0.5 + A_b')$$

$$a_P = a_E + a_W + a_N + a_S + a_T + a_B + \frac{\Delta x \Delta y \Delta z}{\Delta t}$$
$$+ F_w C_w' - D_w C_w'' + F_s C_s' - D_s C_s'' + F_b C_b' - D_b C_b''$$
$$- F_e C_e' + D_e C_e'' - F_n C_n' + D_n C_n'' - F_t C_t' + D_t C_t'' \qquad (50)$$

The variables for the w face are defined as

$$\alpha_w = u_w \delta y_{kw} \delta z_{lw} \qquad \beta_w = v_w \delta x_i \delta z_{lw} \qquad \gamma_w = w_w \delta x_i \delta y_{kw}$$

$$P_{xw} = \frac{|u_w| \delta x_i}{\Gamma_w} \qquad P_{yw} = \frac{|v_w| \delta y_{kw}}{\Gamma_w} \qquad P_{ww} = \frac{|w_w| \delta z_{lw}}{\Gamma_w}$$

$$A_w' = (\cosh(\frac{P_{xw}}{2}) - 1) A_w \qquad A_w'' = S_{uw} (2 \sinh(\frac{P_{xw}}{2}) - P_{xw}) A_w$$

$$B_w' = (\cosh(\frac{P_{xw}}{2}) - 1) B_w \qquad B_w'' = S_{uw} (2 \sinh(\frac{P_{xw}}{2}) - P_{xw}) B_w \qquad (51)$$

$$C_w' = (\cosh(\frac{P_{xw}}{2}) - 1) C_w \qquad C_w'' = S_{uw} (2 \sinh(\frac{P_{xw}}{2}) - P_{xw}) C_w$$

① $\left|\frac{w_w}{v_w}\right| \leqq 1$

$$H_w = |\gamma_w| (1 - \exp(-P_{zw})) \exp(-P_{xw}/2) \exp(-P_{yw})$$
$$+ |\beta_w| (1 - \exp(-P_{yw})) \exp(-P_{xw}/2) + 2 |\alpha_w| \sinh(P_{xw}/2)$$

$$A_w = \frac{\alpha_w - |\beta_w/2| (1 + S_{uw})}{H_w} \qquad B_w = \frac{|\gamma_w|}{H_w} \qquad C_w = \frac{|\beta_w|}{H_w}$$

② $\left|\frac{w_w}{v_w}\right| > 1$

$$H_w = |\beta_w| (1 - \exp(-P_{yw})) \exp(-P_{xw}/2) \exp(-P_{zw})$$
$$+ |\gamma_w| (1 - \exp(-P_{zw})) \exp(-P_{xw}/2) + 2 |\alpha_w| \sinh(P_{xw}/2)$$

$$A_w = \frac{\alpha_w - |\gamma_w/2| (1 + S_{uw})}{H_w} \qquad B_w = \frac{|\beta_w|}{H_w} \qquad C_w = \frac{|\gamma_w|}{H_w}$$

2-4 The results

We calculated for three cases described in the section **2-1**. The number of mashes is 1 6 × 1 6 for the case(a) and case(b), and 1 6 × 1 6 × 1 6 for the case(c). The diffusion coefficient Γ is equal to 0.0 1. In addition, we made two calculations for the case(b); when the number of meshes is 3 2 × 3 2 and when Γ = 0.0 0 2.

For the problems described in the section **2-1**, we can get the exact solution as

$$\phi = 0.5 \left[1 + \mathbf{erf} \left(y_n \sqrt{\left(\frac{V_n}{4 \Gamma x_n} \right)} \right) \right] \tag{52}$$

where **erf** is error function, x_n is parallel to the flow and y_n is normal. Here, we compare calculation values with the exact solution at the section through $(x_n, y_n) = (0.5, 0.5)$.

The results for the case(a) are shown in the figure**7**. All of the calculation values are well coincided with the exact solution. In the case(a), note that the calculation value of SUDS is equal to that of UDS. We calculated about three different conditions in the case(b). The large number of meshes is corresponded to the small cell Peclet number (Pe) and the small Γ means the large Pe. We show the results for mesh number 1 6 × 1 6 and Γ = 0.0 1 in the figure**8**. QUICK and Pseudo-Spectral are well coincided with the exact solution, but UDS and Power-Low aren't for the numerical diffusion. SUDS and SUWDS is roughly equal to the exact solution. However, there is a little different befor and behind at $y_n = 8$. We show the results for the number of meshes 3 2 × 3 2 in the figure**9**. This figure also show the same tendency as figure**8**. Comparing UDS with Power-Low, the calculation value of the latter is improved more than the former and is close to the exact solution. The calculation value of SUWDS is worse than that in the figure**8**. In SUDS the calculation value is coincided with the exact solution. We show the result at Γ = 0.0 0 2 in the figure**10**. The calculation value of SUDS is better than

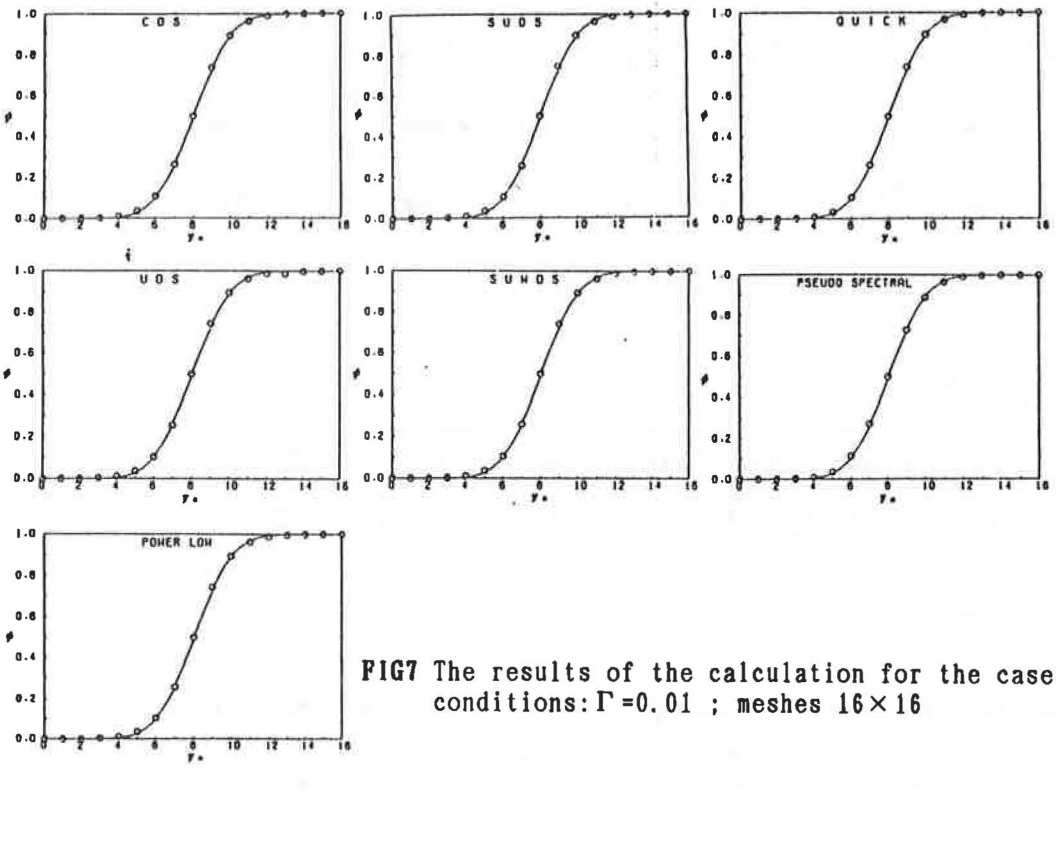
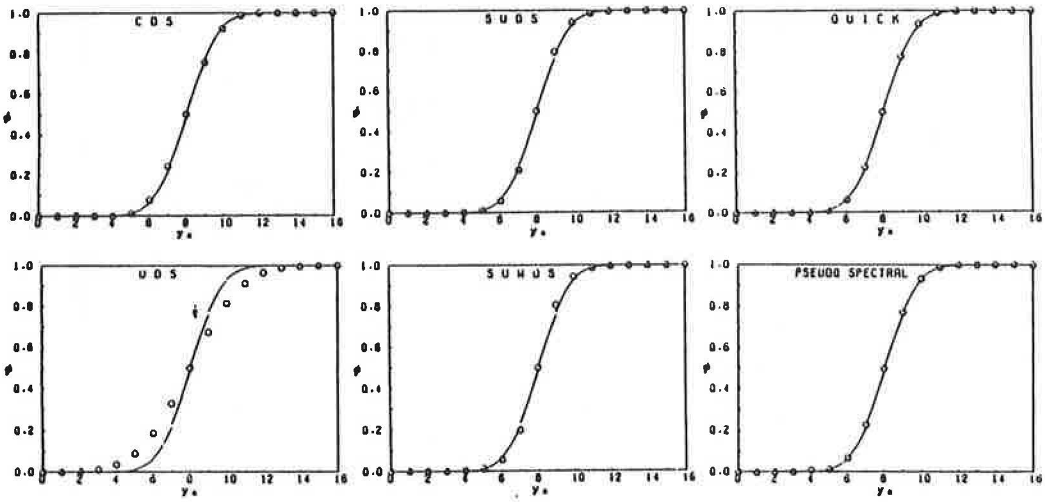


FIG7 The results of the calculation for the case(a) conditions: Γ=0.01 ; meshes 16×16

others. The calculations of QUICK and Pseudo-Spectral occur the oscillation because of the large Pe. UDS and Power-Low become worse. We carry out all of the calculations at time step $\Delta t = 0.01$. But as we couldn't get the result of Pseudo-Spectral at $\Delta t = 0.01$, we set $\Delta t = 0.005$ here. We show the results for the case(c) in the figure11. UDS and Power-Low are bad, because they generate the numerical diffusion. The others are roughly coincided with



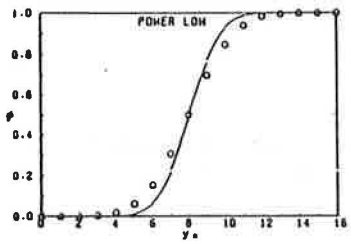
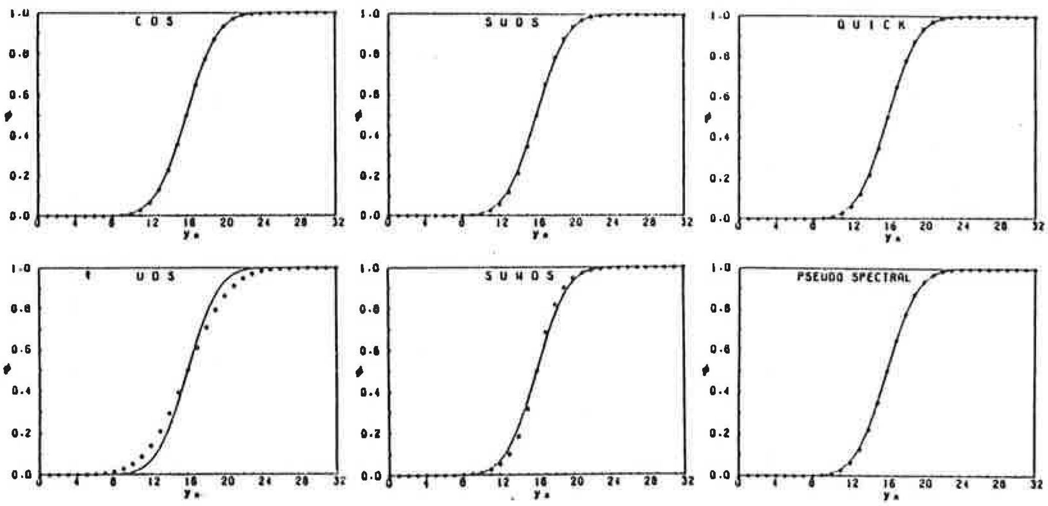


FIG8 The results of the calculation for the case(b) conditions: $\Gamma = 0.01$; meshes 16×16



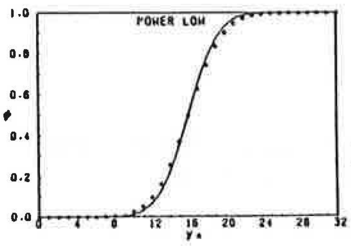


FIG9 The results of the calculation for the case(b) conditions: $\Gamma = 0.01$; meshes 32×32

the exact solution, except for the underestimation and the overestimation in the neighborhood of the region at $y_n = 8$.

For the summary, SUDS and QUICK and Pseudo-Spectral have good accuracy. As Pe is large, SUDS has good accuracy, but QUICK and Pseudo-Spectral occur the oscillation. UDS and Power-Low have relatively poor accuracy because of their numerical diffusion.

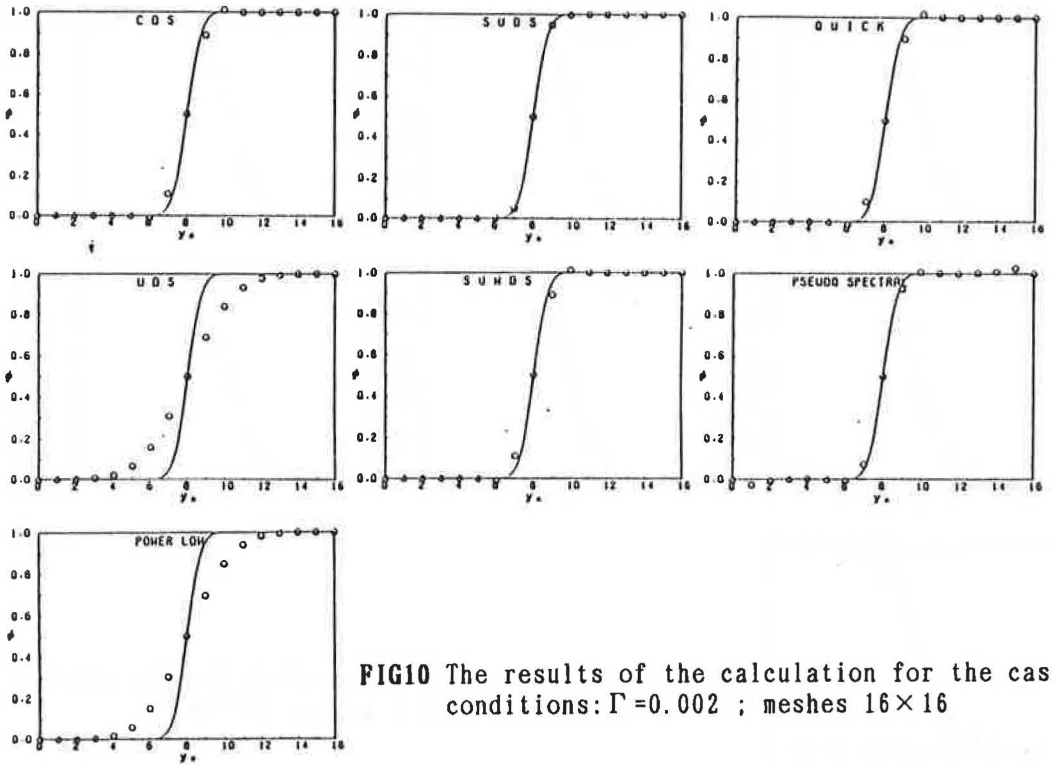


FIG10 The results of the calculation for the case(b) conditions: $\Gamma = 0.002$; meshes 16×16

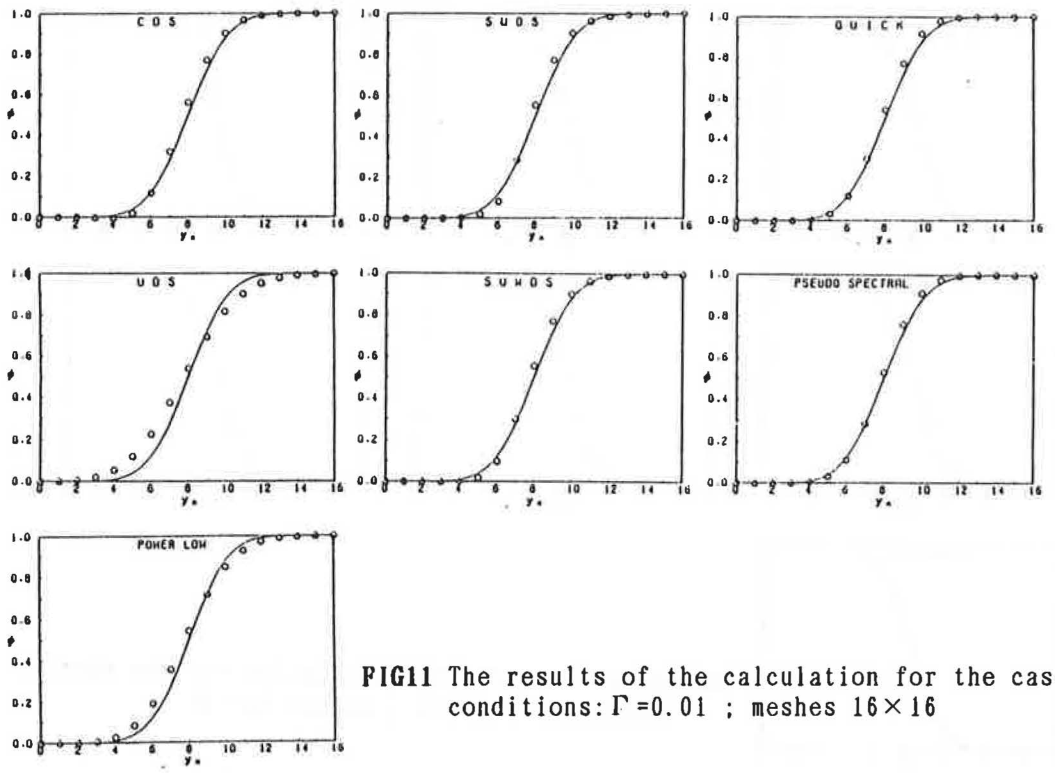


FIG11 The results of the calculation for the case(c) conditions: $\Gamma = 0.01$; meshes 16×16

3. The simulation of atrium airflow with inclined inlet

3-1 The governing equation

We use the three dimensional $k-\varepsilon$ turbulence model for the governing equation. In this calculation we assume the Bussinesq approximation, and the 'Wall Function' based on the logarithmic low of the wall is introduced to set the wall boundary conditions.

3-2 The calculation conditions

We show the floor of the atrium in the figure12. This figure is only half because this atrium is symmetried about AA' axis. It has two column and two outlets. The wall on the right side and partly on the downside in the figure12 and partly on the ceilling are made of glass. Other walls are made of concrete. The temperature on the boundary is assumed to be constant ; 33.75℃ for glass, 26℃ for concrete and 15℃ for the floor. The inlets are located at height 13m. There are five inlets shown in the figure13. They are inclined 45° to z direction, and the inlet's wind has the speed of 6.29m/s and the temperature of 14.5℃. The actual atrium has a magnitude of 19.5m(W)×50.4m(L)×18.5m(H). We made only a half calculation. In this calculation we adopt a non-uniform mesh and set the number of meshes 20×30×18.

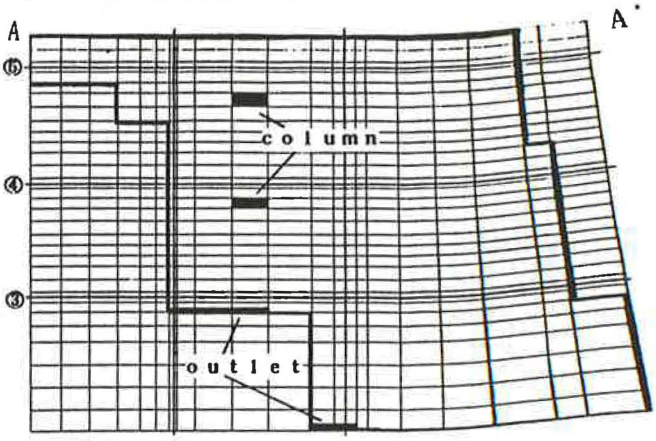


FIG12 The floor of the atrium

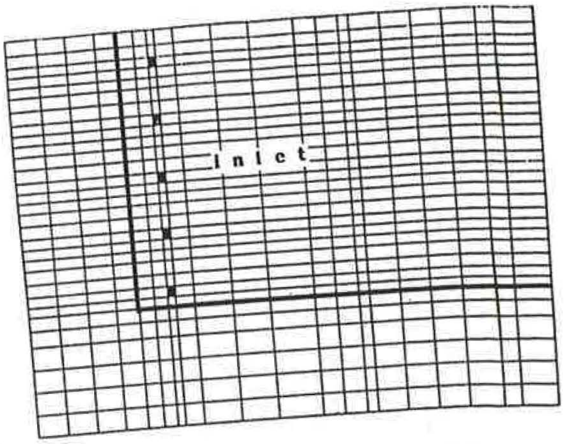


FIG13 The inlets of the atrium

3-3 The results

We made the calculation of the atrium airflow and the temperature distribution with SUDS and Power-Low. The airflow on the six cross sections, where correspond to the floor and ①-⑤ in the figure12, are shown in the figure14 and 15, and the temperature in the figure16 and 17. The top of these figures is the results with SUDS and the bottom is with Power-Low.

At the cross sections③-⑤ including the inlets, the momentums calculated with Power-Low diffuse rapidly, but those with SUDS diffuse only a little and the inlet's wind attains to the floor. The temperature at the same cross sections also diffuse alike. There is a difference about the temperature distribution of the vicinity in cross section①. Seeing the cross sections ③-⑤, though the airflow calculated with Power-low blows horizontally, that with SUDS blows in the direction of the inlet's angle. Accordingly the temperature calculated with SUDS diffuses less than Power-Low(Fig16). We argue in the section **2** that SUDS is much better than Power Low because it generates little numerical diffusion. Applying to the actual atrium, SUDS show the same tendency.

At the cross section⑤, though the airflow with SUDS blows downward in the neighborhood of the inlet, Power-Low doesn't alike. Besides there are a lot of difference between SUDS and Power-Low. Unfortunatery as this atrium is now under the construction, we can't argue in detail which schemes are appropriate. For the future, we'll compare the calculation with the measurement.

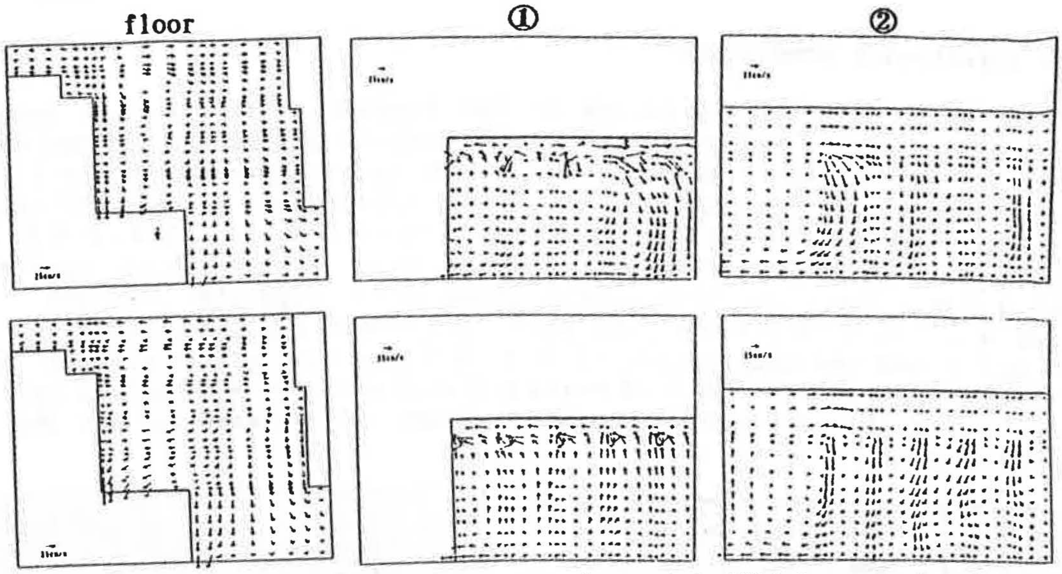


FIG14 The results of the airflow simulation(top, with SUDS; bottom, with Power-Low; the large arrow indicates the wind speed more than 1m/s)

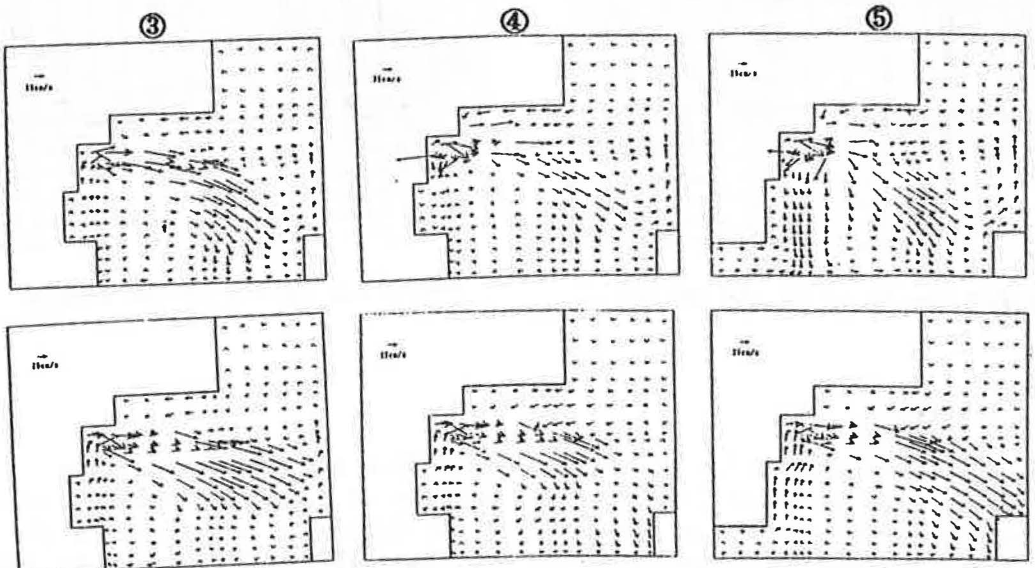


FIG15 The results of the airflow simulation(top, with SUDS; bottom, with Power-Low; the large arrow indicates the wind speed more than 1m/s)

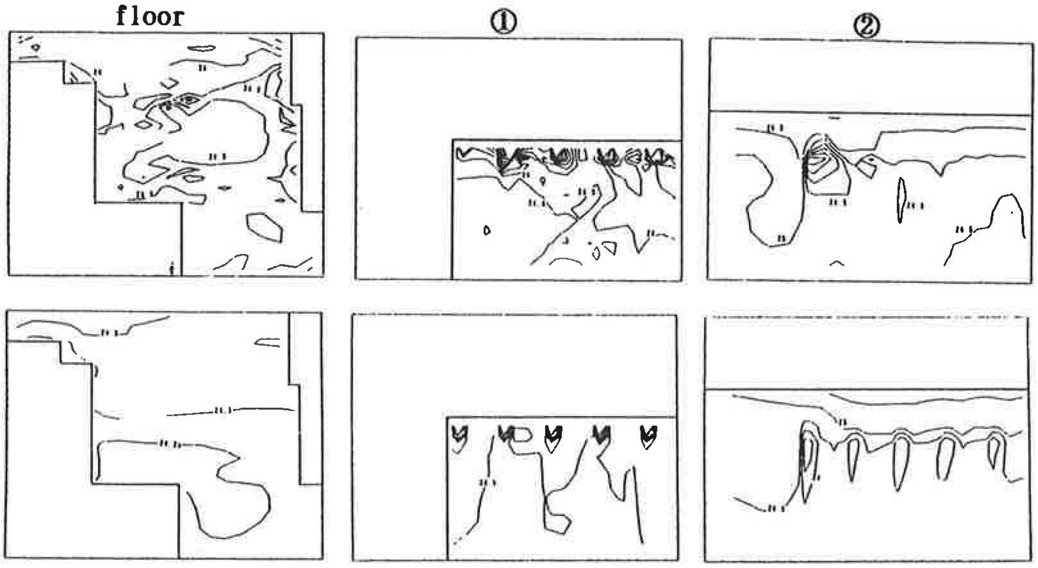


FIG16 The results of the temperature simulation
(top, with SUDS; bottom, with Power-Low)

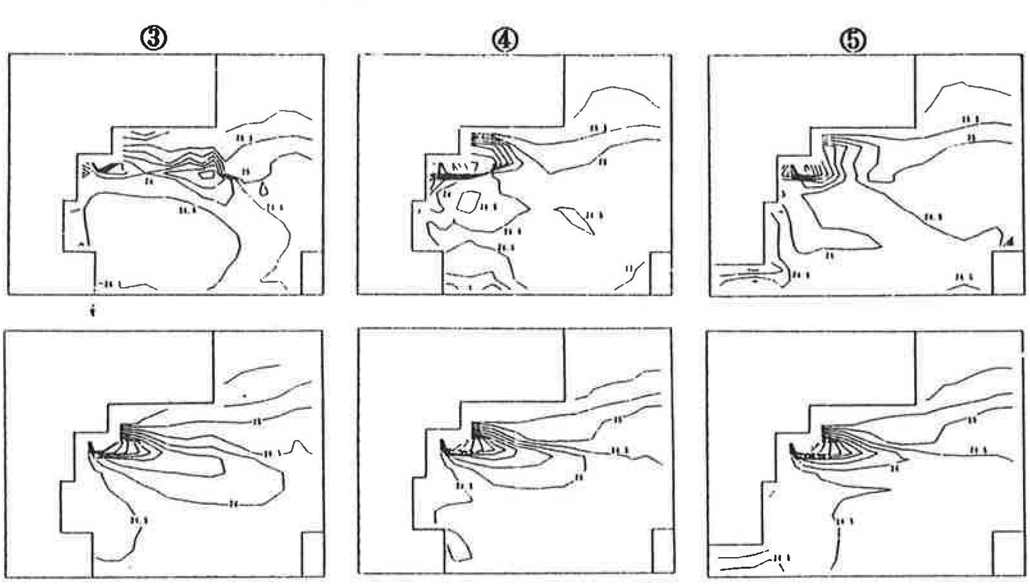


FIG17 The results of the temperature simulation
(top, with SUDS; bottom, with Power-Low)

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