

# A Stochastic Model of User Behaviour Regarding Ventilation

R. FRITSCH\*  
A. KOHLER\*  
M. NYGÅRD-FERGUSON\*  
J.-L. SCARTEZZINI\*

*Airflow rates are directly affected by the amount of open area connecting rooms to the outside and consequently by the inhabitant behaviour with respect to window opening. In this paper, a stochastic model using Markov chains is proposed to generate time series of window angle. It is based on data from four office rooms and a whole heating season (from October to May). The model is then validated by a comparison of the real and generated data. The use of this model within building air infiltration design programmes should significantly improve their realism.*

## 1. INTRODUCTION

THE IMPORTANCE of airflow rates to heating costs and the elimination of pollutants within buildings is an established fact and already much software is available to simulate them [1]. However, it must be pointed out that all these programmes run with unoccupied buildings, even though airflow rates are closely related to the amount of open area and therefore the inhabitant behaviour concerning window opening. For instance, measurements conducted in 25 Danish buildings show that on average the increase in the airflow rate due to occupancy is more than 100% [2].

In order to improve future programmes a model simulating window opening during the winter has been developed and will be presented in this paper. It is based on measured data from four offices of the LESO experimental office building [3]. Using a method similar to that described by Fewkes and Ferris [4], the model generates time series of window opening angles with the same statistics (i.e. average opening angle and time, correlation...) as the measured openings for the heating period.

## 2. DATA USED FOR THE MODEL BUILDING

The model developed is based on measurements taken every half hour in four office rooms facing south in the LESO building (solar units) [3]. The dimensions of the rooms are identical, but the south facades differ. Each one is occupied by one or two persons (Fig. 1).

The facade of the first two solar units considered is a direct gain facade (GDIR facade). It is comprised of double glazed windows sustained by wooden frames covered with aluminium. The breast wall is made of wood and glass wool protected by Eternit panel ( $U = 0.4 \text{ W}$

$\text{m}^{-2} \text{K}^{-1}$ ). There is one side-mounted casement window ( $156 \times 90 \text{ cm}$ ) on the side. For a volume of  $86 \text{ m}^3$  (both rooms), the average air renewal rate due to infiltration is  $0.39 \text{ h}^{-1}$  [5].

The facade of the remaining two rooms is based on a high thermal insulation technique (HIT facade) which consists of double glazed windows with two infra red films (heat mirror) in between, and frames of polyurethane foam in an aluminium profile. The breast wall is also made of polyurethane foam protected by metal sheets ( $U = 0.25 \text{ W m}^{-2} \text{K}^{-1}$ ). There is one side-mounted casement window in the center of  $78 \times 152 \text{ cm}$ . The volume is as before,  $86 \text{ m}^3$ , and the average air renewal rate due to infiltration is very low,  $0.16 \text{ h}^{-1}$ .

The opening angle of the four windows is measured every half hour and stored on magnetic tapes. The winters of 1983-84 for the local HIT and 1984-85 for the local GDIR were used for the model construction and validation. Meteorological variables such as ambient temperature, wind speed or the south vertical solar radiation as well as the inside temperature were also available.

## 3. SETTING UP THE MODEL

### *Autocorrelation functions*

The first approach was to analyse the autocorrelation functions of the measured data. Figure 2 presents the simple autocorrelation as well as the differentiated autocorrelation of the window opening angle. From the simple autocorrelation we can observe that the relationship between two successive measurements (0.5 h delay) is strong: this simply states the fact that a window is usually left in one position for long periods of time. On the other hand, the differentiated autocorrelation function shows clearly that there is not any significative dependance of any greater order. We can deduce from both these graphs that the probability of finding a window in a certain position depends only on its preceding position and not

\* Laboratoire d'Énergie Solaire et de Physique du Bâtiment, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland.

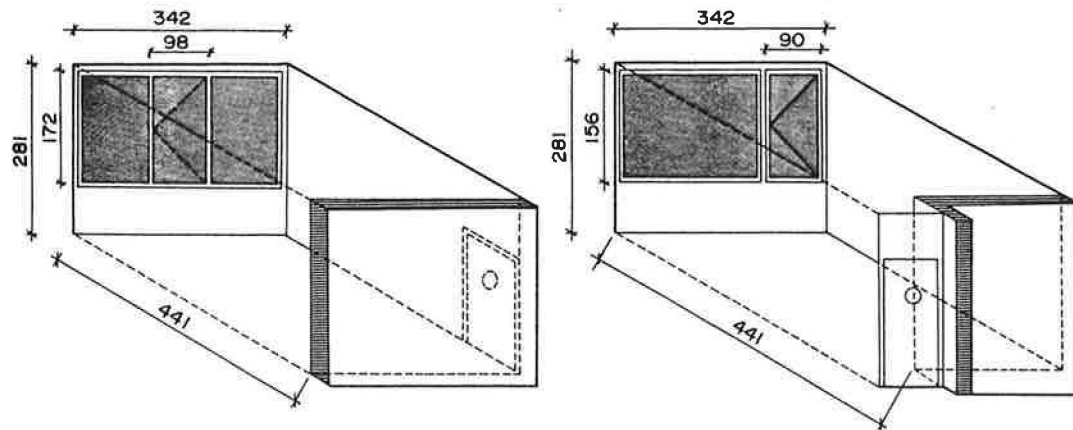


Fig. 1. Solar units equipped with a direct gain facade (right) and high insulation technique facade (left).

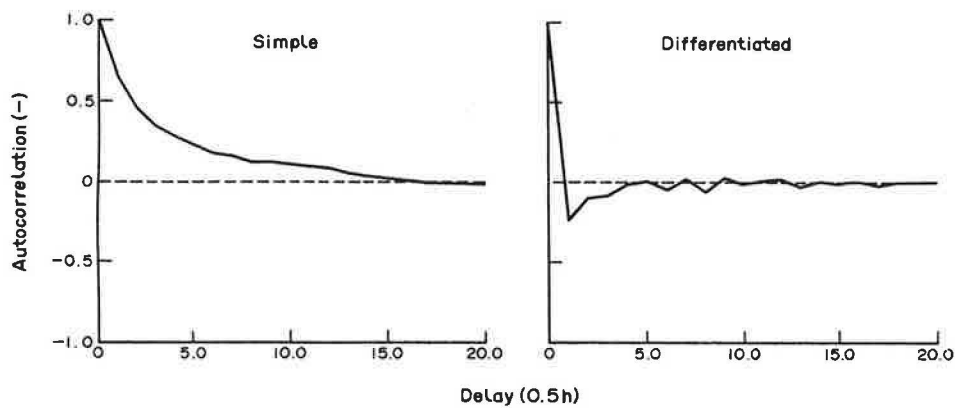


Fig. 2. Simple and differentiated autocorrelations (winter 1984–85, room GDIR-west).

on any others. Therefore we can assume that discrete Markov chains can be used to make a suitable model. A Markovian process has no memory: the next state will depend only on the present state and no others. Thoroughly described in the literature [6], it is rather simple and commonly used.

#### Driving variables

The second logical step was to find out whether the window opening angle is an independent variable and, if not, what are the driving variables.

Different studies [2, 7, 8] tend to prove that there is a multitude of factors influencing window opening (see Table 1) but most of them are very hard to estimate without even considering their relation to window opening. In an attempt to be realistic and simple, we restricted ourselves to the study of the room temperature and a few meteorological variables like wind speed, the south vertical solar radiation and the ambient temperature  $T_a$ . All the values of these variables were available for the considered period and office rooms. Moreover, the literature [2, 7, 8] already gives us some clues as to their importance. Figures 3–5 present the dependance of the percentage of opened windows vs  $T_a$ , wind speed and sunshine.

The wind speed was found to be weakly correlated to the window opening angle as shown by Fig. 6. This is

Table 1. Factors influencing window opening

Table of driving variables	
<i>External</i>	Outdoor temperature Solar radiation Wind speed and direction Rain Noise Odours and pollutants
<i>Internal</i>	Indoor temperature Odours and contaminants Humidity
<i>"Human" parameters</i>	Type of activities Habits

confirmed by Fig. 4: there is a relatively constant percentage of open window for wind speeds less than 5–6  $\text{m s}^{-1}$  which is the case for more than 80% of the observed winters. This is the main reason why we did not retain wind speed as a major driving variable.

The south vertical solar radiation was also discarded although it does have some importance especially in the mid season (October, April and May) by starting increases in the room temperature.

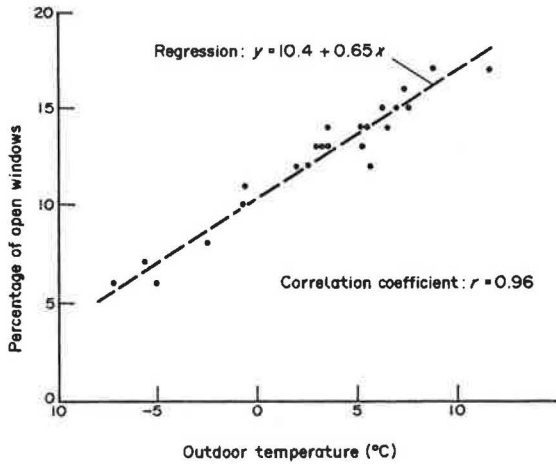


Fig. 3. Relationship between the average use of windows and the average outdoor temperature (AIVC, Technical Notes 23).

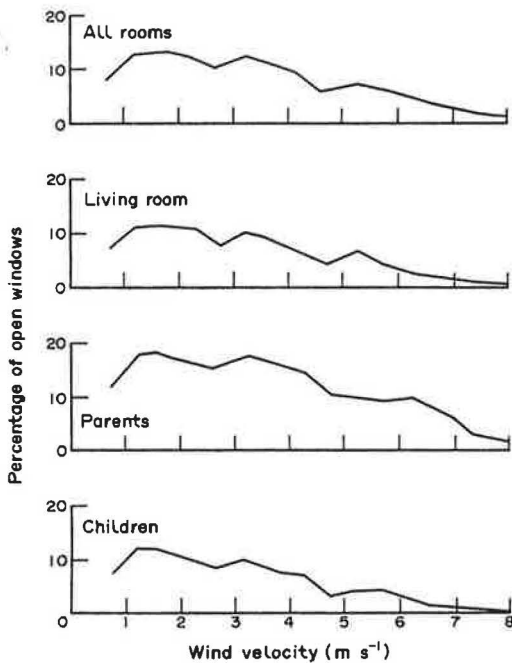


Fig. 4. Percentage of open windows as a function of wind speed (AIVC, Technical Notes 23).

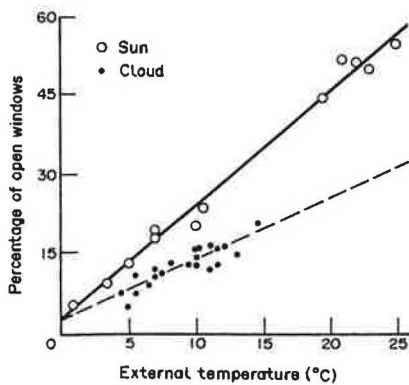


Fig. 5. Window opening as a function of outdoor temperature and sunshine observed on a yearly basis (AIVC, Technical Notes 23).

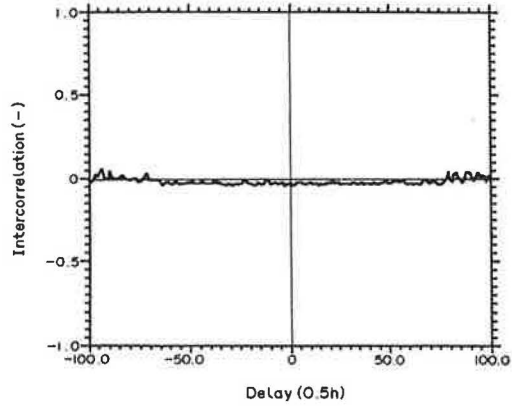


Fig. 6. Intercorrelation: window angle vs wind speed (winter 1983-84, room HIT-west).

However, this compelled us to consider the room temperature even more carefully. Its influence with regards to window opening is complex: several typical types of inhabitant behaviour were pointed out within the framework of an earlier study [7, 8].

Outside the heating period, the inhabitants usually open windows in an attempt to cool the rooms, hoping that  $T_a$  will be less than the room temperature.

During the mid season (October, April and May) windows act as a more convenient heater control than thermostatic valves.

In the LESO building, room temperatures were found to be relatively constant and pleasant (between 19° and 23°C) during the heating period. Therefore there was hardly any noticeable correlation with the window position (Fig. 7). It should also be stressed that as soon as the window is opened, the temperature drops due to fresh air and the angle is then correlated to low room temperature. In brief, this variable usually drives us to open a window. Considering that, on top of this, the room temperature is not easily accessible and that it is necessary to simulate it for non-existent buildings, it seems reasonable to eliminate it.

The remaining driving variable we considered, the out-

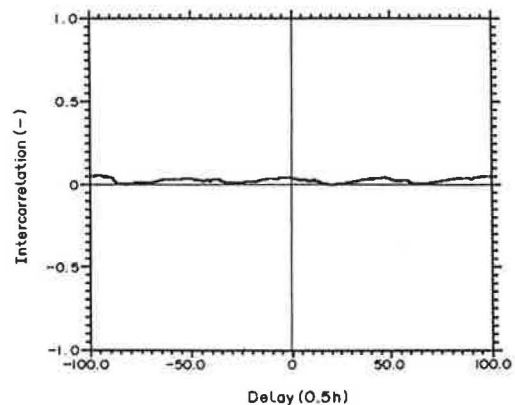


Fig. 7. Intercorrelation: window angle vs room temperature (winter 1983-84, room HIT-west).

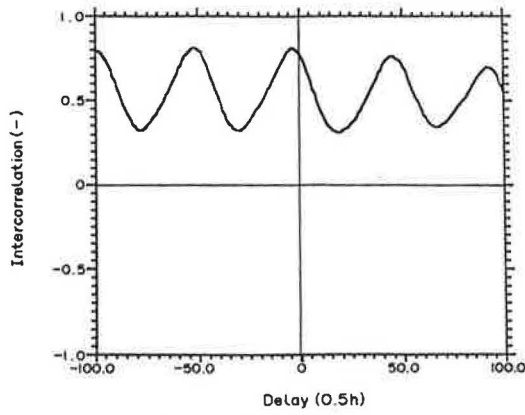


Fig. 8. Intercorrelation : window angle vs ambient temperature (winter 1984-85, room GDIR-west).

door temperature, was found to be the most meaningful one. The intercorrelation function (Fig. 8) of the window angle and  $T_a$  is very strong during the winter. This can also be deduced from the bi-parametric graph (Fig. 9): the cloud of points is clearly drawn higher for increasing temperature. But what is evident for the winter period is no longer true for the summer. As the bi-parametric graph (Fig. 10) shows, for temperature over 18°C, the

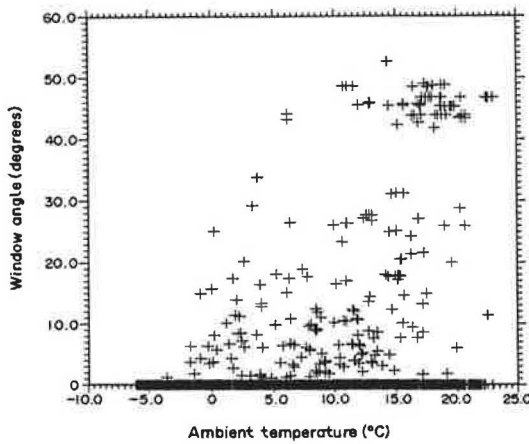


Fig. 9. Bi-parametric graph: window angle vs ambient temperature (winter 1984-85, room GDIR-west).

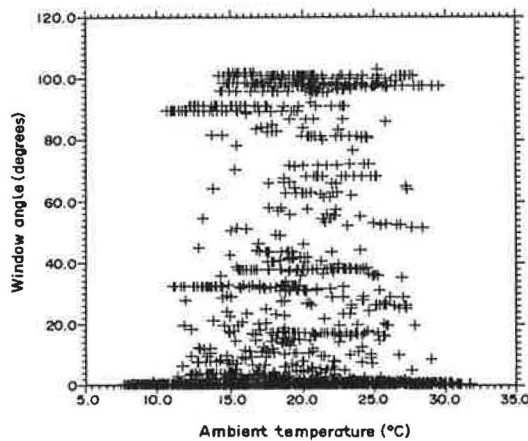


Fig. 10. Bi-parametric graph: window angle vs ambient temperature (summer 1984, room HIT-west).

occurrence of open window as well as the opening angle is independent of the ambient temperature. It appears here that the proposed model for the winter period will not be suitable for the summer period without significant changes.

*Discretisation of  $T_a$  and window opening angle*

Finally it should be stressed out that our model refers to discrete Markov chains.  $T_a$  and the window opening angle were divided into classes. The airflow rate through our single window office rooms vs the opening angle follows a curve described by Warren [9]. In order to obtain meaningful average airflow rates, it is obvious that narrower classes should be chosen at small angles. We set ourselves upon  $[0, 1)$  (closed),  $[1, 15)$ ,  $[15, 35)$ ,  $[35, 60)$ ,  $[60, 90)$ ,  $[90, +\infty)$ . In the model the value taken by a window angle inside a class was the average of the measured angle inside the same class throughout the whole winter. These classes were indicated on the bi-parametric graph. Dense parts of the cloud were isolated and determined the ambient temperature classes:  $(-\infty, 0)$ ,  $[0, 8)$ ,  $[8, 16)$  and  $[16, +\infty)$ . (Fig. 11).

**4. STOCHASTIC MODEL FOR WINDOW OPENING ANGLE DURING WINTER**

*Description of the winter model*

The winter model is based on Markov chains of six states. Each one of the states corresponds to a definite class of window opening angles.

During office hours, that is to say from 8:00 am to 6:00 pm, four different Markov chains realised the link between the ambient temperature and the inhabitant behaviour concerning windows. Each one of them refers to a different class of temperature (taken from  $(-\infty, 0)$ ,  $[0, 8)$ ,  $[8, 16)$ ,  $[16, +\infty)$ ). The four matrices, corresponding to the four chains, were derived for a definite winter and for a precise office room: the matrices elements are the probabilities of moving a window to a certain angle given a certain temperature (Fig. 12) and they depend closely on the inhabitants and the particularities of a room.

During the night and weekend, we have imposed win-

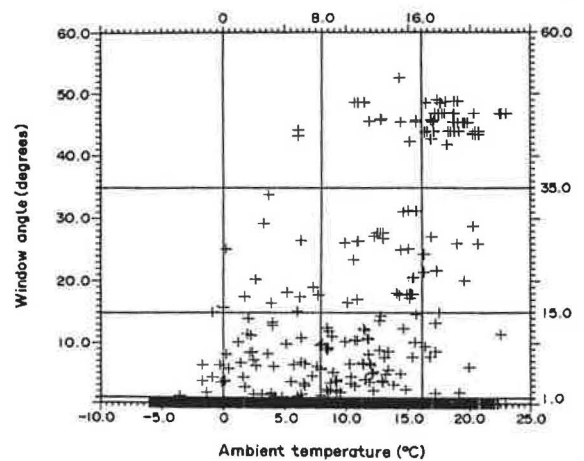


Fig. 11. Window angle classes and temperature classes on the bi-parametric graph.

		room GDIR - east					
		0° C to 8°C class					
output	input	closed	[1,15)	[15, 35)	[35, 60)	[60, 90)	[90, +∞)
closed		8539.10 <sup>-4</sup>	985.10 <sup>-4</sup>	285.10 <sup>-4</sup>	156.10 <sup>-4</sup>	35.10 <sup>-4</sup>	0
[1, 15)		7311.10 <sup>-4</sup>	1103.10 <sup>-4</sup>	1172.10 <sup>-4</sup>	345.10 <sup>-4</sup>	69.10 <sup>-4</sup>	0
[15, 35)		5161.10 <sup>-4</sup>	1774.10 <sup>-4</sup>	1129.10 <sup>-4</sup>	968.10 <sup>-4</sup>	968.10 <sup>-4</sup>	0
[35, 60)		3056.10 <sup>-4</sup>	2778.10 <sup>-4</sup>	833.10 <sup>-4</sup>	1944.10 <sup>-4</sup>	1389.10 <sup>-4</sup>	0
[60, 90)		2857.10 <sup>-4</sup>	357.10 <sup>-4</sup>	1429.10 <sup>-4</sup>	357.10 <sup>-4</sup>	5000.10 <sup>-4</sup>	0
[90, +∞)		.	.	.	.	.	.

Note: . = never reached

		room HIT - east					
		0° C to 8°C class					
output	input	closed	[1,15)	[15, 35)	[35, 60)	[60, 90)	[90, +∞)
closed		9 956.10 <sup>-4</sup>	38.10 <sup>-4</sup>	6.10 <sup>-4</sup>	0	0	0
[1, 15)		7 500.10 <sup>-4</sup>	2 500.10 <sup>-4</sup>	0	0	0	0
[15, 35)		0	1	0	0	0	0
[35, 60)		.	.	.	.	.	.
[60, 90)		.	.	.	.	.	.
[90, +∞)		.	.	.	.	.	.

Note: . = never reached

Fig. 12. Probability matrices for the ambient temperature class [0°C, 8°C] of the rooms GDIR-east and HIT-east.

dow closure. This is due to the fact that only two occurrences of a window opened all night were found during the whole winter and for the four rooms considered.

*Generation of window angle time series*

The technique used to reproduce synthetic data of window opening angle refers to the inverse function method [10]. This method is commonly used with stochastic processes and therefore we will just present it roughly here.

Figure 13 illustrates the inverse function method. It allows the generation of time series of a stochastic process given its distribution function. The only requirement is to dispose of a random number generator with a constant probability density function between 0 and 1. The gen-

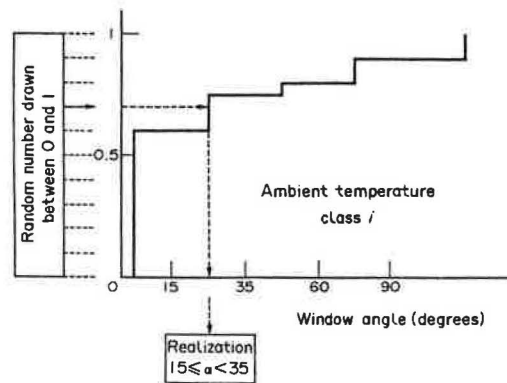


Fig. 13. Generating a new window position for a given temperature class (inverse function method).

erated numbers, between 0 and 1, are compared to the distribution function as shown on the figure: every number given by the generator corresponds to only one class of window opening angles. It has been proved that the distribution function of the generated series using this method is identical to the one used to create them. In our case, these distribution functions are given by the lines of the probabilities matrices as defined in the model description. Figure 14 shows us how to deduce the distribution function given a matrix.

To generate the time series, the procedure is as follows:

The first step is to check the time. If it is during office hours, we choose the appropriate matrix given the ambient temperature and we generate the next state with the inverse function method described above. If it is out of office hours then the next state of the window is closed. The next state becomes the present state and is stored. We then come back to the first step. Table 2 summarises the procedure and Fig. 15 describes it. For comparison purposes, synthetic and real time series of window angle are reproduced in Fig. 16.

5. VALIDATION OF THE MODEL

In order to validate our model, the major characteristics of the generated data were compared to reality.

The first stage was the comparison of the autocor-

Table 2. Procedure for the generation of synthetic time series of window angle

Steps	Operations
# 1	Check the time, if it is not in the office hours the window is closed and go to # 5
# 2	Choose a Markov matrix according to the outdoor temperature
# 3	Build the distribution function from a line of the matrix (# of the line corresponds to # of precedent class of the window angle)
# 4	Generate a new realisation for the window position for the next half hour
# 5	Memorise the window position or window angle class
# 6	Start in # 1 for the next half hour.

relation and intercorrelations calculated from the synthetic and real time series of window opening angle. The general shapes of the autocorrelation are very similar (see Fig. 17). It is therefore possible to conclude that the time dependance was respected: for a given temperature, both the window represented by the synthetic data and the real window stay open the same amount of time. The intercorrelations between the window opening angle and the ambient temperature were also considered. It is clear that the link is very strong in both cases (Fig. 18).

Then we studied the average angle over the winter. Figure 19 represents the histogram of the average of 14

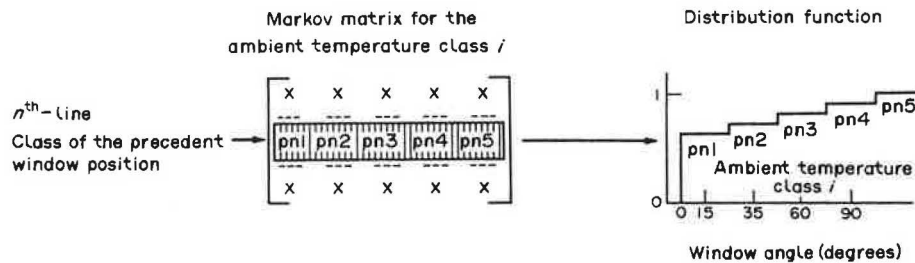


Fig. 14. Building the distribution function from the lines of the Markov probability matrix.

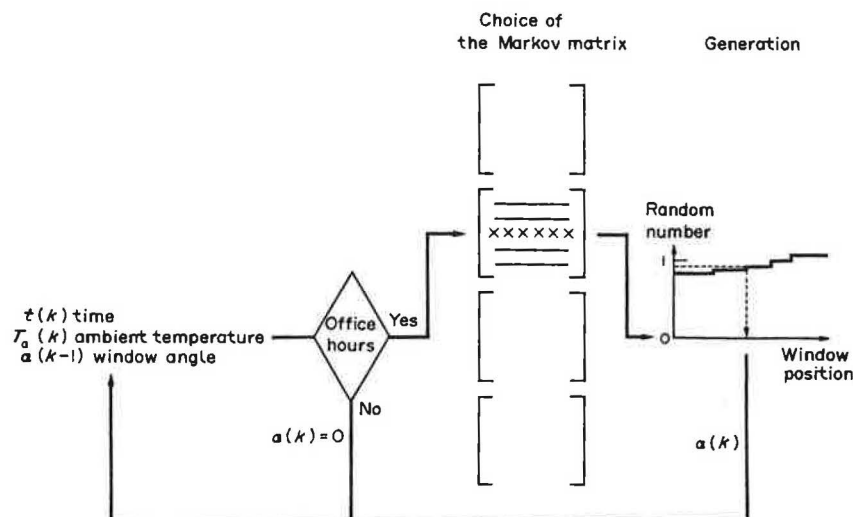


Fig. 15. Procedure for the stochastic generation of window opening angle.

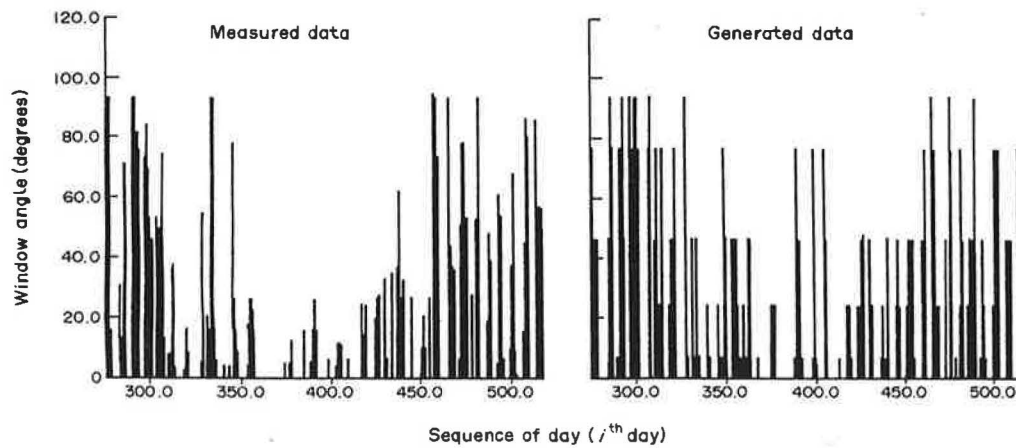


Fig. 16. Real and synthetic time series of window opening angle (winter 1984-85, room GDIR-west).

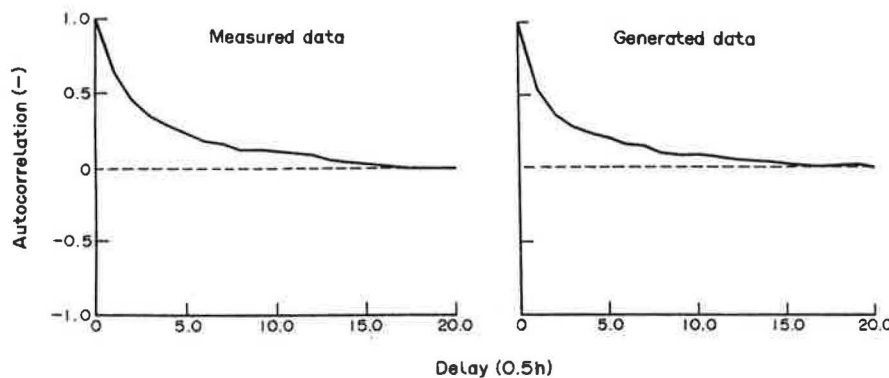


Fig. 17. Autocorrelation function of the measured and generated series (winter 1984-85, room GDIR-west).

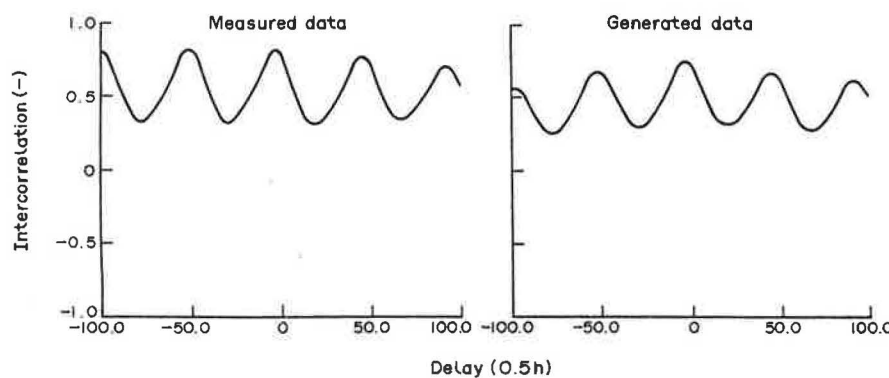


Fig. 18. Intercorrelation function between the window angle and the ambient temperature (winter 1984-85, room GDIR-west).

simulations (14 winters). The mean of this histogram was computed and a 95% confidence interval was estimated [11]. The measured mean was found to be in the interval in all four offices considered.

And last, the histogram of generated (14 simulations for each room) and measured probabilities of finding the window open at an angle within a certain class were compared (Table 3). The comparison is very satisfying.

## 6. CONCLUSION

A stochastic model of the window opening angle for office room has been developed [12].

This model is simple and refers to a basic stochastic process: Markov chains. The day is divided into two periods: office hours and night or weekend. During the night or weekend, all windows are closed. During office

Table 3. Comparison of the probabilities of finding a window angle within a certain class for the real and synthetic data.  $\chi^2$  test

Opening angle Class	Solar Unit HIT-west		Solar Unit HIT-east	
	Measured probabilities	Calculated probabilities	Measured probabilities	Calculated probabilities
Closed	$9786.10^{-4}$	$9791.10^{-4}$	$9938.10^{-4}$	$9938.10^{-4}$
[1, 15)	$111.10^{-4}$	$113.10^{-4}$	$44.10^{-4}$	$43.10^{-4}$
[15, 35)	$45.10^{-4}$	$51.10^{-4}$	$7.10^{-4}$	$7.10^{-4}$
[35, 60)	$58.10^{-4}$	$45.10^{-4}$	$11.10^{-4}$	$12.10^{-4}$
[60, 90)	0	0	0	0
[90, +∞)	0	0	0	0

Opening angle Class	Solar Unit GDIR-west		Solar Unit GDIR-east	
	Measured probabilities	Calculated probabilities	Measured probabilities	Calculated probabilities
Closed	$9605.10^{-4}$	$9608.10^{-4}$	$9235.10^{-4}$	$9252.10^{-4}$
[1, 15)	$164.10^{-4}$	$174.10^{-4}$	$283.10^{-4}$	$280.10^{-4}$
[15, 35)	$84.10^{-4}$	$71.10^{-4}$	$171.10^{-4}$	$166.10^{-4}$
[35, 60)	$79.10^{-4}$	$75.10^{-4}$	$103.10^{-4}$	$104.10^{-4}$
[60, 90)	$69.10^{-4}$	$72.10^{-4}$	$188.10^{-4}$	$182.10^{-4}$
[90, +∞)	0	0	$20.10^{-4}$	$16.10^{-4}$

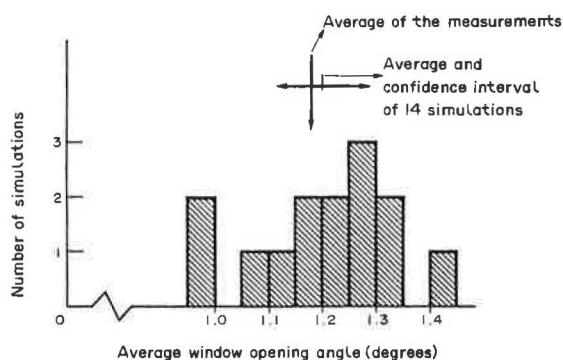


Fig. 19. Histogram of the averages of 14 simulations of the opening angle, compared to the measured average on the whole winter (room GDIR-west).

hours, the outside temperature acts as a driving variable for window opening or closing.

A validation was conducted on the generated series. The major statistic characteristics were compared and found to be similar.

In order to use this model for other offices, it is necessary to have at disposal time series of window openings covering a significant period of time. The Markov matrices computed with these measurements will take into account all the particularities of the room (such as the window size, the volume of the room etc.) and of the users. The only restrictions are different office hours which will affect the length of the night period, as well as the presence of air conditioning devices or mechanical ventilation which could suppress the correlation of the window opening with the ambient temperature.

The lack of adequate measurements and therefore the impossibility to compute the Markov matrices is a difficult problem. In order to transpose the model to such an office and keep a reasonable fit with reality, selected occupant's behaviour and room geometry should be extracted and standard matrices computed. This is planned for a future study.

**Acknowledgement**—This work has been supported by the Swiss Foundation for Energy Research (NEFF). It was carried out as part of the Swiss national research program "ERL—Energie-relevante Luftströmungen in Gebäuden".

## REFERENCES

1. H. Feustel and V. M. Kendon, Infiltration models for multicellular structures: a literature survey. Lawrence Berkeley Laboratory, LBL-17588 (1985).
2. Inhabitant behaviour with respect to ventilation—a summary report of IEA Annex VIII. Technical notes AIVC 23 (1988).
3. J.-L. Scartezzini, A. Faist and J.-B. Gay, Experimental comparison of a sunspace and a water hybrid solar device using the LESO test facility. *Sol. Energy* 38, 355–366 (1987).
4. A. Fewkes and S. A. Ferris, The recycling of domestic waste. A study of the factors influencing the storage capacity and the simulation of the usage patterns. *Bdg Envir.* 17, 209–216 (1982).
5. J.-L. Scartezzini, C. Roecker and D. Quévit, Continuous air renewal measurements in an occupied solar office building. Proceedings of Clima 2000 World Congress on Heating, Ventilation and Air Conditioning, Zurich (1985).
6. J. B. Kemeny and J. L. Snell, *Finite Markov Chains*. Springer, New York (1976).
7. F. Hainard, P. Rossel and C. Trachsel, Attitudes et comportements en matière d'aération en immeuble locatif urbain. Rapport final Office Fédéral de l'Énergie (1986).
8. C. Vezin and N. Lorimy, "Comportement énergétique des usagers en matière d'aération. Rapport final Office Fédéral de l'Énergie (1985).



9. P. R. Warren, Ventilation through openings on one wall only. *Energy Conservation in Heating, Cooling and Ventilating Building*, p. 189. Hemisphere, Washington, D.C. (1978).
10. M. S. Bartlett, *An Introduction to Stochastic Processes: Methods and Applications*. Cambridge University Press (1979).
11. L. L. Chao, *Statistics: Methods and Analysis*. McGraw Hill (1974).
12. J.-L. Scartezzini, R. Fritsch, A. Kohler and M. Nygård Ferguson, Etude Stochastique de Comportement de l'Occupant. Project NEFF 339.5 LESO-PB, EPFL (1990).