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A CRITICAL SURVEY OF MATHEMATICAL MODELS FOR PREDICTING ROOM AIR MOVEMENT

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ABSTRACT

An overview of some of the mathematical models for predicting room air movement is presented. It shows that there is a need for a simplified model which can be used by air conditioning design engineers.

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1.0 INTRODUCTION

A survey by Building Research Establishment in 1972 estimated that in the U.K. more than 40% of the primary energy consumed takes place within the building services sector (1). Against a background of rapidly rising energy prices, the need for energy conservation is obviously important.

Interest in energy conservation has led to the development of a number of computer based dynamic thermal models in the design of buildings. A recent survey by Irving (2) yielded a list of over 40 such methods. However, most of these models are concentrated on simulating the dynamic response of the building fabric with little emphasis placed on modelling environmental services systems.

The energy consumed by environmental services systems is mainly to provide space heating/cooling, water heating, lighting and other sundry usage with space heating/cooling accounts for more than 60% of the total consumption. Provision of space heating/cooling to a built environment can be in various forms; from radiator system in a domestic house to full air conditioning system in an office block. With the ever increasing use of electronic equipment that produces heat, air conditioning systems account for a major share of the energy consumed.

In the U.K. air conditioning systems have traditionally been designed to offset gross heat gains/losses from occupied spaces without sufficient attention being devoted to the local pattern of air movements. Human discomfort can arise due to draught sensations if local air velocities exceed about 0.15 to 0.24 m/s, depending on prevailing air temperature (3-12). Conversely, a feeling of stuffiness may be experienced if the air velocity falls below 0.05 m/s (8-9). As energy consumed by an air conditioning system is mainly to maintain satisfactory environmental standards within the occupied space it is therefore essential that air conditioning engineers understand the air patterns of an occupied space into which conditioned air is being injected.

2.0 AIR MOVEMENT IN VENTILATED ROOMS

In most mechanically ventilated rooms an air jet is used to diffuse air into the required space via opening or slot. The behaviour of the air stream is very similar to that of a classical jet in nominally stagnant surroundings. Near the opening, the outer part of the air stream will exchange momentum with the surrouning room air due to viscous action. The room air is given momentum in the direction of the air stream and outer parts of the streams are correspondingly slowed. Air entrained by the jet is replaced by room air, not in the vicinity of the jet, which flows toward the air stream. This process is a continuous one along the length of the stream, which becomes progressively wider as the forward volume flow increases. At a certain distance downstream of the opening where the stream velocities are comparable with those in the room, the air stream will spread out in all directions and becomes indistinguishable from its surrounds.

To develop a computer-based mathematical model which would simulate the air pattern of a mechanically ventilated room, it is necessary to study the fundamental equations of fluid dynamics and the characteristics of the governing air jets.

2.1 FUNDAMENTAL EQUATIONS OF FLUID DYNAMICS

It is generally accepted that the soluton of any flow problem involves being able to evaluate certain unknown quantities within the flow, in terms of position in space at any instant of time. These unknowns are the three components of velocity u, v and w, the pressure P and the density ρ in terms of the spatial co-ordination x, y and z and time t. The determination of these unknowns in laminar flow involves the solution of;

(1) The law of conservation of mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \qquad (1)$$

(2) The law of conservation of momentum with body forces omitted:

$$\frac{\rho D\phi}{Dt} = - \text{grad } P + \mu \nabla^2 \phi \qquad (2)$$

$$\text{where} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

However, most fluids in motion are turbulent, that is, a motion in which an irregular fluctuation is superimposed on the main stream. This is due to the breakdown of orderly flow into eddies which spread to 'contaminate' a region of the flow with irregular fluctuating motions. Turbulent motion, once established, has a random nature making it difficult to describe exactly. Because of this nature, the components of velocity and pressure at any point can be decomposed into a time-averaged value plus a randomly varying perturbation. When averaged over some finite time interval, the fluctuation components are equal to zero. Thus, if the sum of the time-averaged and fluctuating of both velocity components and pressure are substituted for the instantaneous values, the equations of motion for an incompressible fluid become;

(1) The law of conservation of mass:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad (3)$$

(2) The law of conservation of momentum with body forces omitted:

$$\left.\begin{aligned}
\frac{\rho D\bar{u}}{Dt} + \frac{\partial \rho \overline{u'^2}}{\partial x} + \frac{\partial \rho \overline{u'v'}}{\partial y} + \frac{\partial \rho \overline{u'w'}}{\partial z} &= -\frac{\partial \bar{P}}{\partial x} + \mu \nabla^2 \bar{u} \\
\frac{\rho D\bar{v}}{Dt} + \frac{\partial \rho \overline{u'v'}}{\partial x} + \frac{\partial \rho \overline{v'^2}}{\partial y} + \frac{\partial \rho \overline{v'w'}}{\partial z} &= \frac{\partial \bar{P}}{\partial y} + \mu \nabla^2 \bar{v} \\
\frac{\rho D\bar{w}}{Dt} + \frac{\partial \rho \overline{u'v'}}{\partial x} + \frac{\partial \rho \overline{v'w'}}{\partial y} + \frac{\partial \rho \overline{w'^2}}{\partial z} &= \frac{\partial \bar{P}}{\partial z} + \mu \nabla^2 \bar{w}
\end{aligned}\right] \qquad (4)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

2.2 FUNDAMENTALS OF AIR JETS

When an air jet emanates from an opening into a stagnant environment, the stream behaves similar to that of a turbulent free jet. Due to this similarity the known characteristics of turbulent jet (13-15) have been used by researchers to describe the flow and thermal fields within mechanically ventilated enclosures.

The development of a turbulent free jet consists of zone of flow establishment and zone of established flow. Figure 1.1 illustrates the development of a free jet, where the two zones are identified. The flow establishment zone exists close to the opening and the length of this zone is usually very much smaller than that of the zone of established flow. Due to the differences in velocity between the air jet and the surrounding environment which is stagnant, the kinetic energy of the jet will be steadily dissipated through reaction with the surrounding air. This resulting in the generation of high shear eddies which will immediately give rise to a lateral mixing process which progress both inward and outward with distance from the opening. Owing to this diffusion process, stagnant air is entrained by the jet which spreads out with its velocity decreasing. The limit of this zone is when the diffusion process has penetrated to the centre line of the jet.

Once the entire central part of the jet has become turbulent, the flow may be considered as fully established, the diffusion process continues thereafter without essential change in character. Further entrainment of the surrounding stagnant air by the expanding eddy region is now balanced inertially by a continuous reduction in the velocity of the entire central region.

In the fully developed region, most experimental observations suggest that the longitudinal velocity profiles in any section of the jet lie along a similiar curve. Figure 1.2 illustrates this aspect very vividly. However, the transverse velocity is small when compared with the longitudinal velocity. In most engineering problems involving jet theory the tranverse velocity components are disregarded.

Most workers treatment of the turbulent free jet problems are of a boundary-layer nature. The region of space in which a solution is being sought does not extend far in a transverse direction, as compared with the main direction of flow and that the transverse gradients are large. In this connections, Prandtl's mixing length theory (16) has been widely used as an aid to study the turbulent free jet problems.

Tollmien (17) was the first worker who successfully used the Prandtl's mixing length theory to analyse the problem of turbulent jet mixing of incompressible fluid in 1926. Tollmien used the formula:

$$\tau_{xy} = \pm \rho L^2 \frac{\partial u}{\partial y}\left|\frac{\partial u}{\partial y}\right| \qquad (5)$$

where L - mixing length

obtained a solution of the dimensionless velocity component along the x-axis of the boundary layer of a turbulent free jet,

$$\frac{u_m}{u_o} = \frac{1.21}{(a_1)^{1/2}\,(x/b_o)^{1/2}} \qquad (6)$$

where a_1 is a coefficient to be determined experimentally.

The boundary conditions used by Tollmien are as follows;

(1) At the inner edge of the boundary layer:

$$y = 0;\ \frac{u}{u_m} = 1;\ v = 0;\ \tau_{xy} = 0;\ \frac{y}{a_1 x} = 0$$

(2) At the outer edge of the boundary layer:

$$y = \infty;\ \frac{u}{u_m} = 0;\ \tau_{xy} = 0;\ \frac{y}{a_1 x} = \infty$$

Abramovich (13) used the experimental results of Forthmann (18) and others to obtain a value for the coefficient a_1 which varied from 0.09 to 0.12.

A simpler solution based on Prandtl's hypothesis was obtained by Goertler when he studied the two-dimensional turbulent jet. Gortler (19) used the equation of Prandtl written as

$$\tau_{xy} = \rho \epsilon \frac{\partial u}{\partial y} \qquad (7)$$

where ϵ is known as the coefficient of kinematic eddy viscosity. The boundary conditions Goertler used are as follows;

(1) At the inner edge of the boundary layer:

$$y = 0;\ \frac{u}{u_m} = 1;\ \tau_{xy} = 0;\ v = 0;\ \frac{\delta y}{x} = 0$$

(2) At the outer edge of boundary layer:

$$y = \infty;\ u = 0;\ \tau_{xy} = 0;\ \frac{\delta y}{x} = \infty$$

where δ is another coefficient to be determined experimentally. Goertler obtained two solutions and they are:

$$\frac{u}{u_m} = 1 - \tanh^2\left(\frac{\delta y}{x}\right) \qquad (8)$$

$$\frac{v}{u_m} = \frac{1}{\delta}\left(\frac{\delta y}{x} - \frac{\delta y}{x}\tanh^2\frac{\delta y}{x} - 0.5\tanh\frac{\delta y}{x}\right) \qquad (9)$$

Based on the Reichardt's experimental results (20) obtained, Goertler found that equals 7.67. Both Tollimen and Goertler expressions assumed that the turbulent flow has been established and the virtual origin of the jet is located at the opening itself. This resulting in the axial distance x being measured from the opening.

As most theoretically researchers solved turbulent flow problems by using certain assumptions, some workers initiating the use of semi-empirical hypotheses to evaluate the mean velocity distribution within the turbulent free jet. The advantage of such an approach is that no assumption as to the distribution of turbulence is required.

Albertson et al (21) measured the mean velocity distribution in both two and three dimensional air jets and used simple dimensional considerations to interpret their measurements. The considerations used were based on that if Reynold number for fluid efflux from a submerge boundary outlet is not too low, the mean velocity v at any point shall depend on the co-ordinate x,y,z, on the efflux velocity U_o and on a linear dimension L_o characteris the particular outlet form.

For the zone of established flow for two-dimensional jet:

$$\log_{10} \frac{u}{u_o}\left(x/b_o\right)^{1/2} = 0.36 - 1.84\left(\frac{y}{x}\right)^2 \qquad (10)$$

and for three-dimensional jet:

$$\log_{10} \frac{u}{u_o}\left(x/D_o\right) = 0.79 - 33\left(\frac{r}{x}\right)^2 \qquad (11)$$

The behaviour of isothermal air jets was also studied experimentally by Farquharson (22) . He used the dimensional consideration that the variation of axial velocity with length of travel follows simple hyperbolic law.

$$\frac{u}{u_o} = Kv\left(\sqrt{Ae}/x\right) \qquad (12)$$

Frean et al (23) subsequently extended the original experimental work of Farquharson to deal with the case of a warm air jet injected into a room. Based on the measurements of this study, Frean et al suggested that the value K_v of was 7.2.

As distinct from the work of other researchers which using empirical approach to study turbulent flows, Reichardt (20) postulated an inductive theory of free turbulent flows . The phenomena was based on his examination of experimental data on free turbulent flows which indicated that an analogy exists between the differential equation of thermal conduction and turbulent momentum. Despite the theory is purely phenomenological it makes it possible to change nonlinear flow equations to linear differential equations. Solutions of such linear equations may be superimposed.

In its present form Reichardt's theory does not fully explain the invariance and precision of relationships which may be shown to exist in turbulent flow. After careful study of the experimental data Scott (24) applied the inductive theory to postulate the existence of a single universal constant which appears to govern turbulent flow phenomena.

Scott assumed that for all the flows considered a flow Reynolds Number exists of the form

$$\frac{U_c l_c}{\nu_T} = \frac{1}{\gamma} \qquad (13)$$

where γ is a universal constant, ν_T is the eddy viscosity, U_c and l_c are respectively a characteristic velocity and a characteristic length of the flow system.

Based on this flow Reynolds Number hypothesis Scott demonstrated mathematically that

$$\frac{\partial (u)^2}{\partial x} = \gamma l_c \frac{\partial^2 (u^2)}{\partial y^2} \qquad (14)$$

Solutions to equation may be obtained by using values appropriate to various jet flow configurations and by comparing these solutions with experimental results for the mean velocity distribution, values of the constant γ is 0.0072.

For the two-dimensional jet, Scott used a Gaussian distribution function to describe the velocity distribution of the flow,

$$\frac{U_x}{U_m} = \mathrm{Exp}\left(-\frac{y^2}{8\gamma \ell_c^2}\right) \qquad (15)$$

where $U_m \propto (x-a)^{-1/2}$

$\ell_c \propto b(x-a)$

The values obtained for the empirical constants a and b are 1.57d and 0.5 respectively where d is the slot width.

3.0 MATHEMATICAL MODEL FOR THE TURBULENT FLOW PROBLEMS

All fluids in motion are governed by the 'conservation laws', therefore prediction of the flow pattern requires the solution to these equations.

The calculation procedure generally adopted by most workers are to express the 'conservation laws' in terms of partial differential equations and these equations are integrated over a computational grid which fills the domain of interest. The discretization of the flow domain is obtained by dividing the flow domain into a finite set of small sub-domains, each surrounding a node of the computational grid. The link between the dependent variables of the differential equations and other quantities in these equations, such as fluid properties, exchange coefficients and sources is provided by certain algebraic equations. These algebraic equations are obtained from thermodynamic relationships like the equation of state, from hypotheses about the physical processes, or from generalizations of experimental data. The difference equations are then solved by the use of tri-diagonal matrix algorithm.

Work by Spalding and his team (25-28) on the numerical study of two- and three-dimensional flows have produced computational codes such as TEACH, CHAMPION and SIMPLE which are extensively used by many workers.

Nielsen et al (29) reported measured and calculated values of the velocity characteristics of a scale-model room with high and low level slot arrangements. The calculated values were obtained by using the TEACH computer code (26) to solve the two-dimensional, elliptic, partial-differential equations representing 'conservation laws' and the validity of these values is assessed by comparison with measurements obtained by laser-Doppler anemometry in a model room. They concluded that it is possible to use a two-dimensional calculation procedure to represent the velocity characteristics of a ventilated room.

Gosman et al (30) measured and calculated the isothermal flow field in a rectangular enclosure having a square high side-wall register. The work was similar to that of Nielsen (29) except a three-dimensional version of TEACH computer code was used to include the third velocity component. They reported calculated values were in good agreement with experimental measurements and other available experimental data. However, Gosman et al were concerned that their calculation procedure requires a substantial number of computational grid nodes therefore a simpler flow model might be more appropriate to represent the flow patterns associated with ventilated rooms.

Hammond and his co-workers (31) have developed a high-level model to simulate flow and thermal fields within a warm air heated enclosure. The model, ESCEAT code, is based on the solution methods developed by Spalding and his team (25-28). Not only is the code capable of simulating complex flows, the ESCEAT code also generates the convective heat transfer data within the enclosure. Hammond et al concluded that high-level flow models are capable of providing accurate prediction of the flow and thermal field that would be needed to determine the occupation zone thermal comfort conditions. However, the central processor unit time required per run of the code makes the high-level model unsuitable to be generally used by design engineers. Recognising this, Alamdari (32) developed an intermediate-level model, ROOM-CHT code, to provide a realistic approach to study room air movement.

The ROOM-CHT code was designed to simulate the internal condition of an enclosure into which warm air is being injected. It makes informed estimates of the flow and thermal fields based on two- and three-dimensional turbulent wall-jet characteristics. These jets are assumed to emanate from the supply aperture and then flow sequentally over the room surfaces. The code was orginally designed to provide the heat transfer data necessary for modern building thermal models. Based on the ROOM-CHT code, Chan (33) developed a simple computer code to determine thermal comfort data which accounted for the local flow and thermal fields variation near the cold window, due to the downdraught effect. The flow field data obtained were in agreement with those obtained in a scale-model room using sophisticated measuring techniques. Hammond and his co-workers agreed that intermediate-level models with fairly simple, easy to handle characteristics can be a viable tool for ventilation design purposes.

Hutt (34) has developed a model which divides the flow within enclosed spaces into regions of high and low movement characterized by the inlet jet and induced surrounding air. This theorem is deduced from observations of room air movement under experimental conditions that away from the jet, viscosity and turbulence appeared to have no influence on the distribution of room air velocity. The jet equation used was the Reichardt equations modified by the inclusion of Scott's universal scale constant for turbulence while the rest of the space is modelled by ideal flow. Predictions from the model gave an average absolute error of 30% when compared with those obtained from more complex models. Hutt concluded that it was feasible to use a simplified model to predict room air movement.

4.0 CONCLUDING REMARKS

Generally, research into air movement within an enclosure is concentrated on the application of high-level mathematical models. The models developed so far have been too specific, therefore there is a need for simplified models which can be used by companies with minimum expertise in computational fluid dynamics.

The aim of the present study by the author is to develop such a model for general use in air-conditioning industry. The model to be developed will be very similar to that of Hutt. The air flow field will be predicted by dividing the room into regions of high and low movement. The model will incorporate the boundary conditions for the interface between the areas of high and low movement and for the areas adjacent to the physical boundaries. The inclusion of this arrangement will improve the accuracy of the prediction and provide a realistic simulation of air flow within the room.

The work will also include the prediction of velocity and temperature conditions within a ventilated room and relate the data obtained to a thermal comfort criterion. The resulting information will also be compared with predictions made by more complex models and with experimental results.

Experimental works will be carried out in the test room in the Institute of Environmental Engineering of South Bank Polytechnic. Variation of the type and position of air inlets and outlets as well as room dimensions will be carried out to study the flow variations. Observations obtained from these experimental works will be incorporated into the model to enhance the flexibility of the intermediate-level computational model generated.

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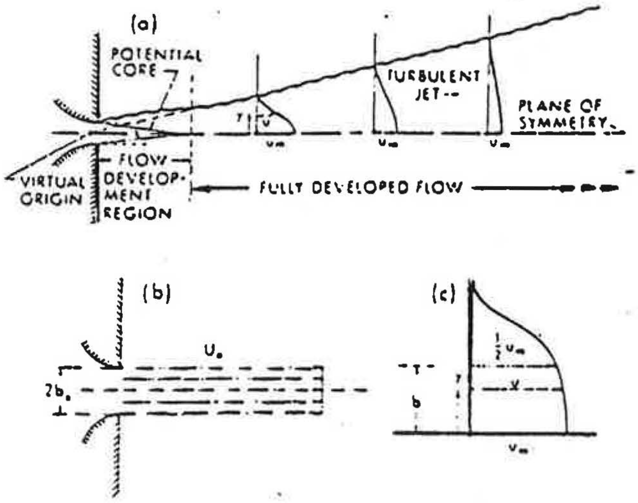

Fig. 1-1. Definition sketch of plane turbulent free jets.

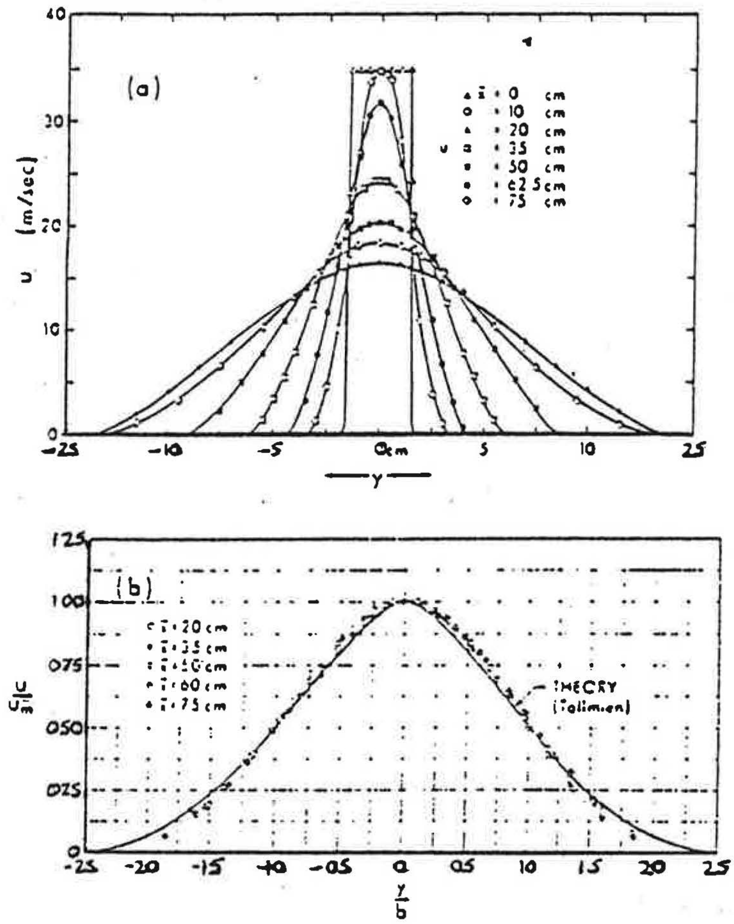

Fig. 1-2. Velocity distribution for plane turbulent free jets (Förthmann, 1934).