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Numerical predictions of three-dimensional flow in a ventilated room using turbulence models

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A three-dimensional recirculation flow in a ventilated room was predicted by the numerical methods in which the turbulence models are applied. The predicted results are compared with the experimental results obtained in a model room in order to estimate the practical utilities of such methods from the viewpoint of engineering. Taking account of the practicability of prediction method which the engineers regard as important, two turbulence models were selected and they were incorporated into the numerical prediction methods respectively. One is the two-equation model, in which transport equations of turbulence energy and its rate of dissipation are adopted. The other is the Deardorff's model, in which the subgrid scale eddy coefficient is utilized. The prediction was made by each numerical method. Consequently, no noticeable difference is recognized between both predicted results. Each result is compared with the experimental results. Generally speaking, each agreement is good with regard to the mean velocity. Thus we can conclude that the numerical method using the two-equation model has more practical utility than that using Deardorff's model, because it can give the solutions in a shorter computer time.

Introduction

Prediction of the distributions of velocity, temperature, and concentration in a room is not only available for designing air-conditioning systems, but is also important in providing amenity in a room. Recent advancements in numerical methods for fluid-mechanics have been accompanied by investigations on numerical prediction of these distributions. However, in that case, because the air flow in a room is turbulent, we must apply the numerical method in which the effects of turbulence on mean flow are taken into account. Thus it is necessary to discuss the numerical methods of predicting the general turbulent flow. It can be considered that those methods which have been developed are roughly divided into the following two types according to the physical meaning of the averaging operator to be used on modelling turbulence. One is a type of method in which one of the turbulence models devised independently of the numerical method is applied. The two-equation model, which was studied by Davidov,[1] Harlow-Nakayama[2] and others, is most typical of such models. The other is a type of method which is called 'large-eddy simulation'. This method was investigated by Smagorinsky *et al.*[3] and Lilly.[4] In the former the averaging operator indicates the ensemble average or the time average for the duration that is longer than the time scale of the largest eddy in the turbulent flow to be solved. But in the latter it denotes the space average over the grid volume of the finite-difference scheme applied to the numerical calculation. Thus the eddies that are larger than the grid scale can be analysed by solving the partial differential equations for the averaged motion. In the earlier formulations of both methods, Reynolds' stresses were assumed to be connected with the strain tensor of the mean flow by a scalar eddy viscosity. Later the investigators' interests have been directed to development of the stress-equation models[5–7] in which Reynolds' stresses themselves are dependent variables, because the relations between

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stresses and strains assumed in the earlier formulations seemed to give rise to erroneous predictions of turbulence quantities.

The purpose of this work is to estimate the practical utilities of those numerical methods from the viewpoint of engineering. Thus the stress-equation models were excluded from the aims of this work as, first they are too complicated to obtain the numerical solutions of the whole flow field in a short computer time and secondly the exact prediction of turbulence quantities is not so much required in the problem of room environment as in fluid-mechanics. The first requirement of this problem is to predict the distributions of velocity, temperature and concentration averaged during the period that is much longer than the time scale of the energy-containing eddy. We must note there are not only jets and boundary layers but also some recirculations in a ventilated room. Consequently the numerical method using the two-equation model and the large-eddy simulation method utilizing the sub-grid scale eddy coefficient were selected as the numerical methods that should be discussed in this work. The transport equations of turbulence energy and its rate of dissipation are adopted in the two-equation model. The latter method was developed by Smagorinsky *et al.*[3] and then was extended to the study of channel flow by Deardorff.[8] The modelled turbulence which was used in this method is referred to as 'Deardorff's model' in this paper. Generally, in the large-eddy simulation, it is necessary to calculate the unsteady solutions for a long period, even when only time mean flow is required. Nevertheless the Deardorff's model was adopted because of its simplicity. That is, in this model a single universal constant appears on formulating turbulence.

The numerical method using the two-equation model has been successfully used to predict various flows, as recommended by Launder-Spalding.[9,10] A few results have already been reported which were obtained by applying this method to the flow in a ventilated room.[11,12] As compared with those studies, this work has the following features. First the three-dimensional numerical method is employed. Secondly two different turbulence models, namely, the two-equation model and the Deardorff's model are applied. Also the results predicted by both models are compared with each other. These results are then compared with the experimental results which were obtained by measuring three components of velocity at many points in the ventilated model room space with a supersonic anemometer. Finally, care must be taken that the problem studied in this work was confined to the isothermal flow for simplification of a problem. The isothermal flow is the most fundamental type of air flow in a ventilated room. To add temperature field to it is far easier than to solve itself.

Mathematical formulation

Conservation equations

For the isothermal incompressible turbulent flow, the equations for conservation of momentum and for continuity can be described as follows, see Nomenclature for definition of terms:

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j}(U_iU_j) = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j}(\overline{u_iu_j}) \quad (1)$$

$$\partial U_i/\partial x_i = 0 \quad (2)$$

where the stresses due to viscous forces are neglected, because Reynolds' stresses $-\rho\overline{u_iu_j}$ are much larger than

Two-equation model of turbulence

According to Hinze,[13] Reynolds' stresses are assumed

$$-\rho\overline{u_iu_j} = \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\rho k\delta_{ij}$$

The transport equations of turbulence energy and its rate of dissipation recommended by Launder-Spalding[9,10] are written:

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j}(kU_j) = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + S - \epsilon \quad (4)$$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x_j}(\epsilon U_j) = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_j}\right) + C_1\frac{\epsilon}{k}S - C_2\frac{\epsilon^2}{k} \quad (5)$$

where molecular diffusion is ignored and:

$$S = \frac{\mu_t}{\rho}\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad (6)$$

The turbulent viscosity μ_t is defined as:

$$\mu_t = C_D\rho k^2/\epsilon \quad (7)$$

In this work, the constants appearing above are given the following values:

$$\sigma_k = 1.0,\ \sigma_\epsilon = 1.3,\ C_1 = 1.59,\ C_2 = 2.0,\ C_D = 0.09$$

Jones-Launder[14] made an attempt at replacing C_1 by an algebraic function of turbulence Reynolds number to analyse more precisely a turbulent boundary layer, which has the weak turbulence region in very close to the wall. But such a modification was not adopted in this work, because the flow in a ventilated room consists largely of injected jets and recirculations. The boundary layers near the walls were assumed to be included in the boundary conditions.

Deardorff's model of turbulence

The overbar $\overline{\quad}$ used in this model implies averaging over the grid volume. Thus the subgrid scale components are filtered out from all dependent variables. Reynolds' stresses are assumed to be similar to (3) in form, namely:

$$-\rho\overline{u_iu_j} = \rho K\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{1}{3}\rho\delta_{ij}\overline{u_ku_k} \quad (8)$$

where K is the subgrid scale eddy coefficient and has a physical meaning different from μ_t/ρ. Smagorinsky *et al.*[3] estimated K as follows:

$$K = (ch)^2\left\{\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right\}^{1/2} \quad (9)$$

where c is a dimensionless constant and h is a grid interval. Deardorff[8] evaluated 0.1 for c, and this value is also used in this work.

Solution procedure

The Marker-and-cell (MAC) method,[15,16] in which three velocity components and pressure are taken as dependent variables, is very convenient to solve the equations for conservation of momentum and for continuity. But the forward finite-difference scheme used for time differential in the original MAC method has a second-order error. When solving the unsteady flow, this error not only makes the solutions incorrect but upsets the computation itself on account of the numerical instability caused by the negative numerical diffusion. Therefore in this work the Adams-Bashforth scheme[17] having a second-order accuracy was employed for the time differential. This scheme, for $\partial U_i/\partial t = R_i$, is:

$$(U_i^{n+1} - U_i^n)/\Delta t = \frac{3}{2} R_i^n - \frac{1}{2} R_i^{n-1} \tag{10}$$

where $n\Delta t = t$. The same finite-difference schemes that are used in the MAC method were applied to space differentials. The schemes of advection terms have the property of conserving the total kinetic energy even on the field of discrete quantity. The partial differential equations for turbulence could be solved by the same numerical technique as was applied to the equations for motion.

Any quantity was made dimensionless by dividing by a representative quantity having the same dimension and composed of the bulk velocity of a supply outlet U_0 and its width L_0, before the partial differential equations were reduced to the finite-difference equations. In the prediction by the two-equation model, time integration was started from the initial conditions of $U_i = 0$ in the room and was continued until the solution was steady. On the other hand, in the prediction by the Deardorff's model, the solutions from dimensionless time $t = 180$ to 780 were regarded as available for the calculation of statistical quantities and were stored on magnetic tape. The initial conditions were the same as in the prediction by the two-equation model and $\Delta t = 0.2$.

Geometry and boundary conditions

Figure 1 illustrates the model room used to predict and measure the velocity distribution. This room is a $2 \times 2 \times 2$ m cube with a supply outlet at the centre of the ceiling and an exhaust inlet at one of the walls. Both the outlet and inlet are $2/9 \times 2/9$ m square. In the predictions the space in the room was divided into $18 \times 18 \times 18$ cells (or grid volumes) having the same shapes as the cubes.

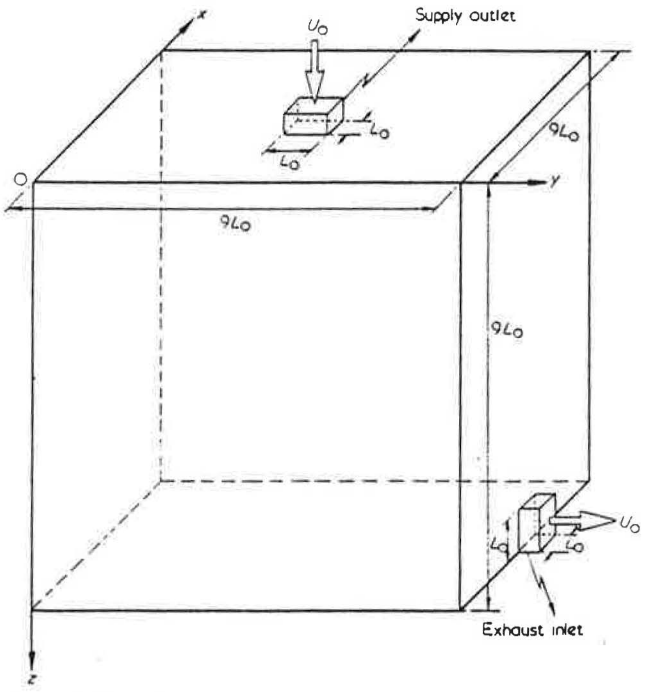

Figure 1 Geometry of an experimental model room

The boundary conditions in both predictions were arranged on the same assumptions. The conditions satisfying the wall law were employed for all wall boundaries. For example, if the wall is located in the x–y plane at $z = 0$, the boundary conditions for U, V, W, k and ϵ are given as:

$$\left(\mu_t \frac{\partial U}{\partial z}\right)_{z=0} = \left(\mu_t \frac{\partial U}{\partial z}\right)_{z=h/2} = \frac{2m}{h} (\mu_t U)_{z=h/2} \tag{11}$$

$$\left(\mu_t \frac{\partial V}{\partial z}\right)_{z=0} = \left(\mu_t \frac{\partial V}{\partial z}\right)_{z=h/2} = \frac{2m}{h} (\mu_t U)_{z=h/2} \tag{12}$$

$$(W)_{z=0} = 0 \tag{13}$$

$$\left(\frac{\partial k}{\partial z}\right)_{z=0} = \left(\frac{\partial k}{\partial z}\right)_{z=h/2} = 0 \tag{14}$$

$$(\epsilon)_{z=h/2} = C_D(k^{3/2}/l)_{z=h/2} = \frac{2C_D^{3/4}}{\kappa h} (k)_{z=h/2}^{3/2} \tag{15}$$

where the turbulent momentum fluxes are assumed to be constant below $z = h/2$. The power laws are applied to the profiles of U and V from $z = 0$ to $h/2$:

$$U = (2z/h)^m (U)_{z=h/2} \tag{16}$$

$$V = (2z/h)^m (V)_{z=h/2} \tag{17}$$

$m = 1/7$ was used for this work. In (15) the following assumption is used:

$$(l)_{z=h/2} = \kappa C_D^{1/4} z = \kappa C_D^{1/4} h/2 \tag{18}$$

Of course (14) and (15) are not necessary in the prediction by the Deardorff's model. These boundary conditions are different from those which Deardorff[8] used.

At the outlet and inlet, the normal velocities to the boundary surface were assumed to have steady and uniform profiles. It seems that the boundary conditions at the corners of outlet and inlet have to be carefully treated when utilizing a numerical method in which vorticities and vector potentials are used as dependent variables.[18] In this case, the corners are singular points on the numerical calculation. The MAC method does not cause such a problem. The boundary condition that all velocity components are zero also holds good at the corners. The turbulence quantities at the supply outlet were set to the values measured with a hot-wire probe. That is, in the prediction by the two-equation model the dimensionless values of k and ϵ at the outlet were 2×10^{-4} and 5.1×10^{-7} respectively, and in the prediction by Deardorff's model $K = 5 \times 10^{-3}$ at the outlet. At the exhaust inlet the normal derivatives of these quantities were assumed to be zero.

Results

The results predicted by the two-equation model and by Deardorff's model and the experimental results are given in *Figures 2–9*. Each is composed of three figures which were obtained from those results respectively. *Figures 2, 3* and *4* show the mean velocity vectors in the x–y plane, and *Figures 5, 6* and *7* show the distributions of $W(=U_3)$. *Figure 8* presents the distributions of the resultant velocity U_r in the y–z plane at $x = 4.5$. All the values are made dimensionless by dividing by U_0. Measurements were performed with a three-direction supersonic anemometer having probes spaced 10 cm apart between the transmitting head and receiving head. Consequently, each component of the velocity vector could be measured correctly except in the

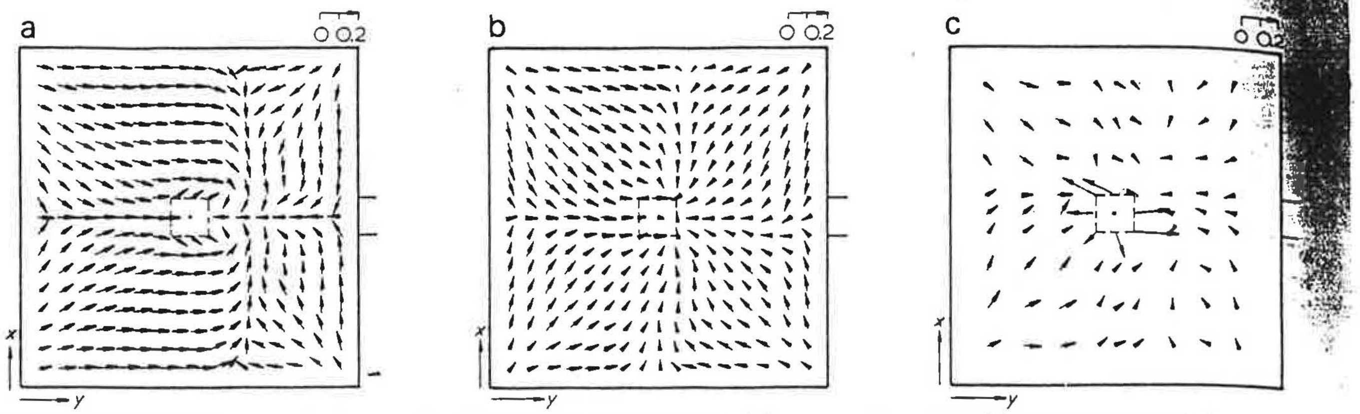

Figure 2 Distributions of dimensionless velocity vectors in $x-y$ plane at $z = 1.0$. (a), two equation model; (b) Deardorff's model; (c) experimental

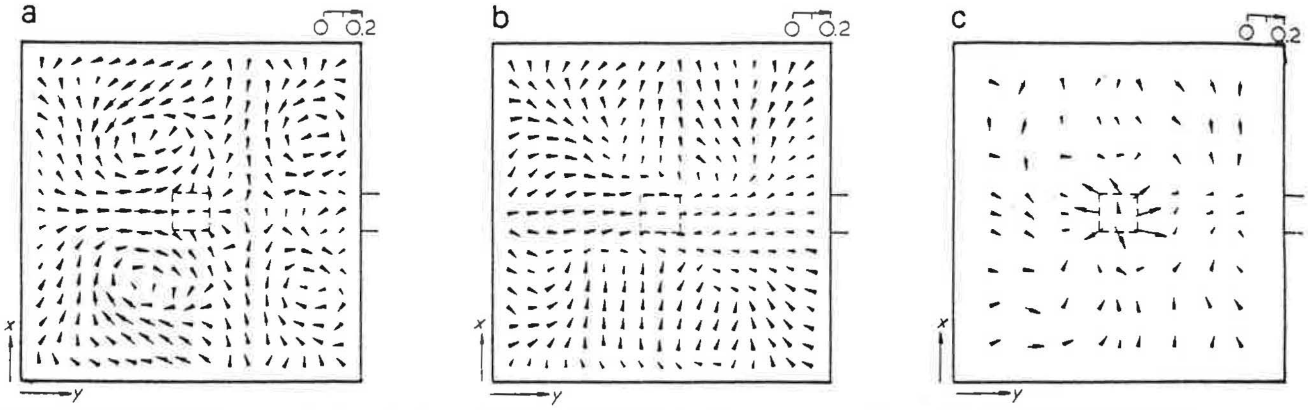

Figure 3 Distributions of dimensionless velocity vectors in $x-y$ plane at $z = 4.5$. (a) two equation model; (b), Deardorff's model; (c), experimental

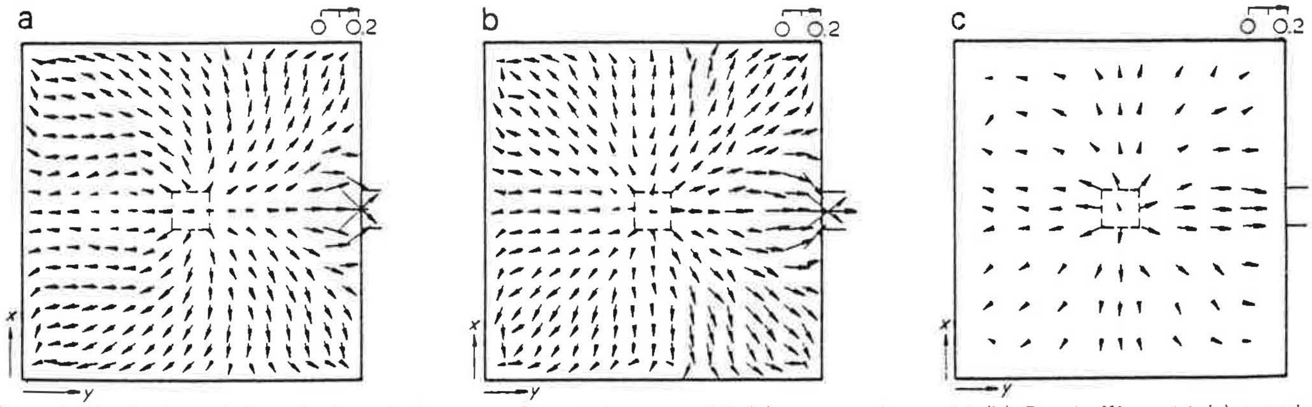

Figure 4 Distributions of dimensionless velocity vectors in $x-y$ plane at $z = 8.0$. (a), two equation model; (b), Deardorff's model; (c), experimental

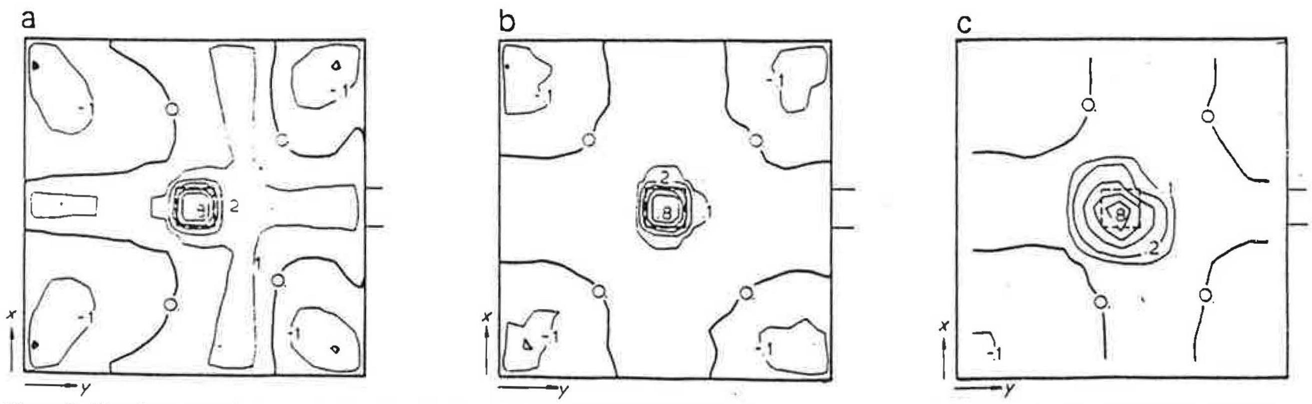

Figure 5 Distributions of dimensionless W-velocity in $x-y$ plane at $z = 1.0$. (a), two equation model; (b), Deardorff's model; (c), experimental

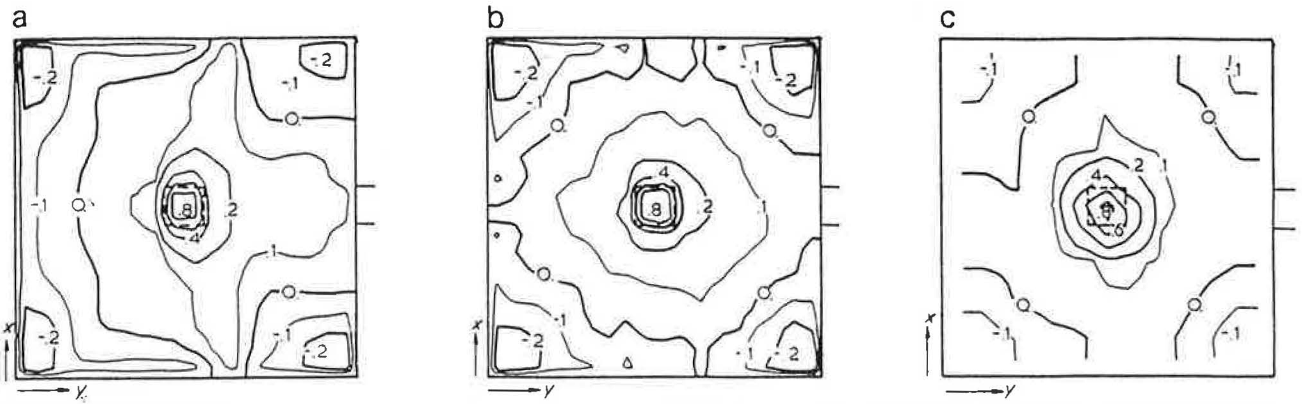

Figure 6 Distributions of dimensionless *W*-velocity in *x–y* plane at z = 4.5. (a), two equation model; (b), Deardorff's model; (c), experimental

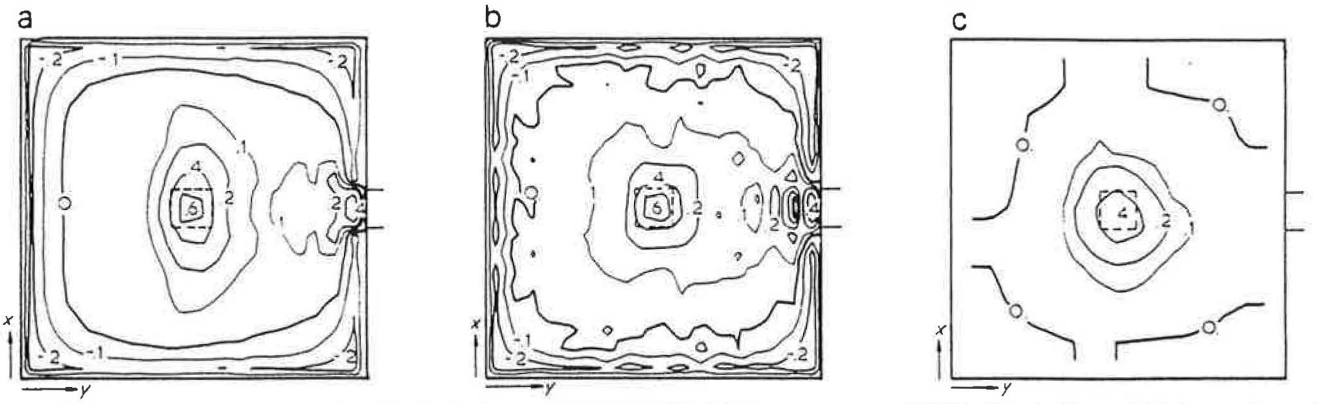

Figure 7 Distributions of dimensionless *W*-velocity in *x–y* plane at z = 8.0. (a), two equation model; (b), Deardorff's model; (c), experimental

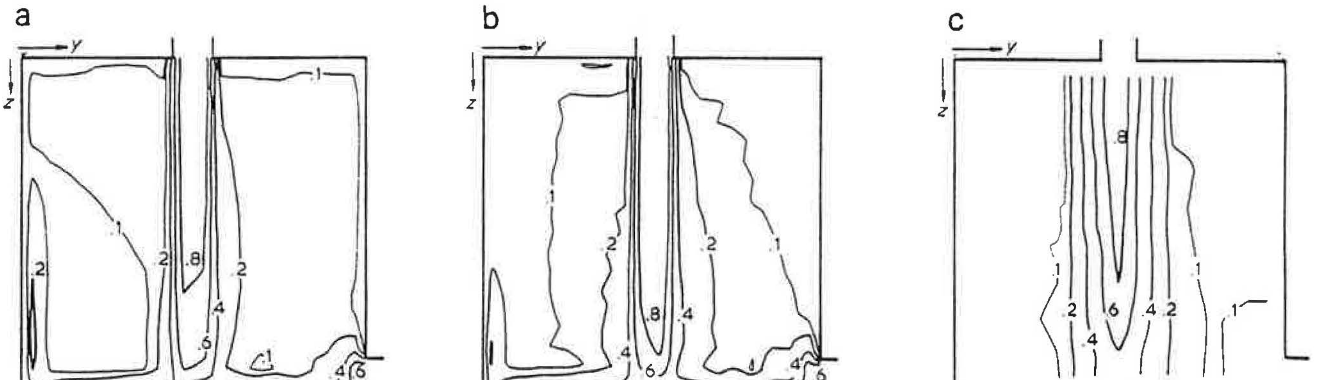

Figure 8 Distributions of dimensionless resultant velocity in *y–z* plane at x = 4.5. (a), two equation model; (b), Deardorff's model; (c), experimental

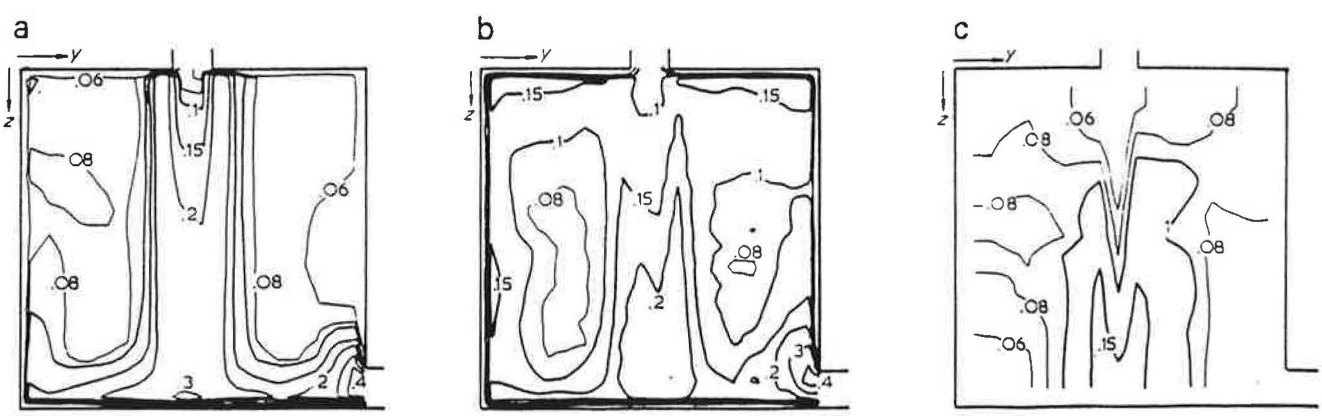

Figure 9 Distributions of dimensionless turbulence velocity in *y–z* plane at x = 4.5. (a), two equation model; (b), Deardorff's model; (c), experimental

jet region and in the vicinity of the walls where the velocity profiles were sharp. The bulk velocity at the outlet was 9.2 m/s during measurements, and the air flows appeared turbulent almost throughout. Sampling duration was 60 s. Each record of the velocity components was sampled every 12 msec, and the statistical quantities were calculated as follows:

$$U_i = \frac{1}{N}\sum_{n=1}^{N} (\tilde{u}_i)_n \qquad (19)$$

$$k = \frac{1}{2}\sum_{i=1}^{3} \frac{1}{N}\sum_{n=1}^{N} \left\{ (\tilde{u}_i)_n - (u_i)_{*} \right\}^2 \qquad (20)$$

where N is the total number of samples. Such statistical treatment was also given to the unsteady solutions calculated by Deardorff's model, although the values of the sampling interval and duration differed from those of the experiment. From the experimental results the jet seems to diffuse as soon as it leaves the outlet, as shown in *Figure 8*. However, this result is suspect, because a supersonic anemometer is not a suitable instrument for measurement of a small jet having a width comparable to the probe span. Except for this discrepancy, the results predicted by both turbulence models agree well with the experimental results. We cannot generalize as to which prediction agrees best with the experimental results. The results predicted by Deardorff's model are not perfectly symetric on the plane at $x = 4.5$. The cause of this may be that the sampling duration was too short. *Figure 9* shows the distributions of dimensionless turbulence velocity u_r/U_0 in the $y-z$ plane at $x = 4.5$. As concerns this quantity, both predicted values are larger in the vicinity of the exhaust inlet than the experimental values. It appears that, around the supply outlet, the result predicted by Deardorff's model corresponds better with the experimental result than that predicted by the two-equation model. In the recirculation regions agreement of both predicted results with the experimental result is fair.

Conclusions

The distribution of mean velocity in a ventilated room could be fairly well predicted by both numerical methods. But there are conspicuous discrepancies between the predictions and the experiment in regard to the distribution of turbulence velocity. It might be inferred that the assumption for Reynolds' stresses, i.e. (3), or the numerical instability caused those discrepancies. However, because distribution of mean velocity is the first requirement in many practical predictions, such discrepancies should not be regarded as so serious. Finally, from the viewpoint of engineering, we can conclude that the numerical methods utilizing such turbulence models are available for the practical predictions of air flows in actual rooms. As regards the two-equation model and Deardorff's model, the difference between both predicted distributions of mean velocity is not so large that the former is more advantageous than the latter in view of the time required for computeration. The numerical method using the former can give the solutions of the whole flow field in less computer time.

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Your Enquiry as promised. No #3922.

Regards

Clare.

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