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Thermal Bridges Across Multilayer Walls: An Integral Approach

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An integral approach has been developed to calculate thermal bridge effects at the junction between dissimilar, multilayer walls. The model is based on the solution of the integrated two-dimensional conduction equation for the main wall and the thermal bridge, with appropriate boundary conditions between the two solutions. The heat transfer coefficients and minimum internal surface temperatures as predicted by the model are shown to compare favourably with a computational solution, for six types of thermal bridges appearing in the International Standard for Thermal Bridges.

INTRODUCTION

THERMAL heat transfer through walls, both steady state and transient, has traditionally been calculated assuming uni-directionality, i.e. neglecting secondary heat transfer in the lateral directions. Such an approach can be justified on the grounds that since the thickness of most building elements is small compared to their lateral dimensions, temperature gradients perpendicular to the wall can also be expected to be much larger than temperature gradients in other directions.

The uni-directionality assumption can lead to errors in the vicinity of the interface between two dissimilar building elements, where lateral temperature gradients may be appreciable. Several methods have been developed to treat lateral heat transfer, the most widely used being the empirically based Zone method, recommended in *ASHRAE Fundamentals* [1]. More recently, thermal bridge calculations for anti-condensation standards were developed by Verhoeven and Liem [2] and Hoffman and Schwartz [3], based on computer solutions of finite difference equations describing the temperature field around a thermal bridge. On the basis of computer solutions, Verhoeven [4] has proposed empirical correlations, suitable for anti-condensation standards, between the thermal bridge effect on overall heat transfer coefficient and minimum surface temperature and the properties of the thermal bridge. These empirical relations have been adopted by the International Standard on Thermal Bridges [5]. Ceylan and Myers [6] have also treated multi-dimensional non-stationary heat transfer by computing the eigenvalues and eigenvectors of the mathematical problem. The efficiency of this method has been improved by Seem *et al.* [7].

It is possible to analyse heat flow patterns around thermal bridges using numerical methods based on finite difference or finite element discretization. This effort, however, is seldom justified if one is interested merely in a first order correction of heat transfer rates and the internal surface temperature that can be obtained assuming uni-directional heat transfer, as for most engineering calculations. In this work an approximate analytical expression for the additional heat transfer across a wall, which is due to the thermal bridge effect is obtained. In a previous work [8], the effect of lateral heat transfer between two homogeneous dissimilar walls has been examined and a simplified expression has been developed for the overall heat transfer coefficient and the internal surface temperature. The expression was obtained using an integral method. In this work, it will be attempted to extend the methodolody to cover thermal bridges between multilayer walls.

THEORY

Consider a multilayer wall of thickness d and thermal conductivity $\lambda(x)$ adjacent to a thermal bridge of width b and thermal conductivity $\lambda'(x)$ the total width of the construct being B (Fig. 1). In the above formalism, films with surface resistances are included and represented as films with zero thickness, but finite resistance.

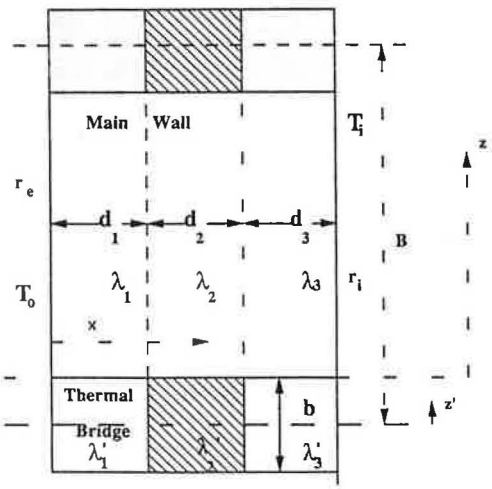

Fig. 1. Configuration of main wall and thermal bridge.

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If one assumes infinite extent in the y direction, the steady-state heat conduction equation for the wall and the thermal bridge is:

$$\frac{\partial}{\partial x}\left(\lambda(x)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(\lambda(x)\frac{\partial T}{\partial z}\right)=0, \tag{1}$$

where T is the temperature. The boundary conditions are as follows.

At $x=0$:

$$T=T_o. \tag{2}$$

At $x=d$:

$$T=T_i. \tag{3}$$

(T_o and T_i being the outside and the inside air temperature, respectively.) Note that in the above equations the influence of incident solar and longwave radiation has been neglected, but those physical processes will be taken account of at a later stage.

A similar set of equations describes the temperature field in the thermal bridge:

$$\frac{\partial}{\partial x}\left(\lambda'(x)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z'}\left(\lambda'(x)\frac{\partial T}{\partial z'}\right)=0. \tag{4}$$

At $x=0$:

$$T=T_o. \tag{5}$$

At $x=d$:

$$T=T_i. \tag{6}$$

Equations (1) and (4) are subject to the following additional boundary conditions.

At the symmetry line of the thermal bridge ($z'=0$):

$$\partial T/\partial z'=0. \tag{7}$$

At the symmetry line of the main wall ($z=(B-b)/2$):

$$\partial T/\partial z=0. \tag{8}$$

At the interface between the wall and the thermal bridge:

$$T|_{z=0}=T|_{z'=b/2}, \tag{9}$$

$$\lambda(x)\,\partial T/\partial z|_{z=0}=\lambda'(x)\,\partial T/\partial z'|_{z'=b/2}. \tag{10}$$

In principle, equations (1)–(10) can be solved using the method of separation of variables. This process, however, is not very efficient for this particular problem: the eigenfunctions for the bridge are different from the eigenfunctions for the main wall and the convergence of the series is rather slow. It is more appropriate to use an approximate integral approach and convert the partial differential equations (1)–(10) into ordinary ones by integrating with respect to x from $x=0$ to $x=d$. Thus, the independent variable would be:

$$F(z)\equiv\int_0^d \lambda(x')T(x',z)\,\mathrm{d}x'. \tag{11}$$

Now, integrating equation (1) from $x=0$ to $x=d$, one obtains:

$$\frac{\mathrm{d}^2F}{\mathrm{d}z^2}+Q_i-Q_o=0, \tag{12}$$

where Q_i and Q_o are the thermal flux in the internal and outside boundaries of the wall, respectively.

Now, for large distances from the thermal bridge, the temperature distribution tends to the two-dimensional solution:

$$T_\infty(x)-T_o=\frac{\Delta T\int_0^x \mathrm{d}x'/\lambda(x')}{\int_0^d \mathrm{d}x'/\lambda(x')}, \tag{13}$$

$$\Delta T\equiv T_i-T_o. \tag{14}$$

From the temperature distribution given by equation (13), a value of F can be derived:

$$F_\infty=\frac{\Delta T\int_0^d \lambda(x')\,\mathrm{d}x'\int_0^{x'}\mathrm{d}x''/\lambda(x'')}{\int_0^d \mathrm{d}x'/\lambda(x')}. \tag{15}$$

Now, for large distances from the thermal bridge:

$$Q_{o\infty}=F_\infty/L_o^2, \tag{16}$$

$$Q_{i\infty}=\left(\Delta T\int_0^d \lambda(x')\,\mathrm{d}x'-F_\infty\right)\Big/L_i^2, \tag{17}$$

where

$$L_o^2\equiv\int_0^d \lambda(x)\,\mathrm{d}x\int_0^x \mathrm{d}x'/\lambda(x')$$

$$=\sum_{i=1}^{N}\lambda_i d_i\left(r_e+\frac{d_i}{2\lambda_i}+\sum_{j=1}^{i-1}\frac{d_j}{\lambda_j}\right), \tag{18}$$

$$L_i^2\equiv\int_0^d \lambda(x)\,\mathrm{d}x\int_x^d \mathrm{d}x'/\lambda(x')$$

$$=\sum_{i=1}^{N}\lambda_i d_i\left(r_i+\frac{d_i}{2\lambda_i}+\sum_{j=i+1}^{N}\frac{d_j}{\lambda_j}\right). \tag{19}$$

In equations (18) and (19), the layers are counted from outside inwards, r_e and r_i are the external and the internal film resistances, respectively, and d_j/λ_j has to be replaced by the film resistance for the case of air gaps.

Now, one can assume that, to the first order, equations (16) and (17) are valid for all z. Thus equation (12) becomes:

$$\frac{\mathrm{d}^2F}{\mathrm{d}z^2}-\frac{F(z)}{L_o^2}+\frac{\Delta T\int_0^d \lambda(x')\,\mathrm{d}x'-F(z)}{L_i^2}=0. \tag{20}$$

One can further simplify equation (20) by replacing $F(z)$ by $f(z)$, in which the temperature in the integrand is replaced by its deviation from the two-dimensional solution:

$$f(z)\equiv\int_0^d \lambda(x')[T(x',z)-T_\infty(x')]\,\mathrm{d}x'. \tag{21}$$

Thus equation (12) becomes:

$$\frac{\mathrm{d}^2f}{\mathrm{d}z^2}-\frac{f(z)}{L_o^2}-\frac{f(z)}{L_i^2}=0. \tag{22}$$

A similar equation can be derived for the thermal bridge:

$$\frac{d^2 f'}{dz'^2} - \frac{f'(z)}{L_o'^2} - \frac{f'(z)}{L_i'^2} = 0, \tag{23}$$

where:

$$f'(z) \equiv \int_0^d \lambda'(x')[T(x',z) - T'_\infty(x')]\,dx', \tag{24}$$

$$T'_\infty(x) - T_o = \frac{\Delta T \int_0^x dx'/\lambda'(x')}{\int_0^d dx'/\lambda'(x')}, \tag{25}$$

$$L_o'^2 \equiv \int_0^d \lambda'(x)\,dx \int_0^x dx'/\lambda'(x') = \sum_{i=1}^{N} \lambda_i' d_i \left(r_e + \frac{d_i}{2\lambda_i'} + \sum_{j=1}^{i-1} \frac{d_j}{\lambda_j'} \right), \tag{26}$$

$$L_i'^2 \equiv \int_0^d \lambda'(x')\,dx' \int_x^d dx''/\lambda'(x'') = \sum_{i=1}^{N} \lambda_i' d_i \left(r_i + \frac{d_i}{2\lambda_i'} + \sum_{j=i+1}^{N} \frac{d_j}{\lambda_j'} \right). \tag{27}$$

Note that:

$$L_o'^2 + L_i'^2 = \Sigma \lambda_i d_i / U. \tag{28}$$

Equations (22) and (23) are subject to the symmetry boundary conditions, which can be derived by integrating equations (9) and (8) from $x = 0$ to $x = d$ and subtracting the influence of the two-dimensional solution.

At $z = (B-b)/2$:

$$\frac{df}{dz} = 0. \tag{29}$$

At $z' = 0$:

$$\frac{df'}{dz'} = 0. \tag{30}$$

From equations (9) and (10), two additional boundary conditions can be derived, expressing the continuity of temperature and flux at the interface between the main wall and the thermal bridge:

$$\frac{F_\infty + f(z=0)}{\int_0^d \lambda\,dx} = \frac{F'_\infty + f'(z' = b/2)}{\int_0^d \lambda'\,dx}, \tag{31}$$

$$\left.\frac{df}{dz}\right|_{z=0} = \left.\frac{df'}{dz'}\right|_{z'=b/2}, \tag{32}$$

where:

$$F'_\infty = \frac{\Delta T \int_0^d \lambda'(x')\,dx' \int_0^{x'} dx''/\lambda'(x'')}{\int_0^d dx'/\lambda'(x')}. \tag{33}$$

It has to be noted that equation (31) is not strictly correct, but the physical justification behind it is that continuity of temperature at the junction of the thermal bridge to the main wall is more important for parts of the configuration in which the conductivity is high, which account for the largest part of the thermal flux.

The solutions of equations (22) and (23), subject to the symmetry boundary conditions (29) and (30) are:

$$f = A \cosh\left[\left(z - \frac{B-b}{2}\right)\Big/ L\right], \tag{34}$$

$$f' = A' \cosh(z/L'), \tag{35}$$

where:

$$L = L_i L_o / \sqrt{(L_i^2 + L_o^2)} = L_i L_o \Big/ \sqrt{\left(\int_0^d \lambda\,dx\right)\left(\int_0^d dx/\lambda\right)}, \tag{36}$$

$$L' = L_i' L_o' / \sqrt{(L_i'^2 + L_o'^2)} = L_i' L_o' \Big/ \sqrt{\left(\int_0^d \lambda'\,dx\right)\left(\int_0^d dx/\lambda'\right)}. \tag{37}$$

Introducing equations (34) and (35) into the boundary conditions at the junction between the main wall and the thermal bridge, i.e. equations (31) and (32):

$$\frac{F_\infty + A\cosh[(B-b)/2L]}{\int_0^d \lambda\,dx} = \frac{F'_\infty + A'\cosh(b/2L')}{\int_0^d \lambda'\,dx} \tag{38}$$

$$-(A/L)\sinh[(B-b)2L] = (A'/L')\sinh(b/2L'). \tag{39}$$

From equations (38) and (39), expressions can be derived for $f(z)$ and $f'(z')$:

$$\frac{f(z)}{\Delta T} = \frac{\left(\frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs}\right)\frac{L}{\sinh[(B-b)/2L]}}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}} \times \cosh\left[\left(z - \frac{B-b}{2}\right)\Big/ L\right], \tag{40}$$

$$\frac{f'(z')}{\Delta T} = -\frac{\left(\frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs}\right)\frac{L'}{\sinh(b/2L')}}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}} \times \cosh(z'/L'), \tag{41}$$

where:

$$s = \int_0^d \lambda(x)\,dx, \tag{42}$$

$$s' = \int_0^d \lambda'(x)\,dx, \tag{43}$$

$$R = \int_0^d dx/\lambda(x), \tag{44}$$

$$R' = \int_0^d dx/\lambda'(x), \tag{45}$$

and F_∞ and F'_∞ are expressed in terms of L_o, s, L_o' and s', using equations (15), (18), (26) and (33).

Two quantities are of particular importance in thermal bridges, the minimum internal surface temperature at the centerline of the thermal bridge, as expressed through the non-dimensional parameter ζ and the average heat

transfer coefficient U_{av}:

$$\zeta \equiv [T_i - T_{is}(z'=0)]/\Delta T = (Q_{in} r_i)/\Delta T = [U' - f'(z'=0)/(L_i'^2 \Delta T)] r_i, \quad (46)$$

$$U_{av} = \left[U'b + U(B-b) + 2\left\{ \left(\int_0^{(B-b)/2} f(z)\,dz \right) \Big/ L_i^2 + \left(\int_0^{b/2} f'(z')\,dz' \right) L_i'^2 \right\} \Big/ \Delta T \right] \Big/ B, \quad (47)$$

where r_i is the internal film resistance and U' is the thermal conductance of the thermal bridge. From equations (40) and (41), the following expressions can be obtained for ζ and U_{av}:

$$\zeta = r_i \left\{ U' + \frac{\left(\frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs} \right) \frac{L'}{L_i'^2 \sinh(b/2L')}}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}} \right\}, \quad (48)$$

$$U_{av} = \frac{U'b + U(B-b)}{B} + \frac{\frac{2}{B}\left(\frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs} \right)^2}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}}. \quad (49)$$

Equations (48) and (49) relate the thermal bridge heat transfer properties to the difference between the conductivity-weighted temperature average in the thermal bridge and the main wall, which appears within parentheses in the numerators of the fractions in their right-hand sides. They are likely to give incorrect results in cases that this difference approaches to zero, when in one part of the thermal bridge–main wall configuration the temperature difference is positive and in the other part it is negative. In such cases, it is proposed to relate the heat transfer between the thermal bridge and the main wall to the difference between conductivity-weighted average temperatures in the internal half of the wall:

$$S \equiv \left[\frac{\int_{d/2}^{d} \lambda'\,dx \int_0^x dx'/\lambda'}{\left(\int_{d/2}^{d} \lambda'\,dx \right)\left(\int_0^d dx/\lambda' \right)} - \frac{\int_0^{d/2} \lambda'\,dx \int_0^x dx'/\lambda'}{\left(\int_0^{d/2} \lambda'\,dx \right)\left(\int_0^d dx/\lambda' \right)} \right]. \quad (50)$$

Thus equations (48) and (49) become:

$$\zeta = r_i \left\{ U' + \frac{S \frac{L'}{L_i'^2 \sinh(b/2L')}}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}} \right\}, \quad (51)$$

$$U_{av} = \frac{U'b + U(B-b)}{B} + \frac{2S^2/B}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}}. \quad (52)$$

Equations (51) and (52) are to be used when:

$$|S| \geqslant \left| \frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs} \right|. \quad (53)$$

There are, admittedly, more rigorous ways to treat thermal bridges for which equation (53) is valid, but it is not thought that the gain in precision is justified by the added complexity.

Equations (48) and (49) can be extended to account for different sol–air temperatures inside and outside the main wall and the thermal bridge. This can be done by modifying the continuity boundary condition at the junction between the main wall and the thermal bridge [equation (31)] to account for these differences:

$$T_{os} + [\Delta T_s L_o^2/R + f(z=0)]/s = T'_{os} + [\Delta T'_b L_o'^2/R + f'(z'=b/2)]/s', \quad (54)$$

where T_{os} is the sol–air temperature outside the main wall, T'_{os} is the sol–air temperature outside the thermal bridge, ΔT_s is the difference between the sol–air temperatures inside and outside the main wall and $\Delta T'_s$ is the difference between the sol–air temperatures inside and outside the thermal bridge.

Introducing equations (34) and (35) into equations (54) and (32), expressions can be derived for ζ and Q_{ar}, the average heat flux through the main wall–thermal bridge system:

$$\zeta = \frac{r_i}{\Delta T'_s} \left\{ U' \Delta T'_s + \frac{\left(\frac{\Delta T'_s L_o'^2}{R's'} - \frac{\Delta T_s L_o^2}{Rs} + T'_{os} - T_{os} \right) \frac{L'}{L_i'^2 \sinh(b/2L')}}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}} \right\}, \quad (55)$$

$$Q_{av} = \frac{U'b\Delta T'_s + U(B-b)\Delta T_s}{B} + \frac{\frac{2}{B}\left(\frac{L_o'^2}{R's'} - \frac{L_o^2}{Rs} \right)\left(\frac{\Delta T'_s L_o'^2}{R's'} - \frac{\Delta T_s L_o^2}{Rs} + T'_{os} - T_{os} \right)}{\frac{L/s}{\tanh[(B-b)/2L]} + \frac{L'/s'}{\tanh(b/2L')}}. \quad (56)$$

COMPARISON WITH COMPUTATIONAL RESULTS

In Figs 2 to 13, the values of U_{av} and ζ for equal external and internal sol–air temperatures at the main wall and the thermal bridge, as calculated using the model are compared with their corresponding values as computed using finite difference digitilization. A 41 × 41 grid was used and the resulting matrix was solved using successive over-relaxation. Six types of thermal bridge were considered, according to the ISO classification [5]: Type a (homogeneous walls), Type b (insulation outside the

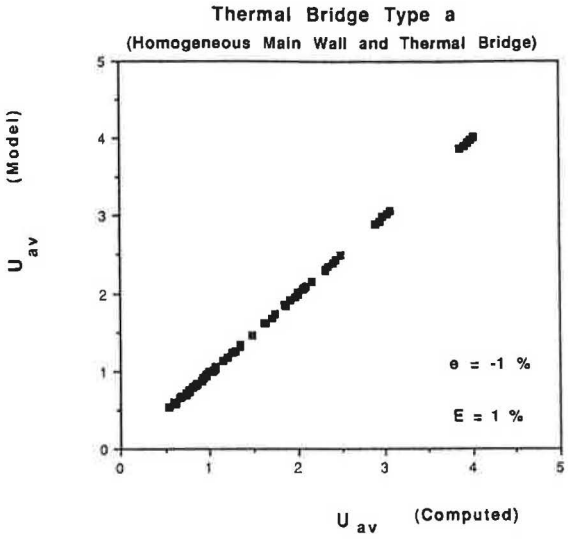

Fig. 2.

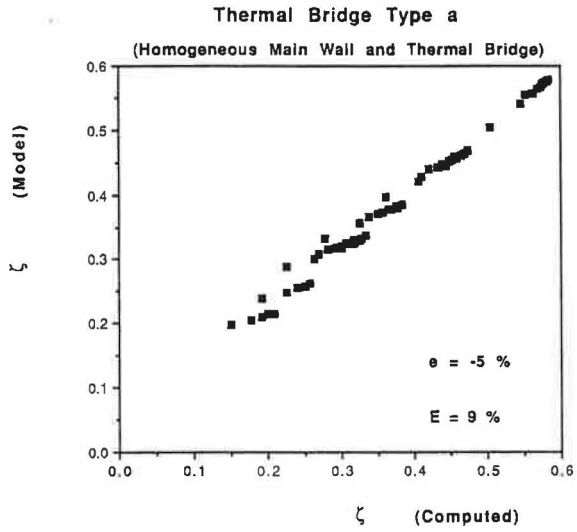

Fig. 3.

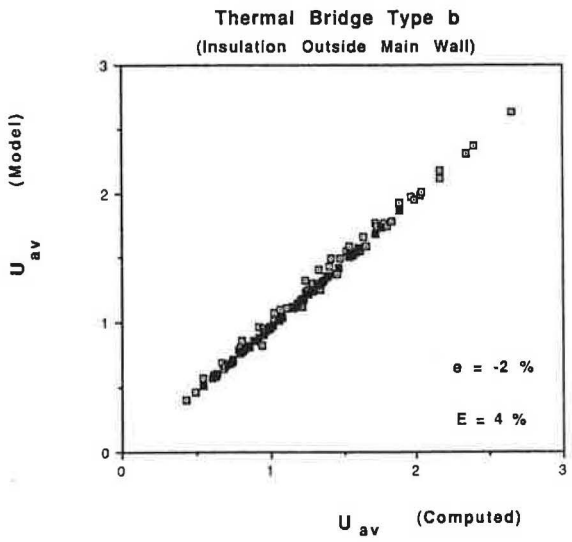

Fig. 4.

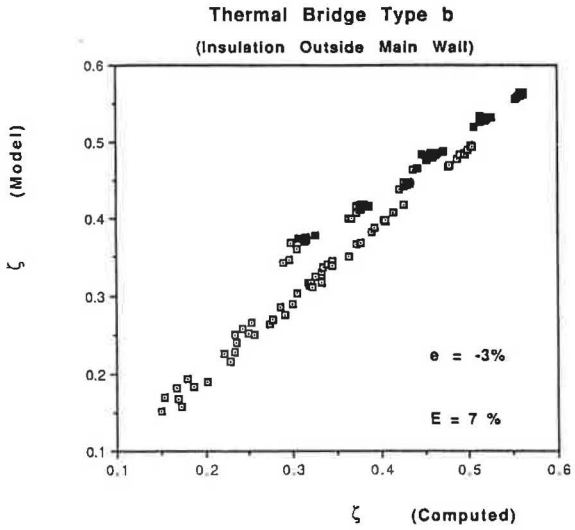

Fig. 5.

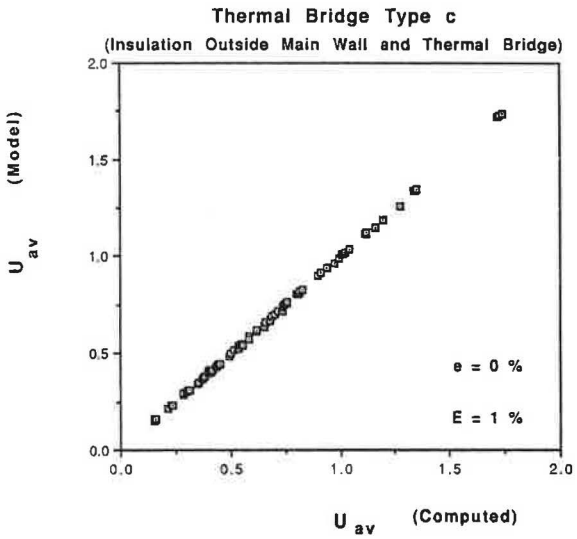

Fig. 6.

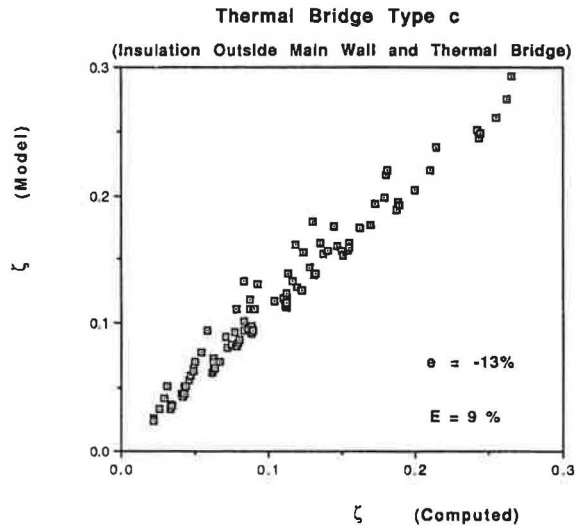

Fig. 7.

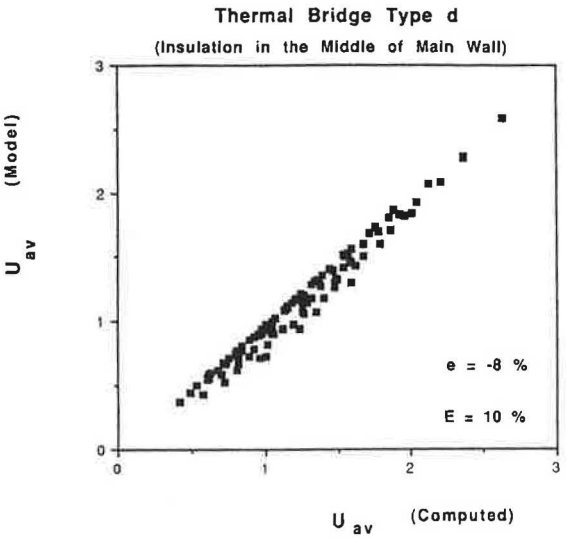

Fig. 8.

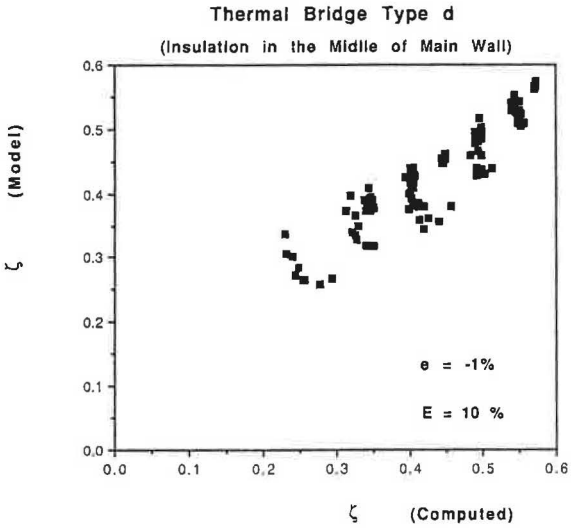

Fig. 9.

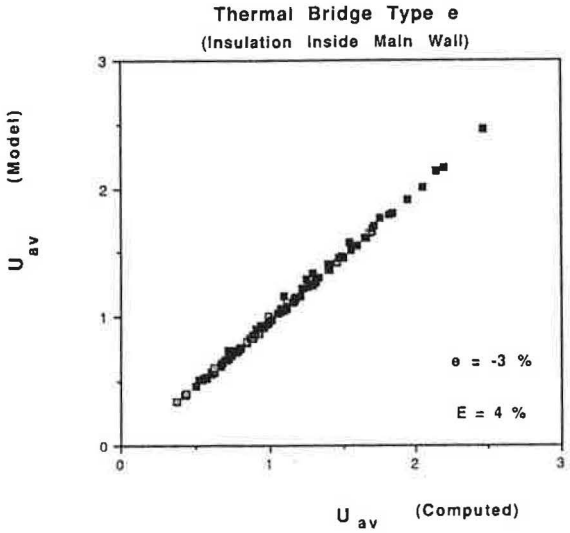

Fig. 10.

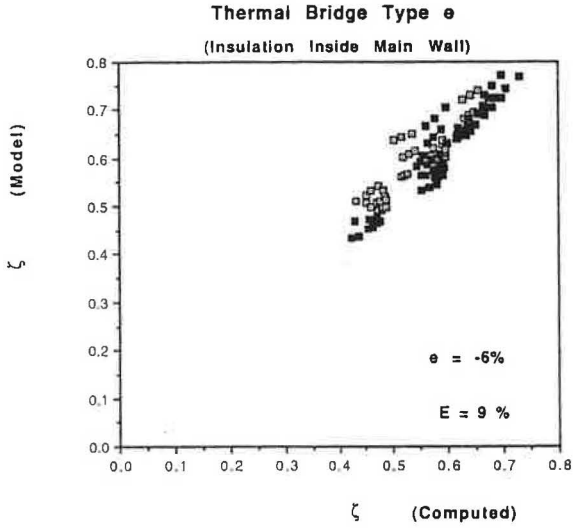

Fig. 11.

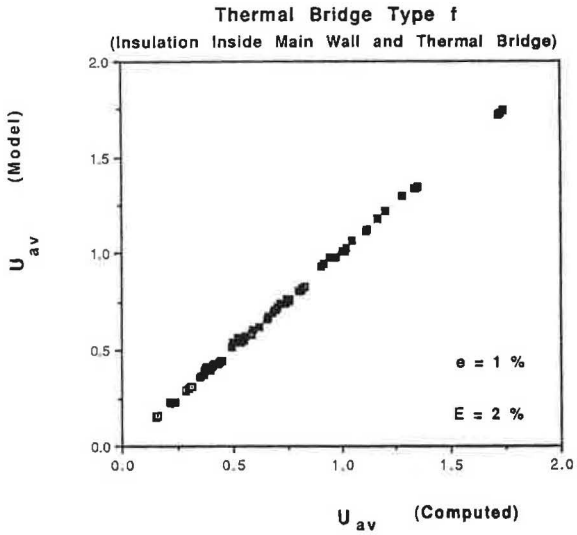

Fig. 12.

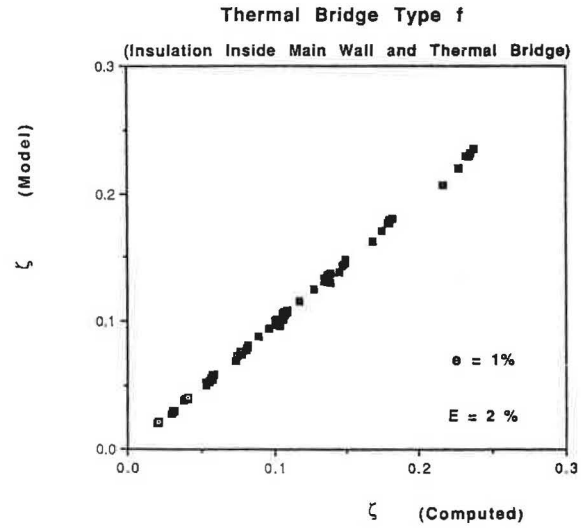

Fig. 13.

main wall), Type d (insulation in the middle of the main wall), and Type e (insulation inside the main wall). In each of the figures, the average relative deviation e1 and the average squared relative deviation are shown. Each combination of the following parameters was considered.

For the thermal bridge width b: 0.05; 0.10; 0.20 m.
For the total construction thickness d: 0.10; 0.20 m.
For the insulation thickness d_{ins}: 0.2 d; 0.4 d; 0.6 d.
For the main wall conductivity λ: 0.2; 2 W (m^{-1} K^{-1}).
For the insulation conductivity λ_{ins}: 0.02; 0.04; 0.06 W (m^{-1} K^{-1}).

For the thermal bridge conductivity λ' and the total construction width B, the values of 2 W (m^{-1} K^{-1}) and 1 m were taken, respectively. For the type a bridges between homogeneous walls, a larger range was tested for b, d, λ and λ'.

Calculations made using equations (48) and (49) are in hollow symbols, whereas calculations made using equations (51) and (52) are shown in full symbols. In the same figures, the average relative error e and the average root square relative error E are shown:

$$e \equiv \frac{1}{M} \sum_{i=1}^{M} \frac{f - f_{4i}}{f_{4i}}, \tag{57}$$

$$E \equiv \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left\{ \frac{f - f_{4i}}{f_{4i}} \right\}^2}, \tag{58}$$

where M is the number of points, f_{4i} is the quantity (U_{av} or ζ) as computed using the finite difference method and f is the quantity as computed using the model.

It can be said that agreement is good for all cases, with exception perhaps, of the predicted ζ for type d bridges. Note also that the values of ζ (but not the value of U_{av}) as calculated by the correlations suggested by the International Standard for Thermal Bridges [5] displays a rather smaller deviation from the computational result than the method suggested in this work for thermal bridges, but the method presented in this work has two advantages: (a) it is uniform for all types of thermal bridges and one does not need to identify the thermal bridge type; (b) it can account for different values of the sol–air temperature inside and outside the wall.

DISCUSSION

A simple model has been devised for the thermal bridge effect, which makes it possible to incorporate this effect in a comprehensive program for energy calculations in buildings, like DOE-2 [9] or ESP [10] with almost no effect on storage and cpu time requirements.

The model can successfully reproduce the main features of the thermal bridge effect. The thermal bridge effect on U_{av} is shown to depend strongly on s, which gives a quantitative expression to the ability of the main wall to transfer heat laterally. The effect is also shown to depend strongly on lateral thermal gradients and is much larger for b, d and e type thermal bridges than for a c and f type ones. The thermal bridge effect predicted by the model tends to zero as the thermal bridge width b tends to zero. For several types of thermal bridges, the minimum internal surface temperature is lower than in a laterally infinite construction identical to the thermal bridge. This is the case for Type e thermal bridges (main wall insulated from the inside), indicating that insulating the main wall from the inside without insulating the thermal bridge may increase the risk of condensation.

Admittedly, the model requires several simplifying assumptions. However, given the relatively small magnitude of the effect on energy and power requirements, a modification that would be more correct, but would severaly increase storage or cpu-time requirements is not justified.

Of course, there are several improvements possible and even necessary. It is necessary to try and relate to the particular problems associated with corner effects, even between similar walls. It is also necessary to extend the method to cover thermal bridge heat transfer around windows. This would necessitate the introduction of a surface resistance between the wall and the thermal bridge (window).

Finally, one should remember that craftsmanship in thermal bridges is a factor that may have a large effect on their thermal performance and that this is a factor that cannot be accounted by any theoretical calculation method. Still, it is felt that the thermal bridge effects may be important enough so that their neglect cannot be justified and therefore a crude method for taking them into account, like the one proposed in this work, is necessary.

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