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Ventilation and aircondition of partial area in high and large workshop

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VENTILATION AND AIRCONDITION OF PARTIAL AREA IN HIGH AND LARGE WORKSHOP

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Summary

We meet often: ventilation and aircondition are required in some local area in huge factory building, but they are not needed in other areas, and because of the restriction of technology, equipment and productive condition, this local area cannot be divided by solid materials. Under the above-mentioned cases, there are following advantages if you use the method: economizing investment in copital construction; decreasing daily operating charge; saving energy and satisfing the special requirement due to the restriction of thechnology, equipment and productive condition. The method is ventilation and aircondition of local area in huge factory buildings. There are three manners: blowing in mode; drawing out mode and mixed mode. To the three manners stated above, author will offer a new method, that is to get analytical solutions by using the concept of conformal mapping in complex function, at last author gave two examples for the sake of illustrating the method by computer.

Introduction

In modern industry, because of the specialization of labour division and the high technological requirements, some new problems have been produced. For example:

1) Ventilation and aircondition are required in some local area in huge factory building, but they are not needed in other areas;

2) Because of the restriction of technology, equipment and productive condition, this local area cannot be divided by solid materials.

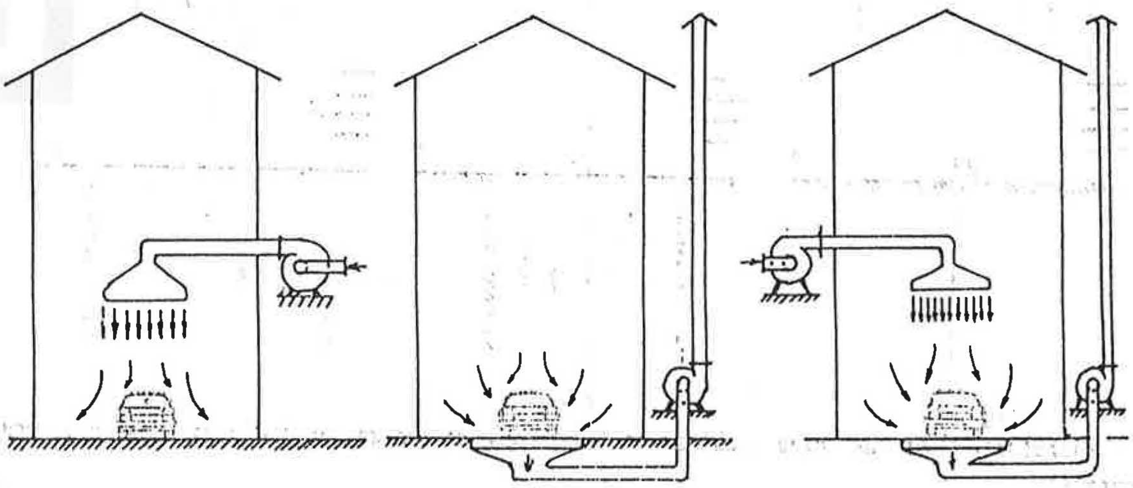

fig.1 fig.2 fig.3

Therefore, a new problem of ventilation and aircondition of local area in huge factory buildings is produced. Solving this kind of problems, will bring about many benefits, such as:

1) economizing investment in capital construction;
2) decreasing daily operating charge;
3) saving energy;
4) satisfing the special requirement due to the restriction of technology, equipment and productive condition.

There are three manners in ventilation and aircondition of local area in huge factory buildings:

1) blowing in mode, see figure 1;
2) drawing out mode, see figure 2;
3) mixed mode, see figure 3.

To solve the three problems stated above, the traditional method is to get experiential formulae by experiments. This paper will offer a new method, that is to get analytical solutions by using the concept of conformal mapping in complex function.

Mathematical Model

In order to set up the model of the problem of ventilation and aircondition of local area in huge factory buildings, first we make some assumptions as follows:

1) Because of the huge factory buildings, the roof and wall can be considered that have no effects on the flow field of the local area.
2) Because of being inside the building, there is not any strong wind, but only some little disturbing airstream caused by people.
3) Because of being inside the building and blowing or drawing, air steadily, the air flow is steady flow and is independent of time t.
4) Because the air is compressed very little, the flow can be considered a uncompressable flow.
5) In engineering practices, the length of local area is often greater than the width, so the flow could be thought as a plane flow.
6) In ventilation and air condition, it is permissible to consider the flow as a flow without vortex .

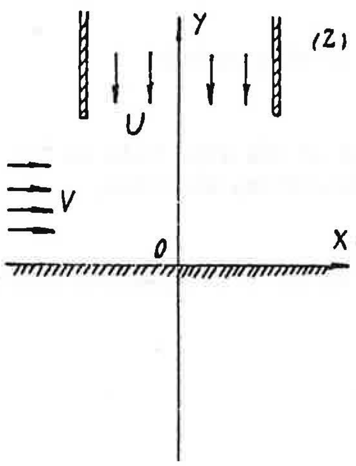

fig.4

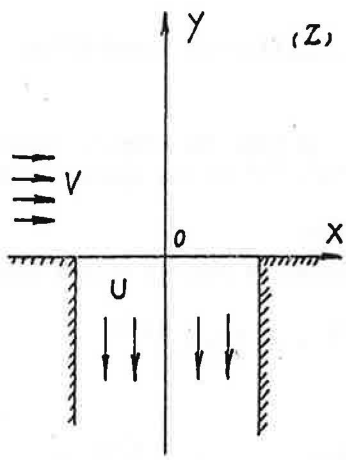

fig.5

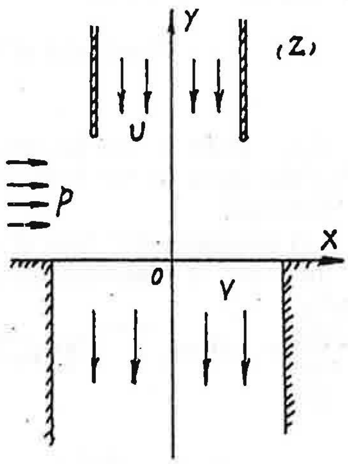

fig.6

Therefore, for the three problems stated above, we can set up their mathematical models separately:

1) Blowing in mode:

X axis is on the ground (see figure 4). The above plane is flow field, "U" is the speed of flowing wind, "V" is the speed of disturbing airstream.

2) Drawing out mode:

X axis is on the ground (see figure 5), The above plane is flow field "U" is the speed of drawing wind, "V" is the speed of disturbing airstream.

3) Mixed mode:

X axis is on the ground (see figure 6), The above plane is the flow field, "U" is the speed of blowing wind, "V" is the speed of drawing wind, "P" is the speed of disturbing airstream.

Complex Velocity, Complex Potential and Superposition

The speed of a point in a plane can be expressed:

$$V_z = V_x - iV_y \quad (1)$$

Here V_z is called complex velocity.

For the whole plane flow field, the general formula can be expressed:

$$V(z) = V_x(x,y) + iV_y(x,y) \quad (2)$$

As stated in hydromechanics, velocity potential and flow function in plane flow without vortex are two harmonic functions that meet Cauchy-Reimann condition. There two functions can constitute a analytical complex function W:

$$W(z) = \phi + i\psi \quad (3)$$

Here W(z) is considered as complex potential.

The relationship between comoplex velocity and complex potential can be written as follows:

$$\frac{dw}{dz} = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x} = V_x - iV_y \quad (4)$$

Because complex potential w(z) is a analytical function, so W(z) have the property of superposition. In this paper, this property will be used to find the analytical solution of a more complex flow field.

Flow Field of Ventilation and Aircondition, in Blowing in Mode.

This problem can be reduced to find the complex potential of the flow field in fig. 7. "u""is the speed of the blowing wind. "v" is the speed of the disturbing airstream.

Solution:

1) Determine the form of $z=f(\xi)$

In order to transform the polygon on plane z into a line, we can use Schwarz-cristalfer formula:

$$\frac{dz}{d\xi} = c(\xi-a_1)^{\alpha_1-1}(\xi-a_2)^{\alpha_2-1}\cdots(\xi-a_n)^{\alpha_n-1} \quad (5)$$

or

$$z = c\int(\xi-a_1)^{\alpha_1-1}(\xi-a_2)^{\alpha_2-1}\cdots(\xi-a_n)^{\alpha_n-1}d\xi+c_1 \quad (6)$$

have c, c_1 are complex numbers, $a_1, a_2, \ldots a_n$ are real constants.

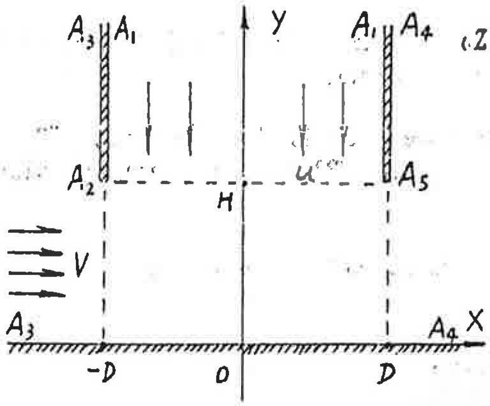

fig. 7

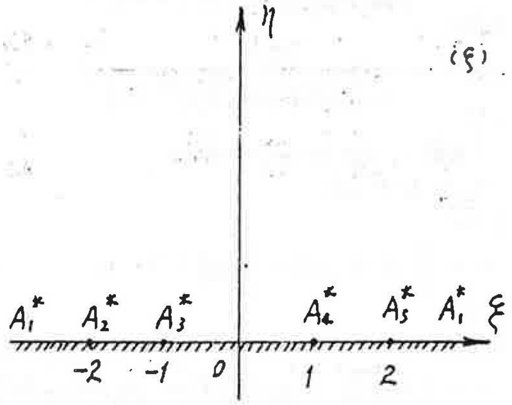

fig. 8

The relationship of all correspond points can be taken as follows:

k	α_k	a_k
1	0	∞
2	2	-2
3	-1/2	-1
4	-1/2	m
5	2	n

Because in mapping, the symmetric points are mapped as symmetric ones still, so m=1, m=2. Therefore, formula (5) can be written as:

$$\frac{dz}{d\zeta} = c(\zeta-a_1)^{\alpha_1-1}(\zeta-a_2)^{\alpha_2-1}\cdots(\zeta-a_5)^{\alpha_5-1}$$

$$= c(\zeta-2)(\zeta+1)^{-\frac{1}{2}}(\zeta-1)^{-\frac{1}{2}}(\zeta-2)$$

$$= c\,(\zeta^2-2^2)(\zeta^2-1)^{-\frac{1}{2}}$$

(7)

Integrate equation (7):

$$z = c\int\frac{(\zeta^2-2^2)}{(\zeta^2-1)^{1/2}}\,d\zeta + c_1$$

$$= c\left[\cosh^{-1}(\zeta) + \frac{3\zeta}{(\zeta^2-1)^{1/2}}\right] + c_1$$

(8)

Here integrate constant c and c_1 can be determined by using boundary condition A_2, A_5 and A_2^*, A_5^*.

Substitute:

z=-D+iH, ζ =-2

and z= D+ih, ζ = 2

into equation (8), we have two equations:

$$-D + iH = c\left[\cosh^{-1}(-2) + \frac{-6}{\sqrt{3}}\right] + c_1$$

(9)

$$D + iH = c\left[\cosh^{-1}(2) + \frac{6}{\sqrt{3}}\right] + c_1.$$

(10)

Solve these simultaneous equations:

Suppose:

$$A = \frac{4[2\sqrt{3}+\ln(2+\sqrt{3})]\ D}{[4\sqrt{3}+2\ln(2+\sqrt{3})]^2+\pi^2}$$

(11)

$$B = \frac{2D\pi}{[4\sqrt{3}+2\ln(2+\sqrt{3})]^2+\pi^2}$$

(12)

thus, c and c_1 can be written as

$$c = A + iB$$

(13)

$$c_1 = \frac{\pi}{2}B + i(H-\frac{\pi}{2}A)$$

(14)

Substitute c and c_1 into equation (8), then:

$$z = (A+iB)\ [\cosh^{-1}(\xi)+\frac{3\xi}{(\xi^2-1)^{1/2}}]+\frac{\pi}{2}B+i(H-\frac{\pi}{2}A)$$

(15)

2) Find $w_1(\xi)$ and $w_1(z)$

Suppose at point A_1 on plane z, there is a flow volume 2DU flowed into area. On the basis of symmetric principle, flow volume DU is flowed into point A_1 and point A_1 separately.

Correspondingly, there is confluence of flow volume Du at point A_1^* and point A_1^* separately on plane ξ. Therefore, the complex potential on plane ξ can be expressed as:

$$W_1(\xi) = -\frac{2DU}{2\pi}\ln(\xi+1) + \frac{-2DU}{2\pi}\ln(\xi-1) = -\frac{DU}{\pi}\ln(\xi^2-1)$$

(16)

Meanwhile; the complex potential on plane z can be written as follows:

$$W_1(z) = -\frac{DU}{\pi}\ln(\xi^2-1)$$

(17)

The relationship between z and ξ is conformable to equation (15).

3) Find $W_2(\xi)$ and $W_2(z)$

There is a flow volume HV flowed into the area at point A_2 on plane z.Correspondingly,there is a source of flow volume HV at point A_2^* on plane ξ. And so, the complex potential $W_2(\xi)$ on plane ξ can be expressed:

$$W_2(\xi) = \frac{2HV}{2\pi}\ln(\xi+1) = \frac{HV}{\pi}\ln(\xi+1)$$

(18)

The complex potential on plane z is:

$$W_2(z) = \frac{HV}{\pi}\ln(\xi+1)$$

(19)

The relationship between z and ξ is conformable to equation (15).

4) Find $W_3(\xi)$ and $W_3(z)$

Using the superposition principle

$$W_3(\xi) = -\frac{DU}{\pi}\ln(\xi^2-1) + \frac{HV}{\pi}\ln(\xi+1)$$

(20)

Meanwhile

$$W_3(z) = -\frac{DU}{\pi}\ln(\xi^2-1) + \frac{HV}{\pi}\ln(\xi+1)$$

(21)

The relationship between z and ξ conforms to equation (15).

5) Find $W_4(z')$ and $W_5(z')$

The obstacle in the flow field on plane z can be simplified as a circle, see figure 9.

The radius is a . The height measured from ground is h.

Now shift the X axis a distance h upper, called it X' axis. Meanwhile, called y axis as y' axis. Then, in the new coordinate system, the complex potential $W_4(z')$ can be written as:

$$z = z' + ih \tag{22}$$

$$W_4(z') = W_3(z'+ih) \tag{23}$$

Considering the circular obstacle, suppose the complex potential in the new coordinate system is $W_5(z')$.

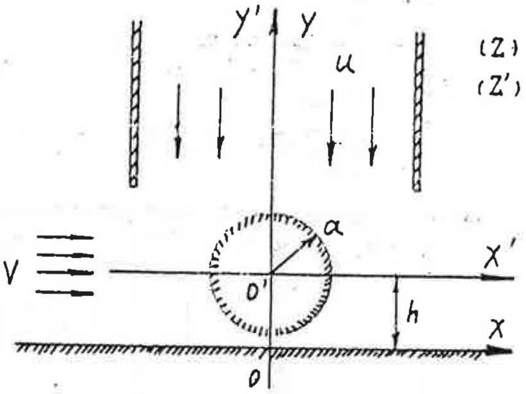

fig. 9

Using circle theorem:

$$W_5(z') = W_4(z') + \overline{W}_4\left(\frac{a^2}{z'}\right) = W_3(z'+ih) + \overline{W}_3\left(\frac{a^2}{z'+ih}\right) \tag{24}$$

Here:

$$W_3(z'+ih) = -\frac{DU}{\pi}\ln(\zeta^2-1)+\frac{HV}{\pi}\ln(\zeta+1)$$

The relationship between z' and ζ is:

$$z'+ih = (A+iB)\left[\cosh^{-1}(\zeta)+\frac{3\zeta}{(\zeta^2-1)^{1/2}}\right]+\frac{\pi}{2}B+i\left(H-\frac{\pi}{2}A\right) \tag{25}$$

$$\overline{W}\left(\frac{a^2}{z'+ih}\right) = -\frac{DU}{\pi}\ln(\zeta^2-1)+\frac{HV}{\pi}\ln(\zeta+1)$$

The relationship between z' and ζ is:

$$\frac{a^2}{z'+ih} = (A+iB)\left[\cosh^{-1}(\zeta)+\frac{3\zeta}{(\zeta^2-1)^{1/2}}\right]+\frac{\pi}{2}B+i\left(H-\frac{\pi}{2}A\right) \tag{26}$$

Flow Field of Ventilation and Aircondition in Drawing out Mode

This problem can be reduced to find the complex potential of the flow field in figure 10. "u"is the speed of the drawing wind. "v" is the speed of the disturbing airstream.

Solution:

1) Determine the form of $z=f(\zeta)$

Using formula(5)

$$\frac{dz}{d\zeta} = c(\zeta-a_1)^{\alpha_1-1}(\zeta-a_2)^{\alpha_2-1}\cdots(\zeta-a_n)^{\alpha_n-1}$$

The relationship of all correspond points can be taken as follows:

k	α_K	a_K
1	-1	∞
2	3/2	-1
3	0	0
4	3/2	m

According to the symmetric property, we have

$$m=1 \tag{27}$$

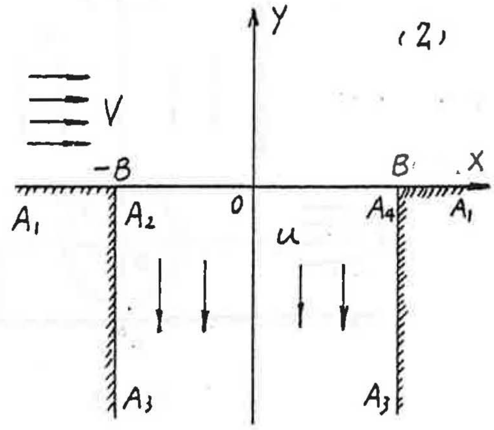

fig.10

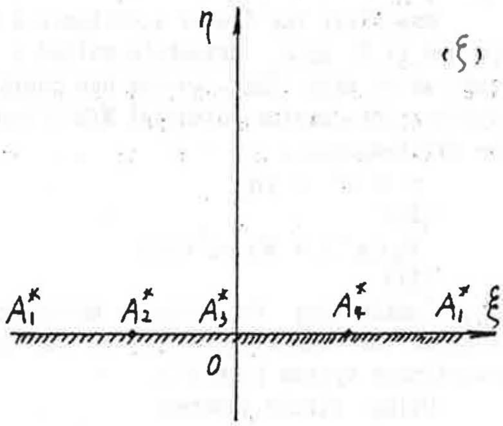

fig.11

Thus the formula (5) can be written as follows:

$$\frac{dz}{d\xi} = c(\xi+1)^{1/2} \cdot \xi^{-1} \cdot (\xi-1)^{1/2} = c\,\frac{(\xi^2-1)^{1/2}}{\xi}$$
(28)

Integrate it:

$$z = c\int \frac{\sqrt{\xi^2-1}}{\xi}\, d\xi + c_1 = c\,(\sqrt{\xi^2-1} - \operatorname{arcsec}\xi) + c_1$$
(29)

Here c, c_1 are complex number, they can be determined by using the correspond boundary condition of A_2, A_4 and A_2^*, A_4^*.

Substitute: $Z=B$, $\xi=1$; $Z=-B$, $\xi=-1$. into equation (29), we have

$$B = c(\sqrt{1^2-1} - \operatorname{arcsec} 1) + c_1$$
(30)

$$-B = c[\sqrt{(-1)^2-1} - \operatorname{arcsec}(-1)] + c_1$$
(31)

Solve these simultaneous equation, we have: $c_1=B$, $c=\frac{2}{\pi}B$. Substitute c and c_1 into equation (29), it can be expressed:

$$z = \frac{2}{\pi}B\,(\sqrt{\xi^2-1} - \operatorname{arcsec}\xi) + B$$
(32)

2) Find $W_1(\xi)$ and $W_1(z)$

On plane z, there is a flow volume $Q=2BU$ flowed into section A_1-A_1. Correspondingly, on plane ξ, there is a flow volume $Q=2BU$ flowed into point A_1^*. This is equivalent to a confluence of strength $4BU$ on point A_1^*. Thus the complex potential on plane ξ is:

$$W_1(\xi) = -\frac{4BU}{2\pi}\ln\xi = -\frac{2BU}{\pi}\ln\xi$$
(33)

And the complex potential on plane z is:

$$W_1(z) = -\frac{2BU}{\pi}\ln\xi$$
(34)

The relationship between z and ξ is conformable to equation (32).

3) Find $W_2(\xi)$ and $W_2(z)$

The disturbing airstream v on plane z, is equivalent to a uniform flow field on plane ξ. Therefore, the complex potential on plane ξ is:

$W_2(\xi)=V\xi$

(35)

While the complex potential on plane z is:

$W_2(z)=V\xi$

(36)

The relationship between z and ξ conforms to equation (32).

4) Find $W_3(\xi)$ and $W_3(z)$

For the same reason, using the superposition principle

$$W_3(\xi) = -\frac{2BU}{\pi}\ln\xi + V\xi$$

(37)

Thus

$$W_3(z) = -\frac{2BU}{\pi}\ln\xi + V\xi$$

(38)

The relationship between z and ξ is conformable to equation(32).

5) Find $W_4(z')$ and $W_5(z')$

The obstacle in the flow field on plane z, can be simplified as a circle, see figure 12. The radius is a. The height measured from ground is h.

Now shift the X axis a distance h upper, called X' axis. Meanwhile call the y axis as y' axis. Then, in the new coordinate system, the complex potential $w_4(z')$ can be written as:

$z=z'+ih$

(39)

$W_4(z')=W_3(z'+ih)$

(40)

Considering the circular obstacle, suppose the complex potential of the flow field in the new coordinate system is $W_5(z')$.

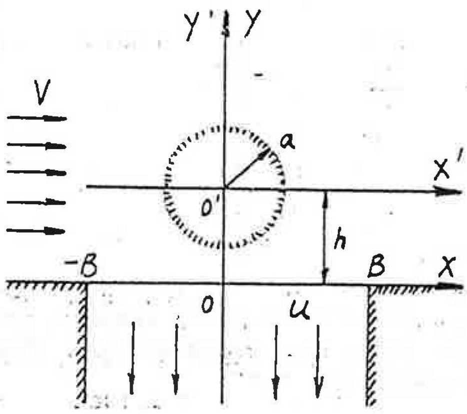

fig. 12

Using circle theorem:

$$W_5(z') = W_4(z') + \overline{W}_4\left(\frac{a^2}{z'}\right) = W_3(z'+ih) + \overline{W}_3\left(\frac{a^2}{z'+ih}\right)$$

(41)

Here:

$$W_3(z'+ih) = -\frac{2BU}{\pi}\ln\xi + V\xi$$

(42)

The velationship between z' and ξ can be expressed as:

$$z'+ih = \frac{2}{\pi}B(\sqrt{\xi^2-1} - \operatorname{arcsec}\xi)+B$$

(43)

$$\overline{W}_3\left(\frac{a^2}{z'+ih}\right) = -\frac{2BU}{\pi}\ln\xi + V\xi$$

(44)

The relationship between z' and can be written as follows:

$$\frac{a^2}{z'+ih} = \frac{2}{\pi}B(\sqrt{\zeta^2-1} - \operatorname{arcsec}\zeta)+B$$

(45)

Flow Field of Ventilation and Aircondition in Mixed Mode

This problem can be reduced to find the complex potential of the flow field in figure 13. " u " is the speed of the blowing wind, "v"is the speed of the drawing wind. "p" is the speed of the disturbing airstream.

Solution:

1) Determine the form of $z=f(\zeta)$, using formula (5)

The relationship of all correspond points can be taken as follows:

k	α_k	a_k
1	0	∞
2	2	-N
3	-1/2	-1
4	3/2	-M
5	0	0
6	3/2	M
7	-1/2	1
8	2	N

On the basis of the symmetric property, the symmetric points will be mapped as symmetric ones still. Thus, suppose $A_2^*=-N$, $A_4^*=-M$, Hence: $A_6^*=M$, $A_8^*=N$. Therefore formula (5) can be expressed as:

$$\frac{dz}{d\zeta} = c(\zeta+N)(\zeta+1)^{-\frac{3}{2}}(\zeta+M)^{\frac{1}{2}}\zeta\cdot(\zeta-M)^{\frac{1}{2}}(\zeta-1)^{-\frac{3}{2}}(\zeta-N) = c\frac{(\zeta^2-N^2)(\zeta^2-M^2)^{\frac{1}{2}}}{\zeta(\zeta^2-1)^{3/2}}$$

(46)

Integrate it:

$$z = c\int\frac{(\zeta^2-N^2)(\zeta^2-M^2)^{1/2}}{\zeta\cdot(\zeta^2-1)^{3/2}}d\zeta + c_1$$

(47)

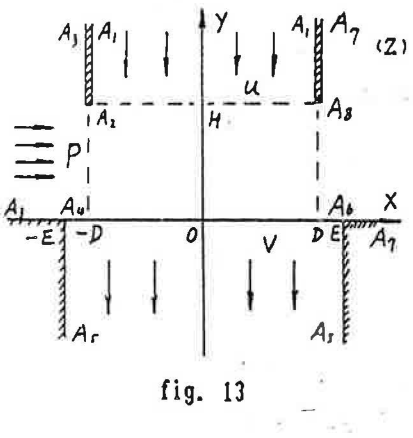

fig. 13

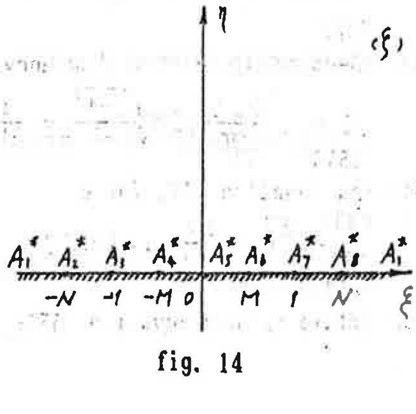

fig. 14

Here, C, C_1 are complex number, M, N are real constants. By integrating, we have:

$$z = c[(N^2-1)\frac{\sqrt{\xi^2-M^2}}{\sqrt{\xi^2-1}}+\ln(\sqrt{\xi^2-1}+\sqrt{\xi^2-M^2})+\frac{MN^2}{2}\ln\frac{\sqrt{\xi^2-M^2}-M\sqrt{\xi^2-1}}{\sqrt{\xi^2-M^2}+M\sqrt{\xi^2-1}}]+c_1$$

(48)

Now, determine c, c_1, N and M. First, determine c and N by using flow condition. On plane ξ, there is a flow volume Q=2EV flowed into section A_1^*. This is equivalent to a confluence of strength 4EV. Thus the complex potential on plane ξ is:

$$W_1(\xi) = -\frac{4EV}{2\pi}\ln\xi = -\frac{2EV}{\pi}\ln\xi$$

(49)

As a reault:

$$\frac{dW}{dz} = \frac{dW}{d\xi}\cdot\frac{d\xi}{dz}$$

$$= -\frac{2EV}{\pi}\cdot\frac{1}{\xi}\cdot\frac{\xi\cdot(\xi^2-1)^{3/2}}{c(\xi^2-N^2)(\xi^2-M^2)^{1/2}} = -\frac{2EV}{\pi c}\cdot\frac{(\xi^2-1)^{3/2}}{(\xi^2-N^2)(\xi^2-M^2)^{1/2}}$$

(50)

Substituting:

$$\xi \longrightarrow 0 \;, \quad \frac{dw}{dz} = -Vi$$

(51)

$$\xi \longrightarrow \infty \;, \quad \frac{dw}{dz} = -ui$$

(52)

into equation (50), we have

$$c = -\frac{2EV}{u\pi}i$$

(53)

$$N^2 = \frac{u}{MV}$$

(54)

Substituting c and N^2 into equation (48), we have

$$z = -\frac{2EV}{u\pi}i[(\frac{u}{MV}-1)\frac{\sqrt{\xi^2-M^2}}{\sqrt{\xi^2-1}}+\ln(\sqrt{\xi^2-1}+\sqrt{\xi^2-M^2}) + \frac{u}{2V}\ln\frac{\sqrt{\xi^2-M^2}-M\sqrt{\xi^2-1}}{\sqrt{\xi^2-M^2}+M\sqrt{\xi^2-1}}] + c_1$$

(55)

Now, c_1 and M can be determind by using boundary condition. Substituting: z=E, =M into equation (55), we have

$$E = -\frac{2EV}{u\pi}i[\ln\sqrt{M^2-1} + \frac{u}{2V}\ln(-1)] + c_1$$

(56)

Because 0 < M < 1, thus above equation can be written as follow:

$$E = -\frac{2EV}{u\pi}i[\ln\sqrt{\frac{1-M^2}{-1}} + \frac{u}{2V}\ln(-1)] + c_1$$

(57)

Solving equation (57) for c_1:

$$C_1 = \frac{EV}{u} + i\frac{EV}{u\pi}\ln(1-M^2)$$

(58)

Substitute c_1 into equation (55):

$$z = -\frac{2EV}{u\pi}i[(\frac{u}{MV}-1)\frac{\sqrt{\zeta^2-M^2}}{\sqrt{\zeta^2-1}}+\ln(\sqrt{\zeta^2-1}+\sqrt{\zeta^2-M^2})$$
$$+\frac{u}{2V}\ln\frac{\sqrt{\zeta^2-M^2}-M\sqrt{\zeta^2-1}}{\sqrt{\zeta^2-M^2}+M\sqrt{\zeta^2-1}}]+\frac{EV}{u}+i\frac{EV}{u\pi}\ln(1-M^2)$$

(59)

Now, Substituting z=D+iH, ζ=N into equation (59), we have

$$D+iH=-\frac{2EV}{u\pi}i[(\frac{u}{MV}-1)\frac{\sqrt{N^2-M^2}}{\sqrt{N^2-1}}+\ln(\sqrt{N^2-1}+\sqrt{N^2-M^2})$$
$$+\frac{u}{2V}\ln\frac{\sqrt{N^2-M^2}-M\sqrt{N^2-1}}{\sqrt{N^2-M^2}+M\sqrt{N^2-1}}]+\frac{EV}{u}+i\frac{EV}{u\pi}\ln(1-M^2)$$

(60)

Considering equqation (54), solving these simultaneous equation, we have

$$(\frac{u}{VM}-1)\frac{\sqrt{u-VM^2}}{\sqrt{u-VM}}+\ln\frac{\sqrt{u-VM}+\sqrt{u-VM^2}}{\sqrt{VM-VM^2}}+\frac{u}{2V}\ln\frac{\sqrt{u-VM^2}-M\sqrt{u-VM}}{\sqrt{u-VM^2}+M\sqrt{u-VM}}+\frac{Hu\pi}{2EV}=0$$

(61)

therefore, $z=f(\zeta)$ is expressed by equation (59) exactly. Here, f(M) =0 is conformable to equation (61).

2) Find $W_1(\zeta)$ and $W_1(z)$

The flow volume Q=2EV flowed into point A_1^* on plane ζ, is equivalent to a confluence of strength 4EV at point A_1^*. Thus the complex potential on plane ζ is:

$$W_1(\zeta) = -\frac{2EV}{\pi}\ln\zeta$$

(62)

and the complex potential on plane z is:

$$W_1(z) = -\frac{2EV}{\pi}\ln\zeta$$

(63)

The relationship between z and ζ conforms to equation (59), here f(M)=0 is conformable to equation (61).

3) Find $W_2(\zeta)$ and $W_2(z)$

The disturbing airstream p on plane z, correspond to a source at point A_1^* on plane ζ. The srength of source is 2PH. So the complex potential on plane ζ is:

$$W_2(\zeta) = \frac{PH}{\pi}\ln\zeta$$

(64)

The complex potential on plane z is:

$$W_2(z) = \frac{PH}{\pi}\ln\zeta$$

(65)

The relationship between z and ζ conforms to equation (59), here f(M)=0 is conformable to equation (61).

4) Find $W_3(\zeta)$ and $W_3(z)$

Using the superposition principle:

$$W_3(\zeta) = \frac{PH}{\pi}\ln\zeta - \frac{2EV}{\pi}\ln\zeta$$

(66)

Therefore:

$$W_3(z) = \frac{PH}{\pi}\ln\xi - \frac{2EV}{\pi}\ln\xi$$

(67)

The relationship between z and ξ conforms to equation (59), and $f(M)=0$ is conformable to equation (61).

5) Find $W_4(z')$ and $W_5(z')$

The obstacle in the flow field on plane z, can be simplified as a circle, see figure 15. The radius is a, and the height measured from ground is h.

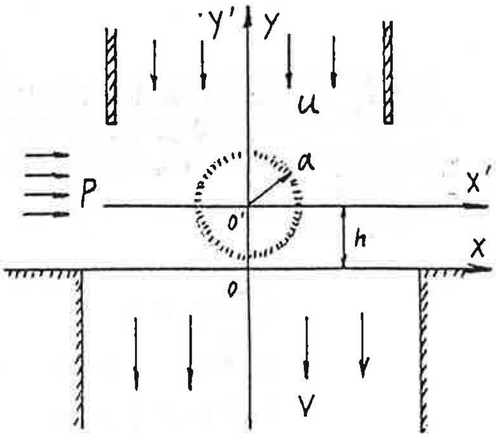

fig. 15

Shifting the axis, we have

$$z=z'+ih$$

(68)

$$W_4(z')=W_3(z'+ih)$$

(69)

Assume $W_5(z')$ is the complex potential in the flow field with a circular obstacle in the new coordinate system.

According to the circle theorem:

$$W_5(z') = W_4(z')+\overline{W}_4\left(\frac{a^2}{z'}\right) = W_3(z'+ih) = W_3\left(\frac{a^2}{z'+ih}\right)$$

(70)

Here

$$W_3(z'+ih) = \frac{PH}{\pi}\ln\xi - \frac{2EV}{\pi}\ln\xi$$

The relationship between z' and ξ is:

$$z'+ih = -\frac{2EV}{u\pi}i\left[\left(\frac{u}{MV}-1\right)\frac{\sqrt{\xi^2-M^2}}{\sqrt{\xi^2-1}} + \ln\left(\sqrt{\xi^2-1}+\sqrt{\xi^2-M^2}\right) + \frac{u}{2V}\ln\frac{\sqrt{\xi^2-M^2}-M\sqrt{\xi^2-1}}{\sqrt{\xi^2-M^2}+M\sqrt{\xi^2-1}}\right] + \frac{EV}{u} + i\frac{EV}{u\pi}\ln(1-M^2)$$

(71)

here $f(M)=0$ is conformable to equation (61). The relationship between z' and ξ is:

$$\frac{a^2}{z'+ih} = \frac{2EV}{u\pi}i\left[\left(\frac{u}{MV}-1\right)\frac{\sqrt{\xi^2-M^2}}{\sqrt{\xi^2-1}} + \ln\left(\sqrt{\xi^2-1}+\sqrt{\xi^2-M^2}\right) + \frac{u}{2V}\ln\frac{\sqrt{\xi^2-M^2}-M\sqrt{\xi^2-1}}{\sqrt{\xi^2-M^2}+M\sqrt{\xi^2-1}}\right] + \frac{EV}{u} - i\frac{EV}{u\pi}\ln(1-M^2)$$

(72)

here $f(M)=0$ is conformable to equation (61).

Using Example

I. Print the flow stream of flow field.

There is a flow field of drawing out mode (see figure 10). U, V and B are given.

The target is to print out the flow stream of flow field.

Solution:

1) Suppose,

$$\eta_1$$
$$\eta_2 = \eta_1 + h$$
$$\vdots$$
$$\eta_i = \eta_1 + (i-1)h$$
$$\vdots$$
$$\eta_n = \eta_1 + (n-1)h$$

Here, i is 1, 2, 3, ...n natural number, h is the step which is given.

2) according to the formular (37)

$$W_3(\zeta) = -\frac{2BU}{\pi}\ln\zeta + V\zeta$$

It can be changed to be

$$W_3(\zeta) = \varphi(\xi,\eta) + i\psi(\xi,\eta)$$

(73)

here

$$\varphi(\xi,\eta) = -\frac{2BU}{\pi}(\xi^2+\eta^2) + V\xi$$

(74)

$$\psi(\xi,\eta) = -\frac{2BU}{\pi}\operatorname{arctg}\frac{\eta}{\xi} + V\eta$$

(75)

$\psi(\xi,\eta)$ is a flow function of the flow field.

3) Suppose: $\psi = \psi_1$

here the ψ_1 is a constant.

So, the formula (75) became

$$\xi = \frac{\eta}{tg(-\frac{\pi\psi}{2BU} - V\eta)}$$

(76)

Using this formula, we can solve ξ_1, ξ_2, ··· ξ_n.

4) Changing the formula (32) to

$$x+iy = f_1(\xi,\eta)+if_2(\xi,\eta)$$

(77)

Here

$$f_1(\xi,\eta) = \frac{\sqrt{2}}{\pi}B\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2+\xi^2-\eta^2-1}$$

$$-\frac{2}{\pi}B[\operatorname{arctg}\frac{\sqrt{\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2}+\xi^2-\eta^2-1}}{\sqrt{2-\sqrt{\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2}-(\xi^2-\eta^2-1)}}}-\operatorname{arctg}\frac{\eta}{\xi}]+B$$

(78)

$$f_2(\xi,\eta) = \frac{\sqrt{2}}{\pi}B\sqrt{\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2}-(\xi^2-\eta^2-1)} - \frac{2}{\pi}B[\ln\sqrt{\xi^2+\eta^2} - \ln(1-\sqrt{2}\sqrt{\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2}-(\xi^2-\eta^2-1)}+\sqrt{(\xi^2-\eta^2-1)^2+4\xi^2\eta^2})] \tag{79}$$

5) according to

$x=f_1(\xi,\eta)$

$y=f_2(\xi,\eta)$

and $\eta_1, \eta_2, \cdots\cdots \eta_n$

$\xi_1, \xi_2, \cdots\cdots \xi_n$

solving out $x_1, x_2 \ldots\ldots x_n$

$y_1, y_2 \ldots\ldots y_n$

6) Link up the dots from the first dot (x_1, y_1) to the next $(x_2, y_2) \ldots\ldots (x_n, y_n)$ as a curve, thus we have got the flow stream of $\psi = \psi_1$.

7) Set the $\psi=\psi_2, \psi=\psi_3, \ldots$ we can get a group of flow streams. Although this is a complicate process, but when a computer is used, the flow streams of flow field are easy to be printed out.

II, Solve the drawing out velocity

There is a flow field of drawing out mode (see figure 12). V, a, h, B is given. The control dot Zo (Xo, Yo) is also given.

Target: Solve the drawing out velocity of flow field —u.

Solution:

1) Because Xo, Yo is given, according to formula (43), solve the ξ_1, η_1.

2) Because Xo, Yo is given, according to formula (45), solve the ξ_2, η_2.

3) The same: formula (37) can be changed to formula (73), formula (74) and formula (75).

so, $\psi_1 = f(\xi_1, \eta_1)$, $\psi_2 = f(\xi_2, \eta_2)$

4) Change the formula (41) to $W_5(\xi)=\varphi_1+\varphi_2+i(\psi_1+\psi_2)$

here, $\psi_1+\psi_2$ is the flow function of this flow field.

5) Set $X_1=B-\varepsilon; Y_1=\varepsilon$.

here, ε is a very little positive number which is given. Redo the process 1) to 4), then

$$\psi_{b1} = (\xi_{b1}, \eta_{b1}) , \qquad \psi_{b2} = (\xi_{b2}, \eta_{b2})$$

6) Because Zo, Z_1 must in one line, so

$$\psi_1(\xi_1, \eta_1) + \psi_2(\xi_2, \eta_2) = \psi_{b1}(\xi_{b1}, \eta_{b1}) + \psi_{b2}(\xi_{b2}, \eta_{b2})$$

Solve out the u.

This process is complicate, it is necessary to use computer for getting the solution. The program is omitted, because of the limited space.