LASL SIMILARITY STUDIES: PART I

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HOT ZONE/COLD ZONE: A QUANTITATIVE STUDY OF NATURAL HEAT DISTRIBUTION MECHANISMS IN PASSIVE SOLAR BUILDINGS*

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ABSTRACT

Interzone distribution of solar heat gains by natural convection provides a viable low-cost alternative to forced air systems. A procedure for developing a general empirical law for interzone transport rates based on data from a small laboratory model and the principles of similitude is outlined. Preliminary results are presented and discussed.

1. INTRODUCTION

One of the basic problems confronting the designer of a passive solar heated building is to achieve reasonably uniform temperatures within a structure whose southern zones are preferentially heated by the sun. If care is not exercised, there is a danger that the temperature differential between north and south zones will be so large that occupants will either be too hot in the south zones or too cold in the north zones (or both) even though sizing calculations indicate that solar gains are well matched to the building's overall load and thermal storage characteristics. In general, four strategies may be applied by designers, either separately or in combination, as a means of assuring adequate thermal uniformity:

a. distributed solar gains,
b. shallow construction,
c. forced air heat distribution, and
d. natural convective heat distribution.

Distributed solar gains are achieved by employing clerestory windows or roof apertures which allow direct solar heating of north zones. This strategy can be very effective, but, due to the associated structural complexity, it costs more than ordinary windows[1] and may be too expensive for the mass housing market. Shallow construction refers to building configurations which are one zone deep in the north-south direction such that each room of the structure has a south facing wall that can provide solar heat. Although many attractive designs employ the concept of shallow construction, the geometric limitations imposed on building floorplans may hinder widespread application.

Many passive solar designs intended for the mass housing market attempt to escape the added cost of distributed gains and the geometric limitations of shallow construction by relying on forced air distribution systems to transport thermal energy from southern zones to northern zones. This approach yields a low first cost because it entails the least departure from conventional construction. However, electric power is required to run the necessary fans, and the solar heating system is, therefore, not independent of the local utility company, i.e., a power failure would not only deprive the occupant of the benefit of any electrical backup system, but would significantly impair normal operation of the "passive" solar system. Energy used to power the blowers must, of course, be charged against the solar heat gains.

The fourth design strategy is the subject of this paper. Heat transfer by natural convection can prevent excessive thermal non-uniformity in most buildings provided interior doorways and other apertures are properly sized and distributed. Indeed, buildings which are extremely well insulated and tightly constructed tend to be thermally uniform even though no specific consideration has been given to aperture design.[2] However, decades of reliance on forced air distribution systems

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has stifled research on free convective systems and left us with a lack of confidence in their viability. The purpose of this paper is to outline a method which is being used at LASL to develop quantitative design tools for the architect/builder interested in natural heat distribution strategies. We are motivated by the desire to provide an inexpensive alternative (or supplement) to the concepts of distributed gains and shallow construction that can function properly in the absence of electromechanical support.

The general methodology selected is to develop an empirical law relating interzone transport rates to the geometric properties of the enclosure and the temperature fields in the separate zones. The law will be based on a large body of data obtained in a small-scale laboratory test chamber in which physical processes occurring in full-scale enclosures are simulated. Results obtained in the test chamber (model) are related to the performance of actual buildings (prototypes) through the principles of similitude. The theoretical basis for this approach and the associated formulae are presented in the next section.

2. THEORETICAL CONSIDERATIONS

2.1. The Similarity Conditions

Similarity is defined as the relationship between analogous physical processes that can be described by a single set of dimensionless conservation equations. Writing the conservation equations in terms of non-dimensional forms of the associated independent and dependent variables yields dimensionless groups of parameters that appear as coefficients. All physical processes for which the dimensionless groups have the same value are said to be similar. The dimensionless groups that appear in the equations governing free convective motion and heat transfer in a fluid medium are the Grashof number,

$$Gr = \frac{g\beta \ell^3 \Delta T_o}{\nu^2} \tag{1}$$

and the Prandtl number,[3]

$$Pr = \frac{\nu}{a} \tag{2}$$

where g = gravitational constant;
ℓ = characteristic length;
β = coefficient of expansion;
ΔT_o = characteristic temperature difference;
ν = kinematic viscosity; and
a = thermal diffusivity.

For a perfect gas, $\beta = 1/\bar{T}$, where $\bar{T}$ is the reference or average temperature of the gas. The Grashof number in a perfect gas, therefore, becomes

$$Gr = \frac{g\ell^3 \Delta T_o}{\nu^2 \bar{T}} . \tag{3}$$

Now, if we wish to simulate natural convection currents occurring in a full-scale structure (referred to as the prototype) within a small well-controlled laboratory model, it is only necessary to make sure that the Grashof and Prandtl numbers are matched, i.e.,

$$Gr_m = Gr_p,$$

$$Pr_m = Pr_p,$$

where the subscripts m and p refer to model and prototype respectively.

The Prandtl number is a fluid property that is always of the order of unity in gases and generally much larger and more variable in liquids. The model Prandtl number is, therefore, easily matched to the prototype air system by employing almost any gaseous medium. The Grashof number is more complex, involving geometric and thermal parameters as well as fluid properties. The parameters in the Grashof number are given the following interpretation with respect to the two-zone geometry of interest in this study (see Fig. 1).

ℓ = H, interior height of the two-zone space,

$\Delta T_o = \bar{T}_1 - \bar{T}_2$,
$= \Delta\bar{T}$ average temperature difference between zones,

where $\bar{T}_1$ = average temperature of Zone 1, and
$\bar{T}_2$ = average temperature of Zone 2.

$\bar{T} = \frac{\bar{T}_1 + \bar{T}_2}{2}$, average temperature of combined zones.

The Grashof number can now be written as

$$Gr = \frac{gH^3 \Delta\bar{T}}{\nu \bar{T}} . \tag{4}$$

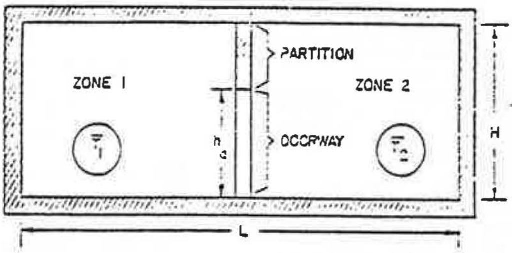

Fig. 1. Vertical cross section of two-zone building geometry.

(Note: There is no a priori basis for the above interpretation of the Grashof number, and final acceptance must be based on analysis of experimental data.)

Since the two-zone laboratory model is considerably smaller than the full-scale prototype, equality of the Grashof number can only be maintained by using a fluid other than air. According to Eq. 4, Grashof number equality implies

$$\frac{H_m}{H_p} = \left[\frac{\Delta\bar{T}_p}{\Delta\bar{T}_m} \cdot \frac{\bar{T}_m}{\bar{T}_p} \cdot \left(\frac{\nu_m}{\nu_p}\right)^2\right]^{1/3} \qquad (5)$$

and the subscripts m and p again refer to model and prototype.

Now, if a one-fifth scale model is selected ($H_m/H_p = .2$) and the temperatures in the model are to be held approximately equal to those in the prototype in order to minimize differences in radiation transport rates, Eq. 5 reduces to

$$\frac{\nu_m}{\nu_p} = 0.0894 \approx 0.1 \quad .$$

So, complete similitude can be obtained in a one-fifth scale model by using a gas that has a kinematic viscosity that is roughly one-tenth that of air. Any required fine tuning can be accomplished by allowing slight variations in scale or by allowing the temperatures in the model to depart from those in the prototype.

2.2. The Effective Conductance for Interzone Transport of Thermal Energy by Free Convection.

For the purposes of building loads analysis, heat transport rates are usually represented by the following functional form in thermal network codes

$$Q = UA\Delta T \quad , \qquad (6)$$

where Q = the heat flow rate between nodes;
U = the conductance;
ΔT = the temperature difference between nodes; and
A = area of the conductance path between nodes.

Equation 6 is used to represent heat flows due to conduction, convection, and radiation by selection of appropriate constants or functions for the conductance. However, for the case of interzone transport by free convection, a general law has not yet been developed. We propose to develop such a law and for the sake of compatibility with existing thermal network models, a format based on Eq. 6 is selected.

$$Q_{12} = U_{12}A_a(\bar{T}_1 - \bar{T}_2) \quad , \qquad (7)$$

where Q_{12} = net rate at which thermal energy is convected from a hot zone (#1) to a cold zone (#2),
U_{12} = the effective conductance; and
A_a = area of the aperture through which heat is convected.

The effective conductance, U_{12}, will, of course, be a function of the geometric and thermal characteristics of the two-zone enclosure. The exact functional form will be determined on the basis of many steady-state experiments performed with the small-scale laboratory model in which the geometric and thermal parameters can be varied in a carefully controlled and systematic manner. At present, we expect the conductance to be most strongly dependent on the Grashof number and the ratio of door height to ceiling height. We, therefore, have

$$U_{12} = U_{12}(Gr, R_a, \ldots) \quad , \qquad (8)$$

where $R_a = h_a/H$, h_a is the aperture height,

and allowance for future addition of other independent variables is indicated by the three dots.

2.3. The Generalized Interzone Transport Rate

As represented in Eq. 8, the empirical law for interzone transport is not yet in a general form. In order to develop results for full-scale structures in a general manner, it is necessary to invoke the similarity conditions. Having matched the Grashof and Prandtl number of model and prototype, we know that dimensionless velocities in the two structures will be equal, i.e.,

$$V_m^* = V_p^* \quad , \qquad (9)$$

where $V^* = \frac{V}{(\nu/H)}$ (10)

and V is the local fluid velocity at any point.

(Note: This nondimensional form of the velocity is the one that yields dimensionless free convective conservation equations in which only the Grashof and Prandtl numbers appear as coefficients.)

The ratio of prototype to model velocity is, therefore,

$$\frac{V_p}{V_m} = \frac{(\nu_p/H_p)}{(\nu_m/H_m)} \quad (11)$$

When we adopt a method similar to that used by Shaw,[4] the heat convection rate from Zone 1 to Zone 2 can be written as

$$Q_{12} = \rho c_p \bar{V}_{12} A_{12}(\bar{T}_1 - \bar{T}_2) - \rho c_p \bar{V}_{21}(A_a - A_{12})(\bar{T}_2 - \bar{T}_1) \quad (12)$$

where $\bar{V}_{12}$ = average velocity from Zone 1 to Zone 2;
$\bar{V}_{21}$ = average velocity from Zone 2 to Zone 1;
A_{12} = aperture area over which flow velocity from Zone 1 to Zone 2 is positive;
ρ = fluid density;
c_p = fluid specific heat at constant pressure.

In terms of the area-weighted average exchange velocity,

$$\bar{V} = \frac{A_{12}\bar{V}_{12} + (A_a - A_{12})\bar{V}_{21}}{A_a} \quad (13)$$

Equation 12 becomes

$$Q_{12} = \rho c_p \bar{V} A_a(\bar{T}_1 - \bar{T}_2) \quad (14)$$

Eliminating Q_{12} from Eqs. 7 and 14 yields

$$U_{12} = \rho c_p \bar{V} \quad (15)$$

Based on Eqs. 11 and 15, the ratio of interzone transport coefficients for model and prototype is

$$\frac{(U_{12})_p}{(U_{12})_m} = \frac{\rho_p (c_p)_p}{\rho_m (c_p)_m} \cdot \frac{(\nu_p/H_p)}{(\nu_m/H_m)} \quad (16)$$

Equation 16 may be used to determine the transport coefficient in the prototype, $(U_{12})_p$, when that in the model, $(U_{12})_m$ is known. It is more convenient, perhaps, to express all results in terms of the heat transfer number, J, where on the basis of Eq. 16,

$$J = \frac{U_{12}}{\rho c_p (\nu/H)} \quad (17)$$

This dimensionless group has the same form as the heat transfer number reported by Landau and Lifshitz[5], except that ν/H appears in place of the characteristic velocity that is of unknown magnitude in interior flows. The following equivalence should also be noted:

$$J = \frac{Nu}{Pr} \quad (18)$$

$$\text{where } Nu = \frac{U_{12}}{k/H} \quad (19)$$

is the frequently used Nusselt number and k is the thermal conductivity. Representing interzone transport rates with the Nusselt number is equivalent to using the heat transfer number provided the Prandtl number is constant, i.e., provided full similitude is maintained. Based on Eq. 19 the interzone transport rate for a building (prototype) is give by

$$U_{12} = (k/H)Nu(Gr, R_a, \ldots) \quad , \quad (20)$$

where the functional form of Nu is determined by test chamber data.

3. EXAMPLE BASED ON PRELIMINARY RESULTS

A description of the experimental appratus and technique used in this study is presented in "LASL Similarity Studies: Part II," in these proceedings. The preliminary results are presented in Fig. 2 of that paper where the Nusselt number appears as a function of the Grashof number for two distinct aperture configurations. One configuration has a door-to-ceiling height ratio of 0.82 and the second has a ratio of 1.0, i.e., the doorway extends all the way to the ceiling. In this paper we will discuss a specific example that makes use of the data reported in general form in Part II.

In Fig. 2 below, the interzone transport rate, Q_{12}, is plotted as a function of the difference between average zone temperatures. The lower curve with data points indicated by circles represents a standard 36" x 80" doorway in a partition separating two zones with eight-foot ceilings. The higher curve with data points indicated by squares represents a

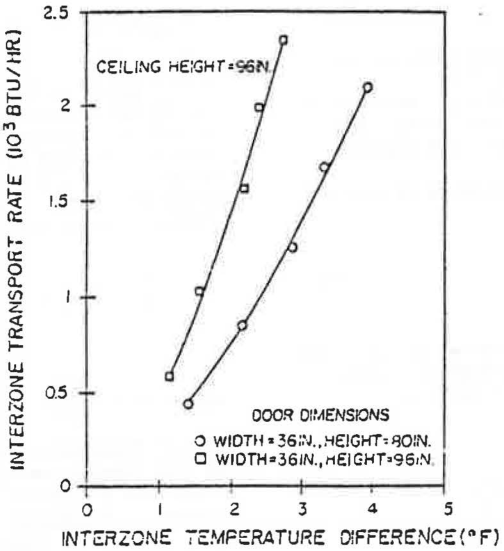

Fig. 2. Convective heat flow through aperture vs interzone temperature difference for standard and full-ceiling height doorways.

modified doorway whose width is held constant at 36" but whose height is extended up to the 96" high ceiling. Thus, configuration number 2 has a door area of 24 ft^2 and configuration number 1 has only 20 ft^2. On the basis of area increase alone we might therefore expect an enhancement of interzone transport rate by about 20% for the larger doorway. One is, therefore, startled to find that the enhancement factor for full ceiling height doorways exceeds 100% for interzone temperature differences approaching 3°F. Thus, a relatively minor alteration in doorway geometry can significantly strengthen interzone coupling by free convection. Note also that the larger the interzone temperature difference becomes, the greater the advantage of the full ceiling height configuration. The phenomenon that is revealed and quantified by these results is that hot air pockets formed by stratification and trapped near the ceiling of the hot zone are released by full-height doorways, thereby enhancing interzone mixing in a non-linear manner.

4. DESIGNING FOR THERMAL UNIFORMITY

In order to assure adequate thermal uniformity in a building, the designer must adhere to the following procedure.

1) Select a reasonable interzone temperature difference.

2) Select an aperture configuration and size that provides interzone transport rates (at the selected temperature differential) at least as large as the intended net rate of heat addition in the south zone and the intended net rate of heat loss from the north zone.

Future work at LASL will be directed to generating a large data base for interzone transport rates that can serve as the basis for a general empirical law. The designer will then have the tools required for effective use of free convective design. The results presented in this paper are preliminary and are intended primarily for illustration.

5. CONCLUSION

A technique for determining interzone transport rates has been developed and demonstrated. We have shown that a minor alteration in aperture geometry can greatly improve interzone coupling, thereby improving thermal uniformity in most buildings. In the future, we will continue to explore variations of geometric and thermal parameters until all significant independent variables have been identified. At that time a general interzone transport law will be formulated.

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