#3545

Development of a Dynamic Model for Simulating Indoor Air Temperature and Humidity

C.G. Barringer, P.E.

C.A. McGugan
Associate Member ASHRAE

ABSTRACT

A project was undertaken to develop a computer program to predict air temperature and humidity within buildings. Mathematical models were developed for heat recovery ventilators, both with and without moisture recovery, moisture transport through basement walls and floors, window condensation and evaporation, and moisture storage in interior materials. These models were inserted into a commercially available dynamic simulation program that previously had only rudimentary methods for estimating indoor humidity. The upgraded program was used to investigate the role of various types of ventilators in maintaining acceptable indoor humidity. This investigation produced recommendations for selection guidelines for residential enthalpy recovery ventilation systems.

INTRODUCTION

Building simulation has been almost entirely concerned with the transport of sensible heat in buildings. Recently, however, several factors have focused more attention on indoor humidity. Excessive indoor humidity can significantly increase the energy consumption of air conditioners and can cause deterioration in building materials. Furthermore, due to its effects on allergens and the infectivity of airborne bacteria, indoor humidity is an important factor in the health of occupants.

Building simulation programs have recently been developed to model the transport of moisture. One of the first models to accurately predict indoor humidity was developed by Tsuchiya (1980). His approach was to find the indoor humidity through the solution of a system of three equations consisting of: an equation for the humidity ratio at the surface of each material, an equation for changes in air humidity ratio, and an equation for changes in material moisture content. The predictions of this model were in close agreement with experimental data from a test house. The main drawback with this model is that it contains several mass transfer coefficients that are not commonly available.

Another model for moisture transport in buildings was developed by Miller (1984). His approach was to incorporate moisture transport into a postprocessing procedure that was applied to the output of an hourly sensible load program. The moisture transport model included simple equations for dehumidification, infiltration/ventilation, and moisture storage in building materials. Miller's equation for moisture storage was based on an analogy to a resistor-capacitor circuit. Consequently, the moisture flow into materials could be characterized with a time constant and the hygroscopic storage constant. Miller's model was found to be in close agreement with experimental data from a mobile home.

To date, the most sophisticated program for simulating the indoor enthalpy in buildings has been developed by Fairey et al. (1986). This advanced program, known as MADTARP (Moisture Adsorption/Desorption Thermal Analysis Research Program), is a detailed dynamic simulation program. It utilizes very sophisticated models for analyzing simultaneous thermal and mass transfer. It has been used quite extensively to investigate the energy consumption of various cooling strategies. MADTARP has also undergone extensive laboratory and field validation.

MADTARP is extremely detailed and requires a large amount of time and effort in assembling the input files. There was no simple program available to obtain predictions of indoor humidity in houses. As a result, the objective of this project was to develop a simple computer program to predict indoor air temperature and humidity.

METHOD OF APPROACH

Moisture transport in buildings involves several time-dependent mechanisms. Consequently, accurate prediction of indoor moisture levels requires the dynamic or time-dependent approach to building simulation.

The effect of the various moisture sinks and sources on the dynamics of indoor humidity is illustrated in the equation below:

$dRh / dt =$		rate of increase in indoor relative humidity in a particular zone	(1)
	dRh_m / dt	addition from moisture stored in internal materials	
+	dRh_v / dt	addition from outdoor air to the zone	

C.G. Barringer is a Project Engineer and **C.A. McGugan** is Manager of Product Evaluation Services, ORTECH International, Mississauga, Ontario, Canada.

- $+ \; S \, (w_{sat} \, \rho_{air} \, V_{ol})^{-1}$ addition from internal sources (people, appliances, etc.) and from other zones
- $+ \; dMc / dt \, (w_{sat} \, \rho_{air} \, V_{ol})^{-1}$ addition due to moisture diffusion through basement concrete
- $+ \; dMg / dt \, (w_{sat} \, \rho_{air} \, V_{ol})^{-1}$ addition due to evaporation from windows

where

Rh = indoor relative humidity
Rh_m = contribution to indoor relative humidity from moisture stored in internal materials
Rh_v = contribution to indoor relative humidity from ventilation or infiltration
w_{sat} = saturation humidity ratio at indoor temperature
V_{ol} = house volume
ρ_{air} = dry air density
S = moisture generation rate from internal sources
Mc = moisture contribution (to air) from diffusion through basement concrete
Mg = moisture contribution (to air) from glass

For unrealistically simple models of houses, numerical solutions to this equation can be obtained quite easily. However, the time-dependent simulations required for realistic modeling are very complex. Therefore, rather than develop a complete indoor enthalpy simulation program from first principles, the new simulation was developed from a commercially available program.

The selected commercial dynamic simulation program had the following features:

— modeling of up to four zones;
— very simple models for indoor humidity based on ventilation and internal gains (no storage);
— displays of frequency distributions for indoor humidity and temperature; and
— displays of monthly totals for solar gains, internal gains, and energy use for space heating and cooling.

This program had only rudimentary models for moisture transport. It was necessary to modify the program by adding models corresponding to most of the terms in the above equation. Therefore, the following models were developed:

— a model for the absorption and desorption of moisture in walls, floors, ceilings, and furnishings;
— an enthalpy recovery ventilator model;
— a model for the diffusion of moisture through basement concrete; and
— a window condensation and evaporation model.

The sections below describe the development of each of these models. The final section describes a solution to the above equation for a simple building over a single day.

MOISTURE ABSORPTION AND DESORPTION IN WALLS, FLOORS, CEILINGS, AND FURNISHINGS

The model for moisture sorption into walls, floors, ceilings, and furnishings was based on several measurements by Kusuda (1985), Stamm (1965), and Martin et al. (1986) of the response of a number of materials to step changes in room humidity. These measurements indicate that the rate of change in the moisture content is very similar to an exponential decay. Consequently, the following equation was used to model the flow of moisture into walls, furnishings, floors, and other internal materials.

$$dm_i / dt = (C_i \, M_i \, Rh - m_i) / \tau_{m,i} \tag{2}$$

where

C_i = hygroscopic constants for the ith material
Rh = room relative humidity
M_i = dry weight of the ith material
m_i = moisture mass in the ith material
$\tau_{m,i}$ = ith material mass change time constant

The maximum mass flow rate occurs at zero humidity and $m_i = C_i M_i$, or $Rh = 100\%$ and $m_i = 0$. Therefore, the maximum possible mass flow rate is:

$$(dm_i / dt)_{max} = \pm C_i \, M_i \, / \, \tau_{m,i} \tag{3}$$

The maximum mass flow rate corresponds to a maximum vapor velocity. The expression for this is:

$$u_{max,i} = \pm C_i \, M_i \, / \, (\tau_{m,i} \, \rho_{air} \, w_{sat} \, A_i) \tag{4}$$

where

$u_{max,i}$ = the maximum vapor velocity of the ith material

Since the mass of building materials is frequently defined in terms of area density and surface area, the maximum vapor velocity into the material can be simplified to:

$$u_{max,i} = C_i \, \rho_{m,i} \, / \, (\tau_{m,i} \, \rho_{air} \, w_{sat}) \tag{5}$$

where

$\rho_{m,i}$ = ith material mass per unit area (e.g., gypsum: 8.5 kg/m^2)

(According to the above equations, this maximum flow can occur in either direction. A positive sign implies that the vapor is flowing into the material.)

Inserting the maximum vapor velocity equation into the mass flow rate equation yields:

$$dm_i / dt = u_{max,i} \, \rho_{air} \, w_{sat} \, A_i \, (Rh - m_i \, / \, (C_i \, M_i)) \tag{6}$$

The change in room relative humidity due to changes in the moisture content of materials is:

$$dRh_m / dt = - (\Sigma \, dm_i / dt) \, / \, (\rho_{air} \, w_{sat} \, V_{ol}) \tag{7}$$

The above two equations can be combined to form a new equation for the flow of moisture into the room air from interior materials:

$$dRh_m / dt = - \Sigma \, [(Rh - m_i \, / \, (C_i \, M_i)) \, u_{max,i} \, A_i \, / \, V_{ol}] \tag{8}$$

Examination of the above equation indicates that the time constant governing the rate of change of the relative humidity due to the ith material is:

$$\tau_{rh,m,i} = V_{ol} / (u_{max,i} A_i) \tag{9}$$

where

$\tau_{rh,m,i}$ = humidity change time constant for the ith material

The combined time constant for the transport of moisture from all storage materials to the air is therefore given by:

$$\tau_{rh,m} = 1 / (\Sigma \ 1 / \tau_{rh,m,i}) \tag{10}$$

where

$\tau_{rh,m}$ = humidity change time constant for all materials

Therefore, the equation for the flow of moisture into the room from moisture storage materials is:

$$dRh_m / dt = - Rh / \tau_{rh,m} + \Sigma [m_i / (C_i M_i \tau_{rh,m,i})] \tag{11}$$

At this point it is convenient to draw some conclusions on the above derivations:

1. Since the indoor air temperature is roughly constant, the indoor air density and the saturation humidity ratio are virtually constant. Consequently, the maximum vapor velocity is effectively a function of the material properties.
2. The $\tau_{rh,m,i}$ time constant is a function of many variables: the material's surface area, the room volume, the material area density, the hygroscopic constant, and the $\tau_{m,i}$ time constant. As a result, $\tau_{rh,m,i}$ can be quite different from $\tau_{m,i}$. Furthermore, widely differing materials may have very similar $\tau_{rh,m,i}$ time constants.
3. An examination of the equation for $\tau_{rh,m}$ reveals that $\tau_{rh,m}$ will be less than the shortest $\tau_{rh,m,i}$. This indicates that the materials with the shortest $\tau_{rh,m,i}$ have a dominant influence on the fluctuations in room relative humidity.
4. Since the internal moisture gains have rise times on the order of one hour or less, moisture storage will not significantly dampen the indoor humidity fluctuations unless the humidity change time constant $\tau_{rh,m}$ is less than about one hour.

MODELING OF ENTHALPY RECOVERY VENTILATORS

The rate of increase of the indoor relative humidity due to the ventilator is given by:

$$dRh_v / dt = (1 - \varepsilon_m) (w_1 - w_3) V' / (V_{ol} w_{sat}) \tag{12}$$

where

ε_m = mass transfer effectiveness of the enthalpy recovery ventilator
V' = volume flow rate of the ventilator
V_{ol} = volume of the house's interior
w_1 = outdoor air humidity ratio
w_3 = indoor air humidity ratio

The above equation can be rewritten in the following form:

$$dRh_v / dt = (w_1 / w_{sat} - Rh) / \tau_v \tag{13}$$

where

$$\tau_v = V_{ol} / [V' (1 - \varepsilon_m)] \tag{14}$$

= ventilation time constant

This time constant can be compared directly to the humidity change time constant developed in the previous section. Like the case of the humidity change time constant, indoor humidity fluctuations will not be significantly dampened by ventilation unless the ventilation time constant is less than about one hour. This will be discussed further in a later section.

Models for the following three types of residential enthalpy recovery ventilators were developed:

— non-desiccant wheel enthalpy recovery ventilators
— desiccant wheel enthalpy recovery ventilators
— porous plate enthalpy recovery ventilators

To permit comparison of conventional ventilators to the above units, a model for a sensible heat recovery ventilator was also developed. The subsections below describe the models for each of these types of equipment.

Non-Desiccant Wheel Heat Recovery Ventilators

The non-desiccant wheel heat recovery ventilator has a rotating core that picks up and stores heat from the exhaust stream and releases it to the fresh airstream. If the core is made of a non-desiccant material, moisture is transferred only if it condenses from the warm airstream on the core and then evaporates in the colder stream. This will happen only if the cold stream is below the dew point of the hot stream.

The model for the non-desiccant wheel heat recovery ventilator is based on a theoretical study by Holmberg (1977), which predicted performance through a detailed component-by-component analysis of the heat exchange core. Results were obtained through numerical solutions.

The method used to formulate a model for the non-desiccant wheel enthalpy recovery unit consists of extending the general characteristics evident in Holmberg's results to conditions beyond the range of Holmberg's input data. A plot of Holmberg's moisture transfer efficiency results vs. $w_3 - w_1$ is shown in Figure 1. Since the moisture transfer is based on condensation, the moisture transfer efficiency is zero at non-zero $w_3 - w_1$.

At this point it is convenient to introduce a new dependent variable, the condensation threshold humidity ratio, w_{cdt}. At this humidity ratio, the moisture transfer efficiency goes to zero:

$$\varepsilon_m (w_{cdt} - w_1) = 0 \tag{15}$$

Values for w_{cdt} can be obtained from Figure 1.

In Figure 2 the moisture transfer efficiency is plotted vs. $w_{cdt} - w_1$. It can be seen that the efficiency curves have a similar shape. Fitting a curve to these data yields the following equation for the efficiency:

$$\varepsilon_m = \begin{cases} 119.07 \, (w_{cdt} - w_1)^{0.7691} \varepsilon_{m,max} & \text{for } 0 \le w_{cdt} - w_1 \le 0.002 \\ \varepsilon_{m,max} & \text{for } 0.002 < w_{cdt} - w_1 \end{cases} \tag{16}$$

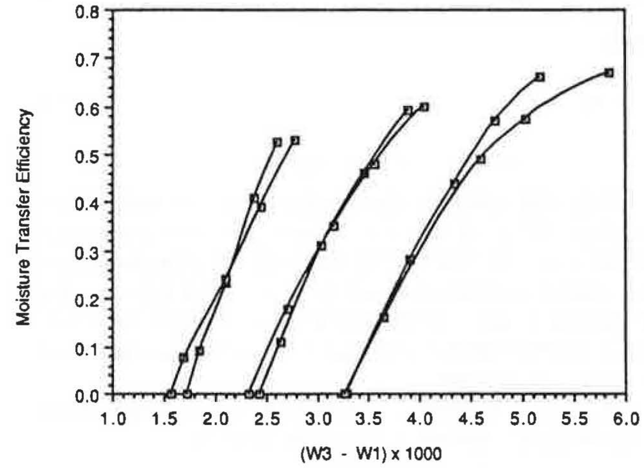

Figure 1 *Non-desiccant HRV: efficiency vs. indoor/outdoor humidity ratio difference*

where

$\epsilon_{m,max} = 0.7127$

A relationship for the condensation threshold humidity ratio is now needed. It was postulated in the work by Barringer and McGugan (1988) that $w_3 - w_{cdt}$ is a function of $DP_3 - T_1$. Figure 3 shows a graph of $w_3 - w_{cdt}$ based on Holmberg's data. Based on this graph, the following equation is postulated for $w_{cdt} - w_1$:

$$\begin{aligned} w_{cdt} - w_1 &= w_3 - w_1 - 0.001241\,(5.119 - (DP_3 - T_1)) && \text{for } 0 < DP_3 - T_1 < 5.119 \\ &= w_3 - w_1 && \text{for } 5.119 < DP_3 - T_1 \end{aligned} \qquad (17)$$

Another factor in the performance of the non-desiccant wheel heat recovery ventilator is frosting. The curves in Figure 1 each terminate at the onset of frosting. Holmberg's results indicated that the frosting threshold outdoor temperature rises with increasing exhaust temperature and with increasing exhaust humidity. This is illustrated by plotting the frosting threshold outdoor temperature (obtained from Holmberg's paper) against exhaust dew point in Figure 4. The following equation is therefore proposed for the frosting threshold temperature:

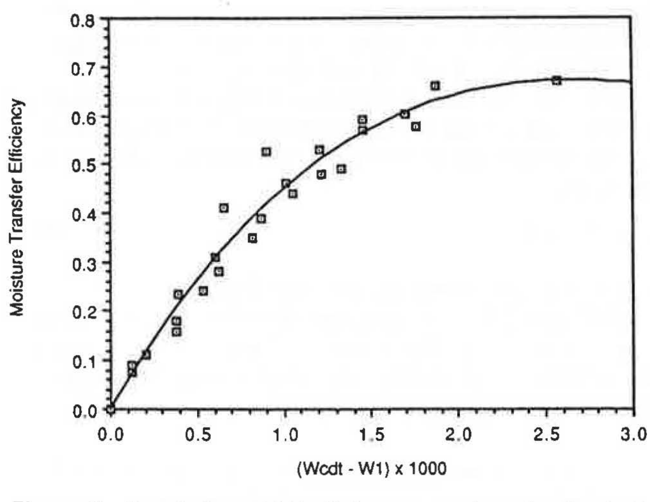

Figure 2 *Non-desiccant HRV: efficiency vs. condensation threshold humidity ratio*

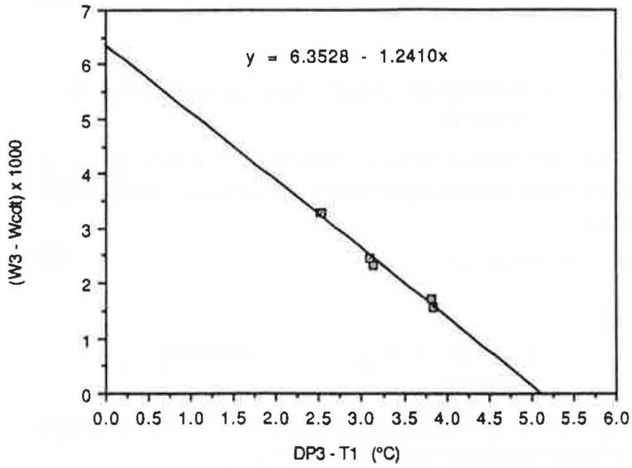

Figure 3 *Non-desiccant HRV: condensation threshold humidity ratio*

$$\begin{aligned} T_{fst} &= -12.5283 + 0.8609\,DP_3 && \text{for } DP_3 < 6.42 \\ &= -7 && \text{for } DP_3 \geq 6.42 \end{aligned} \qquad (18)$$

The power requirements for preheating the outdoor air to avoid defrosting are given by:

$$\begin{aligned} Q_h &= 0 && \text{for } T_1 > T_{fst} \\ &= m\,C_p\,(T_{fst} - T_1) && \text{for } T_1 \leq T_{fst} \end{aligned} \qquad (19)$$

Desiccant Wheel Enthalpy Recovery Ventilators

In this type of rotary heat exchanger, the wheel contains a desiccant material which will absorb moisture from the humid airstream and then release the moisture into the less humid stream. The desiccant wheel will also transfer moisture through condensation.

The model for the desiccant wheel enthalpy recovery ventilator was obtained from a paper by Hoagland (1986). In this work, the enthalpy and moisture transfer efficiency (ϵ_e and ϵ_m) of a residential enthalpy recovery unit were found to be constant at 0.75. This did not include the effects of frosting.

The preheater input was assumed to be given by the above relationship for the non-desiccant wheel. The frost threshold temperature has not been thoroughly investi-

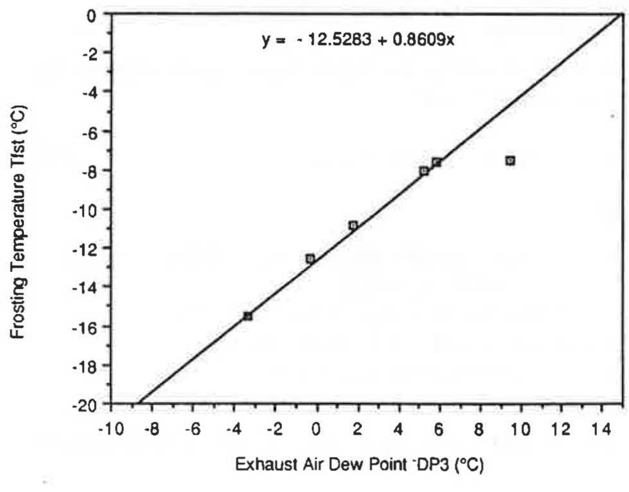

Figure 4 *Non-desiccant HRV: frosting threshold temperature*

gated. However, the paper by Hoagland (1986) suggests that frosting occurs at conditions ranging from an outdoor temperature of 0°F (−17.8°C) at 20% indoor relative humidity, to an outdoor temperature of 15°F (−9.4°C) at 60% indoor relative humidity. If it is assumed that there is a linear relationship between the indoor relative humidity and the outdoor temperature frosting threshold, then the previous conditions fit the following equation:

$$T_{fst} = 0.20833 \ (Rh_3 - 105.33) \qquad (T_{fst} \text{ in °C}) \tag{20}$$

where

Rh_3 = indoor relative humidity (%).

Porous Plate Enthalpy Recovery Ventilators

The plates in a porous plate enthalpy recovery ventilator are normally made from a specially treated paper that has good sensible heat transfer characteristics and a high moisture permeability. The performance data on porous plate enthalpy recovery ventilators were obtained from product literature of a major manufacturer of this equipment. These data show sensible and enthalpy efficiency as a function of volume flow rate and the ratio of exhaust to supply volume flow. It was assumed that the efficiency data from the literature do not include the effects of fans or preheaters.

It was found that the equations below could be used to represent the sensible efficiency, the heating enthalpy efficiency, and the cooling enthalpy efficiency:

$$\varepsilon_s' = 1.111 - 0.1209 \ m' + 0.0118 \ m'^2 \tag{21}$$

$$\varepsilon_{e,heat}' = 1.1241 - 0.1271 \ m' + 0.0045 \ m'^2 \tag{22}$$

$$\varepsilon_{e,cool}' = 1.1882 - 0.2238 \ m' + 0.0357 \ m'^2 \tag{23}$$

where

ϵ_s' = $\epsilon_s(m) / s(m_{min})$
$\epsilon_{e,heat'}$ = $\epsilon_{e,heat}(m) / \epsilon_{e,heat}(m_{min})$
$\epsilon_{e,cool'}$ = $\epsilon_{e,cool}(m) / \epsilon_{e,cool}(m_{min})$
$\epsilon_{e,heat}$ = heating enthalpy efficiency
$\epsilon_{e,cool}$ = cooling enthalpy efficiency
m' = m / m_{min}
m = mass flow rate of the enthalpy recovery ventilator
m_{min} = minimum supply mass flow rate indicated in the model's performance charts
$\epsilon_s(m)$ = sensible efficiency at supply mass flow rate m
$\epsilon_s(m_{min})$ = sensible efficiency at the minimum mass flow rate

The information on the porous plate heat exchanger did not include moisture transfer efficiency. However, the sensible and enthalpy efficiencies can be used to calculate the humidity ratio of the fresh air entering the home. The contribution to the change in indoor humidity due to ventilation can be written as:

$$dRh_v / dt = (w_2 / w_{sat} - Rh) \ V' \ / \ V_{ol} \tag{24}$$

where

w_2 = humidity ratio of fresh air entering the house interior

An equation for w_2 can be derived by rearranging the enthalpy equation from ASHRAE (1985, Figure 6.9):

$$w_2 = (h_2 / 1000 - T_2) \ / \ (2501 + 1.86 \ T_2) \tag{25}$$

where

h_2 = enthalpy of fresh air entering the house interior
T_2 = temperature of fresh air entering the house interior

Values for h_2 and T_2 can be obtained using the above equations for sensible and enthalpy efficiency, and the basic definitions for these efficiencies (from the Canadian Standards Association 1985):

$$h_2 = [\ \varepsilon_e \ (m \ (h_3 - h_1) + Q_{sf}) + Q_{ef} + Q_h] \ / \ m + h_1 \tag{26}$$

$$T_2 = [\ \varepsilon_s \ (m \ C_p \ (T_3 - T_1) + Q_{sf}) + Q_{ef} + Q_h] \ / \ (m \ C_p) + T_1 \tag{27}$$

where

C_p = specific heat of air
h_1 = outdoor air enthalpy
h_3 = indoor air enthalpy
T_1 = outdoor air temperature
T_3 = indoor air temperature
Q_{ef} = heat addition to exhaust air due to exhaust fan
Q_h = heat addition to outdoor air from the preheater
Q_{sf} = heat addition to outdoor air from the supply fan

The manufacturer recommends that the outdoor air be preheated to prevent both condensation and frosting. The strategy for accomplishing this can be illustrated by referring to the psychrometric chart in Figure 5. Warm and cold air enter the heat exchange core at condition $W(T_3,w_3)$ and $C(T_1,w_1)$, respectively. If a straight line between W and C intersects the saturation line, then condensation may occur. To avoid this, the manufacturer recommends adding sensible heat to the cold air so that it is at condition P.

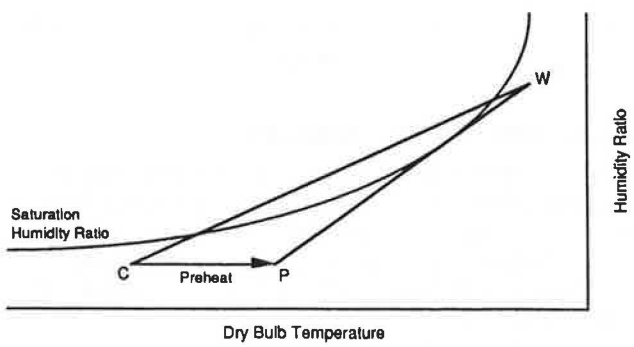

Figure 5 *Porous plate heat exchanger: strategy for preventing condensation*

Based on the above strategy, an equation for the condensation threshold temperature can be developed. Referring to Figure 5, it can be seen that the line joining W and P is tangent to the saturation line. Therefore, two equations can be derived for the slope of this line:

$$dw_s(T_{tg}) / dT = (T_3 - T_{cdt}) / (w_3 - w_1) \quad (28)$$

and

$$dw_s(T_{tg}) / dT = (T_3 - T_{tg}) / (w_3 - w_s(T_{tg})) \quad (29)$$

where

T_{tg} = temperature at the intersection of the tangent line and the saturation line

$dw_s(T_{tg}) / dT$ = slope of the saturation line at T_{tg}

By combining the above equations, an expression can be derived for the condensation threshold temperature, T_{cdt}. The only unknown in this new equation is T_{tg}. This can be eliminated using the following approximate equation (based on a regression fit, valid for $-20°C < T_{tg} < 10°C$) for the saturation humidity ratio w_s.

$$w_s(T_{tg}) = 0.001 (0.0073 T_{tg}^2 + 0.3022 T_{tg} + 3.8285) \quad (30)$$

Using this equation, an additional equation for the slope of the tangent line can be derived:

$$dw_s(T_{tg}) / dT = 0.001 (0.0146 T_{tg} + 0.3022) \quad (31)$$

Combining the above equations produces the following equations for T_{tg} and T_{cdt}:

$$T_{tg} = T_3 - (T_3^2 + 136.986 (0.3022 T_3 + 3.8285 - 1000 w_3))^{0.5} \quad (32)$$

$$T_{cdt} = T_3 - (w_3 - w_1) / (dw_s(T_{tg}) / dT) \quad (33)$$

If the above calculation indicates that $T_1 > T_{tg}$, then there is no need to calculate the condensing threshold temperature. If this is not the case, then T_{tg} should be calculated and inserted into the T_{cdt} equation.

Based on this analysis, the preheat power requirement is given by:

$$Q_h = \begin{cases} m\, C_p (T_{cdt} - T_1) & \text{for } T_{cdt} > T_1 \\ 0 & \text{for } T_{cdt} \leq T_1 \end{cases} \quad (34)$$

Sensible Heat Recovery Ventilator

The model for the sensible heat recovery ventilator was obtained through a curve fit to experimental data obtained from Energy Mines and Resources Canada (1985). In order to allow comparison of the various units, the values of the sensible efficiency and the mass flow were normalized in the following manner:

$$\varepsilon_s' = \varepsilon_s / \varepsilon_{s,o} \quad (35)$$

$$m' = m / m_o$$

where

$\epsilon_{s,o}$ = sensible efficiency of the unit's core at the R2000 rating point

m = mass flow rate (balanced)

m_o = mass flow rate at the R2000 rating point

The R2000 rating conditions are:

indoor temperature: 22°C
outdoor temperature: 0°C
indoor humidity ratio: 0.0066
outdoor humidity ratio: 0.0019 to 0.0038
supply airflow: 0.055m³/sec

The normalized sensible efficiency data are plotted in Figure 6. Linear regression yields for the following equation for these data:

$$\varepsilon_s' = (-0.1135\, m' + 1.1068) \quad \text{for } 0.5 < m' < 2 \quad (36)$$

These test data also featured sensible efficiency measurements at 0.055 m³/sec for much colder outdoor conditions (T_1 = $-25°C$, w_1 = 0.0002 to 0.0004). These measurements showed that the sensible efficiency varied little with outdoor temperature.

MOISTURE DIFFUSION THROUGH BASEMENT CONCRETE

Properly designed homes are designed such that liquid water in the ground is not in direct contact with basement concrete. Under these conditions, the moisture levels in an unfinished basement are governed by moisture storage processes, transfers from other zones, and water vapor diffusion from the ground.

The moisture transport through basement concrete can be calculated using the following formula:

$$m' = \mu A \Delta P / L \quad (37)$$

where

m' = mass flow rate from the concrete

μ = permeability of the concrete to water vapor

A = area of the concrete

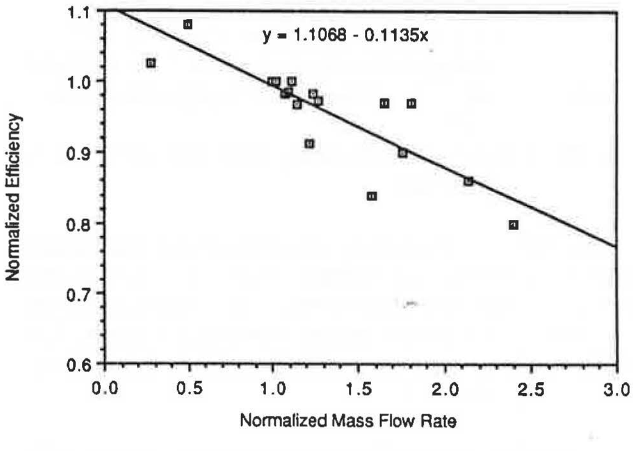

Figure 6 *Sensible HRV: efficiency vs. mass flow rate*

ΔP = vapor pressure difference across the concrete
L = thickness of the concrete

In order to calculate the mass flow, it is necessary to obtain the pressure difference across the concrete and the concrete's permeability to water vapor. If the soil beneath the concrete floor is damp, then vapor pressure in this area is the saturation vapor pressure at the temperature of the soil adjacent to the concrete. Based on this, the vapor pressure difference is:

$$\Delta P = P_{sat}(T_{subbase}) - RH\, P_{sat}(T_b) \tag{38}$$

where

P_{sat} = water vapor saturation pressure
$T_{subbase}$ = temperature of the soil adjacent to the concrete
RH = relative humidity inside the basement
T_b = temperature in the basement

It has been estimated by Barringer and McGugan (1988) that:

$$T_b - T_{subbase} = 1.4 \text{ to } 0.2\ ^{\circ}C \tag{39}$$

According to ASHRAE (1985, Figure 22.5) the concrete permeability (for a 1:2:4 mix used for foundations) is about 4.7 ng/Pa/s/m. Further information on concrete permeability coefficients was obtained from a study conducted by Brewer (1965) for the Portland Cement Association on the moisture transport through concrete slab-on-grade floors. Using the data from this study, the concrete permeability coefficient was estiated to be 14.24 to 3.65 ng/Pa/s/m. However, researchers such as Timusk (1988) consider the permeability of the basement to be higher. Timusk believes a value of 30 ng/Pa/s/m to be more realistic. It can therefore be concluded that the permeability coefficient for concrete can have a wide range of values.

Sample calculations on the moisture flow through basement concrete have been done by Barringer and McGugan (1988). It was found that the vapor diffusion can add up to 2 to 3 kg/day of water to the basement. This is consistent with the values report by Quirouette (1983). It was also found that the moisture flow through the concrete is not strongly dependent on the basement floor temperature drop, or on the basement temperature.

WINDOW AND SILL MOISTURE TRANSPORT MODEL

Glass Condensation and Evaporation Model

The condensation and evaporation model is based on the following:

- the flow over the glass is free convection in which both heat and mass transfer occur;
- the analogy between heat transfer and mass transfer is valid. This analogy is valid if (from Bird 1960): fluid and material properties are constant; mass transfer rates are low; there are no chemical reactions; there is no radiant heat transfer; and there is no pressure diffusion, thermal diffusion, or forced diffusion.

The sensible heat gain rate per unit area to the building from the glass may be written as:

$$q_s = h\,(T_g - T_i) \tag{40}$$

The mass addition to the glass per unit area is:

$$dm_g''/dt = k_{mg}\,(x_i - x_{sat}(T_g)) \tag{41}$$

where

q_s = sensible heat gain rate per unit area
h = heat transfer coefficient
T_i = interior temperature
T_g = glass interior surface temperature
dm_g''/dt = mass addition to the glass per unit glass area from the house
k_{mg} = mass transfer coefficient (mass per unit area per unit time)
x_i = mole fraction of water vapor in the building interior
$x_{sat}(T_g)$ = mole fraction of water vapor at saturation at the glass surface

The heat and mass heat coefficients are found from equations for the Nusselt (Nu) number and the Sherwood number (Sh):

$$\begin{aligned} Nu = h L_g / k &= 0.56\,(Gr\,Pr)^{.25} \quad \text{for } Gr\,Pr < 8.473 \times 10^7 \text{ (laminar)} \\ &= 0.13\,(Gr\,Pr)^{.33} \quad \text{for } Gr\,Pr \geq 8.473 \times 10^7 \text{ (turbulent)} \end{aligned} \tag{42}$$

$$\begin{aligned} Sh_g = k_{mg} L_g / (\rho_{air} D_v) &= 0.56\,(Gr_{mg}\,Sc)^{.25} \quad \text{for } Gr_{mg}\,Sc < 8.473 \times 10^7 \\ &= 0.13\,(Gr_{mg}\,Sc)^{.33} \quad \text{for } Gr_{mg}\,Sc \geq 8.473 \times 10^7 \end{aligned} \tag{43}$$

where

L_g = characteristic length (the glass height)
k = air conductivity
Gr = Grashof number
Pr = Prandtl number
ρ_{air} = air density
D_v = mass diffusivity of water vapor in air at room temperature
Gr_{mg} = Grashof number for binary diffusion for the glass surface
Sc = Schmidt number

The fact that the Nusselt and Sherwood numbers have the same form is a consequence of the analogy between heat and mass transfer. Equations for the Grashof, Prandtl, and Schmidt numbers are (from Bird 1960) given below:

$$\begin{aligned} Gr &= |L^3 g \beta \Delta T / \nu^2| & Gr_{mg} &= |L_g^3 g \zeta \Delta x_g / \nu^2| \\ \Delta T &= T_g - T_i & \Delta x_g &= x_{sat}(T_g) - x_i \\ Pr &= Cp\,\mu / k & Sc &= \nu / D_v \end{aligned} \tag{44}$$

where

g = gravitational acceleration
β = thermal coefficient of volumetric expansion
ν = kinematic viscosity at room temperature
ζ = concentration coefficient of volumetric expansion
Cp = heat capacity of air at room temperature
μ = absolute viscosity of air at room temperature

It can be shown that for all normal humidity conditions:

$$\beta = 1 / (T_i + 273.16) \qquad \zeta = -1 \tag{45}$$

Sill Moisture Transport Model

When the moisture condensed on the glass surface exceeds the maximum capacity of the surface, the excess moisture is considered to be deposited on the window sill. According to Tanaka (1975), the maximum amount of water that can be held on a clean vertical surface is about 0.26 kg/m^2.

The moisture gain to the sill is given by:

$$dm_s'' / dt = dm_{se}'' / dt + dm_{sg}'' / dt \tag{46}$$

where

dm_s'' / dt = moisture gain rate to the sill
dm_{se}'' / dt = moisture gain rate to the sill from the house
dm_{sg}'' / dt = moisture transferred from the glass to the sill

The moisture gain from the house is governed by equations similar to those for glass. The moisture addition per unit area is:

$$dm_{se}'' / dt = k_{mse} (x_i - x_{sat}(T_s)) \tag{47}$$

where

T_s = sill surface temperature
dm_{se}'' / dt = mass addition to the sill from the house
k_{mse} = mass transfer coefficient (mass per unit area per unit time)
$x_{sat}(T_s)$ = mole fraction of water vapor at saturation at the sill surface

As before, the mass transfer coefficient is found from equations for the Sherwood number:

$$\begin{aligned} Sh_s &= k_{ms} L_s / (\rho_{air} D_v) \\ &= 0.56 (Gr_{ms} \; Sc)^{.25} \qquad \text{for } Gr_{ms} \, Sc < 8.473 \times 10^7 \\ &= 0.13 (Gr_{ms} \; Sc)^{.33} \qquad \text{for } Gr_{ms} \, Sc \geq 8.473 \times 10^7 \end{aligned} \tag{48}$$

L_s = characteristic length (the sill width)
Gr_{ms} = Grashof number for binary diffusion for the sill

Equations for the Grashof and Schmidt numbers are the same as above with the appropriate characteristic length and surface temperature.

As before, the equation for the Sherwood number can be developed from the Nusselt number equation. According to ASHRAE (1985, Figure 3.13), the heat transfer coefficient for free confection over a horizontal plate with the upper surface heated from the environment is:

$$h = 1.32 (\Delta T / L)^{.25} \qquad \text{(air at 294 K)} \tag{49}$$

Converting this to a Nusselt number equation (using the properties of air at 294 K):

$$Nu = 2.856 \times 10^{-3} (Gr \; Pr)^{.25} \tag{50}$$

As a consequence of the heat/mass transfer analogy, the Sherwood number is:

$$Sh_s = 2.856 \times 10^{-3} (Gr_{ms} \; Sc)^{.25} \tag{51}$$

Glass/Sill System

The moisture gain to the house from the glass and sills is:

$$\begin{aligned} dMg / dt &= - \Sigma (A_{gi} \, dm_{gi}'' / dt) \\ dMs / dt &= - \Sigma (A_{si} \, dm_{sei}'' / dt) \end{aligned} \tag{52}$$

where

dMg / dt = total moisture gain rate to the house from the glass
A_{gi} = ith glass area
dm_{gi}'' / dt = moisture gain rate per unit glass area to the ith pane (positive for condensation)
dMs / dt = total moisture gain rate to the house from the sills
A_{si} = ith sill area
dm_{sei}'' / dt = moisture gain rate per unit sill area from the ith sill (negative for evaporation)

Evaporation cannot take place if no water is present. Therefore,

$dm_g'' / dt = 0$ for $m_g'' = 0$

and

$dm_g'' / dt < 0$ (evaporation).

COMBINED MOISTURE STORAGE AND VENTILATION TIME CONSTANTS

The rate of change of the relative humidity due to moisture storage and ventilation is given by:

$$\begin{aligned} dRh_m / dt + dRh_v / dt = & - Rh / \tau_{rh,m} + \Sigma [m_i / (C_i M_i \tau_{rh,m,i})] \\ & + (w_1 / w_{sat} - Rh) / \tau_v \end{aligned} \tag{53}$$

Examination of this equation reveals that this equation can be rewritten as:

$$\begin{aligned} dRh_m / dt + dRh_v / dt = & - Rh / \tau_{rh,m,v} + \Sigma [m_i / (C_i M_i \tau_{rh,m,i})] \\ & + w_1 / w_{sat} / \tau_v \end{aligned} \tag{54}$$

where

$$\tau_{rh,m,v} = (1 / \tau_{rh,m} + 1 / \tau_v)^{-1} \tag{55}$$

= combined storage/ventilation time constant

The above derivations indicate that the time constant for the dampening of relative humidity fluctuations is due to the combination of the ventilation time constant and the humidity change time constant.

As mentioned earlier, since the rise time of internal moisture gains is less than one hour, the relative humidity fluctuations will not be significantly dampened unless the combined storage/ventilation time constant is less than about one hour.

TABLE 1
House Specifications and Conditions for Modeling

House Dimensions				**Ventilation System**				
occupied volume	cu.ft. (cu.m.)	7763	(220.0)	maximum speed RH set point			40%	
number of zones		1						
					volume flow flow rate		moisture transfer effectiveness	ventilation time constant
floor area	sq.ft. (sq.m.)	968	(90.0)					
ceiling area	sq.ft. (sq.m.)	968	(90.0)					
walls area	sq.ft. (sq.m.)	1022	(95.0)		cfm	(L/s)		hours
Total Area	sq.ft. (sq.m.)	2959	(275.0)	maximum flow	85	(40)	0.62	4.020
				minimum flow	42	(20)	0.70	10.19
Temperature Conditions								
indoor temperature	°F (°C)	70	(21.1)	Average Int. Moisture Gain	lb/day (kg/day)		15.4	(7.00)
outdoor dew point	°F (°C)	32	(0)					

MODELING OF A SIMPLE HOUSE FOR A SINGLE DAY

This section presents the predictions of indoor humidity for a 24-hour period in a single-zone house. The predictions are numeric solutions to the system of equations consisting of: the overall indoor relative humidity equation (Equation 1), equations for material moisture content (Equation 2), equations for moisture flow into the room from the materials (Equation 11), and the equation for the ventilator (Equation 12). The latent load profile used in the calculations was obtained from Fairey et al. (1986, pp. 3-28). Window condensation and evaporation were not included in the modeling. These predictions include an illustration of the effect of moisture storage on daily fluctuations in indoor humidity.

The specifications for the house used in the modeling are listed in Table 1. Five cases were simulated, each with a different set of moisture storage materials. The various cases and moisture storage conditions are described in Table 2.

Very different materials have been used in Cases 4 and 5. However, the areas of the materials were adjusted so that the combined moisture storage time constant was the same.

Several observations can be made on the various time constants in Table 2.

TABLE 2
Moisture Storage Characteristics Used in Modeling

		Painted Gypsum Dry Wall	Drapery	Pre-Finished Plywood Panel	Acoustic Ceiling Tile	Vinyl Floor Tile	Rug	Concrete Block	Combined Moisture Storage Time Constant, All Materials	Combined Storage/ Ventilation Time Constant*, All Materials
Hygroscopic Storage Constant		0.016	0.118	0.019	0.036	0.0055	0.050	0.0012		
Air-to-Mass Time Constant	hours	2	1	12	4	12	12	24		
Area Density	kg/sq.m.	8.5	0.34	5.5	3.2	6.4	3.2	113		
Maximum Vapor Velocity	m/hour	3.563	2.111	0.462	1.518	0.154	0.704	0.308		
Case 1: No Moisture Storage										
Area	sq.m.	0	0	0	0	0	0	0		
Moisture Storage Time Constant	hours	-	-	-	-	-	-	-	-	4.020
Case 2: Long Time Constant										
Area	sq.m.	0	0	0	0	0	0	275		
Moisture Storage Time Constant	hours	-	-	-	-	-	-	2.60	2.60	1.578
Case 3: Short Time Constant										
Area	sq.m.	121.0	19.0	0.0	45.0	90.0	0.0	0.0		
Moisture Storage Time Constant	hours	0.51	5.48	-	3.22	15.88	-	-	0.391	0.357
Case 4: Intermediate Time Constant										
Area	sq.m.	0.0	9.5	85.5	90.0	0.0	90.0	0.0		
Moisture Storage Time Constant	hours	-	10.97	5.57	1.61	-	3.47	-	0.848	0.700
Case 5: Intermediate Time Constant										
Area	sq.m.	62.2	0.0	0.0	0.0	180.0	0.0	32.8		
Moisture Storage Time Constant	hours	0.99	-	-	-	7.94	-	21.79	0.848	0.700

* The combined mass/ventilation time constant combines the mass storage and ventilation time constants. The ventilation time constant used here (4.020 hours) is the time constant at the maximum ventilation rate.

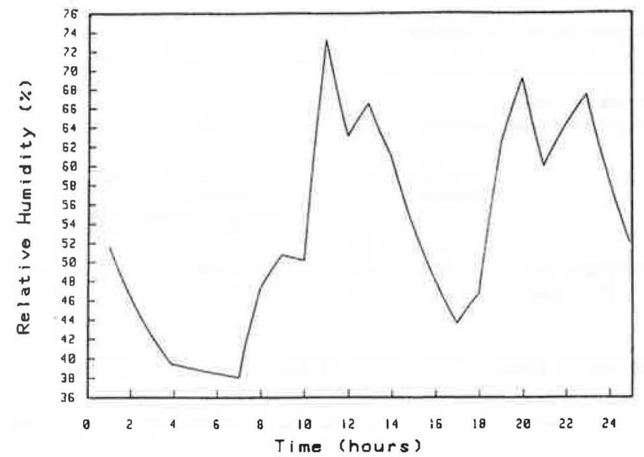

Figure 7a *Daily variation of indoor relative humidity. Case 1: No moisture storage. Combined storage/ventilation time constant: 4.020 hours.*

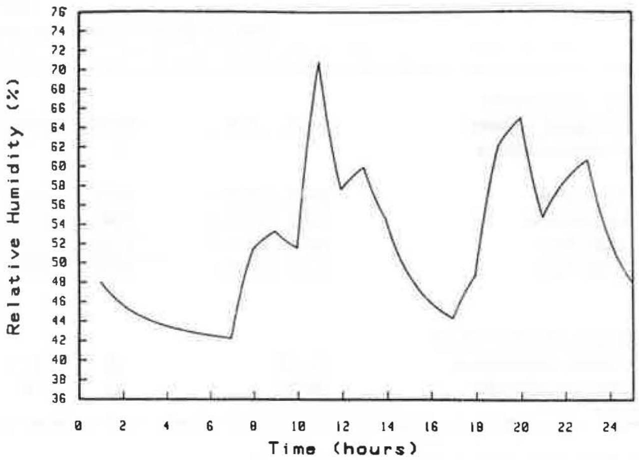

Figure 8a *Daily variation of indoor relative humidity. Case 2: Long time constant. Combined storage/ventilation time constant: 1.578 hours.*

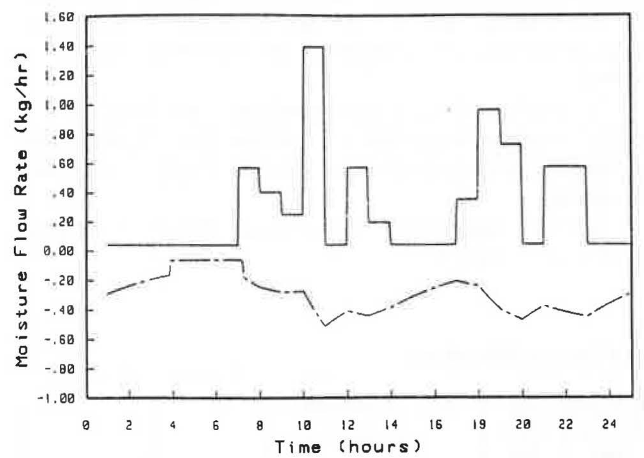

Figure 7b *Daily variation of moisture flow into the house. Case 1: No moisture storage. Combined storage/ventilation time constant: 4.020 hours.*
——— *Internal moisture gains.*
— - — *Moisture flow through the ventilator.*

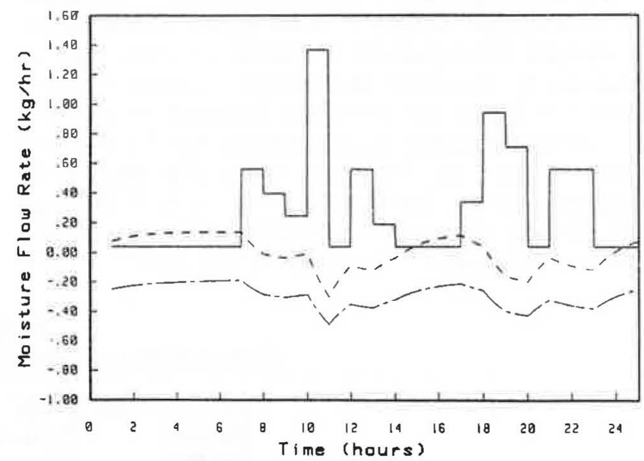

Figure 8b *Daily variation of moisture flow into the house. Case 2: Long time constant. Combined storage/ventilation time constant: 1.578 hours.*
——— *Internal moisture gains.*
- - - - - - *Moisture flow out of internal materials.*
— - — *Moisture flow through the ventilator.*

1. According to the analysis, materials with similar maximum vapor velocities can have similar effects on the humidity dynamics. This implies that materials as different as ceiling tile and draperies can have a very similar effect on indoor humidity fluctuations.
2. The moisture storage time constants frequently differ markedly from the air-to-mass time constants. In the various cases, the moisture storage time constants are frequently less than the air-to-mass time constants.

The results for the modeling are presented in Figures 7 through 11. Figure 7a shows the indoor relative humidity when there is no moisture storage (Case 1). The relative humidity fluctuates between about 74% and 38%. Figure 7b shows the moisture addition and removal due to the internal moisture gains and the enthalpy recovery ventilator. It should be noted that the ventilator switches to low speed when the relative humidity goes below 40%.

Figure 8a shows the indoor relative humidity of Case 2 (long time constant). This graph shows that a long time constant produces a slight reduction in the relative humidity fluctuations. Figure 8b shows moisture flowing in and out of the material.

The indoor humidity for the short time constant case (Case 3) is presented in Figure 9a. In this case the fluctuations in relative humidity are greatly reduced. The moisture flowing in and out of the material (shown in Figure 9b) is greatly increased.

Cases 4 and 5 have the same combined time constant. Comparisons between Figures 10a and 11 indicate that the relative humidity fluctuations for the two cases are very similar.

In general it should be noted that time constants on the order of one hour or less produce significant dampening of the relative humidity fluctuations. This is because many fluctuations in the internal gains have rise times that are less than one hour.

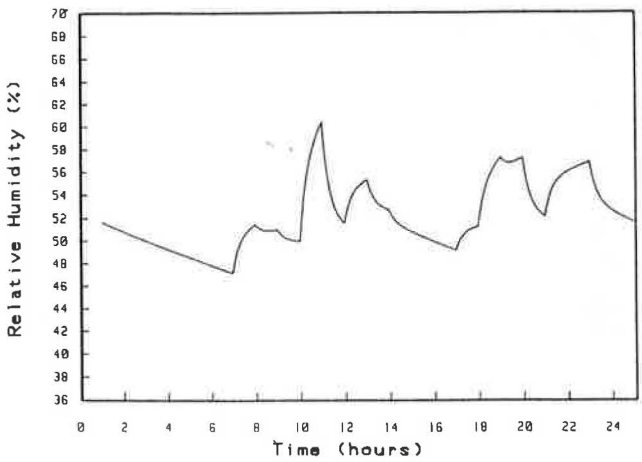

Figure 9a *Daily variation of indoor relative humidity. Case 3: Short time constant. Combined storage/ventilation time constant: 0.357 hours.*

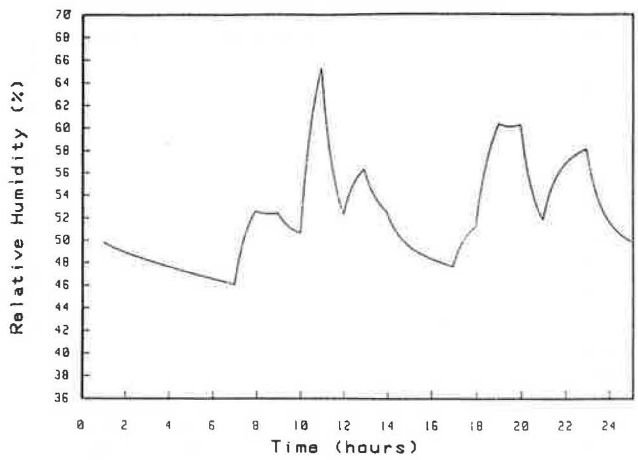

Figure 10a *Daily variation of indoor relative humidity. Case 4: Intermediate time constant. Combined storage/ventilation time constant: 0.700 hours.*

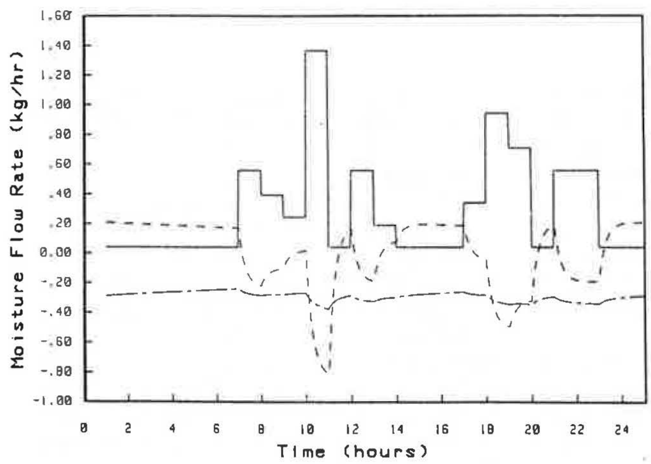

Figure 9b *Daily variation of moisture flow into the house. Case 3: Short time constant. Combined storage/ventilation time constant: 0.357 hours.*
Internal moisture gains.
Moisture flow out of internal materials.
Moisture flow through the ventilator.

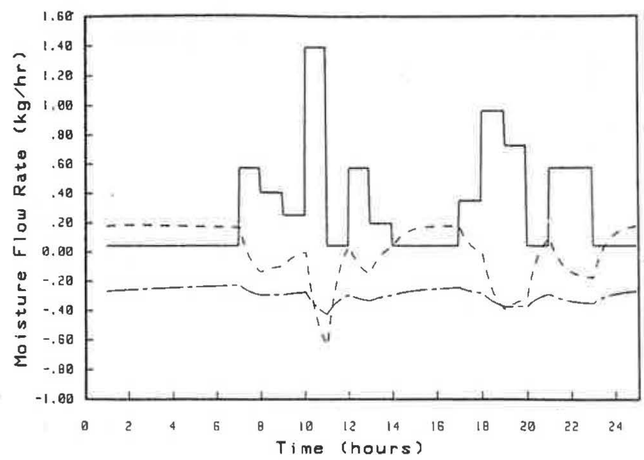

Figure 10b *Daily variation of moisture flow into the house. Case 4: Intermediate time constant. Combined storage/ventilation time constant: 0.700 hours.*
Internal moisture gains.
Moisture flow out of internal materials.
Moisture flow through the ventilator.

CONCLUSIONS

A simple model was developed for moisture storage in building materials. Use of this model requires only three properties of the material: the hygroscopic storage constant, the time constant for the rate of change of moisture mass, and the density of the material. Using this model, a new time constant was derived for the effect of moisture storage on relative humidity. This new time constant accounts for the effect of room volume and material surface area, as well as the material's physical properties.

Three models for enthalpy residential heat exchangers have been developed. These models account for defrosting energy consumption. It was found that a ventilation time constant could be derived from the equations for the rate of moisture addition or removal.

Models were developed for water vapor diffusion through basement concrete, and for condensation and evaporation from windows.

The individual models for moisture transport were combined into a single equation for the indoor relative humidity. Examination of this equation and several simple simulations revealed that fluctuations in indoor relative humidity (generated by varying internal gains) were dampened by moisture storage in building materials, and by ventilation. The magnitude of the dampening was found to be a very strong function of the combined storage/ventilation time constant.

REFERENCES

ASHRAE. 1985. *ASHRAE handbook—1985 fundamentals.* Atlanta: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc.

Barringer, C.G., and McGugan, C.A. 1980. "Investigation of enthalpy residential air-to-air heat exchangers." Prepared by Ontario Research Foundation for ASHRAE, ORF Report EEE/ESC-88-08.

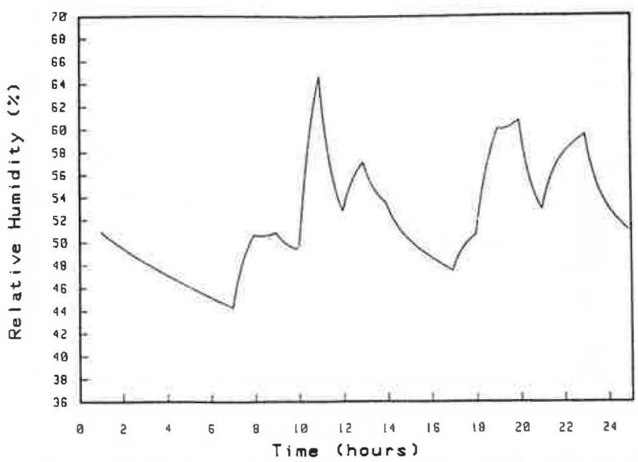

Figure 11a *Daily variation of indoor relative humidity. Case 5: Intermediate time constant. Combined storage/ventilation time constant: 0.700 hours.*

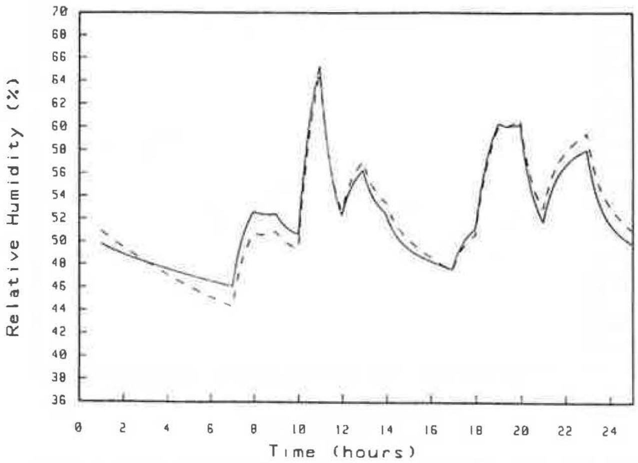

Figure 12 *Daily variation of moisture flow into the house. Cases 4 and 5: Intermediate time constant. Combined storage/ventilation time constant: 0.700 hours.*
———— Case 4
- - - - - - Case 5.

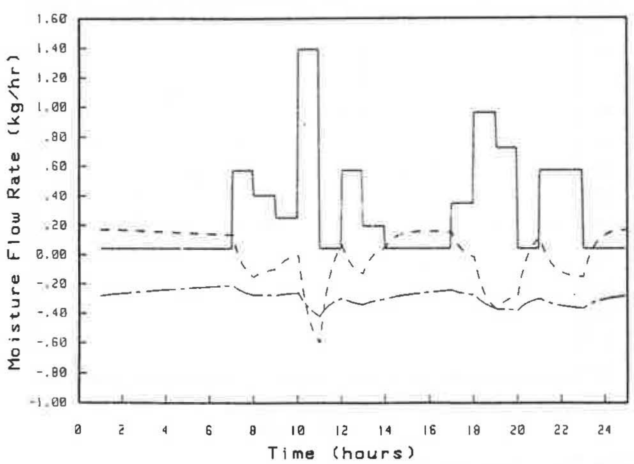

Figure 11b *Daily variation of moisture flow into the house. Case 5: Intermediate time constant. Combined storage/ventilation time constant: 0.700 hours.*
———— Internal moisture gains.
- - - - - - Moisture flow out of internal materials.
— - — Moisture flow through the ventilator.

Bird, R.B.; Stewart, W.E.; and Lightfoot, E.N. 1960. *Transport phenomena*, p. 645. New York: John Wiley & Sons, Inc.

Brewer, H.W. 1965. "Moisture migration; concrete slab-on-ground construction." Portland Cement Association, Research and Development Laboratories, Development Department Bulletin D89.

Canadian Standards Association. 1985. "Standard methods for rating the performance of heat recovery ventilators." Preliminary Standard C439-M1985, p. 13.

Energy Mines and Resources Canada. 1985. "HRV design specification sheet." Ottawa: Canadian Home Builders' Association.

Fairey, P.; Kerestecioglu, A.; Vieirfa, R.; Swami, M.; and Chandra, S. 1986. "Latent and sensible load distributions in conventional and energy efficient residences, final report." Prepared by the Florida Solar Energy Center for the Gas Research Institute, FSEC Report FSEC-CR-153-86, GRI Report GRI-86/0056.

Hoagland, L.C. 1986. "Moisture transfer characteristics of heat recovery ventilation systems employing rotary heat wheels; winter and summer conditions." First draft of paper presented at BTECC Conference on Air Infiltration, Ventilation and Moisture Transfer, December 2-6, Ft. Worth, TX.

Holmberg, R. 1977. "Heat and mass transfer in rotary heat exchangers with nonhygroscopic rotor materials." *Transactions of the ASME Journal of Heat Transfer*, Vol. 99, May, pp. 196-202.

Kusuda, T. 1985. "Measurement of moisture content for building interior surfaces." *Moisture and Humidity 1985 Measurement and Control in Science and Industry, Proceedings of the 1985 Symposium on Moisture and Humidity*, Washington, DC, April 15-18, pp. 297-302.

Martin, P.C., and Verschoor, J.D. 1986. "Investigation of dynamic latent heat storage effects of building construction and furnishing materials." Prepared by Manville Service Corporation for Oak Ridge National Laboratory, ORNL/Sub/86-22016/1.

Miller, J.D. 1984. "Development and validation of a moisture mass balance model for predicting residential cooling energy consumption." *ASHRAE Transactions*, Vol. 90, Part 2B, pp. 275-293.

Quirouette, R.L. 1983. "Moisture sources in houses." *Humidity, Condensation and Ventilation in Houses; Proceedings of the Building Science Insight '83*, Proceedings No. 7, National Research Council Canada, NRCC 23293, pp. 15-27.

Stamm, A.J. 1965. *Wood and cellulose science*, p. 146. New York: Ronald Press.

Tanaka, H. 1975. "A theoretical study of dropwise condensation." *Transactions of the ASME, Journal of Heat Transfer*, February, pp. 72-78.

Timusk, J. 1988. Department of Civil Engineering, University of Toronto, personal communication, January.

Tsuchiya, T. 1980. "Infiltration and indoor air temperature and moisture variation in a detached residence." *Journal of the Society of Heating, Air Conditioning and Sanitary Engineers of Japan*, Vol. 54, No. 11.