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The Coupled Airflow and Thermal Analysis Problem in Building Airflow System Simulation

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ABSTRACT

The Indoor Air Quality and Ventilation Group at the National Institute for Standards and Technology (NIST, formerly the National Bureau of Standards) has developed a method of building airflow analysis, based upon element assembly techniques, that has been successfully applied to the determination of the macroscopic characteristics of infiltration, exfiltration, and interzonal airflows in complex building airflow systems driven by wind pressures, buoyant forces, and the building HVAC system. This analytical method was formulated to be compatible with a discrete thermal analysis method, also based on element assembly techniques and developed earlier, which may be applied to problems of building thermal analysis.

This paper will review the theoretical bases of these two related methods and present a theoretical framework for integrating the flow with the thermal analysis methods to solve the coupled airflow and thermal analysis problem in building airflow system simulation. Formulation of the coupled airflow-thermal analysis problem will be presented and numerical methods for the solution of this problem will be outlined.

INTRODUCTION

One may approach the problem of modeling interzonal heat and mass transfer from a macroscopic or a microscopic point of view.[1] Macroscopic modeling techniques are based upon the application of mass, momentum, and energy balances, expressed in terms of ordinary differential equations, to one or more discrete control volumes chosen to idealize the building system under consideration. These techniques are computationally straightforward and efficient, consequently, they may be used for whole-building simulation. Microscopic modeling techniques, on the other hand, are based upon solutions of partial differential equations that define the transport of mass, momentum, and energy in the building system (e.g., the Boussinesq simplification of the Navier-Stokes equations). As microscopic approaches are computationally demanding, their application must be limited to the analysis of small portions of building systems. Although both approaches have their merits, this paper will place an emphasis on the development of macroscopic techniques. The approach taken to the formulation of the governing macroscopic equations presented in this paper is, however, based upon finite element solution techniques used for microscopic analysis, eventually allowing the integration of microscopic and macroscopic approaches.

Although a large variety of macroscopic models for multi-zone heat transfer and for multi-zone airflow analysis in buildings have been developed, few researchers have integrated thermal and airflow analysis models to directly address the coupled airflow/thermal problem. Two exceptions, however, deserve special note. Walton (1982) integrated a simple network airflow analysis technique with the conduction transfer function approach to multi-zone building thermal analysis to solve the coupled airflow/thermal problem and later Clarke (1985) described a similar airflow analysis technique and briefly outlined a computational solution strategy for the coupled problem that has been implemented as part of the ESP building thermal simulation program (ABACUS 1986). In both cases the nonlinear flow problem was simply inserted into the time-stepping scheme used to solve the dynamic thermal analysis problem. Walton provided two options. In the first option, at each step in time, the nonlinear flow problem was formed and solved, given the current estimate of system temperatures, then the thermal problem was formed and solved. In the second option the sequential solution of the flow and thermal problems were repeated, in an iterative manner, until convergence was realized. Clarke's approach is equivalent to Walton's first option. Although Walton's second option provides an implicit means to account for the nonlinearity introduced into the thermal problem by the temperature dependence of convective heat transport, this dependency is not explicitly accounted for in the thermal system of equations used in the model. Clarke's approach and Walton's first approach ignore this nonlinearity altogether. These formulations of the coupled problem are reasonable for the analysis of building systems controlled to maintain near-constant interior air temperatures but may not be appropriate for the analysis of building systems with floating interior air temperatures.

This paper will present an approach to modeling the coupled problem that involves integrating element-assembly formulations of both airflow analysis and thermal analysis techniques to form a coupled set of equations that accounts both for the nonlinearity of the airflow analysis problem (i.e., nonlinearity with respect to pressure) and the thermal analysis problem (i.e., nonlinearity with respect to temperature). This approach has grown out of an informal

1 The use of "microscopic" and "macroscopic" here follows the chemical engineering convention, as clearly defined and elaborated by Bird (1960).

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and formal collaboration between the authors and George Walton at NIST and is the basis of a program, DTFAM (Discrete Thermal-Flow Analysis Method), presently being developed at NIST by the second author of this paper, Grot.

GENERAL CONSIDERATIONS

We begin by considering a building system to be a three-dimensional continuum within which we seek to completely describe the temporal, t, and spatial, x,y,z, variation of the *state* of the system at all points within the continuum. The state of solid portions of the continuum will be defined by temperature, T, and the state of the air portions of the continuum will be defined by the temperature, pressure, P, and velocity, v, of infinitesimal air parcels within this portion of the building system.

The determination of the spatial and temporal variation of the temperature field will be referred to as *thermal analysis,* and the determination of spatial and temporal variation of the flow field will be referred to as *flow analysis.* The solution of both analysis problems involves replacing the continuously defined state variables:

$$T(x,y,z,t),\ P(x,y,z,t),\ \vec{v}(x,y,z,t)$$

by a finite set of *discrete state variables* that are meant to approximate, in some sense, the values of the continuous variables at discrete points or regions, identified by *nodes,* in the building system.

For the purposes of the present discussion the temperature and pressure fields will be approximated by spatially discrete, but temporally continuous, sets of temperature and pressure variables (organized as vectors) but the velocity vector field will be replaced by a collection of discrete mass flow rates, **w**, (i.e., having units of mass per time) corresponding to mean mass flow rates through discrete flow paths connecting well-mixed zones within the building airflow system[2]:

$$\{\mathbf{T}(t)\},\ \{\mathbf{P}(t)\},\ \text{and}\ \{\mathbf{w}(t)\}$$

It must be emphasized, however, that limiting the discussion to building idealizations consisting of well-mixed zones linked by discrete flow paths is simply a matter of convenience here and not a fundamental assumption in the basic modeling approach. In principle, the airflow portion of the building system could be modeled in part or in whole using finite element techniques that would involve approximating the flow field, $\vec{v}(x,y,z,t)$, by a spatially discrete but temporally continuous set of flow velocity variables $\{\vec{\mathbf{v}}(t)\}$.

Both the thermal and airflow analysis problems will be formulated using element assembly techniques, wherein equations approximating the behavior of the macroscopic system as a whole—the system equations— will be assembled from equations that describe the behavior of discrete elements of the system model. These element equations will be defined in terms of subsets of the discrete system state variables:

$$\{\mathbf{T}^e(t)\},\ \{\mathbf{P}^e(t)\}\ \text{and}\ \{\mathbf{w}^e(t)\}$$

which will be referred to as the *element* state variables: vectors of discrete temperature, pressure, and mass flow rates associated with a given element "e."

2 Column vector quantities will be expressed by bold-faced variables enclosed in braces, { }, and matrix quantities by bold-face variables enclosed in brackets, [ ].

DISCRETE THERMAL ANALYSIS

Building energy simulation has been approached using a variety of methods including methods based upon resistance-capacitance network analysis techniques, LaPlace transform techniques (e.g., conduction response function techniques), Fourier transform techniques (e.g., harmonic transmission matrix methods), finite difference techniques, etc. The authors have favored an approach to building energy simulation that is based upon *element assembly techniques* used in other fields of discrete system simulation, including those based on the finite element method, because it is felt that such an approach may serve to unify the various and diverse simulation methods presently used within a single theoretical framework and, importantly, because it allows the inclusion of the powerful finite element method, and the numerical techniques associated with it, within the repertoire of techniques that may be applied to building energy simulation.

The details of this approach are discussed elsewhere (Axley 1986, 1988); here, we will consider the general features of the formulation and consider only those specific details necessary to explore the essential features of the formulation of the coupled airflow/thermal problem.

The element assembly formulation is based upon the assertion that *building thermal systems may be idealized by assemblages of discrete thermal elements chosen to model specific instances or aspects of thermal transport that occur within the building system.* The program DTAM1, developed to provide a demonstration of the basic approach, provides five thermal elements including a simple thermal resistance element and a well-mixed zone or "lumped" capacitance element (i.e., the elements of the RC network analysis approach), a fluid flow loop element, and a 1D and 2D conduction element based on isoparametric finite element formulations. Equations describing a variety of other elements for radiant and fluid transfer have been presented (Axley 1986, 1988) but not yet implemented.

We distinguish flow elements from nonflow elements and describe the behavior of these subclasses of elements by equations of the general forms given below:

Nonflow Elements

$$\{\mathbf{q}^e_{net}\} = \mathbf{L}^e_q(\{\mathbf{T}^e\}) - \{\mathbf{q}^e\} \qquad (1)$$

Flow Elements

$$\{\mathbf{h}^e_{net}\} = \mathbf{L}^e_q(\{\mathbf{T}^e\}) - \{\mathbf{h}^e\} \qquad (2)$$

where

- $\{\mathbf{q}^e_{net}\}$ is a vector of element net flow rates
- $\{\mathbf{q}^e\}$ is a vector of element-derived heat generation rates
- $\{\mathbf{h}^e_{net}\}$ is a vector of element net enthalpy flow rates
- $\{\mathbf{h}^e\}$ is a vector of element-derived enthalpy generation rates

$$\mathbf{L}^e(\{\mathbf{T}^e\}) \equiv [\mathbf{k}^e]\{\mathbf{T}^e\} + [\mathbf{c}^e]\frac{d\{\mathbf{T}^e\}}{dt}$$

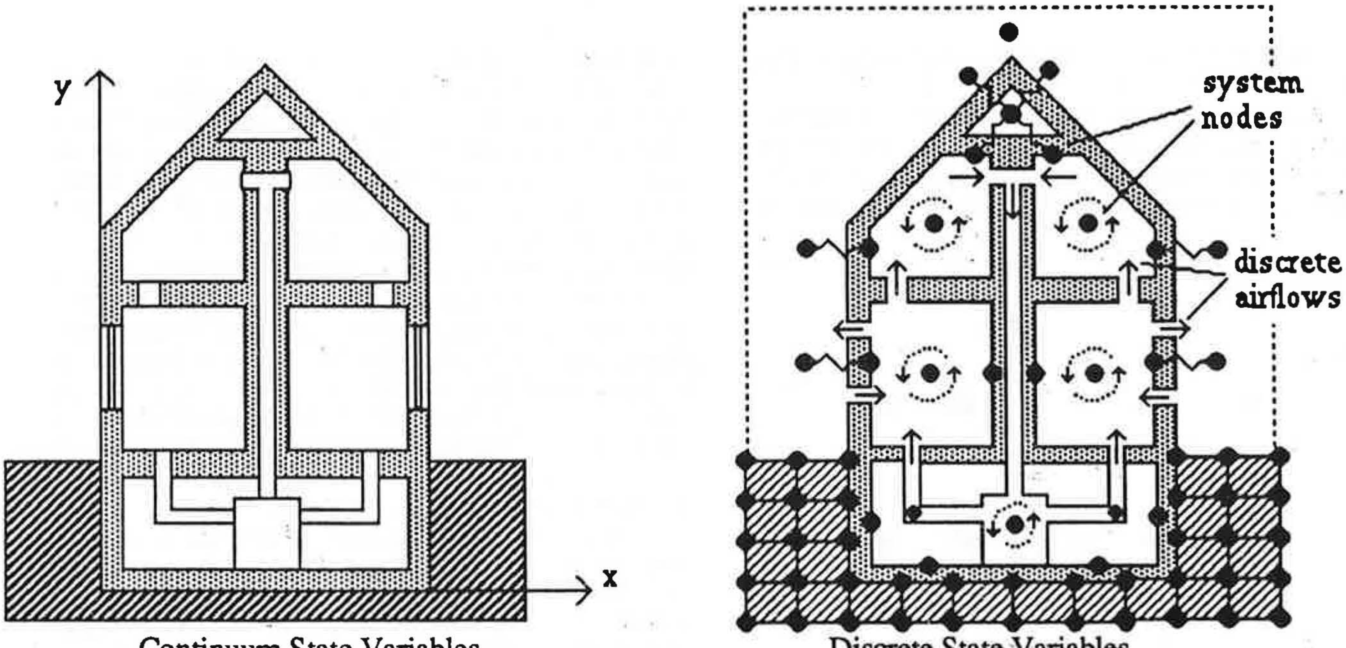


Figure 1 *State variables*

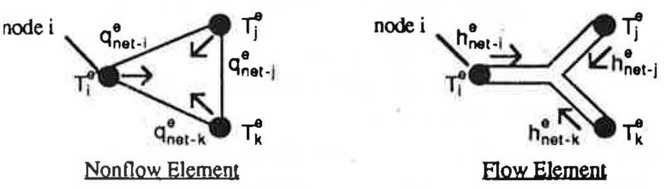
$L^e(\{T^e\})$ is a transformation of $\{T^e\}$ that has the *form* of a linear transformation, specific to a given element type, where $[k^e]$ and $[c^e]$—the element *conductance* and *capacitance* matrices, respectively—are square transformation matrices that may, in general, vary with time (i.e., be nonsteady) or temperature (i.e., be nonlinear).

The meaning of the element variables employed in these general element expressions may be clarified by the diagrammatic representations of hypothetical nonflow and flow elements shown in Figure 2. An element (equation) defines the nature of heat transfer between specific nodes in the system corresponding to a specific heat transfer process being modeled. Nodal temperature and either nodal heat flow rates or enthalpy flow rates are associated with each node with the convention assumed that flow into the element is positive.

The three simplest element equations follow directly from fundamental considerations. These are:

The 1-Node Well-Mixed Zone or Simple Capacitance Element: A single-node element, say element e associated with node i, that models the (ideal) capacitance of a well-mixed zone enclosing a mass of air m^e having a specific heat capacity of C^e_p:

$$\{h^e_{net-i}\} = m^e C^e_p [1] \frac{d\{T^e_i\}}{dt}; \text{ or } [c^e_h] = m^e C^e_p [1] \qquad (3)$$



gure 2 *Thermal element variables*

The 2-Node Simple Resistance Element: A two-node element, say element e with nodes i and j, that models one-dimensional heat transfer through a material having a resistance of R^e and an area available for heat transfer of A^e:

$$\begin{Bmatrix} q^e_{net-i} \\ q^e_{net-j} \end{Bmatrix} = (A^e/R^e)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T^e_i \\ T^e_j \end{Bmatrix}; \text{ or}$$

$$[k^e_q] = (A^e/R^e)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (4)$$

The 2-Node Simple Flow Element: A two-node element, say element e with nodes i and j, that models heat transfer due to (practically) instantaneous flow of rate $\mathbf{w}^e$ of air of specific heat capacity C^e_p through a discrete airflow path:

$$\begin{Bmatrix} h^e_{net-i} \\ h^e_{net-j} \end{Bmatrix} = (\mathbf{w}^e C^e_p)\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}\begin{Bmatrix} T^e_i \\ T^e_j \end{Bmatrix}; \text{ or}$$

$$[k^e_h] = (\mathbf{w}^e C^e_p)\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}; \mathbf{w}^e \geqslant 0 \qquad (5)$$

Although a variety of other element equations could be presented, these simple element equations will be sufficient to discuss the essential features of the coupled airflow/thermal problem. Even with these simple elements building thermal systems of considerable complexity may be modeled.

Two points should be noted at this time. First, the resistance element, being representative of those elements that may be used to model conduction in solids, is defined by a symmetric system of equations while the simple flow element, as other more complex flow elements, is defined by a nonsymmetric system of equations. Second, due to thermally induced buoyancy, the air mass flow rate, $\mathbf{w}^e$, will in general be dependent upon the nodal temperatures,

$\mathbf{w}^e = \mathbf{w}^e(\{\mathbf{T}^e\})$; thus, the simple flow element equations will be nonlinear.

Demanding the conservation of thermal energy at each of the system nodes, the element equations may be directly *assembled* to yield the *system equations* that describe heat transfer in the building system as a whole:

$$\boxed{[\mathbf{K}]\{\mathbf{T}\} + [\mathbf{C}]\frac{d\{\mathbf{T}\}}{dt} = \{\mathbf{E}\}} \tag{6a}$$

where

$$[\mathbf{K}] = \underset{e=a,b,\ldots}{\mathbf{A}} [\mathbf{k}_q^e] + \underset{e=\alpha,\beta,\ldots}{\mathbf{A}} [\mathbf{k}_h^e],$$
the *system conductance matrix* (6b)

$$[\mathbf{C}] = \underset{e=a,b,\ldots}{\mathbf{A}} [\mathbf{c}_q^e] + \underset{e=\alpha,\beta,\ldots}{\mathbf{A}} [\mathbf{c}_h^e],$$
the *system capacitance matrix* (6c)

$$[\mathbf{E}] = \{\mathbf{Q}\} + \underset{e=a,b,\ldots}{\mathbf{A}} [\mathbf{q}_q^e] + \underset{e=\alpha,\beta,\ldots}{\mathbf{A}} [\mathbf{h}_h^e],$$
the *system excitation* (6d)

$a, b, \ldots$ = nonflow element indices
$\alpha, \beta, \ldots$ = flow element indices

A, above, is the assembly operator, a generalization of the conventional summation operator, Σ. The system excitation, $\{\mathbf{E}\}$, is the sum of direct contributions or generation at each of the system nodes, $\{\mathbf{Q}\}$, and the element-derived contributions.

To apply this system of equations to the solution of practical problems, prescribed temperature conditions and the possibility of zero capacitance system nodes must be accounted for. When this is done, one is left with a reduced set of equations of the same form as Equation 6; hence, we shall simply consider operations with this equation and not consider these details here. Consideration of temperature boundary conditions and zero capacitance nodes become, however, key issues when considering computational strategies for implementing this approach to thermal analysis.

The system conductance matrix, $[\mathbf{K}]$, being an element assembly sum of element matrices like those presented above in Equations 4 and 5, will, in general, be nonsymmetric and nonlinear (e.g., due to flow element contributions) $[\mathbf{K}] = [\mathbf{K}(\{\mathbf{T}\})]$. It may be shown[3] however, that $[\mathbf{K}]$ will be a nonsingular M-matrix that may be factored into lower and upper triangular form by LU decomposition without pivoting when one or more prescribed temperature boundary conditions have been imposed. This fact may be used to advantage in developing efficient computational strategies to solve these equations.

STEADY AIRFLOW ANALYSIS

Macroscopic approaches to steady airflow analysis, based upon idealizing building airflow systems by collections of well-mixed zones linked by discrete airflow paths, have been developed by several groups (Liddament 1982, 1983; Feustel 1985). These *multi-zone airflow network models* share a close relationship to water piping network analysis models (Jeppson 1976). In airflow network models, flow along the discrete flow paths is, most commonly, described by power-law *pressure-flow models*, wind pressures are modeled via pressure coefficients and the dynamic pressure variation of the wind, and buoyant effects are accounted for. Axley (1987) and Walton (1988) have shown that these models can be reformulated on an element assembly basis so that multiple pressure-flow models may be considered in a single system model. A variety of flow elements have been introduced that together may be used to model flow within both the building construction and through the HVAC system.

Airflow network models have placed whole-system airflow analysis on a rational basis and are, consequently, welcomed in the indoor air quality and building energy simulation communities. Although preliminary attempts to validate these models against field measurements have been encouraging (Persily 1985), these models can only be expected to provide rough estimates of flows (Etheridge 1988) due to uncertainties in a) the topology of the discrete flow network, b) the pressure-flow correlations used to model these discrete flows, c) the wind-driven pressure coefficients and the wind environment itself, and d) the temperature field within the building system.

Details of the element assembly formulation of the macroscopic approach are discussed elsewhere (Axley 1987; Grot 1987; Walton 1988). The general features of this approach follow that presented above for thermal analysis. The building airflow system is idealized by assemblages of flow elements that model the pressure flow characteristics of discrete flow paths in the building/HVAC system. Flow element equations are formulated and, for each specific system idealization, element equations are assembled to form the system equations.

Although there is much work to be done to refine existing flow element models and to develop additional ones, the basic procedure to do so is in hand. They are developed using the Bernoulli equation for incompressible flow between an entry, subscript "1," and an exit, subscript "2," of a flow path:

$$\Delta P_{loss} = (P_1 + \rho\bar{v}_1^2/2g) - (P_2 + \rho\bar{v}_2^2/2g) + \rho g(z_1 - z_2) \tag{7}$$

where

ΔP_{loss} = pressure "loss" (i.e., dissipation of mechanical energy) due to dynamic and frictional affects along the flow path
ρ = density of air in the flow path
$\bar{v}$ = mean or bulk fluid velocity at the entry and exit
g = acceleration of gravity
z = vertical height from an arbitrary datum at the entry and exit

and one of several different expressions that account for frictional and dynamic losses, ΔP_{loss}, along the type of flow paths being considered. These expressions include

3 Axley (1988) demonstrates this for the closely related system matrices associated with contaminant dispersal analysis.

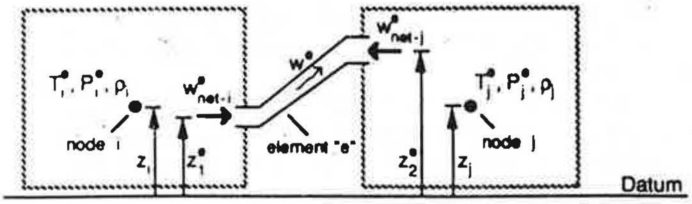


Figure 3 *Two-node flow element variables*

pressure-law correlations for building constructions, friction factor correlations for ducts, orifice, and crack correlations, expressions for flow through large openings, and fan pressure-flow curves. The resulting element equations may be represented by systems of nonlinear algebraic equations of the general form:

$$\{\mathbf{w}^e_{net}\} = [\mathbf{a}^e]\{\{\mathbf{P}^e\} + \{\mathbf{P}^e_B\}\} + \{\mathbf{w}^e_o\} \qquad (8)$$

where

$\{\mathbf{w}^e_{net}\}$ = a vector of air mass flow rates

$[\mathbf{a}^e]$ = $[\mathbf{a}^e(\{\mathbf{P}^e\}, \{\mathbf{P}^e_B\})]$, the element pressure-flow coefficient matrix, nonlinear with respect to pressure

$\{\mathbf{w}^e_o\}$ = a vector of zero-ΔP air mass flow rate terms (e.g., the free-delivery mass flow rate for fans)

$\{\mathbf{P}^e_B\}$ = a vector of buoyancy-induced pressure terms dependent on air densities associated with element nodes. For a two-node flow element:

$$\{\mathbf{P}^e_B\} = g[z^e]\begin{Bmatrix}\rho^e_i\\ \rho^e_j\end{Bmatrix} ; [\mathbf{z}^e] = \begin{bmatrix}(z_i - z^e_k) & 0\\ 0 & (z_j - z^e_k)\end{bmatrix} ;$$

k = 2 for flow from i to j
k = 1 for flow from j to i

The vector of air mass flow rates, $\{\mathbf{w}^e_{net}\}$, is defined analogously to the vectors of net heat and enthalpy flow rates defined above for the thermal problem with individual terms representing mass flow rates from each of the element's nodes into the element. These and the other element variables used in the expressions above are illustrated in Figure 3 for a two-node flow element.

In addition to the element equations, equations defining the matrix of partial derivatives, $\partial\{\mathbf{w}^e_{net}\}/\partial\{\mathbf{P}^e\}$, needed for Newton-Raphson solution methods of the flow problem may be developed for each type of element. One must be careful to ensure that this matrix of partial derivatives will be bounded (i.e., to avoid numerical instability) for all airflow rates, consequently, for some of the elements developed, it has been necessary to provide separate element equations and partial derivative expressions for laminar flow and non-laminar flow regimes.

Finally, equations defining the partial derivatives, $\partial\mathbf{w}^e/\partial\{\mathbf{T}^e\}$, needed for Newton-Raphson solution of the nonlinear thermal problem may also be developed from these element equations by employing the ideal gas law:

$$\rho = \frac{P}{RT} \qquad (9)$$

where R is the gas constant, which for air is approximately equal to 286.8 J/kg·°K (53.34 ft·lb/lb·°R), and T is the absolute temperature of the gas.

Demanding the conservation of mass flow at each of the system nodes, element equations corresponding to a specific system idealization may be *assembled* to form *system equations* that govern the behavior of the system as a whole:

$$\boxed{\{\mathbf{W}\} = [\mathbf{A}]\{\mathbf{P}\} + \{\mathbf{W}_B\} + \{\mathbf{W}_o\}} \qquad (10)$$

where

$$[\mathbf{A}] = \underset{e=a,b,...}{\mathbf{A}} [\mathbf{a}^e] ; \{\mathbf{W}_B\} = \underset{e=a,b,...}{\mathbf{A}} [\mathbf{a}^e]\{\mathbf{P}^e_b\} ;$$

$$\{\mathbf{W}_o\} = \underset{e=a,b,...}{\mathbf{A}} \{\mathbf{w}^e_o\}$$

$\{\mathbf{W}\}$ is a vector of the direct generation rates of air mass at each of the systems nodes. It is reasonably assumed to be a zero vector for the usual cases of building thermal or indoor air quality analysis. For building fire analysis, on the other hand, this vector will be non-zero.

The airflow equations, Equation 10, may be solved by a variety of methods (i.e., to determine the system pressure vector, $\{\mathbf{P}\}$) although variants of the Newton-Raphson method appear to be most effective (Walton 1988). The Newton-Raphson method is an iterative scheme based upon Taylor's expansion of Equation 10 written in residual form:

$$\{\mathbf{R}(\{\mathbf{P}\})\} \equiv [\mathbf{A}]\{\mathbf{P}\} + \{\mathbf{W}_B\} + \{\mathbf{W}_o\} - \{\mathbf{W}\} = \{0\} \qquad (11)$$

that leads to the following iterative algorithm:

$$\left[\frac{\partial\{\mathbf{R}(\{\mathbf{P}\})\}}{\partial\{\mathbf{P}\}}\bigg|_{\{\mathbf{P}\}^k}\right]\{\Delta\mathbf{P}\}^{k+1} = -\{\mathbf{R}(\{\mathbf{P}\}^k)\} \qquad (12a)$$

$$\{\mathbf{P}\}^{k+1} = \{\mathbf{P}\}^k + \{\Delta\mathbf{P}\}^{k+1} \qquad (12b)$$

With an initial estimate of the system pressure vector, $\{\mathbf{P}\}^k$, one forms and solves Equation 12a to obtain $\{\Delta\mathbf{P}\}^k$, which is then substituted into Equation 12b to obtain a better estimate of the system pressure vector, $\{\mathbf{P}\}^{k+1}$. This process is repeated until the system pressure estimates converge. Element flow rates can then be determined from the element equations using the solution for the system pressure vector.

The solution of Equation 12a will require the specification of one nodal pressure, typically the outside air node pressure, or, for those cases where the system is composed of uncoupled groups of zones, a single-node pressure must be specified for each group.

The square matrix on the left-hand side of Equation 12a is known as the system *Jacobian*. It follows from Equation 10 that the Jacobian may be directly assembled from the element expressions for the partial derivatives $\partial\{\mathbf{w}^e_{net}\}/\partial\{\mathbf{P}^e\}$ as:

$$\left[\frac{\partial\{\mathbf{R}(\{\mathbf{P}\})\}}{\partial\{\mathbf{P}\}}\bigg|_{\{\mathbf{P}\}^k}\right] =$$

$$\underset{e=a,b,...}{\mathbf{A}} \left[\partial\{\mathbf{w}^e_{net}\}/\partial\{\mathbf{P}^e\}\big|_{\{\mathbf{P}\}^k}\right] + \left[\frac{\partial\{\mathbf{W}\}}{\partial\{\mathbf{P}\}}\bigg|_{\{\mathbf{P}\}^k}\right] \qquad (13)$$

(The last term on the right-hand side may be ignored when $\{\mathbf{W}\}$ is the zero vector.)

COUPLED AIRFLOW-THERMAL ANALYSIS

Numerical methods for solving algebraic systems of nonlinear equations of the form of Equation 10 and first-order systems of nonlinear equations of the form of Equation 6 have been treated by many authors. Dhatt and Touzot (1984) provide an especially useful review of these methods, outline key computational strategies, and provide FORTRAN subroutines to implement them.

Depending on the nature of the thermal excitation (i.e., steady, steady harmonic, or dynamic) and the nature of the building system being studied (i.e., linear or nonlinear), a variety of solution options for the thermal equations, Equation 6, may be considered including a) steady linear and nonlinear analysis, b) steady linear harmonic analysis, and c) dynamic linear and nonlinear analysis. The latter case is often of greatest interest, thus the subsequent discussion will be limited to it.

The full dynamic problem defined by Equation 6 (i.e., after accounting for temperature-prescribed boundary conditions) may be solved numerically using one of several finite difference schemes. A general semi-implicit method has been employed by the authors for the solution of the linear problem (Axley 1988) and is presently under investigation for solution of the nonlinear coupled airflow/thermal problem. This method is based upon the difference approximation:

$$\{\mathbf{T}\}_{n+1} \approx \{\mathbf{T}\}_n + (1-\alpha)\delta t\{d\mathbf{T}/dt\}_n + \alpha\delta t\{d\mathbf{T}/dt\}_{n+1} \qquad (14)$$

where

$0 \leqslant \alpha \leqslant 1$

$\alpha = 0$ corresponds to the *forward difference scheme*

$\alpha = 1/2$ corresponds to the *Crank-Nicholson scheme*

$\alpha = 2/3$ corresponds to the *Galerkin scheme*

$\alpha = 1$ corresponds to the *backward difference scheme*

where the time domain has been divided into discrete steps, $t_{n+1} \equiv t_n + \delta t$, and an abbreviated notation has been introduced:

$$\{\mathbf{T}\}_n \equiv \{\mathbf{T}(t_n)\}\ ;\ \{d\mathbf{T}/dt\}_n \equiv \left.\frac{d\{\mathbf{T}\}}{dt}\right|_{t_n}$$

Substituting Equation 14 into Equation 6 leads to the following time-stepping algorithm:

$$[\alpha\delta t[\mathbf{K}(\{\mathbf{T}\}_{n+1})] + [\mathbf{C}]]\{\mathbf{T}\}_{n+1}$$
$$\approx \alpha\delta t\{\mathbf{E}\}_{n+1} + (1-\alpha)\delta t\{\mathbf{E}\}_n + [\mathbf{C}]\{\mathbf{T}\}_n$$
$$- (1-\alpha)\delta t[\mathbf{K}(\{\mathbf{T}\}_n)]\{\mathbf{T}\}_n \qquad (15a)$$

or

$$[\hat{\mathbf{K}}(\{\mathbf{T}\}_{n+1})]\ \{\mathbf{T}\}_{n+1} \approx \{\hat{\mathbf{E}}\}_{n+1} \qquad (15b)$$

where

$[\hat{\mathbf{K}}(\{\mathbf{T}\}_{n+1})] \equiv [\alpha\delta t[\mathbf{K}(\{\mathbf{T}\}_{n+1})] + [\mathbf{C}]]$

$\equiv$ *the dynamic conductance matrix*

$\{\hat{\mathbf{E}}\}_{n+1} \equiv \alpha\delta t\{\mathbf{E}\}_{n+1} + (1-\alpha)\delta t\{\mathbf{E}\}_n + [\mathbf{C}]\{\mathbf{T}\}_n - (1-\alpha)\delta t[\mathbf{K}(\{\mathbf{T}\}_n)]\{\mathbf{T}\}_n$

This algorithm is self-starting (i.e., given initial conditions the right-hand side is determined) and is, as written, nonlinear due to the dependency of $[\mathbf{K}]$ or $[\hat{\mathbf{K}}]$ on $\{\mathbf{T}\}$. In those cases when this nonlinear dependency can be ignored the algorithm will be unconditionally stable for $\alpha \geqslant 1/2$ (Dhatt 1984).

With a given initial system temperature vector specified, $\{\mathbf{T}\}_0$, Equation 15 may be solved to determine the system temperature vector at the next time step, $\{\mathbf{T}\}_1$. Repeating this process, in a step-wise manner, provides an approximate solution for the thermal response of the building system, $\{\mathbf{T}(t)\}$, to an arbitrary thermal excitation, $\{\mathbf{E}(t)\}$. At each time step, however, the airflows in the building system must be determined to form the system conductance matrix used on the left hand side of Equation 15. If it can be assumed that the airflows in the building system are not changing rapidly one may reasonably use the steady airflow equations (Equation 10) for this determination.

Combining Equation 10 with Equation 15, then, yields the following system of equations that defines the coupled airflow-thermal problem to be solved at each time step:

$$\begin{bmatrix}[\mathbf{A}]_{n+1} & [0] \\ [0] & [\hat{\mathbf{K}}]_{n+1}\end{bmatrix}\begin{Bmatrix}\{\mathbf{P}\}_{n+1} \\ \{\mathbf{T}\}_{n+1}\end{Bmatrix} = \begin{Bmatrix}\{\hat{\mathbf{W}}\}_{n+1} \\ \{\hat{\mathbf{E}}\}_{n+1}\end{Bmatrix} \qquad (16)$$

where

$$\{\hat{\mathbf{W}}\}_{n+1} \equiv \{\mathbf{W}\}_{n+1} - \{\mathbf{W_B}\}_{n+1} - \{\mathbf{W_o}\}_{n+1}$$

The matrix of flow coefficients, $[\mathbf{A}]_{n+1}$, will be dependent on the system pressure vector and the system temperature vector:

$$[\mathbf{A}]_{n+1} = [\mathbf{A}(\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T)]$$

The dependency on the system pressure vector was discussed above. The dependency on the system temperature vector results from the dependency of the coefficients on air density which, in turn (i.e., by Equation 9), is dependent on nodal temperatures.

The *dynamic system conductance matrix*, $[\hat{\mathbf{K}}]_{n+1}$, will also be dependent on the system pressure and temperature vectors when the thermal system includes flow elements. Thermal flow elements depend on the flow through the elements which, in turn, will be dependent on nodal pressures and temperatures, as above:

$$[\hat{\mathbf{K}}]_{n+1} = [\hat{\mathbf{K}}(\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T)]$$

As a result, Equation 16 is, in general, a system of nonlinear equations coupled implicitly through the nonlinear dependencies of the $[\mathbf{A}]_{n+1}$ and $[\hat{\mathbf{K}}]_{n+1}$ matrices.

One may approximate a solution to Equation 16 by forming the system matrices using past estimates of the system temperature and pressure vectors and solving the upper and lower halves of Equation 16 independently following the three-step procedure below:

Solution Strategy 1 (at time step t_{n+1})

Step 1: Solve $[\mathbf{A}]_{n+1}\{\mathbf{P}\}_{n+1} = \{\hat{\mathbf{W}}\}_{n+1}$, using $\{\mathbf{T}\}dn$ and equations (12).

Step 2: Form $[\hat{\mathbf{K}}]_{n+1} \approx [\hat{\mathbf{K}}(\{\{\mathbf{P}\}_n \mid \{\mathbf{T}\}_n\}^T)]$

Step 3: Solve $[\hat{\mathbf{K}}]_{n+1}\{\mathbf{T}\}_{n+1} \approx \{\hat{\mathbf{E}}\}_{n+1}$, using Gauss Elimination or variant

Alternatively, one may improve the above procedure by updating the pressure estimate at the second step:

Solution Strategy 2 (at time step t_{n+1})

Step 1: Solve $[\mathbf{A}]_{n+1}\{\mathbf{P}\}_{n+1} = \{\hat{\mathbf{W}}\}_{n+1}$, using $\{\mathbf{T}\}_n$ and equations (12)

Step 2: Form $[\hat{\mathbf{K}}]_{n+1} \approx [\hat{\mathbf{K}}(\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_n\}^T)]$

Step 3: Solve $[\hat{\mathbf{K}}]_{n+1}\{\mathbf{T}\}_{n+1} \approx \{\hat{\mathbf{E}}\}_{n+1}$, using Gauss Elimination or variant

This second solution strategy is essentially equivalent to the strategy employed by Clarke (1985) and the first of two strategies employed by Walton (1982) discussed above. If the solution is computed using very small time steps these solution strategies may prove sufficiently accurate.

One may further improve the accuracy of the prediction, at the expense of computational effort, by iteratively repeating these solution strategies until an appropriate measure of convergence (e.g., temperature, pressure, or element mass flow rate) reaches an acceptable value. The iterative variation of solution strategy 2 above would involve the following five-step procedure:

Solution Strategy 3 (at time step t_{n+1})

Step 1: Set initial estimate equal to solution from previous time step; $\{\{\mathbf{P}\}^1_{n+1} \mid \{\mathbf{T}\}^1_{n+1}\}^T = \{\{\mathbf{P}\}_n \mid \{\mathbf{T}\}_n\}^T$

Initialize iteration counter; $k = 0$

Until convergence is realized repeat Steps 2, 3, and 4:

Step 2: Increment iteration counter; $k = k + 1$

Solve $[\mathbf{A}]^{k+1}_{n+1}\{\mathbf{P}\}^{k+1}_{n+1} = \{\hat{\mathbf{W}}^k_{n+1}$, using $\{\mathbf{T}\}^k_{n+1}$ and equations (12)

Step 3: Form $[\hat{\mathbf{K}}]^{k+1}_{n+1} \approx [\hat{\mathbf{K}}(\{\{\mathbf{P}\}^{k+1}_{n+1} \mid \{\mathbf{T}\}^k_{n+1}\}^T)]$

Step 4: Solve $[\hat{\mathbf{K}}]^{k+1}_{n+1}\{\mathbf{T}\}^{k+1}_{n\$1} \approx \{\hat{\mathbf{E}}\}_{n+1}$, using Gauss Elimination or variant

Step 5: Report solution for time step as: $\{\{\mathbf{P}\}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T = \{\{\mathbf{P}\}^{k+1}_{n+1} \mid \{\mathbf{T}\}^{k+1}_{n+1}\}^T$

Continue to next time step.

Walton's second solution strategy is, essentially, equivalent to this third solution strategy.

Finally, one may attack the problem using the Newton-Raphson method discussed above for isothermal steady flow analysis. Rewriting Equation 16 in residual form:

$$\left\{\mathbf{R}\left(\left\{\begin{matrix}\{\mathbf{P}\}_{n+1}\\ \{\mathbf{T}\}_{n+1}\end{matrix}\right\}\right)\right\} \equiv \begin{bmatrix}[\mathbf{A}]_{n+1} & [0]\\ [0] & [\mathbf{K}]_{n+1}\end{bmatrix}\left\{\begin{matrix}\{\mathbf{P}\}_{n+1}\\ \{\mathbf{T}\}_{n+1}\end{matrix}\right\} - \left\{\begin{matrix}\{\hat{\mathbf{W}}\}_{n+1}\\ \{\hat{\mathbf{E}}\}_{n+1}\end{matrix}\right\} = \left\{\begin{matrix}[\mathbf{A}]_{n+1}\{\mathbf{P}\}_{n+1} - \{\hat{\mathbf{W}}\}_{n+1}\\ [\hat{\mathbf{K}}]_{n+1}\{\mathbf{T}\}_{n+1} - \{\hat{\mathbf{E}}\}_{n+1}\end{matrix}\right\} \quad (17)$$

the Newton-Raphson method may be directly represented by the following iterative algorithm:

Solution Strategy 4 (at time step t_{n+1})

Step 1: Set initial estimate equal to solution from previous time step: $\{\{\mathbf{P}\}^1_{n+1} \mid \{\mathbf{T}\}^1_{n+1}\}^T = \{\{\mathbf{P}\}_n \mid \{\mathbf{T}\}_n\}^T$

Initialize iteration counter; $k = 0$

Until convergence is realized repeat Steps 2, 3, and 4:

Step 2: Increment iteration counter; $k = k + 1$

Form $[\mathbf{J}]^k_{n+1} \equiv \left[\dfrac{\partial \mathbf{R}\left(\left\{\begin{matrix}\{\mathbf{P}\}\\ \{\mathbf{T}\}\end{matrix}\right\}\right)}{\partial\left\{\begin{matrix}\{\mathbf{P}\}\\ \{\mathbf{T}\}\end{matrix}\right\}} \Bigg|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right]$

the coupled system Jacobian

Step 3: Solve $[\mathbf{J}]^k_{n+1}\left\{\begin{matrix}\{\Delta\mathbf{P}\}^{k+1}_{n+1}\\ \{\Delta\mathbf{T}\}^{k+1}_{n+1}\end{matrix}\right\} = -\left\{\mathbf{R}\left(\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}\right)\right\}$,

using Gauss Elimination or variant

Step 4: Update

$$\left\{\begin{matrix}\{\mathbf{P}\}^{k+1}_{n+1}\\ \{\mathbf{T}\}^{k+1}_{n+1}\end{matrix}\right\} = \left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\} + \left\{\begin{matrix}\{\Delta\mathbf{P}\}^{k+1}_{n+1}\\ \{\Delta\mathbf{T}\}^{k+1}_{n+1}\end{matrix}\right\}$$

Step 5: Report solution for time step as:

$$\{\{\mathbf{P}_{n+1} \mid \{\mathbf{T}\}_{n+1}\}^T = \{\{\mathbf{P}\}^{k+1}_{n+1} \mid \{\mathbf{T}\}^{k+1}_{n+1}\}^T$$

Continue to next time step.

Again, convergence evaluation may be based upon system pressures and temperatures, element mass flow rates, or a combination of these.

From Equation 17 it follows that the coupled system Jacobian matrix, $[\mathbf{J}]^k_{n+1}$, consists of the following four submatrices:

$$[\mathbf{J}]^k_{n+1} \equiv \left[\begin{matrix}\left[\dfrac{\partial\{[\mathbf{A}]\{\mathbf{P}\} - \{\hat{\mathbf{W}}\}\}}{\partial\{\mathbf{P}\}}\Bigg|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right] & \left[\dfrac{\partial\{[\mathbf{A}]\{\mathbf{P}\} - \{\hat{\mathbf{W}}\}\}}{\partial\{\mathbf{T}\}}\Bigg|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right]\\ \left[\dfrac{\partial\{[\hat{\mathbf{K}}]\{\mathbf{T}\} - \{\hat{\mathbf{E}}\}\}}{\partial\{\mathbf{P}\}}\Bigg|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right] & \left[\dfrac{\partial\{[\hat{\mathbf{K}}]\{\mathbf{T}\} - \{\hat{\mathbf{E}}\}\}}{\partial\{\mathbf{T}\}}\Bigg|_{\left\{\begin{matrix}\{\mathbf{P}\}^k_{n+1}\\ \{\mathbf{T}\}^k_{n+1}\end{matrix}\right\}}\right]\end{matrix}\right] \quad (18)$$

The upper left submatrix is seen to be identical to the steady flow system Jacobian (Equation 12a) evaluated for the discrete temperature field $\{\mathbf{T}\}^k_{n+1}$. It therefore follows that this first Jacobian submatrix may be directly assembled from the element expressions as defined by Equation 14. In a similar manner, each of the other three submatrices may be assembled from the individual flow and thermal element contributions.

Although a complete discussion of the details of implementation of these strategies is beyond the scope of this paper three key considerations warrant special mention:

1. From a computational point of view, solution strategies 1, 2, and 3 may be considered to be special cases of strategy 4 that involve ignoring the off-diagonal Jacobian submatrices (i.e., replacing them by null matrices) and, for strategies 1 and 2, avoiding iteration. For strategy 1 the sys-

tem pressure vector would not be updated as the solution progressed and for strategies 2 and 3 it would be. It is, therefore, possible to develop a single general purpose set of computational routines to implement all of the strategies considered.

2. Nonflow linear thermal elements will contribute only to the lower-right submatrix of the coupled system Jacobian and will remain constant. Therefore, their contribution may be assembled initially once and for all and simply added to the nonlinear contributions, as appropriate, during the solution process and, thereby, avoiding unnecessary computation.

3. The solution of the coupled problem will require the specification of (at least) one system pressure variable and those temperature variables corresponding to prescribed temperature conditions (that may be time varying). It is useful to isolate these prescribed system variables in a lower or upper partition of the coupled system variables, $\{\{\mathbf{P}\} \mid \{\mathbf{T}\}\}^T$, to facilitate the imposition of these conditions on the solution process.

These approaches to the formation and solution of the coupled airflow/thermal analysis problem are presently under investigation at the National Institute of Standards and Technology as part of an ongoing project by Grot to develop methods for indoor air quality analysis.

CONCLUSION

The theoretical bases of a building airflow analysis technique and a building thermal analysis technique, developed at the National Institute of Standards and Technology, have been reviewed and an approach to integrate these methods to solve problems of coupled airflow and thermal analysis has been outlined. The individual techniques and, hence, the integrated approach is based upon element assembly techniques that allow the analyst to consider a practically unlimited variety of system idealizations of arbitrary complexity. These element assembly techniques also lead to highly modular computer programs facilitating development and future changes.

The integrated approach presented involves the incremental solution of two implicitly coupled nonlinear problems that may be solved using a variety of solution strategies. Four candidate strategies have been outlined. Developing guidelines for the identification of appropriate computational strategies is a major focus of the current investigation.

The basic approach to the solution of the *dynamic* coupled airflow/thermal problem presented in this paper must be consider to be a quasi-dynamic, or alternatively, quasi-steady, approach as a steady airflow solution is integrated with a dynamic thermal solution procedure. Although this is justified when airflows are not changing rapidly within the building system, this assumption must be critically evaluated to determine its range of applicability and nonsteady flow formulations should be considered.

The airflow analysis portion of the solution procedure is based upon element pressure-flow laws that have been developed, primarily, for isothermal situations. An evaluation of the applicability of these laws to nonisothermal conditions is, therefore, warranted.

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