

A Versatile Procedure for Calculating Heat Transfer through Windows

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ABSTRACT

Advances in window technologies and the desire to standardize the reporting of standard window heat-transfer indices have necessitated the development of a comprehensive analytical procedure for calculating heat transfer through windows. This paper shows how complete window heat transfer can be considered as the area-weighted sum of the three window component areas: the center-of-glass area, the edge-of-glass area, and the frame area. Algorithms for calculating heat transfer through each of these areas and for combining these to calculate total window indices are presented.

INTRODUCTION

Advances in glass-coating technology, the use of low-conductivity gases in insulated glass, and alternative window-edge details have necessitated the development of a flexible and accurate analytical procedure for calculating heat transfer through windows. Laboratory and field testing, while useful for basic product evaluation, are too expensive for initial product design and for reporting on all possible combinations of available products. For these reasons, the availability of an accurate and unbiased analytical procedure to calculate window heat transfer performance indices is necessary if windows are to be compared against each other and evaluated for their overall impact in buildings.

Such a procedure was initially documented in Rubin (1982). This procedure, as compared to its predecessor presented in ASHRAE Fundamentals (ASHRAE 1977), offers added flexibility. The method is not limited to one or two layers but can be written for an arbitrary number of layers. The layers can be partially transparent to long-wave radiation, can be coated with solar control or low-emissivity coatings, and can be separated by low-conductivity gases. For a given set of environmental conditions, which include inside and outside temperatures and air velocities as well as incident solar radiation, the temperature distribution across the window system can be calculated. From this temperature distribution, overall window performance indices such as U-value, shading coefficient, and relative heat gain can be calculated. Knowing these indices is the first step toward predicting the energy performance and assessing the comfort of windows in both residential and

commercial buildings. The algorithms can also be used in whole-building energy analysis programs for increased accuracy in studying the effects of windows in buildings.

Since the introduction of this method (Rubin 1982), several improvements to the model have been made. These include the ability to specifically model conduction through solid layers, the inclusion of some gas mixtures, a procedure to analyze spectrally selective (in the solar spectrum) glazing layers, updated natural convection algorithms, the inclusion of two-dimensional glazing edge and frame heat transfer correlations, and the ability to handle tilted windows.

The procedure presented in Rubin (1982) was turned into a PC-compatible computer program (LBL 1985, 1988) and distributed to the glass and window industries in 1986. Comparisons between experimentally measured and calculated heat-transfer rates for a variety of glazing systems indicated a very good correlation between experiment and theory (Arasteh et al. 1987). Comments from users and the desire to be able to accurately model new products led to the additions to the original calculational procedure (Rubin 1982). This paper describes the revised procedure for calculating center-of-glass, edge-of-glass, and frame heat transfer, culminating in complete window system indices.

WINDOW SYSTEM HEAT TRANSFER

A double-glazed window is typically constructed as shown in Figure 1a. Windows with more than two glazing layers are constructed similarly. The total heat transfer through a window system can be considered as the area-weighted sum of heat transfer through the center-of-glass, edge-of-glass, and frame areas (Petersen 1987). Glazing layers are separated by a spacer (usually metallic) and then sealed to form an insulated glass (IG) unit. The IG unit is then sealed inside a wood, aluminum, vinyl, or fiberglass frame. In the center of the IG unit, a one-dimensional process accurately approximates the heat transfer. However, since the spacer's conductivity is generally much higher than the effective conductivity of the airspace it bridges, the spacer acts as a thermal bridge between the edge areas of the glazing layers it separates. Therefore, two-dimensional heat transfer in these areas must be considered. The edge area is considered to be that area of the glass within 2.5 in (63.5 mm) of the window's sight line (Petersen 1987;

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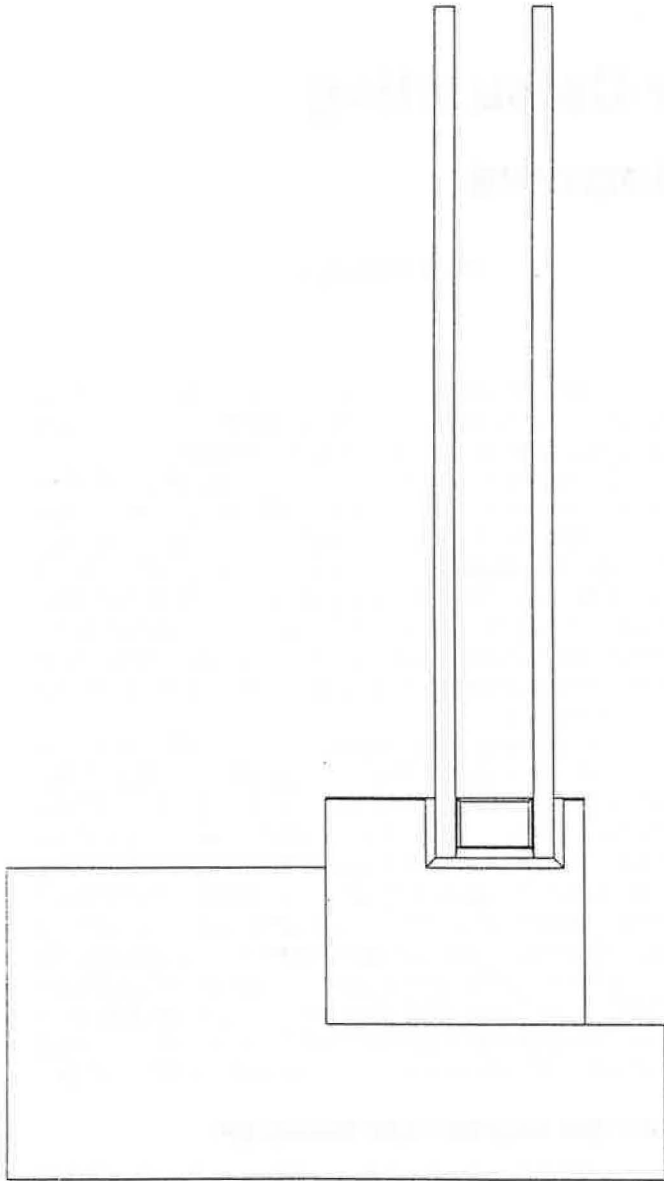


Figure 1a Cross section of a window edge and frame. Shown are two glazing layers, separated by a desiccant-filled metal spacer, sealed inside a wood sash that rests on a wood frame.

ASHRAE 1989). Heat transfer through frames is primarily one-dimensional. Figure 1b shows the frame and edge effects and the magnitude and direction of heat transfer through the typical double-glazed window cross section of Figure 1a.

CENTER-OF-GLASS PERFORMANCE: ONE-DIMENSIONAL STEADY-STATE ENERGY BALANCE

Given boundary conditions (wind, solar radiation, temperature) that remain constant over the response time of the window, the temperature of each layer is determined by the condition that no net energy is absorbed or released by any layer. Figure 2 illustrates this. Q represents energy flux, and energy leaving a layer or surface is a positive quantity. An individual layer is denoted by the subscript n ; corresponding layer surfaces are denoted by $2n-1$ and $2n$. Layers and surfaces are numbered from the outside in. Conduction across layer n is denoted by Q_n^k . The radiative

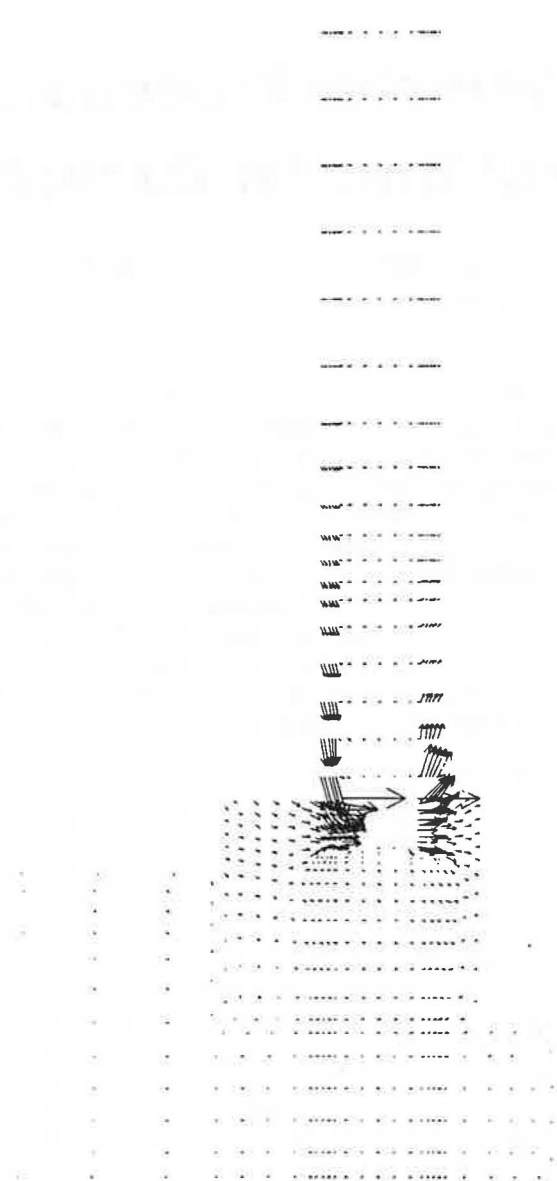


Figure 1b Vector plot based on finite element modeling of two-dimensional heat transfer through the window cross section of Figure 1a. The warm interior is on the left, the cold exterior on the right. The size of the vectors denotes the magnitude of heat transfer; the arrow denotes the direction. Small vectors may appear as dots. Note that all glass area two-dimensional heat transfer occurs within the bottom 2.5 in (63.5 mm) of the sightline.

flux leaving the individual layer surfaces is denoted by Q_{2n-1}^r and Q_{2n}^r , and the conductive/convective flux leaving the individual layer surfaces is denoted by Q_{2n-1}^c and Q_{2n}^c . Note that equations "a" and "b" refer to inch-pound units and SI units, respectively.

For a system of N layers, the temperature of each layer surface and the temperature at the center of each layer is desired. This results in $2N+N$ unknown temperatures. Initially, a system of N nonlinear equations for the N center of layer (or average layer) temperatures is solved. An energy balance is performed at each layer node and the equations are written as:

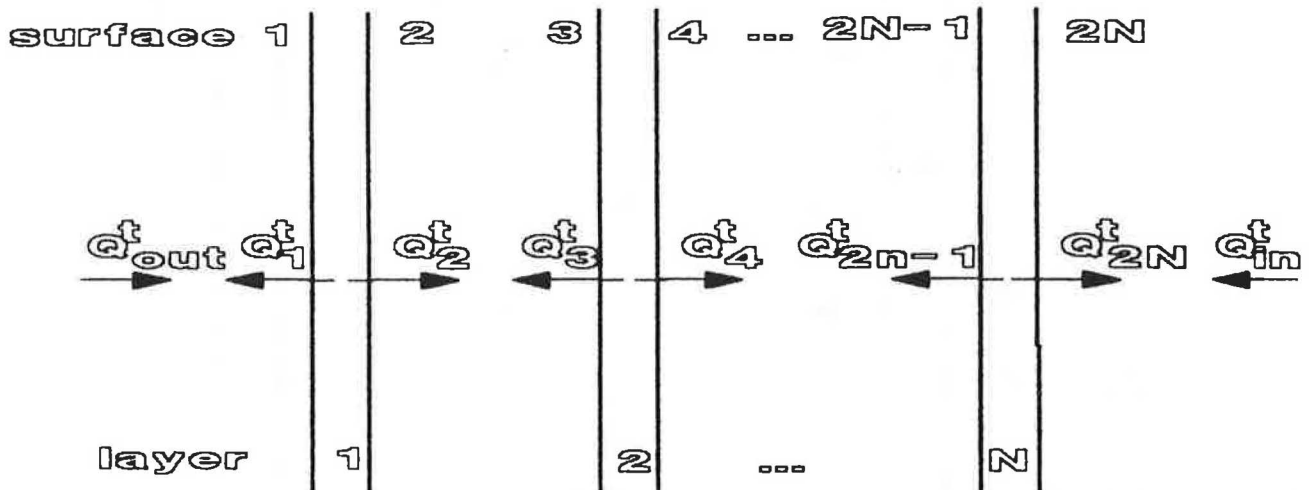
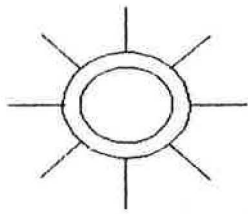


Figure 2 Net energy balance for a window of N solid layers. Q_k^t is the total (radiative and conductive/convective) energy flux from surface k . Q_{out}^t and Q_{in}^t are the energy fluxes from the environment and the room, respectively.

$$\Delta_n(\{\theta_k, k = 1, N\}) = 0 \quad (1)$$

The functions Δ_n are given by:

$$\Delta_n = Q'_{2n-1} + Q'_{2n} - Q'_{2n-2} - Q'_{2n+1} + Q^c_{2n-1} + Q^c_{2n} - Q^c_{2n-2} - Q^c_{2n+1} + Q^k_{n-1} + Q^k_n - Q^k_{n+1} - A_n I \quad (2)$$

where I is the solar intensity outside the window and A_n is the fraction of incident solar energy absorbed in layer n . The calculation is initialized by guessing a linear progression of the temperatures from the outside to the inside. Thereafter, the exact temperatures are written as

$$\theta_k = \theta_k^0 + \delta\theta_k \quad (3)$$

where θ_k^0 is the temperature from the previous iteration. For $\delta\theta_k$ small, we expand to first order

$$\begin{aligned} \Delta_n(\{\theta_k\}) &= \Delta_n(\{\theta_k^0 + \delta\theta_k\}) = 0 \\ &\approx \Delta_n(\{\theta_k^0\}) + \sum_j \left[\frac{\partial \Delta_n}{\partial \theta_j} \right] \delta\theta_j \end{aligned} \quad (4)$$

This equation can be solved for the $\delta\theta_k$ by inverting the matrix $[\partial \Delta / \partial \theta]$.

$$\delta\theta_n^1 = - \sum_j \left[\frac{\partial \Delta}{\partial \theta} \right]_{nj}^{-1} \Delta_n(\{\theta_k^0\}) \quad (5)$$

If our guess was close, then a better approximation to the temperature distribution is

$$\theta_k^1 = \theta_k^0 + \delta\theta_k^1 \quad (6)$$

From these N center-of-layer temperatures, the remaining $2N$ layer surface temperatures are found by performing similar energy balances around each surface node and solving for the surface temperatures. The above procedure is repeated, beginning with Equation 4, until the

TABLE 1
Glazing Material Conductivities

Material	Conductivity Btu · in/h · ft ² · °F (W/m ² · °C)
Glass	6.25 (0.9)
Polyester film	0.97 (0.14)
Acrylic or polycarbonate sheet	1.32 (0.19)
Air	0.17 (0.024)

components of the solution converge to the desired accuracy. Once the final temperature distribution is determined, the exact fluxes ("Q's" and A_n) can be determined. The remainder of this section presents methods or algorithms for calculating these fluxes and/or heat-transfer coefficients.

Conduction through Solid Layers

In most window systems, the glazing layers contribute very little toward the overall resistance to heat transfer. Table 1 gives the conductivities of common glazing materials and, for comparison, that of air (Rubin 1982). Generally, because most windows are made up of 0.125 in or 0.25 in (3 mm or 6 mm) glass, the glazing layer resistance is negligible and usually ignored. However, for solid plastic glazings and thick glass, including the glazing layer resistance can make small differences in a window's overall U -value and glazing surface temperatures. The energy conducted from the center of a layer to one surface is

$$Q_n^k = K_n \Delta\theta_n \quad (7)$$

The glazing layer thermal conductivity divided by half the thickness of the glazing layer equals the solid conductance, K_n , and $\Delta\theta_n$ corresponds to the temperature difference between the solid layer centerline temperature and one of the layer's surface temperatures.

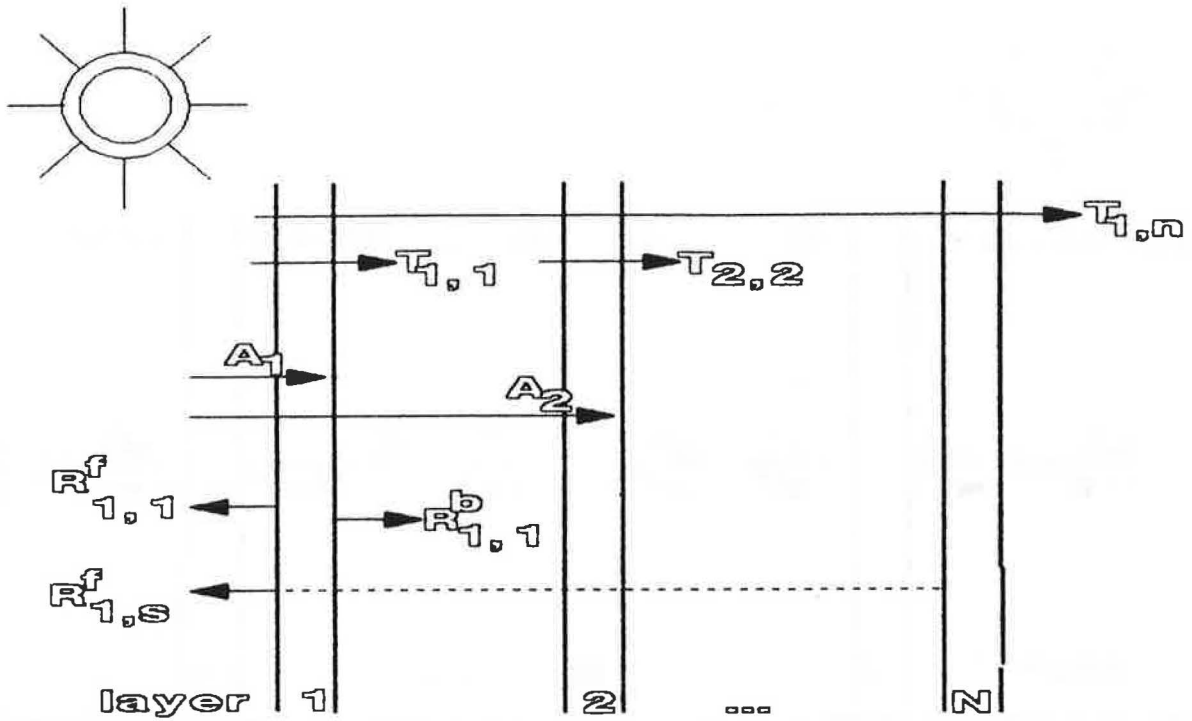


Figure 3 General structure of the layers in a window with the numbered layers separated by non-absorbing gas layers. $T_{ij}(\lambda)$ is the transmittance through layers i to j ; $R_{ij}(\lambda)$ is the reflectance from layers i to j . The subscripts f and b denote front and back, respectively. $A_j(\lambda)$ is the absorption in layer j as part of the whole window structure.

Solar and Visible Optical Properties

Solar radiation absorbed by glazing layers alters layer temperatures and thus the heat transfer. Some of this absorbed solar radiation as well as solar radiation directly transmitted into the space ends up as heat gain into the interior space. This heat gain depends on the solar and infrared optical properties of the individual layers. In addition, the overall visible optical properties for the window system, which constitute a subset of the solar optical properties, are an important design consideration. They reveal how a window appears and how the window may visually affect its surroundings. The following discusses the solar and visible optical properties; the infrared region is covered later.

Generally, glazing material transmittance and reflectance data are measured as a function of angle of incidence, θ , and wavelength, λ . For a specific angle of incidence and assuming unpolarized light, layer transmittance and reflectance are measured over the range of the solar spectrum as a function of wavelength. For each spectral property, $P(\lambda)$, the following individual layer properties are defined:

P_s , the total solar property:

$$P_s = \int P(\lambda) E_s(\lambda) d\lambda \quad (8)$$

P_v , the total visible property:

$$P = \int P(\lambda) E_s(\lambda) R(\lambda) d\lambda \quad (9)$$

P_{uv} , the total damage-corrected "ultraviolet" property:

$$P_{uv} = \int P(\lambda) E_s(\lambda) D(\lambda) d\lambda \quad (10)$$

where $E_s(\lambda)$ is the solar spectral irradiance function, $R(\lambda)$ is the photopic response of the eye (IES 1972), and $D(\lambda)$ is the Krochman damage function (Krochman 1983).

The overall properties of a system of multiple glazing layers are also computed from spectrophotometric data. Given the window system of Figure 3, $T_{ij}(\lambda)$ and $R_{ij}(\lambda)$ refer to the transmittance through and reflectance from elements i to j as a function of wavelength as if they were standing alone. Reflection is denoted from the incident or front side by an "f" superscript and from the backside by a "b" superscript. The superscripts "s" and "v" refer to solar and visible properties.

For window heat-transfer calculations, we need to know the total solar absorptance in each layer within the given system, $A_1^s \dots A_n^s$. Of interest to architects and HVAC engineers are the total solar transmittance through the system of n layers, $T_{1,n}^s$, and the total solar and visible reflectance of both the front and back of the glazing system, $R_{1,n}^s$, $R_{n,1}^s$, $R_{1,n}^v$, and $R_{n,1}^v$. These quantities can be obtained by first solving the following recursion relations for $T_{i,n}(\lambda)$, $A_i(\lambda)$, $R_{1,n}^f(\lambda)$, and $R_{n,1}^b(\lambda)$ and then inputting these quantities into Equations 8 and 9:

$$T_{ij}(\lambda) = \frac{T_{i,j-1}(\lambda) T_{j,j}(\lambda)}{1 - R_{i,j-1}^f(\lambda) R_{j-1,j}^b(\lambda)} \quad (11)$$

$$R_{ij}^f(\lambda) = R_{i,j-1}^f(\lambda) + \frac{T_{i,j-1}^2(\lambda) R_{j,j}^f(\lambda)}{1 - R_{i,j-1}^f(\lambda) R_{j-1,j}^b(\lambda)} \quad (12)$$

$$R_{ij}^b(\lambda) = R_{j,j}^b(\lambda) + \frac{T_{ij}^2(\lambda) R_{j-1,j}^b(\lambda)}{1 - R_{j-1,j}^b(\lambda) R_{ij}^f(\lambda)} \quad (13)$$

$$A_j(\lambda) = \frac{T_{i,j-1}(\lambda) A'_j(\lambda)}{1 - R'_{i,N}(\lambda) R^b_{j-1,1}(\lambda)} + \frac{T_{i,j}(\lambda) R'_{j+1,N}(\lambda) A^b_j(\lambda)}{1 - R^b_{j,1}(\lambda) R'_{j+1,N}(\lambda)} \quad (14)$$

where

$$A'_j(\lambda) = 1 - T_{i,j}(\lambda) - R'_{i,j}(\lambda) \quad (15)$$

$$A^b_j(\lambda) = 1 - T_{i,j}(\lambda) - R^b_{i,j}(\lambda) \quad (16)$$

For some common glazing materials, solar optical properties are roughly constant with wavelength. For these cases, the wavelength-dependent properties $T_{i,j}(\lambda)$, $R'_{i,j}(\lambda)$, $R^b_{i,j}(\lambda)$, and $A_j(\lambda)$ in Equations 11 through 16 can be replaced with the constants $T_{i,j}$, $R'_{i,j}$, $R^b_{i,j}$, and A_j (calculated with Equations 8 and 9).

Thermal Radiation

In the thermal infrared spectrum, as in the solar spectrum, the wavelength-averaged radiant properties of the glazing materials must be known to calculate the net radiation balance of a window. This integrated average is weighted by the emissive power of a black body at ambient temperature (the result of the average does not depend strongly on the choice of source temperature). Even though the surfaces may be characterized as specularly reflecting in the infrared, only the hemispherical average infrared properties are required to calculate the heat fluxes, because all of the sources of thermal radiation are diffusely emitting. Furthermore, for the special geometry of parallel planes with high aspect ratios, the radiative interchange does not depend on the nature of the surfaces.

In the case of opaque materials, we need only the hemispherical reflectivity or emissivity of the two surfaces, but for partially transparent materials, such as some plastic films less than 0.012 in (0.3 mm) thick, both the hemispherical reflectance and transmittance are required. The transmittance measured from either side is the same. For an asymmetric material, such as a thin-film-coated pane, the reflectance values generally differ from one side to the other. A procedure for converting normal to hemispherical data is presented in (Rubin et al. 1987).

Consider a system composed of N solid layers, or $2N$ surfaces, numbered consecutively from the outdoor layer or outdoor-facing surface (see Figure 2). For nonabsorbing gas fills between glazing layers, no energy is lost in transit between layers. We define the net infrared energy flux leaving the k th surface as Q'_k , where the superscript r denotes radiation. Then, for the n th layer, which is bounded by surfaces $2n-1$ and $2n$, we have:

$$Q'_{2n} = S_{2n} + R_{2n} Q'_{2n+1} + T_n Q'_{2n-2} \quad (17)$$

$$Q'_{2n-1} = S'_{2n-1} + R_{2n-1} Q'_{2n-2} + T_n Q'_{2n+1} \quad (18)$$

where R_k is the infrared reflectance of the layer measured from the k th surface, and T_n is the transmittance (which is the same from either side) of the n th layer. The emitted energy flux from the k th surface, S_k , given by:

$$S_k = \epsilon_k \sigma \theta_k^4 \quad (19)$$

where ϵ_k is the emissivity of the k th surface, σ is the Stefan-Boltzmann constant, θ_k the temperature of the k th surface, and k is $2n$ or $2n-1$.

For layers 1 and N , the terms Q'_0 and Q'_{2N+1} appear in Equations 17 and 18. These terms represent the infrared sources from the outdoor environment and the room, which we will rename Q'_{out} and Q'_{in} , respectively. For the room, $Q'_{in} = \epsilon_{in} \sigma \theta_{in}^4$, where θ_{in} is taken to be the indoor air temperature and $\epsilon_{in} = 1$, since the room enclosure is assumed to look like a black body to the aperture of the window. Caution should be exercised when making these assumptions about a room in a real building. Direct sunlight on an interior wall may raise the wall temperature well above air temperature, or large areas of exterior glazing or uninsulated wall may be close to the outside temperature. If the geometry of the room is such that the window in question receives a large fraction of the radiation emitted by these areas, then a detailed radiation balance must be performed to determine the net flux.

On overcast days, Q'_{out} is a similar function of the outside air temperature, θ_{out} ; however, clear skies have a lower emissive power than cloudy skies because of the transparency of the atmosphere. Swinbank (1963) reports the following correlation between the radiation falling on a horizontal surface, S_{sky} , in Btu/h·ft² (20a) or W/m² (20b), and the dry-bulb temperature, θ_{out} , in R (20a) or K (20b),

$$S_{sky} = 4.80 \times 10^{-15} \theta_{out}^6 \quad (20a)$$

$$S_{sky} = 5.31 \times 10^{-13} \theta_{out}^6 \quad (20b)$$

Equation 20 is found to be independent of location and, based on theoretical considerations, is expected to be independent of altitude. We define a sky emittance as:

$$\epsilon_{sky} = \frac{S_{sky}}{\sigma \theta_{out}^4} = 2.80 \times 10^{-6} \theta_{out}^2 \quad (21a)$$

$$\epsilon_{sky} = \frac{S_{sky}}{\sigma \theta_{out}^4} = 9.27 \times 10^{-6} \theta_{out}^2 \quad (21b)$$

(θ_{out} is in R [K]). Then the effective outside emittance, ϵ_{out} , is just the weighted average of ϵ_{sky} and the emittance of the rest of the environment (ground, obstructions, and cloudy sky), ϵ_{env} , which we take to be one. If F_{sky} is the view factor of the sky from the window ($F_{sky} = 1$ for a horizontal skylight or $1/2$ for an unobstructed vertical window), and F_c is the fraction of sky that is clear,

$$\epsilon_{out} = F_{sky} F_c \epsilon_{sky} + (1 - F_{sky} F_c) \epsilon_{env} \quad (22)$$

F_c can be obtained from standard meteorological reports of fractional cloud cover. Most standard environmental conditions (i.e., ASHRAE) do not specify the amount of cloud cover. The assumed default is usually a cloudy ($\epsilon_{out} = 1$) sky.

All of the terms in the N pairs of equations (Equations 17 and 18) have now been defined. These equations form an inhomogeneous linear set, which we arrange in the matrixform:

$$\sum_j M_{ij} Q'_j = S_i \quad (23)$$

The $2N \times 2N$ matrix $[M]$ has the structure

$$M_{ii} = 1 \quad (24)$$

$$M_{2n-1, 2n-2} = -R_{2n-1} \quad (25)$$

$$M_{2n-1, 2n+1} = -T_n \quad (26)$$

$$M_{2n, 2n-2} = -T_n \quad (27)$$

$$M_{2n, 2n+1} = -R_{2n} \quad (28)$$

All other elements are zero. Given the temperatures of each layer, one can compute the sources, S_k , and therefore the net fluxes by

$$Q_i = \sum_j (M^{-1})_{ij} S_j \quad (29)$$

Infrared Absorbing Gases

Radiant exchange between layers separated by infrared absorbing gases, such as SF_6 and CO_2 , further complicates the window thermal analysis. The absorptance of these gases is spectrally dependent and generally a gray gas assumption—one that assumes constant optical properties across the spectrum—is inadequate. The absorptance also depends on the partial pressure (for mixtures of gases) and the temperature of the infrared absorbing gas.

For simplicity, consider two glazing layers separating an absorbing gas. Calculating the temperature distribution between two solid layers bounding an infrared absorbing gas involves dividing the gas layer into $N-2$ nodes and assigning a node to each bounding surface. An energy balance is performed on each of the N nodes by first summing the radiation contributions within each absorption band and then that outside these bands from each node. The conduction/convection component between successive nodes is then added to the radiation component. The equation of conservation for this system is

$$\frac{d}{dy} (h_{eff} \frac{d\theta}{dy}) + \frac{d}{dy} (q_r) = 0 \quad (30)$$

where h_{eff} refers to an effective heat-transfer coefficient that accounts for conduction and convection, and q_r is the radiation component. This effective heat-transfer coefficient, h_{eff} , may differ from node to node depending on the slant of the window and the direction of heat transfer through the window. $d\theta$ represents the temperature difference between two successive nodes; the distance separating the nodes is dy . An iterative procedure is used to solve for the temperature distribution and the window performance indices can then be found.

Presently, Edwards' wide-band model (Edwards and Balakrishnan 1973) is used to find the transmittance of the infrared radiation absorption bands of SF_6 and CO_2 . The wide-band model accounts for the effects of total pressure, partial pressure, and temperature on the absorptance of the gas. Therefore, mixtures of absorbing gases with nonabsorbing gases can be handled directly by this model. The wide-band parameters for CO_2 can be found in Edwards and Balakrishnan (1973), and the parameters for SF_6 have recently been determined through research (Reilly et al. 1988). In addition, those for N_2O can be found in Tien et al. (1972), and those for NH_3 in Tien (1973).

The net radiation within the absorbing regions leaving the individual gas layers is calculated by applying Equations 17 through 19 and Equations 23 through 29 over the absorbing regions. The reflectance of the gas layer surfaces is zero, and the emitted energy flux is calculated using Planck's Law over each absorption band. The net radiation outside the absorption bands only contributes to

the glazing layer energy balances. This is calculated by considering the fraction of black body energy emitted outside the absorbing regions. Reilly et al. (1988) provides more details.

Indoor Surface Film Coefficient

The conductive/convective heat-transfer coefficient in $Btu/h \cdot ft^2 \cdot ^\circ F$ (31a) or $W/m^2 \cdot ^\circ C$ (31b) for natural convection between room air and a vertical plate is based on experimental data and is given by ASHRAE (1985):

$$h_c = 0.27 (\Delta \theta)^{0.25} \quad (31a)$$

$$h_c = 1.77 (\Delta \theta)^{0.25} \quad (31b)$$

where $\Delta \theta$ is the temperature difference between the glass surface and the still air. For the case of forced air in front of a window (i.e., an air-conditioning system), where v is the velocity in mph (32a) or m/s (32b), the following relation holds (ASHRAE 1985):

$$h_c = 0.99 + 0.3v \quad (32a)$$

$$h_c = 5.6 + 3.8v \quad (32b)$$

Konrad and Larsen (1978) present the following indoor convective film coefficients:

$$\text{for } \Phi = 90^\circ \quad h = 0.5354842 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ F \quad (33a)$$

$$\text{for } \Phi = 90^\circ \quad h = 3.0415502 \text{ W/m}^2 \cdot K \quad (33b)$$

for surfaces heated from below:

$$\text{for } \Phi = 0^\circ \quad h = 0.7051849 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ F \quad (34a)$$

$$\text{for } \Phi = 0^\circ \quad h = 4.0054502 \text{ W/m}^2 \cdot K \quad (34b)$$

$$\text{for } \Phi = 45^\circ \quad h = 0.6752377 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ F \quad (35a)$$

$$\text{for } \Phi = 45^\circ \quad h = 3.8353502 \text{ W/m}^2 \cdot K \quad (35b)$$

for surfaces heated from above:

$$\text{for } \Phi = 0^\circ \quad h = 0.1661355 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ F \quad (36a)$$

$$\text{for } \Phi = 0^\circ \quad h = 0.9436496 \text{ W/m}^2 \cdot K \quad (36b)$$

$$\text{for } \Phi = 45^\circ \quad h = 0.3957306 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ F \quad (37a)$$

$$\text{for } \Phi = 45^\circ \quad h = 2.2477496 \text{ W/m}^2 \cdot K \quad (37b)$$

These values can be normalized at 90° with Equations 31 and 32, and values for angles other than those listed determined through linear interpolation.

Exterior Surface Film Coefficient

Forced convection (wind) dominates the exterior film coefficient, h_{out} , given in units of $Btu/h \cdot ft^2 \cdot ^\circ F$ (Equations a) or $W/m^2 \cdot K$ (Equations b). This film coefficient is a function of wind speed (v in m/s) and the direction of the wind with respect to the window's azimuth (Φ). Tilt does not significantly affect this film coefficient (Shakerin 1987). For many years the following correlation of Ito and Kimura (1972), based on measurements three stories above ground level, has been used for building heat-transfer studies. On the windward side of the building ($|\Phi| < \pi/1$),

$$\text{for } v > 4.5 \text{ mph} \quad h_{out} = 0.87 v^{0.605} \quad (38a)$$

TABLE 2a
Gas Thermophysical Properties

Gas	M lb/mole	ρ lb/ft ³	$\frac{d\rho}{dT}$ 10 ⁻⁴ lb/ft ³ · °R	λ 10 ⁻² Btu/h · ft · °R	$\frac{d\lambda}{dT}$ 10 ⁻⁵ Btu/h · ft · °R ²	μ 10 ⁻⁷ lb · s/ft ²	$\frac{d\mu}{dT}$ 10 ⁻⁹ lb · s/ft ² · °R	Pr	$\frac{dPr}{dT}$ 10 ⁻³ 1/°R
Air	0.064	0.081	-1.4	1.39	2.4	6.57	1.2	0.72	1.0
Argon	0.088	0.106	-2.1	0.94	1.6	4.43	0.7	0.68	0.3
Krypton	0.184	0.234	-4.9	0.50	0.9	4.79	1.0	0.66	0.0
CO ₂	0.167	0.124	-0.0	0.84	2.4	2.92	0.6	0.79	0.7
SF ₆	0.322	0.419	-8.4	0.75	2.3	3.04	0.5	0.69	1.5

TABLE 2b
Gas Thermophysical Properties

Gas	M g/mole	ρ kg/m ³	$\frac{d\rho}{dT}$ 10 ⁻² kg/m ³ · K	λ 10 ⁻² W/m · K	$\frac{d\lambda}{dT}$ 10 ⁻⁵ W/m · K ²	μ 10 ⁻⁵ kg/m · s	$\frac{d\mu}{dT}$ 10 ⁻⁷ kg/m · s · K	Pr	$\frac{dPr}{dT}$ 10 ⁻³ 1/K
Air	28.96	1.29	-0.4	2.41	7.6	1.73	1.0	0.72	1.8
Argon	39.95	1.70	-0.6	1.62	5.0	2.11	0.6	0.68	0.6
Krypton	83.70	3.74	-1.4	0.86	2.8	2.28	0.8	0.66	0.0
CO ₂	76.01	1.98	-0.0	1.46	7.4	1.39	0.5	0.79	1.3
SF ₆	146.05	6.70	-2.4	1.38	7.8	1.45	0.4	0.69	2.7

$$\text{for } v < 4.5 \text{ mph } h_{out} = 2.16 \quad (39a)$$

$$\text{for } v > 2 \text{ m/s } h_{out} = 8.07 v^{0.605} \quad (38b)$$

$$\text{for } v < 2 \text{ m/s } h_{out} = 12.27 \quad (39b)$$

while on the leeward side (abs value of $\Phi > \pi/2$),

$$h_{out} = 3.28 (0.3 + 0.02v)^{0.605} \quad (40a)$$

$$h_{out} = 18.64 (0.3 + 0.05v)^{0.605} \quad (40b)$$

Note that at a wind speed of 0, the exterior film coefficient defined by this correlation is much higher than the interior film coefficient. This raises doubts about this correlation's accuracy at low wind speeds. Current research is aimed at developing a new correlation for the exterior film coefficient as a function of wind speed that is accurate at both high and low wind speeds and for ground-level building surfaces.

Natural Convection/Conduction in Gaps

A conductance, h , defines the net heat flux due to conduction and convection between layers n and $n+1$ as:

$$Q = h(\theta_n - \theta_{n+1}) \quad (41)$$

The heat flux is somewhat artificially divided into two parts to parallel the convention used for infrared fluxes.

$$Q_{2n}^c = h \theta_n \quad (42)$$

$$Q_{2n+1}^c = h \theta_{n+1} \quad (43)$$

The conductance is given by

$$h = \frac{(\lambda)}{w} Nu \quad (44)$$

where λ is the thermal conductivity of the gas in the gap, w is the width of the gas gap, and Nu is the gap Nusselt number. Elsherbiny et al. (1982) presents an experimentally determined correlation for the Nusselt number for vertical windows and specific conditions:

$$Nu = [1 + (0.0303Ra^{0.402})^{11}]^{0.091} \text{ for } Ra < 2 \times 10^5 \quad (45)$$

where Ra , the Rayleigh number, is the product of the Grashoff (Gr) and Prandtl (Pr) numbers. The Grashoff number is defined by

$$Gr = \frac{g \beta \rho^2 w^3 \Delta\theta}{\mu^2} \quad (46)$$

where g is the gravitational acceleration (32.2 ft/s² or 9.8 m/s²), β is the coefficient of thermal expansion and is approximated by the inverse of the absolute temperature, ρ is the density, μ is the viscosity, and $\Delta\theta$ is the temperature difference across the gap. Prandtl numbers for various gases are defined in the literature.

The above correlation is given for an aspect ratio (height/width) of 40, typical for common windows. These expressions do not change significantly for larger, or slightly smaller, aspect ratios. Others in the past have given similar expressions for the Nusselt number, also based on experimental studies. Most of these are not considered appropriate for analyzing window heat transfer since they are based on aspect ratios between 1 and 15.

Correlations for Nu for tilted windows heated from below are taken from Hollands et al. (1976) for windows inclined between 0° and 60°, and from Elsherbiny et al. (1982) for windows inclined between 60° and 90°. Hollands et al. (1976) found that for Ra greater than 10⁵ and high aspect ratios, Nu could be approximated by

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra \cos \Phi} \right] \cdot \left(\times - \frac{(\sin 1.8 \Phi)^{1.6} 1708}{Ra \cos \Phi} \right) + \left[\left(\frac{Ra \cos \Phi}{5830} \right)^{1/3} - 1 \right] \cdot \quad (47)$$

$$[X] = (|X| + X)/2 \quad (48)$$

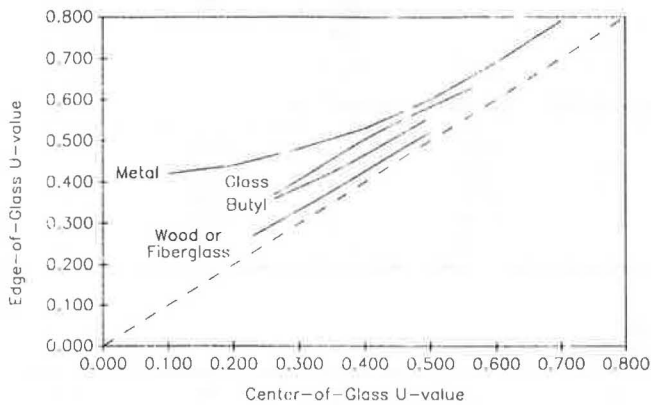


Figure 4 Edge-of-glass U -values (U_{eog}) as a function of center-of-glass U -values (U_{cog}) for insulated glass units manufactured with different spacers.

and X signifies any quantity. This equation agrees with experimental data to within 5%. The experimental data are based on an aspect ratio of 48 and an 0.5 in (13 mm) air layer between two surfaces.

Elsherbiny et al. (1982) conducted their work for air layers inclined at 60° and at 90° over a wide range of aspect ratios. The horizontal aspect ratio, which is the width of the layer divided by the thickness, is kept large in order to eliminate any dependence Nu has on the horizontal aspect ratio. For a 60° inclination angle, Nu is given by

$$Nu_1 = [1 + 0.0936 Ra^{0.318}/(1 + G)^{1.7}]^{1.7} \quad (49)$$

$$G = 0.5/[1 + (Ra/3160)^{20.6}]^{0.1} \quad (50)$$

$$Nu_2 = (0.104 + 0.175/A)Ra^{0.283} \quad (51)$$

$$Nu_{60} = [Nu_1, Nu_2]_{max} \quad (52)$$

For layers tilted between 60° and 90° , linear interpolation is recommended between the relations for 60° and 90° (Elsherbiny et al. 1982). This method matches data to within 5%, although errors up to 7% occur.

The application of these equations is extended to all gas fills heated from below. For gas layers heated from above (i.e., a warmer exterior), the following relation is assumed (Ferguson and Wright 1984):

$$Nu = 1 + (Nu_{90} - 1)\sin \Phi \quad (53)$$

where Nu_{90} represents the Nusselt number for a vertical layer (Equation 45).

The gas thermophysical properties necessary for the above equations are listed in Tables 2a and 2b. These were compiled from a variety of sources (Chemical Rubber Co. 1985; Hirschfelder et al. 1969; Andrussow 1962; DiPippo and Kestin 1985; Liley 1985; Glaser 1977; Kestin and Imaishi 1985).

It is often desirable to mix gases in an insulated glass unit. Using a chemical kinetics program (Kee et al. 1983), the thermophysical properties (P) of Air/Ar/Kr and Ar/SF₆ mixtures were determined. These are expressed as:

$$P(X_{air}, X_{ar}, X_{kr}) = K(A_1 + A_2 X_{air} + A_3 X_{ar} + A_4 X_{kr} + A_5 X_{air}^2 + A_6 X_{ar}^2 + A_7 X_{kr}^2) \quad (54)$$

$$P(X_{ar}, X_{SF_6}) = K(B_1 + B_2 X_{ar} + B_3 X_{SF_6}^2) \quad (55)$$

where

X_{air} = volumetric fraction of air

X_{ar} = volumetric fraction of argon

X_{kr} = volumetric fraction of krypton

X_{SF_6} = volumetric fraction of sulfur hexafluoride.

and Tables 3, 4, and 5 list A, B, and K for Equations 54 and 55.

EDGE-OF-GLASS U-VALUES

The previous section presented a discussion of heat transfer through the center-of-glass area of a window system. Heat transfer at the edge of a system of parallel glazing layers can be significantly different than at the center. Often, the materials used as spacers at the edge of a system of glazing layers have thermal conductivities many orders of magnitude greater than the glazing's air space; as shown in Figure 1, this leads to large two-dimensional heat transfer effects.

Performing a two-dimensional heat-transfer analysis for every configuration would greatly complicate the task of evaluating a window's thermal performance, so simplified correlations have been developed. As shown in Figure 1b, two-dimensional heat transfer in the glazed area begins at the sightline and extends toward the center-of-glass area. The two-dimensional effects gradually diminish (the arrows in Figure 1b become horizontal) until heat transfer can be categorized as primarily one-dimensional. Petersen (1987) and ASHRAE (1989) have defined an edge-of-glass area that incorporates all glazed area, two-dimensional heat-transfer effects for all common window designs as that area within 2.5 in (63.5 mm) of an insulated glass unit's sightline. The heat transfer over this entire edge-of-glass area can be determined experimentally (Petersen 1987) or numerically using finite-element

TABLE 3
Coefficients for Use in Equation 54

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
ρ	0	1.29	0	1.70	0	3.74	0
$d\rho/dT$	0	-4.4e-3	0	-6.0e-3	0	-13.7e13	0
$\lambda \times 10^2$	0.857	1.438	0.957e-01	0.897	-0.790e-01	-0.272	0.282
$d\lambda/dT \times 10^5$	2.123	4.969	-0.235	2.723	0.406	0.813	-0.157
$\mu \times 10^5$	1.061	0.855	-0.166	0.958	0.131	1.580	-0.335
$d\mu/dT \times 10^7$	0.280	0.280	-0.066	0.341	0.046	0.676	-0.170
Pr	0	0.72	0	0.68	0	0.66	0
dPr/dT	0	1.8e-3	0	0.7e-3	0	0.1e-3	0

TABLE 4
Coefficients for Use in Equation 55

	B_1	B_2	B_3
ρ	6.000	4.900	0
dp/dT	-24.0	18.0	0
$\lambda \times 10_2$	1.303	0.153	0.212
$d\lambda/dT \times 10_5$	7.1	-2.1	0
$\mu \times 10_5$	1.417	0.197	0.520
$d\mu/dT \times 10_7$.4	.2	0
Pr	0.69	-0.01	0
dPr/dT	2.70	-2.03	0

TABLE 6
Frame U-Values

Frame Type	U-Value Btu/h · ft ² · °F (W/m ² · °C)
Thermally unbroken aluminum	1.90 (10.79)
Thermally broken aluminum	1.00 (5.68)
External Flush Glazed Aluminum	0.70 (3.97)
Wood with or without cladding	0.40 (2.27)
Vinyl	0.3 (1.7)

TABLE 5
K for Use in Equations 54 and 55

	M	ρ	$\frac{d\rho}{dT}$	λ	$\frac{d\lambda}{dT}$	μ	$\frac{d\mu}{dT}$	Pr	$\frac{dPr}{dT}$
K for I-P	0.0022	0.0625	0.035	0.59	0.33	0.021	0.012	1.0	0.56
K for SI	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

techniques (Arasteh and Wolfe 1988; Carpenter 1987), and expressed simply as a function of the area's boundary conditions (i.e., center-of-glass *U-value*, spacer material, and location).

Figure 4 presents several simple correlations for edge-of-glass *U-values* (U_{eog}) as a function of center-of-glass *U-values* and spacer type; more accurate correlations are the subject of current research (Arasteh and Wolfe 1988).

FRAME U-VALUES

Frames are the final window component for which heat transfer must be calculated. Conduction is the primary mode of heat transfer through all common frame types. Frame and edge heat transfer have been shown to be roughly independent of one another (Petersen 1987; Arasteh and Wolfe 1988; Carpenter 1987). Table 6 presents experimentally determined *U-values* for generic frame types found in the United States (Petersen 1987; ASHRAE 1989).

Frame design, type, size, and placement of thermal breaks, as well as frame material specifications, can alter these values. Generating an expanded table of frame *U-values* through computer analyses and testing is the subject of current research.

WINDOW INDICES

Given the final steady-state temperature distribution obtained by solving Equations 1 through 6, the resistance between each pair of adjacent nodes can be calculated:

$$R_{n,n+1} = \frac{1}{h_{n,n+1}} = \frac{\Delta T_{n,n+1}}{Q_{n,n+1}} \quad (56)$$

In a system of N glazing layers with the outside denoted as 0 and the inside as $N+1$, the total center of glass *U-value* (U_{cog}) is thus:

$$U_{cog} = \frac{1}{\sum_{k=0}^N R_{k,k+1}} \quad (57)$$

The total solar heat gain through a glazing system can be written as:

$$Q_{1,N}^s = (T_{1,N}^s + \sum_{i=1}^N N_i A_i) I \quad (58)$$

where I is the incident solar radiation and N_i is the inward-flowing fraction of absorbed solar energy for layer i :

$$N_i = \frac{\sum_{k=0}^{i-1} R_{k,k+1}}{\sum_{k=0}^N R_{k,k+1}} \quad (59)$$

The glazing system's shading coefficient (SC) is defined as:

$$SC = \frac{Q_{1,n}^s}{Q_{ds}^s} \quad (60)$$

where Q_{ds}^s refers to the solar heat gain through double-strength clear glass. In addition to the above defined *U-values*, other window system indices are used to describe the thermal performance of a window system. These are:

$$U_{ig} = \text{insulated glass unit } U\text{-value} \\ = \frac{(U_{cog} A_{cog} + U_{eog} A_{eog})}{(A_{cog} + A_{eog})} \quad (61)$$

$$U_w = \text{overall window } U\text{-value} \\ = \frac{(U_{cog} A_{cog} + U_{eog} A_{eog} + U_f A_f)}{(A_{cog} + A_{eog} + A_f)} \quad (61)$$

$$\text{Relative Heat Gain} = SC \cdot 200 \text{ Btu/h} \cdot \text{ft}^2 \\ + (14.9^\circ\text{F})(U_{ig}) \quad (63a)$$

$$\text{Relative Heat Gain} = SC \cdot 630 \text{ W/m}^2 \\ + (7.8^\circ\text{C})(U_{ig}) \quad (63b)$$

where A_{cog} , A_{eog} , and A_f are center-of-glass, edge-of-glass, and frame areas, respectively.

ASHRAE (1989) recommends two standard-sized windows, a residential window measuring 3.0 ft (0.91 m) by 4.0 ft (1.22 m) with a center divider and a commercial window measuring 4.0 ft (1.22 m) by 6.0 ft (1.83 m). Aluminum

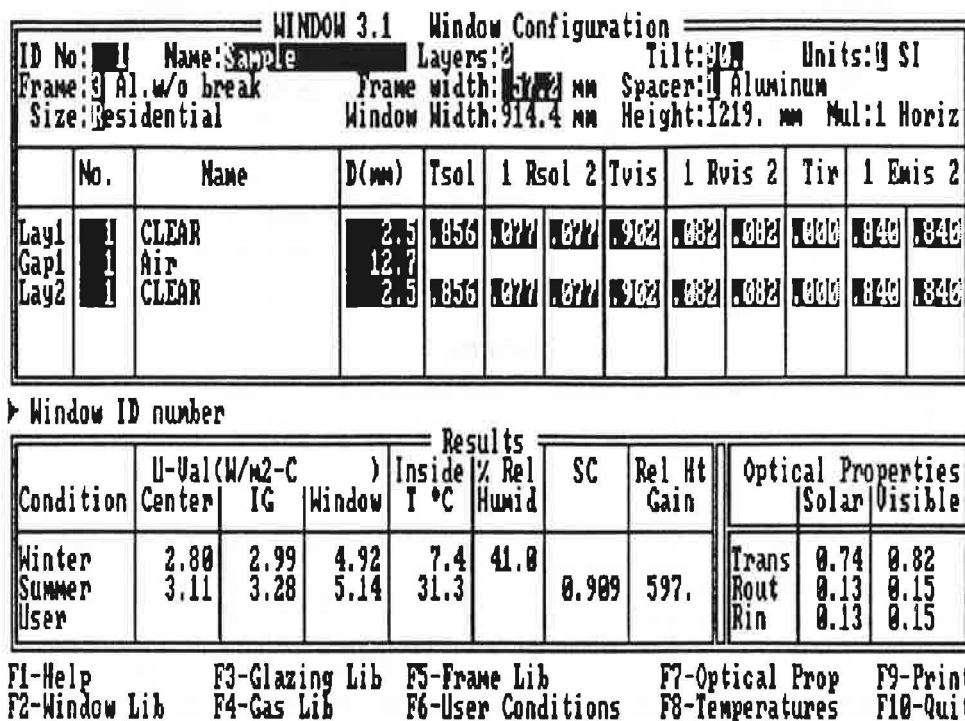


Figure 5 WINDOW 3.1 main screen from which a window can be described and results computed.

frames are assumed to have a thickness of 2.24 in (57 mm); wood and vinyl frames a thickness of 2.76 (70 mm). Given the temperature distribution across the window, the percent relative humidity at which condensation occurs on the interior glazing surface can also be calculated (ASHRAE 1989).

DISCUSSION AND CONCLUSIONS

The procedure presented here can be used to calculate the thermal performance indices of common residential and commercial window systems. Because of the component approach presented, new algorithms or correlations that more accurately predict heat transfer for existing or developing products can be easily incorporated into this procedure. This procedure allows the user to specify all design and environmental parameters that influence window heat transfer or default to standard material properties and conditions. Such a versatile approach is more accurate than procedures based on simple look-up tables, which approximate material temperatures and the influence of solar radiation (ISO 1988).

Center-of-glass heat transfer calculations have been shown to agree well with experimental measurements (Arasteh et al. 1987; Reilly et al. 1988). A comparison between complete window U-values calculated using this procedure for a limited number of window products indicates relatively good agreement (given measurement uncertainties) between this calculational procedure, field tests, and lab tests (Klems 1989). This same study also points out the need for an updated exterior-film coefficient and a more accurate analysis of frame and edge heat transfer. This work is under way at the time of this writing. Ultimately, the aim of this research is to show that this calculational procedure is consistent with standard lab and field tests. The procedure presented here has been

incorporated into a micro-computer program (Reilly and Arasteh 1988). This is an updated, more user-friendly version of the program (LBL 1985), which the insulated glass and window industries have used since 1986. A sample screen is shown in Figure 5. The program is used to design and develop new products, to assess and compare performance characteristics of all types of window products, to assist educators in teaching window heat transfer, and to help public officials in developing building codes.

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REFERENCES

- Andrussow, L. 1962. "Diffusion, viscosity, and conductivity of gases." In *Progress in international research on thermo-physical and Transport Properties*. New York: Academic Press.
- Arasteh, D., and Wolfe, J. 1988. "An analysis of frame and edge heat transfer in residential windows" (draft). Windows and Daylighting Group, Lawrence Berkeley Laboratory, Berkeley, CA, December.
- Arasteh, D.; Hartmann, J.; and Rubin, M. 1987. "Experimental verification of a model of heat transfer through windows." *ASHRAE Transactions*, Vol. 93, Part 1, pp. 1425-1431.
- ASHRAE. 1977. *ASHRAE handbook — 1977 fundamentals*. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- ASHRAE. 1985. *ASHRAE handbook — 1985 fundamentals*. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- ASHRAE. 1989. *ASHRAE handbook — 1989 fundamentals*. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

- Carpenter, S. 1987. "The effect of frame design on window heat loss, phase I." Waterloo, ON: Enermodal Engineering, September.
- Chemical Rubber Company. 1985. CRC handbook of chemistry and physics. CRC Press Inc.
- DiPippo, R., and Kestin, J. 1985. "The viscosity of seven gases up to 500C and its statistical interpretation." In Proceedings of the Fourth Symposium on Thermophysical Properties, April 1985. New York: The American Society of Mechanical Engineers.
- Edwards, D.K., and Balakrishnan, A. 1973. "Thermal radiation by combustion gases." *International Journal of Heat Transfer*, Vol. 16, pp. 25-40.
- Elsherbiny, S.M.; Raithby, G.D.; and Hollands, K.G.T. 1982. "Heat transfer by natural convection across vertical and inclined air layers." *ASME Transactions*, Vol. 104, February, p. 96.
- Ferguson, J.E., and Wright, J.L. 1984. "Vision: A computer program to evaluate the thermal performance of super windows." Solar Energy Program, National Research Council Canada, Report No. Passive-10, June.
- Glaser, H.J. 1977. "Verbesserung der wärmedämmung bei isoliergläsern." *Glastechnische Berichte*, Vol. 50, pp. 248-256.
- Hirschfelder, J.O.; Curtis, C.F.; and Bird, R.B. 1969. *Molecular theory of gases and liquids*. New York: John Wiley and Sons, Inc.
- Hollands, K.G.T.; Unny, T.E.; Raithby, G.D.; and Konicek, L. 1976. "Free convective heat transfer across air layers." *Journal of Heat Transfer*, May, p. 189.
- IES. 1972. *IES lighting handbook*. New York: Illuminating Engineering Society.
- ISO. 1988. "Thermal insulation—windows—calculation rules for determining the thermal transmittance." Fourth Draft, ISO/TC59-160-162/WG2, Thermal Transmission Properties of Windows, Doc. N67, February.
- Ito, N., and Kimura, K. 1972. "A field experiment study on the convective heat transfer coefficient on the exterior surface of a building." *ASHRAE Transactions*, Vol. 78, Part 2, p. 184.
- Kee, R.J.; Warnatz, J.; and Miller, J.A. 1983. "A fortran computer code package for the evaluation of gas-phase viscosities, conductivities, and diffusion coefficients." Sandia National Laboratories, SAND83-8209, March.
- Kestin, J., and Imaishi, N. 1985. "Thermal conductivity of sulfur hexafluoride." *International Journal of Thermophysics*, Vol. 6, No. 2, pp. 107-118.
- Klems, J.H. 1989. "U-values, solar heat gain, and thermal performance: recent studies using the MoWITT." *ASHRAE Transactions*, Vol. 95, Part 1.
- Konrad, A., and Larsen, B.T. 1978. "Encore—Canada: computer program for the study of energy consumption of residential buildings in Canada." Proceedings, 3rd International Symposium on Buildings, Banff, Canada.
- Krochman, J. 1978; *Lichttechnik* 1972, (I), 2 (1978) 66; *Lichttechnik* (II), 3 (1978) 104. Cited by E. Neeman in the Proceedings of the 1983 International Daylighting Conference, February 1983, p. 71.
- LBL. 1985. Window: a computer program for calculating U-values and shading coefficients of windows—user's guide. Berkeley, CA: Windows and Daylighting Group, Lawrence Berkeley Laboratory, University of California.
- LBL. 1988. Window 3.1: IBM-PC program for analyzing window thermal performance—user's guide. Berkeley CA: Windows and Daylighting Group, Lawrence Berkeley Laboratory, University of California.
- Lilley, P.E. 1985. "The thermal conductivity of 46 gases at atmospheric pressure." In Proceedings of the Fourth Symposium on Thermophysical Properties, April. New York: The American Society of Mechanical Engineers.
- Petersen, C.O. 1987. "How is low-E performance criteria determined?" *Glass Digest*, January 15, pp. 70-76.
- Reilly, S., and Arasteh, D. 1988. "Window 3.1: a computer tool for analyzing window thermal performance." Presented at the 13th National Passive Solar Conference, Cambridge, MA, June 18-24.
- Reilly, S.; Arasteh, D.; and Rubin, M. 1988a. "The effects of infrared absorbing gases on window heat transfer" (draft). Windows and Daylighting Group, Lawrence Berkeley Laboratory, Berkeley, CA, December.
- Reilly, S.; Rubin, M.; and Tien, C.L. 1988b. "Infrared thermal properties of sulfur hexafluoride," (draft). Berkeley, CA: Windows and Daylighting Group, Lawrence Berkeley Laboratory, December.
- Rubin, M. 1982. "Calculating heat transfer through windows." *Energy Research*, Vol. 6, pp. 341-349.
- Rubin, M.; Arasteh, D.; and Hartmann, J. 1987. "A correlation between normal and hemispherical emissivity of low-emissivity coatings on glass." *International Communications in Heat and Mass Transfer*, Vol. 14, No. 5, pp. 561-565.
- Shakerin, S. 1987. "Wind-related heat transfer coefficient for flat-plate solar collectors." *Journal of Solar Energy Engineering*, Vol. 109, May, pp. 108-110.
- Swinbank, W.C. 1963. "Long-wave radiation from clear skies." *Journal of the Royal Meteorological Society*, Vol. 89, pp. 349-348.
- Tien, C.L. 1973. "Band and total emissivity of ammonia." *International Journal of Heat Transfer*, Vol. 16, p. 856.
- Tien, C.L.; Modest, M.F.; and McCreight, C.R. 1972. "Infrared radiation properties of nitrous oxide." *Journal of Quantitative Spectroscopic Radiation Transfer*, Vol. 12, pp. 267-277.