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Summary The most basic calculation carried out by the building services engineer is probably that to determine the loss of heat from a room under steady-state conditions. The 1959 *IHVE Guide* made this a very simple process. The passage of time has seen the introduction of what are claimed to be more accurate methods. This improvement in accuracy complicated the calculations and now a new method has been proposed, which is claimed to be easy to use, accurate and give a good representation of the physical processes involved. One way to test different methods is to compare them on a number of example cases. This is done here, for the 1959, 1970, 1986 *CIBSE Guide* methods, the new method (Davies's binary star method) and a more complex method developed in the paper as a basis for comparison. It is demonstrated that great benefit may not accrue from the use of very complicated ways to calculate design heat loss.

Heat loss from rooms: Comparison of determination methods

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1 Introduction

Energy, comfort and reputation depend on the correct sizing and control of heating and cooling plant. This requires a certain level of competence and understanding by the services engineer of the behaviour of buildings, plant and control systems. Controls are often considered to be a specialist topic, and nowadays there is perhaps a tendency to assume that sophisticated methods can be used to overcome design deficiencies, in particular the oversizing of plant. This is probably not the case, and even if it were, such an approach can lead to systems with high first cost and high running costs. The objective of the design must be to ensure that the plant is suitable for the intended purpose. This means that it must be capable of providing specified comfort conditions over a specified range of operating conditions. There will be a failure rate because it is not considered practical to design for the complete spectrum of weather to which the building will be subjected. Approaches to defining an acceptable failure rate are given in Section A2 of the *CIBSE Guide*[1]. It is therefore accepted that while the plant should not fail to achieve its design objective it may on occasions not come up to the user's expectations. In such circumstances it is important that the failure was not due to the method used to size the plant but because of external conditions outside the design parameters. The design method must therefore be both theoretically sound and accepted as the 'right way' to do the job by the industry as recommended by the CIBSE.

Plant sizing usually comprises a number of tasks (excluding the selection of design conditions):

- Sizing the room heat emitters
- Corrections for heat gains
- Corrections for dynamic response
- Selection of the supply system
- Sizing the supply system.

The first of these relates most closely with the building user, and also directly affects the other stages. Heat gain effects are normally only considered relevant with air conditioning. This is because where there is only heating then, assuming intermittent plant operation, unless there is significant storage within the structure they will not be present during the pre-heat time. Storage of heat and intermittent operation lead naturally to a consideration of dynamic response.

An accurate assessment of the dynamic response of plant and buildings requires sophisticated numerical models. Such models are not the concern of this paper; however, it is most important to recognise the relationship between them and the calculation of steady-state heat loss (or gain). The selection of the best way to represent building dynamics[2] has been and will continue to be discussed at great length. Most discussion in the past has centred around numerical representations of unsteady heat transfer through walls. Such considerations tend to gloss over other equally important factors. In particular:

- Distribution of heat gains without the space
- Radiant heat transfer
- Convection coefficients.

Unless the relationship between these and the response of the building is understood, there is little point in arguing about the best approximation to the dynamic behaviour of walls. It might be said that prolonged discussions on that subject have resulted in a decrease in general appreciation of the heat transfer within rooms. The modes of heat transfer and heat distribution within the space link the dynamic and the steady-state models. It is irrelevant as to whether the heat flows directly through a wall or into storage: the internal analysis is the same. The only exception might be the admittance method. In this case standard values have been calculated for a particular heat transfer temperature, which depends upon assumptions concerning the distribution of radiant energy within the space[3]. The adoption of mean and alternating solar gain factors also presupposes a heat flow distribution[4]. It can be seen that whatever the object it is important to understand the calculation of steady-state heat loss from a room.

Time has seen a number of different 'methods' presented by CIBSE[5-7], and because each is based on different assumptions it is not unreasonable to expect each to give a different answer. More recently Davies[8] has questioned the validity of the concept of environmental temperature and presented a new approximation to heat transfers within a room. Should this be adopted? Even more important: If the older (and simpler) methods worked, why do we need to invent more complicated ways of doing the same thing? It is difficult to justify this. Some reasons might be:

— Originally plant was run continuously, so an error in sizing might not be apparent.

— When plant is operated intermittently it is essential to increase the capacity to allow for storage. Thus initial oversizing will be proportionately increased (increasing cost) and undersizing will result in client dissatisfaction (failure to achieve set point at time of occupation).

— In the past user expectations were lower.

— Current design conditions are much closer to comfort boundaries. A 10% error (reduction) in heat emitter size will result in a space temperature reduction of 0–10% (depending on airchange rate and heat gains). If the design temperature is 19°C a 5% error will usually mean discomfort (unless of course the heating medium temperature is increased).

— Concern with energy consumption.

The theoretical comparison of different methods to determine the heat loss from rooms is usually only possible through absurd limit cases. This paper does not attempt to do that but uses a number of realistic examples to indicate where there might be differences. Such an approach can only touch the surface of the problem, but hopefully it puts some perspective on this fundamental design calculation.

The methods discussed are:

(*a*) The current *CIBSE Guide* method[7]

(*b*) The environmental temperature method[6]

(*c*) The air temperature method[5]

(*d*) The binary star method[8].

These are compared with a reference method developed in this paper.

2 Theoretical background

The exact calculation of heat loss is not possible. It would require a complete numerical description of the building and its surroundings in total, down to mortar joints and ties. This is not practical, and so approximations are used. The problem with approximations is that something must be left out and what that something should be can cause argument. This is why a number of different 'methods' have been developed. Additionally machine aids have changed considerably over the last 25 years. This has made it possible to take more factors into consideration. The advent of the powerful personal computer should soon make many simple methods redundant. This is important, not because more detailed representation of the thermal processes necessarily results in a more accurate answer, but because the chance of error is reduced. There is also the advantage that unusual situations can be handled confidently.

This section describes each of the methods used for the comparisons, emphasising the approximations inherent in each method. To put some perspective on these approximations the 'reference' method is presented first.

2.1 Reference method

In this case reference means basis for comparison; it will be seen that it is still necessary to make many approximations. In addition the method could only be proposed now that cheap computing is available to all.

The loss of heat from a room is the sum of that transmitted through the walls (fabric loss) and that required to overcome losses due to any ventilation (air loss). The latter is often a significant unknown, but with increased use of mechanical ventilation and improved standards of construction it is becoming more easily quantified. The value of the air loss is usually approximated as:

$$Q_a = \dot{m}_a C_{pa} (t_{ai} - t_{ao}) \tag{1}$$

where Q_a is the air loss (W), $\dot{m}_a$ is the air mass flow rate (kg s^{-1}), C_{pa} is the specific heat capacity of air (J kg^{-1} K^{-1}), t_{ai} is the internal air temperature (°C), and t_{ao} is the external air temperature (°C).

A more exact representation would replace the specific heat and temperatures by enthalpy at the point of supply and extract. This will not be done, so the usual approach to air loss is adopted. (It is trivial to make some assumptions about the internal distribution of temperature and so correct t_{ai} for different air movement patterns).

2.2 Fabric loss

This can be written (for the steady state) as

$$Q_f = \oint \frac{t_s - t_o}{R} \, dS \tag{2}$$

That is the integral over the whole of the building surface of the difference between internal surface temperature t_s and external temperature t_o divided by a thermal resistance R. The first approximation is to rewrite equation 2 as:

$$Q_f = \sum_{n=1,N} A_n (t_{sn} - t_{on})/R_n \tag{3}$$

where Q_f is the fabric heat loss (W), A_n is the area of surface element n (m^2), R_n is the thermal resistance from internal surface to outside (m^{2}K W^{-1}), t_{sn} is the inner surface temperature of surface n (°C), t_{on} is the external heat transfer temperature appropriate to surface n, and N is the total number of surface elements.

The external heat transfer temperature for design purposes will usually be either the sol–air temperature (summertime temperature predictions, cooling) or the dry bulb (heating).

This fabric loss is equal to the sum of the heat entering the surface from within the room, that is:

$$Q_f = \sum_{n=1,N} A_n[(t_{an} - t_{sn})h_{an} + q_n] \tag{4}$$

where t_{an} is the air temperature near surface n (°C), h_{an} is the convection heat transfer coefficient (W m^{2}K^{-1}) and q_n is the radiant heat flow into surface n (W). (The convention is that here heat flow into surface is negative.)

Again some approximations are necessary; for example it is usual but not essential to assume that t_{an} is the mean room air temperature.

The radiant term represents the transmission of heat from other surfaces and any input due to radiant sources within the space. The reference method will be developed via an analysis of radiant heat flows within a room. The techniques used are not new, but are presented here so that those unfamiliar with them can follow the development of the method. It is also possible to develop a number of different equation sets from those presented; the method here is intended to be fairly easy to follow.

Assuming non-specular radiation, then the rate at which radiation leaves a surface can be expressed as:

$$\mathcal{J}_n = \rho_n G_n + \varepsilon_n E_{bn} \quad (5)$$

where $\mathcal{J}_n$ is the rate at which radiation leaves the surface (radiosity) (W m^{-2}), G_n is the incident radiation intensity (W m^{-2}), E_{bn} is the black-body emissive power (W m^{-2}), ρ_n is the reflectance and ε_n is the emissivity $(1 - \rho_n)$.

The heat flow entering the surface is therefore:

$$q_n = A_n(\mathcal{J}_n - G_n) \quad (6)$$

or

$$q_n = A_n \alpha_n (E_{bn} - \mathcal{J}_n) \quad (7)$$

where

$$\alpha_n = \frac{\varepsilon_n}{\rho_n} = \frac{\varepsilon_n}{1 - \varepsilon_n} \quad (8)$$

The incident radiation G_n has two components; that from other surface G_{sn} and any radiation from heat sources Q_n. The radiation from other surfaces is obtained from:

$$A_n G_{sn} = \sum_{j=1,N} \mathcal{J}_j A_j F_{jn} \quad (9)$$

where F_{jn} is the radiation shape factor from surface j to surface n, and because $A_n F_{nj} = A_j F_{jn}$:

$$A_n G_n = \sum_{j=1,N} A_n F_{nj} \mathcal{J}_j - Q_n \quad (10)$$

This can now be substituted into equation 6 and equated with equation 7 to give a set of simultaneous equations in $\mathcal{J}_n$. (The negative sign arises because heat flow into a surface is negative by convention.)

Once the $\mathcal{J}$'s have been obtained back substitution will give the heat flows. Unfortunately there are two complications:

— Black body emissive power is proportional to the absolute surface temperature raised to the power 4.

— Heat flows through surfaces are not contained explicitly in the equations.

To overcome these it is necessary to linearise the radiation and introduce a relationship between surface temperature and heat flow.

After substitution of equation 10 into equation 6 and equating this to equation 3 the equation set for the radiosity is:

$$\mathcal{J}_1(1 + \alpha_1 - F_{1,1}) - \mathcal{J}_2 F_{1,2} \ldots \mathcal{J}_n F_{1,N} = \alpha_1 E_{b1} - Q_1/A_1$$

or because $F_{n,n}$ is usually zero,

$$\mathcal{J}_n(1 + \alpha_n) - \sum_{j=1,N} F_{n,j} \mathcal{J}_j \ldots = \alpha_n E_{bn} - Q_n/A_n \quad (11)$$

A linear relationship between black-body emissive power and temperature can be used over the small temperature range encountered in buildings, so:

$$E_{bn} = a + b t_{sn} \quad (12)$$

The flow of heat through the building fabric can be written as:

$$-q_n + t_{an} h_{an} + K_n t_{on} = (h_{an} + K_n) t_{sn} \quad (13)$$

(where the negative sign is necessary because a positive q means heat flow from the surface).

K_n is the thermal conductance from the wall surface to the outside heat transfer temperature.

Substitution of equations 7, 12, 13 into equation 11 gives the following set of equations for surface and air temperatures:

$$\frac{t_{sn}}{\varepsilon_n}(h_{an} + K_n + b) - \sum_{m=1,N} \frac{t_{sm} F_{n,m}}{\varepsilon_m}[(h_{am} + K_m) \times (1 - \varepsilon_m) + \varepsilon_m b] - \frac{t_{an} h_{an}}{\varepsilon_m} + \sum_{m=1,N} t_{am} F_{n,m} h_{am} \frac{1 - \varepsilon_m}{\varepsilon_m} = t_{on} K_n - \sum_{m=1,N} K_m F_{n,m} \frac{1 - \varepsilon_m}{\varepsilon_m} t_{om} - Q_n/A_n \quad (14)$$

In deriving equations 14 it is assumed that no surface can 'see' itself, so that $F_{n,n}$ is zero, and that the shape forms a complete enclosure so that the sum of form factors for any single surface is unity.

A further equation is necessary to determine air temperatures. If all t_{an} are assumed to have the same value this is:

$$-\sum_{n=1,N} h_{an} t_{sn} A_n + t_a \left(\dot{m}_a C_{pa} + \sum_{n=1,N} A_n h_{an} \right) = Q_c + \dot{m}_a C_{pa} t_{ao} \quad (15)$$

where $\dot{m}_a$ is the mass flow rate of air into the space (kg s^{-1}), C_{pa} is the specific heat capacity of air (J kg^{-1} K^{-1}), t_{ao} is the temperature at which the air enters the space (°C), and Q_c is a convective heat gain (W). The use of a single air temperature will also simplify equations 14.

If temperature gradients are incorporated then it will be necessary to write down some rules linking each of the air temperatures to a reference value.

The solution of equations 14 and 15 looks formidable but is trivial on any computer. The only problem is to specify the correct form of equation 14 for the particular arrangement of heat inputs and the determination of the form factors. The former is mainly common sense, and standard values of the latter for most normal room shapes are available from the literature[9].

2.3 Heat loss models

The model developed in the previous section cannot be used directly to determine the heat loss from a room. It is necessary to rearrange equations 14 and 15, specify the control temperature, and solve for the appropriate heat input. Thus for a convective input the value of Q_c will be required from equation 15, and if totally radiant the appropriate surface radiant flows (Q_n in equation 14) need to be determined. Combinations of convective and radiant heating can be obtained by specifying the source and fraction of total input at each surface or the air point.

For example, to calculate the convective load Q_c required to heat a room to t_a with all external temperatures equal to t_o, equations 14 and 15 become:

$$\frac{t_{s1}}{\varepsilon_1}(h_{a1}+K_1+b)-t_{s2}F_{1,2}[(h_{a2}+K_2)(1-\varepsilon_2)+b\varepsilon_2]/\varepsilon_2$$
$$-t_{s3}F_{1,3}[(h_{a3}+K_3)(1-\varepsilon_3)+b\varepsilon_3]/\varepsilon_3-\ldots$$
$$=t_oK_1+\frac{t_a h_{a1}}{\varepsilon_1}-F_{1,2}\frac{1-\varepsilon_2}{\varepsilon_2}(K_2t_o+h_{a2}t_a)-F_{1,3}\ldots \tag{16}$$

and

$$t_{s1}h_{a1}A_1+t_{s2}h_{a2}A_2+t_{s3}h_{a3}A_3+\ldots$$
$$=m_aC_{pa}(t_a-t_{ao})+t_a(h_{a1}A_1+h_{a2}A_2+h_{a3}A_3\ldots) \tag{17}$$

2.4 *Heat transfer temperature model*

Most room surfaces have an emissivity of around 0.9 and unless great care is taken in cleaning, lower emissivities will tend towards this value—so it would not be unreasonable to set all emissivities ε_n to unity. In this case equation 16 becomes:

$$t_{s1}(h_a+k_1+b)-t_{s2}F_{12}b-t_{s3}b\ldots$$
$$=t_oK_1+t_ah_{a1} \tag{16a}$$

Further, if the room is assumed cubic then all form factors are equal to 0.2. So with a slight rearrangement:

$$(t_{s1}-t_o)K_1=h_{a1}(t_a-t_{s1})$$
$$+0.2(t_{s2}+t_{s3}+t_{s4}+t_{s5}+t_s)b-bt_{s1} \tag{16b}$$

The term $(t_{s1}-t_o)K_1$ is of course the heat lost through surface 1, and if the concept of mean radiant temperature t_R is introduced,

$$t_R=(t_{s1}+t_{s2}+t_{s3}+t_{s4}+t_{s5}+t_{s6})/6 \tag{18}$$

The expression for heat loss through the surface becomes:

$$\text{Heat loss}=h_1(t_a-t_{s1})+\tfrac{6}{5}(t_R-t_{s1})b \tag{19}$$

The term b is the rate of change of radiant heat flux with surface temperature. That is if Q_R is defined as a radiant heat flow then:

$$b=\frac{dQ_R}{dt_s}\quad \text{at } t_s \tag{20}$$

and because

$$Q_R=\sigma(t_s+273)^4 \tag{21}$$

if $t_s=20$, then

$$b=5.7\ \mathrm{W\,m^{-2}\,K^{-1}} \tag{22}$$

(σ is the Stephan–Boltzman constant = $5.678\times10^{-8}\ \mathrm{W\,m^{-2}\,K^{-1}}$.)

This may not be the best value of b for all calculations. It may be better to use a linear approximation covering a wider temperature range. A fairly good fit giving errors in radiant flow of around 5% over a temperature range from −10°C to +35°C is:

$$Q_R=5.26\,t_s+311.46 \tag{23}$$

or

$$b=5.26 \tag{24}$$

Whatever the numerical value of b, it can be called a radiant heat transfer coefficient h_R and if a heat transfer temperature t_{ei} is defined then equation 19 becomes

$$(h_1+\tfrac{6}{5}h_R)(t_{ei}-t_{s1})=h_1(t_a-t_{s1})+\tfrac{6}{5}h_R(t_R-t_{s1}) \tag{19a}$$

or more familiarly:

$$t_{ei}=\frac{h_1t_a}{h_1+\frac{6}{5}h_R}+\frac{6h_R/5}{h_1+\frac{6}{5}h_R} \tag{25}$$

the environmental temperature found in the *CIBSE Guide*. Thus the assumptions which appear to be implicit in the environmental temperature method are:

(*a*) All surfaces have an emissivity of unity.

(*b*) The surface is a complete wall of a cubic room.

(*c*) Radiant heat transfer can be represented by a simple heat transfer coefficient.

(*d*) Air temperature is constant over each room surface.

The assumption of unit emissivity appears to be necessary to derive equation 25 from equations 16. Examination of the *CIBSE Guide* suggests that the radiant coefficient h_R be replaced by Eh_R, where

$$\frac{1}{E}=\frac{1-\varepsilon_1}{\varepsilon_1}+\frac{1-\varepsilon_2}{5\varepsilon_2} \tag{25a}$$

Here ε_1 is the emissivity of the exposed and ε_2 that of the non-exposed surface.

In this case it seems that heat loss can only be through a single surface, while the previous working allows heat loss through any number of surfaces of the cube. The validity of equation 25a will not be discussed further here as it will be the subject of a future publication.

2.5 *Air temperature model*

The previous model made use of the assumption that the emissivity of the room surfaces was not very different from unity and that the room was a cube. Another, not unreasonable, assumption is that radiant heat loss is to a temperature not too different from the air temperature.

In this case the surface temperatures in equation 16 could be replaced by the air temperatures. Then, if all emissivities are equal and, remembering that the sum of all form factors in an enclosure is unity, equation 16 becomes:

$$t_{s1}(K_1+h_1+\varepsilon b)-\varepsilon bt_a-t_ah_1=t_oK_1 \tag{16c}$$

or the surface heat loss is:

$$K_1(t_{s1}-t_o)=(h_1+\varepsilon b)(t_a-t_{s1}) \tag{26}$$

This means that if the heat transfer temperature is the air temperature then the surface coefficient must be defined as:

$$h=h_1+\varepsilon b \tag{27}$$

where h_1 is the convective heat transfer coefficient, ε is the emissivity and b is as previously defined.

2.6 *Other approximations*

The environmental temperature and air temperature models are approximations to avoid the necessity of calculating the form factors for radiant heat transfer. Another method might be to find a better approximation to the form factors. One way of doing this is proposed by Davies in what he calls the *binary star method*[8]. Essentially air and radiant nodes are separated, and by an approximation to the true radiant transfers a system of equations is derived which are claimed to represent more satisfactorily the heat exchanges within a space than either the environmental temperature or air temperature models.

3 Comparisons

The previous section has presented the background to a number of different ways of calculating the heat loss from rooms. This section presents an outline of the equations necessary to use them (without detailed explanations since that can be found in the references) and a comparison of predictions on four room types, representing a large factory, the ideal cubic building, a ground floor corner office and an atrium style building. It must again be stressed that none of the methods can be considered to be accurate because of the number of assumptions which have to be made, in particular:

— a uniform air temperature throughout the space

— standard heat transfer coefficients

— one-dimensional heat transfer

— perfect and uniform construction.

These are the standard assumptions made when sizing heating systems. In addition it is usual to make a correction to allow for temperature gradients. Such corrections are fairly arbitrary and because they will not affect the comparisons they have been omitted. Full details of standard values and the room constructions are given in Section 3.6.

3.1 Selection of methods

Past *CIBSE/IHVE Guides* have contained what appear to be three different methods for calculating heat loss:

— 1959: an air temperature based method

— 1970: the environmental temperature method

— 1986: the dry resultant temperature method.

For the purpose of this paper the 1970 and 1986 methods can be considered identical as they use the same fundamental theory (and consequently approximations). The only difference is in the manipulation of equations to make use of different index temperatures. A new method has now been proposed for the next 'Guide method': the 'binary star method'. The 1959, 1970 and binary star methods are therefore used in these comparisons.

3.2 1959 IHVE Guide

In this case no distinction was made between air and radiant temperatures (1.5) so the steady state heat loss is:

$$Q = \sum_{n=1,N} UA_n(t_i - t_{on}) + \dot{m}_a C_{pa}(t_i - t_o) \tag{28}$$

where Q is the steady state heat loss (W), UA_n is the product of thermal transmittance and area for surface n (W K^{-1}), t_i is the internal design temperature (°C), t_{on} is the temperature on the other side of wall n (°C), N is the number of surfaces in the room, $\dot{m}_a C_{pa}$ is the product of the ventilation (and infiltration) mass flow rate and the specific heat capacity of air (W K^{-1}), and t_o is the outside air temperature (°C).

3.3 1970 IHVE Guide

In this case radiant and air temperatures are recognised as different and combined to form a heat transfer temperature (environmental temperature). The assumptions involved have already been discussed (1.4). The mechanics of the heat loss calulation are:

Fabric heat loss:

$$Q_f = \sum_{n=1,N} AU(t_{ei} - t_{on}) \text{ (W)} \tag{29}$$

Air heat loss:

$$Q_a = \dot{m}_a C_{pa}(t_{ai} - t_o) \tag{30}$$

where t_{ei} is the internal environmental temperature (°C), and t_{ai} is the internal air temperature (°C).

The relationship between environmental and air temperature is said to depend upon the type of heating system, and while there are some conceptual difficulties applying the method (which will not be discussed here as Davies[8] has been quite eloquent on this subject!), the relevant relationships for convective and radiant heating systems are:

Convective heating:

$$t_{ei} = t_{ai} - \frac{Q_f}{4.8\Sigma A} \tag{31}$$

where A is the room surface area (m^2), 4.8 is the thermal conductance from environmental to air temperature (W m^{-2} K^{-1}).

The fabric heat loss in equation 31 can be removed by substitution of equation 29 resulting in the expression:

$$t_{ei} = (t_{ai} + \Sigma(AUt_o)_n/4.8\,A)/(1 + \Sigma(AU)_n/4.8\Sigma A) \tag{31a}$$

It is important that the products $(AUt_o)_n$ and $(AU)_n$ include all surfaces, with t_o the appropriate temperature to which heat is lost (even if it is the same environmental temperature as that in the room under consideration).

Radiant heating:

$$t_{ei} = t_{ai} + \frac{Q_a}{4.8\Sigma A} \tag{32}$$

3.4 Binary star method

This method separates the radiant and convective transfers in a room and essentially follows the 'reference method' with approximation to the radiant heat transfers. It is necessary to solve the following set of equations:

$$\begin{pmatrix} (\Sigma A_n h_{cn} + m_a C_{pa}) + 0 - A_1 h_{c1} - A_2 h_{c2} & \ldots \\ 0 + \Sigma A_n E_n^\star h_r - A_1 E_1^\star h_r - A_2 E_2^\star h_r & \ldots \\ -A_1 h_{c1} - A_1 E_1^\star h_r + (A_1 h_{c1} + A_1 E_1^\star + A_1 U_1^\star) - 0 & \ldots \\ -A_2 h_{c2} - A_2 E_2^\star h_r + 0 + (+A_2 h_{c2} + A_2 E_2^\star + A_2 U_2^\star) & \ldots \\ & \end{pmatrix} \begin{pmatrix} t_{ai} & = \\ t_r & = \\ t_{s1} & = \\ t_{s2} & = \\ \text{etc.} & \end{pmatrix} \begin{pmatrix} Q_a + t_o \dot{m}_a C_{pa} \\ Q_r \\ Q_1 + t_o A_1 U_1^\star \\ Q_1 + t_o A_2 U_2^\star \\ \text{etc.} \end{pmatrix} \tag{33}$$

Table 1

Method	Hardware	Specification
1959 *Guide*	Four-function calculator	With memory
1970 *Guide*	Four-function calculator	With memory
Davies	ATARI 800XL personal computer	8-bit word, 64 kbyte memory, BASIC
Reference	Apricot Xi	16-bit word, 1 Mbyte core, FORTRAN 77

where h_{cn} is the convective heat transfer coefficient for surface n (W m^{-2} K^{-1}), h_r is the radiant heat transfer coefficient (W m^{-2} K^{-1}), t_r is the mean radiant temperature (°C), t_s is a surface temperature (°C), $U_n^\star$ is the conductance from the wall surface to outside (W m^{-2} K^{-1}):

$$U_n^\star = \frac{U_n(E_n^\star h_r + h_{cn})}{E_n^\star h_r + h_{cn} - U_n}$$

U_n is the thermal transmittance of surface n (W m^{-2} K^{-1}), $E_n^\star$ is a corrected emissivity which includes a radiation form factor:

$$E_n^\star = \varepsilon_n/(1 - \varepsilon_n + \beta_n\varepsilon_n)$$

β_n for a six-sided enclosure is given by Davies[8] as:

$$\beta_n = 1 - f_n[1 + 3.53(f_n - 0.5) - 5.04(f_n^2 - 0.25)]$$

$$f_n = A_n/\Sigma A$$

These equations are assumed here to hold true for all room shapes so that a simple method can be derived. Some rearrangement of equations is necessary if a design method is to be derived. For example, to obtain the convective heat input required to hold a constant air temperature the air temperature terms need to be moved to the right-hand side and replaced by the convective heat terms (−1 in the first equation, zero in all others). In addition because the surface conductance $U^\star$ is calculated from standard U-values using separate radiant and convective heat transfer coefficient, the standard 'Guide' radiant coefficient of 5.7 W m^{-2} K^{-1}) must be used instead of the wider range value 5.26 W m^{-2} K^{-1} suggested in this paper. In practice differences are small.

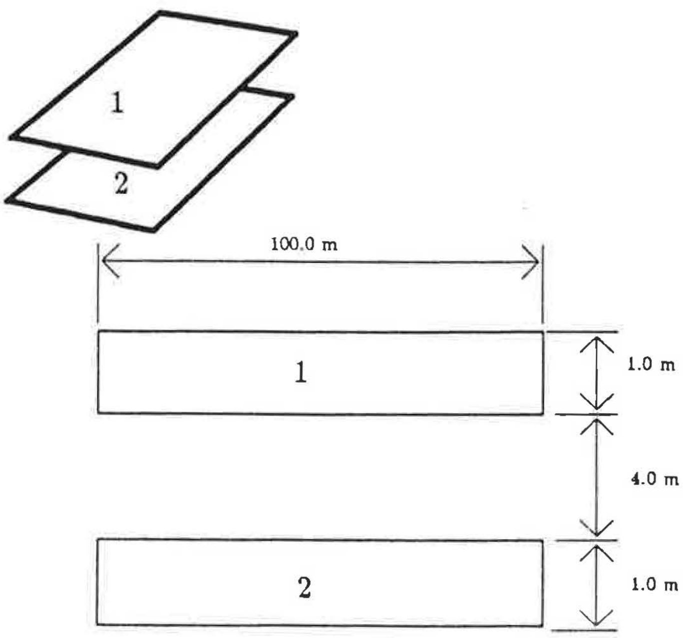

Figure 1 2D building

3.5 Calculation aids

Before going into detail it is worth recording the calculation aids used in each of the methods (Table 1).

Calculation aids were chosen for convenience, but in hindsight they probably represent the minimum necessary to carry out each method with any reasonable degree of efficiency. In particular a machine to the Apricot Xi standard is probably essential for the calculation of form factors for complex room shapes (not those considered here).

3.6 Examples for comparison

Dimensions and U values are given in Figures 1–4 and Tables 2 and 3.

The following convection coefficients were used as appropriate (Table 4):

A radiant coefficient of 5.7 W m^{-2} K^{-1} was used in the Davies and 5.26 W m^{-2} K^{-1} in the Reference method (which is a wide average to cover the range of temperature that might be found in buildings). The effect of changes in the convection coefficients is not relevant here because standard values are built into the U-values together with a standard radiant coefficient. It is a simple matter to vary the coefficients in the reference method. This has not been done

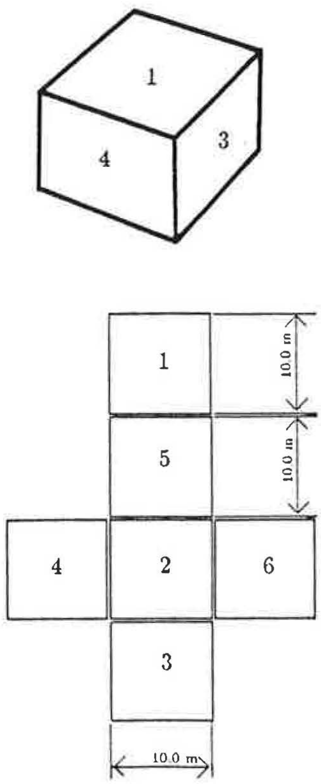

Figure 2 Cubic building

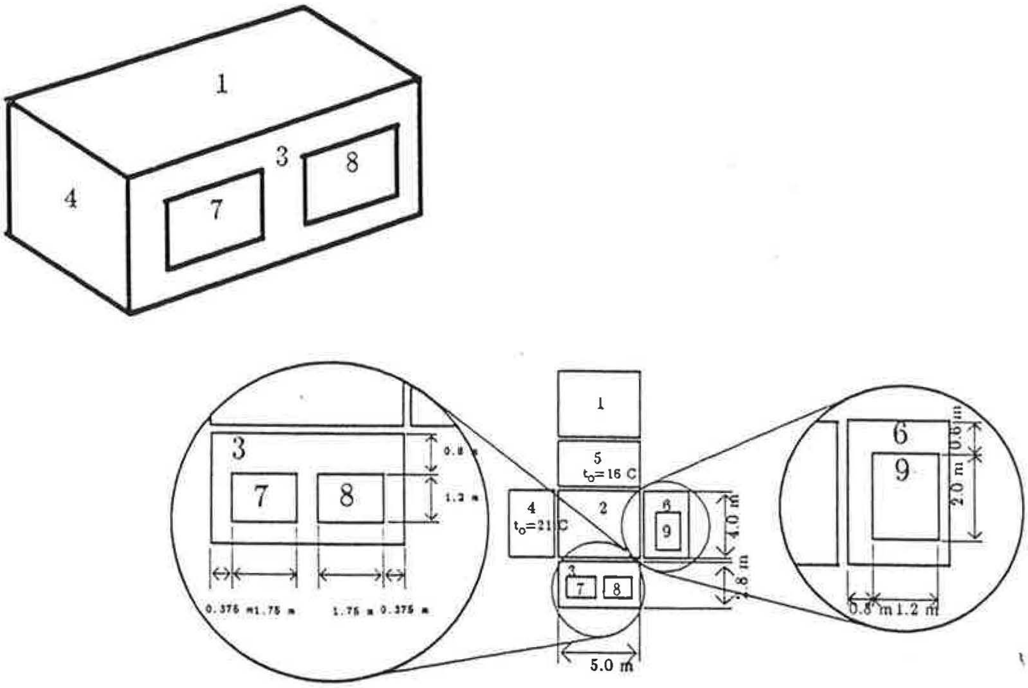

Figure 3 Office building

here because the sole objective is to compare different ways of calculating a standard design heat loss.

3.7 Test conditions for comparison

Two types of heating system are used: 100% convective, and a heated floor (50% radiant heating according to the 1986 *CIBSE Guide* Section A9). In both cases the design temperature is the air temperature and all heat losses are calculated for an air temperature of 21°C. The external air temperature is −1°C with any other temperatures as shown in Figures 1–4.

To bring some reality into the comparisons it is assumed that a very well insulated floor would be used with the radiant heating system so the floor U-values are reduced to 0.01 W m^{-2} K^{-1} for that case.

Table 2 U-values for 2D building of Figure 1

Surface no.	U-value (W m^{-2} K^{-1})
1	1.2
2	0.1

Table 3 U-values for office building† of Figure 3

Surface no.	U-value (W m^{-2} K^{-1})
1	0.4
2‡	0.7
3	0.45
4	2.0
5	2.0
6	0.45
7	5.6
8	5.6
9	5.6

† 1.17 air changes per hour
‡ U = 0.01 Wm^{-2}K^{-1} for radiant heating

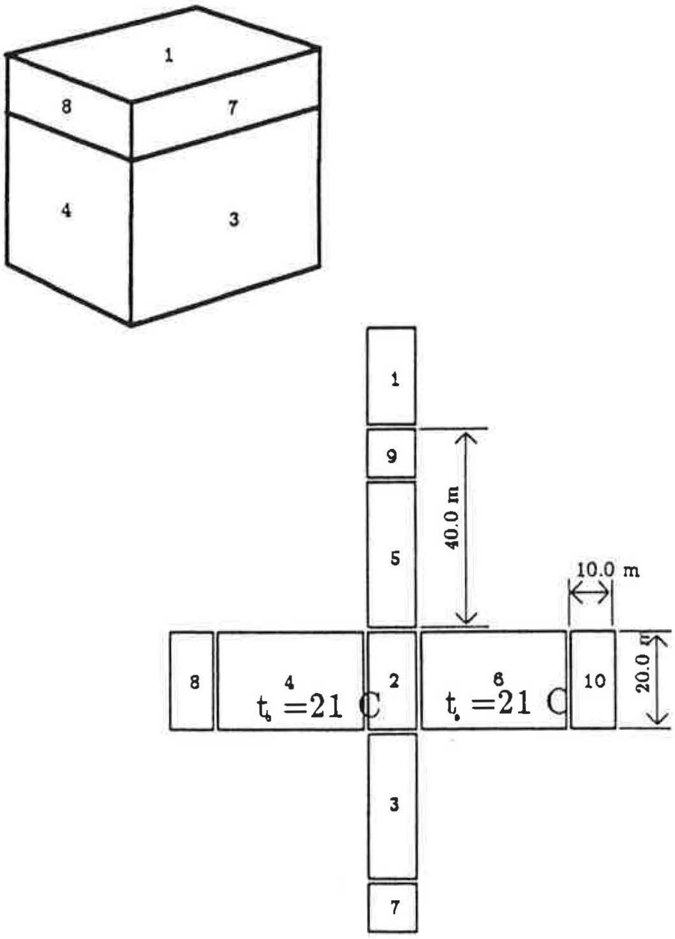

Figure 4 Atrium building

Table 4

Heat flow direction	Convection coefficient ($W\ m^{-2}\ K^{-1}$)
Horizontal	3.0
Vertical upwards	4.3
Vertical downwards	1.5

Table 6 Heat loss predicted by reference method

Building	Fabric loss (W)		Air loss (W)
	Convective heating	Radiant heating	
2D	3305	3593	752
Cube	9712	10269	3758
Office	1682	1834	452
Atrium	39718	47468	12019

3.8 Results and discussion of comparison

Table 5 gives the percentage difference between each of the methods and the reference method for convective and radiant heating, with and without the air change heat loss. Table 6 gives heat loss as determined by the reference method, and Tables 7 and 8 compare surface temperature predicted by each of the methods.

The first conclusion from Table 5 is that the air temperature method of 1959 gives significantly different results from the 1970 environmental temperature method and the binary star method, and that there is little difference between the latter two. The smallest differences for these are, as expected, to be found with the cubic building—the only shape for which their theories are true. The heat losses given in Table 6 are only included for interest; the surface temperatures listed in Tables 7 and 8 are more significant. Again with the exception of the air temperature method (1959) all predictions are similar, with one significant difference: floor temperatures with radiant heating. It may be possible to adapt the 1970 guide method to provide a more realistic floor temperature, but this has not been done because the objective has been to apply the standard methods as presented, not to interpret how they might be used. Both the reference and binary star methods have heat source modelling as an inherent feature, so are more capable of modelling reality. It must also be said that the reference method is most likely to yield realistic surface temperatures. The objective of this study is, however, not to find an accurate method to calculate room surface temperatures but to compare ways of calculating heat loss. It would appear that the 1970 *IHVE Guide* method is the best of the simple methods and gives as good results as the more complex binary star method. The difference between both of these and the reference method is exaggerated by building form; that is, the further the form departs from a uniform cube the greater the probable error.

4 Design for comfort

The comparisons presented in the previous sections were for a constant air temperature of 21°C. It has been established that a reasonable measure of comfort at low air speeds is the dry resultant temperature, equal to the average of air and mean radiant temperatures. To take this into consideration a modified version of the 1970 *IHVE Guide* method was published in 1979. The difference between this and the previous method is the ability to use dry resultant temperature directly in the heat loss calculation. This was done by introducing correction factors for the differences in radiant and convective heat transfers as between different system types. The binary star and reference methods are easily adapted to use dry resultant temperature. In the binary star method, air temperature is replaced by $(2t_{dry} - t_R)$. In the reference method, air temperature is replaced by

$$2t_{dry} - \frac{A_1 t_1}{\Sigma A} - \frac{A_2 t_2}{\Sigma A} \cdots$$

where t_{dry} is the dry resultant temperature, t_R is the mean radiant temperature, and t_1 etc. are room surface temperatures.

The air temperature method does not involve the use of surface temperatures, but because it is necessary to assume that air and surface temperatures are the same in the derivation of the heat loss equation (see Section 2.5) it is possible that predicted heat losses are near to those appro-

Table 5 Differences between methods

Method	% Difference from reference method†							
	2D		Cube		Office		Atrium	
	Con‡	Rad§	Con‡	Rad§	Con‡	Rad§	Con‡	Rad§
1959 *Guide*	+9.3	−5.0	+6.7	−3.2	+23.0	−3.5	+12.8	−7.8
	+12.0	−6.3	+10.8	−5.1	+32.0	−4.6	+18.3	−10.5
1970 *Guide*	−1.1	−2.3	+0.2	+0.4	+2.6	+2.3	−5.7	−0.6
	−1.3	−3.0	+0.4	+0.6	+3.7	+3.2	−8.2	−0.8
Binary star	−1.7	−3.0	−0.3	−0.7	+1.0	+0.5	−6.2	−4.1
	−2.2	−3.8	−0.4	−1.2	+1.4	+0.7	−8.9	−5.5

† % Difference = (Method—Reference) × 100/Reference
‡ Con = Convective heating
§ Rad = Radiant (floor heating)
The first row of figures for each method includes the ventilation loss, the second is for fabric loss alone.

Table 7 Surface temperature prediction with convective heating

Building	Method	Surface temperature (°C) (Figures 1–4)									
		1	2	3	4	5	6	7	8	9	10
2D	Reference	16.4	17.3								
	Binary star	16.5	17.3								
	1970	16.1	18.1								
	1959	18.4	20.7								
Cube	Reference	18.3	17.8	17.4	17.4	17.4	17.4				
	Binary star	18.3	18.3	17.4	17.4	17.4	17.4				
	1970	18.1	18.4	17.5	17.5	17.5	17.5				
	1959	20.1	20.4	19.4	19.4	19.4	19.4				
Office	Reference	17.5	15.5	17.3	18.7	17.4	17.0	5.1	5.0	5.0	
	Binary star	17.4	15.5	16.9	18.6	17.4	16.9	5.0	5.0	5.0	
	1970	17.3	16.2	17.0	18.7	17.6	17.0	5.3	5.3	5.3	
	1959	20.1	18.8	19.8	21.0	19.8	19.8	6.2	6.2	6.2	
Atrium	Reference	17.4	15.7	14.8	19.5	17.6	19.5	14.2	16.9	14.2	16.9
	Binary star	18.1	18.0	14.9	19.4	17.7	19.4	14.7	17.7	14.7	17.7
	1970	17.9	18.4	14.8	19.6	17.7	19.6	14.8	17.7	14.8	17.7
	1959	19.7	20.2	16.2	21.0	19.4	21.0	16.2	19.4	16.2	19.4

priate for a design based on dry resultant temperature. This hypothesis is tested in Table 9 for convective heating.

The results shown in Table 9 were obtained by the reference method using a convective heat input equal to that predicted by the 1959 method.

The heat load required to obtain a specified dry resultant temperature is probably more significant. Predictions for each method are given in Table 10, with the 1979 method replacing the 1970 method.

Table 10 would tend to confirm the assumption that the air temperature model gives heat loss predictions close to those required to hold a constant dry resultant temperature. It is obvious that there must be limitations to this statement, and that any such restriction is likely to depend upon the ratio of heat flow through surface to heat flow into the surface based on convection alone. That is the ratio $\Sigma AU/\Sigma A$.

The final comparison is made for the most extreme case which might be proposed, a cube with all surfaces at the same temperature. This means that there is no radiant heat exchange within the enclosure. The various predictions are given in Table 11, for convective heating to an air temperature of 21°C. In addition all surfaces have the same U

Table 9 Dry resultant temperature predicted by the air temperature method

Building	t_{dry} (°C)
2D	20.75
Cube	20.66
Office	21.22
Atrium	21.16

Table 8 Surface temperature prediction with radiant heating

Building	Method	Surface temperature (Figures 1–4)									
		1	2	3	4	5	6	7	8	9	10
2D	Reference	19.7	24.1								
	Binary star	19.6	24.2								
	1970	19.0	21.8								
	1959	18.4	21.0								
Cube	Reference	21.0	31.1	20.5	20.5	20.5	20.5				
	Binary star	20.9	31.7	20.4	20.4	20.4	20.4				
	1970	21.4	22.3	20.6	20.6	20.6	20.6				
	1959	20.1	21.0	19.4	19.4	19.4	19.4				
Office	Reference	21.1	29.6	21.1	21.7	20.2	20.8	6.3	6.2	6.2	
	Binary star	20.8	30.2	20.5	21.6	20.4	20.5	6.2	6.2	6.2	
	1970	21.1	22.0	20.8	21.8	20.6	20.8	6.6	6.6	6.6	
	1959	20.1	21.0	19.8	21.0	19.8	19.8	6.2	6.2	6.2	
Atrium	Reference	18.7	44.7	17.5	22.1	20.9	22.1	15.1	17.9	15.0	17.9
	Binary star	20.0	44.9	16.7	21.4	19.8	21.4	16.5	19.8	16.5	19.8
	1970	20.5	21.9	16.9	21.7	20.2	21.7	16.9	20.2	16.9	20.2
	1959	19.7	21.0	16.2	21.0	19.4	21.0	16.2	19.4	16.2	19.4

Table 10 Design heat loss for a dry resultant temperature of 21°C with the outside air at −1°C

Method	2D		Cube		Office		Atrium	
	Con	Rad	Con	Rad	Con	Rad	Con	Rad
1959	3611	3414	10358	9940	2072	1769	44807	43450
1979	3600	3419	10517	9934	2028	1805	46142	43721
Binary star	3771	3420	10490	9895	2020	1775	43478	44035
Reference	3653	—	10520	—	2003	—	44112	—
Air loss	833		4071		510		13030	

Radiant heating implies a well insulated floor and the 50% convective/radiant split suggested as appropriate for a floor heating system in the 1979 *CIBSE Guide*. The air loss is included in all figures and listed separately in the last row (calculated at the air temperature predicted by the reference method).

value and surface resistance (0.12 W m^{-2} K^{-1}, convective coefficient = 3 W m^{-2} K^{-1})

Only the air temperature method of 1959 differs significantly from the reference.

5 Conclusions

To draw specific conclusions from tests on a limited number examples would be foolish; in particular the influence of more representative convection coefficients has been ignored. The objective of this paper was to outline different approaches which might be used to calculate the design steady-state heat loss from a room. It is apparent that the method described in the 1959 *IHVE Guide*, which is based on air temperature, gives significantly higher losses than any of the current proposed methods. This meant that if air temperature was used as the design parameter then there would in many cases be a considerable factor of safety inherent in that method. This was removed in the 1970 method for heat losses calculated for a given air temperature. The introduction of dry resultant temperature in 1979 meant that systems would be of a similar size to those designed to meet a specified air temperature in 1959.

The use of more sophisticated calculation techniques has not been justified, but there is a hint that the 'halfway house' binary star method is not a significant improvement over the existing *Guide* methods for simple rooms and is inadequate for complex rooms. The advantage of the binary star method is that heat can be injected at air and surface nodes, as opposed to the convoluted approach implied in the current *CIBSE Guide*. It must be concluded here that on the basis of a limited number of comparisons the current *CIBSE Guide* method is adequate for the purpose of calculating steady-state heat loss.

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Table 11 Idealised cubic enclosure: Fabric heat loss and dry resultant temperatures

Surface U-value (W m^{-2} K^{-1})	$\frac{\Sigma AU}{A}$ (W/K)	1959 Loss (W)	1959 T_{dry} (°C)	1970 Loss (W)	1970 T_{dry} (°C)	Binary star Loss (W)	Binary star T_{dry} (°C)	Reference Loss (W)	Reference T_{dry} (°C)
5	5	660	14.4	323	12.2	313	12.1	319	12.1
4	4	558	15.7	288	13.1	280	13.0	284	13.1
3	3	396	17.0	244	14.3	238	14.2	242	14.3
2	2	264	18.4	186	15.9	183	15.8	185	15.9
1	1	132	19.7	109	18.0	108	17.9	109	18.0
0.5	0.5	66	20.3	60	19.4	59	19.2	60	19.4
0.25	0.25	33	20.7	31	20.0	31	20.0	31	20.0