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Summary The factors which determine the moisture content of the air in a room are discussed briefly, and simple differential equations are obtained for the balance of water vapour flow through a ventilated or non-ventilated room. The differential equations are solved for a number of cases, showing the transient, dynamic behaviour of the moisture content of the inside air in response to various time-related patterns of excitation. Examples show the application of the solutions to practical cases. The same solutions can also apply to transients in the concentration of other airborne gases such as carbon dioxide or tracer gases.

Room air moisture content: Dynamic effects of ventilation and vapour generation

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List of symbols

N	Air change rate (ac h^{-1})
R	Recirculation ratio
V	Room volume (m^3)
g_i	Inside air moisture content (kg kg^{-1})
g_o	Outside air moisture content (kg kg^{-1})
$\dot{g}$	Rate of change of moisture content (kg kg^{-1} h^{-1})
$\dot{m}$	Rate of generation of water vapour (kg h^{-1})
t	time (h)
ρ	Air density (kg m^{-3})
$\Delta g(t)$	$g_i - g_o$ at time t (kg kg^{-1})

1 Introduction

The moisture content of the air inside occupied buildings is generally greater than that of the outside air. This may be due to processes which generate water vapour, as well as the activities of the occupants: breathing, cooking, washing and drying clothes all generate quite large amounts of water vapour which is mostly released freely into the inside air[1]. The excess of moisture content is associated with a rise of the inside air vapour pressure above that of the outside air, and promotes diffusion of vapour through permeable building materials. Most of the flow of vapour to the outside, however, is carried by infiltration air, and in traditional construction (say before 1960), a number of factors operated to promote ventilation and to reduce the likelihood of dampness problems: open flues, the requirement for the provision of air bricks, generally 'leaky' construction, etc. In modern dwellings most of these factors are absent or reduced, and the problem is often intensified by the deliberate installation of such features as draught-stripping, double glazing, cavity wall insulation, showers and unvented clothes driers. The result is that dampness and condensation problems affect more than two million dwellings in the UK at present.

Some processes and appliances which release water vapour produce relatively short-term effects, and it is sometimes desirable to assess the dynamic, transient response of the inside air moisture content (and hence vapour pressure) to changes in conditions. This paper presents an analytical treatment of this form of response, based on simple assumptions. The effects of changes in vapour generation rate are examined, and solutions are presented for the time response of the inside air moisture content. The same solutions can also be applied to the time-related pattern of the concentration of other airborne gaseous pollutants, such as carbon dioxide. Some solutions in this area have been published by Jones[2]. Experimentally, the technique is well established as a method of measuring infiltration rate by observing the decay in concentration of pulses of tracer gases.

In practice, the relative humidity has to be determined in order to assess the risk of mould growth. This can be found from the combination of moisture content and dry-bulb temperature.

2 Sealed room

As an initial example, we take the hypothetical case of a room which is perfectly sealed and with an impermeable lining so that there is neither any diffusion of vapour through the structure nor any transfer of air or vapour by infiltration.

In this case any vapour which is generated inside the room is assumed to mix uniformly and instantaneously with the room air as long as the room air is unsaturated, and all the room surface temperatures are above the dewpoint.

A simple mass flow balance on the moisture can then be written as follows; Rate of generation = Rate of accumulation in the room air:

$$\dot{m}(t) = \rho V \, \mathrm{d}g_i(t)/\mathrm{d}t \qquad (1)$$

The air density ρ is in fact a weak function of the moisture content, but this is a small effect and can usually be neglected. At ordinary temperatures the density can be assumed to be constant at 1.2 kg m^{-3}. In the UK the outside air density in winter will be up to about 10% greater than that of inside air; in summer it will be up to about 5% lower than inside air.

Assuming a stepwise function for $\dot{m}(t)$, starting at $\dot{m} = 0$ and taking the steady-state value $\dot{m}_1$ at and after time $t = 0$,

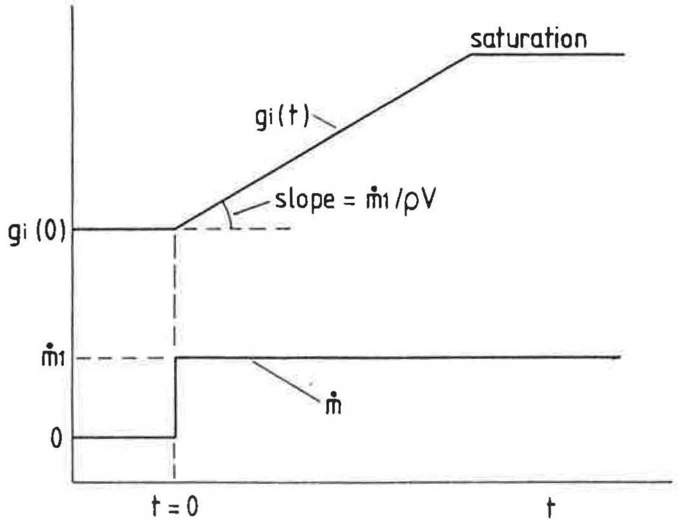

Figure 1 The rise in moisture content in a sealed room due to a step rise in vapour generation rate

equation 1 can be solved by rearranging and integrating:

$$g_i(t) = (\dot{m}_1/\rho V)t + g_i(0) \quad \text{for } 0 \leq t \leq t_2 \qquad (2)$$

where $g_i(0)$ is the initial value of $g_i(t)$. The value of g_i will continue to rise at a steady rate until $t = t_2$, when saturation is reached, or until the air dew point reaches the lowest inside surface temperature, when condensation will start to occur as shown in Figure 1.

The time t_1 taken for g_i to reach a specified value g_{i1} can easily be calculated by rearranging equation 2.

$$t_1 = (\rho V/\dot{m}_1)(g_{i1} - g_i(0)) \quad \text{for } 0 \leq t_1 \leq t_2$$

This is not of course a practically realisable case, as it would be extremely difficult to seal perfectly a room of any normal size and construction. However, to take a simple illustrative example, we can consider a room of volume 60 m^3 with a steady internal moisture generation rate of 0.35 kg h^{-1}. The rate of rise of moisture content in the room air will be 0.35/(1.2 × 60) = 0.0049 kg kg^{-1} h^{-1}. If the air temperature is steady at 23°C and the initial moisture content is 0.006 kg kg^{-1} then the time taken to reach saturation moisture content (0.0178 kg kg^{-1} at 23°C) will be (0.0178 − 0.006)/0.0049 = 2 h 27 min. This assumes that no room surface temperature is lower than 23°C. If one surface is at, say, 18°C, then the time taken for condensation to start on that surface will be (0.0130 − 0.006)/0.0049 = 1 h 26 min, where 0.0130 is the saturation moisture content at 18°C.

3 Ventilated room

A room which exchanges some air with the outside is a more realistic case than the previous one. The vapour generated in the room is partly accumulated in the (unsaturated) room air and partly lost to the outside. Because of the ventilation exchange, some vapour also enters with the outside air. The mass flow balance then becomes: Rate of generation in the room + Rate of gain from outside = Rate of loss to outside + Rate of accumulation in the room air:

$$\dot{m}(t) + \rho V N g_o(t) = \rho V N g_i(t) + \rho V \, \mathrm{d}g_i(t)/\mathrm{d}t \qquad (3)$$

This equation does not include the losses due to adsorption of water onto the room surfaces. It is not an easy effect to quantify, as very few data are available on the moisture adsorption properties of building and furnishing materials. Kusuda[3] has reported some measurements by Tsuchiya[4] on the kitchen and living room of an experimental house. The stated result was a value of 3 g m^{-2} h^{-1} per g kg^{-1} room air moisture content for the mean moisture adsorption factor of the surfaces of the test rooms. What these surfaces were made of is not stated. The diffusion of vapour through permeable building materials is also neglected in equation 3 as being small in magnitude, although it may be very important in the long term because of the danger of internal (i.e. interstitial) condensation. The size of this vapour flow is relatively easy to calculate; it is part of the standard condensation prediction method[5]. The magnitude of this effect has been analysed by Becker and Jaegermann[6], who state that, for a moderately airtight dwelling of medium size (N = 0.5 ac h^{-1}, V = 150 m^3), the rate of diffusion through the permeable surfaces is generally less than 10% of the rate of vapour flow by infiltration. It therefore seems reasonable to neglect this route of vapour transfer. Further research in this area is needed, especially as the trends in modern construction referred to in the Introduction must lead to an increase in the relative importance of these paths of vapour transfer by adsorption and diffusion.

Accepting these approximations and assumptions, equation 3 may be solved to find the response of g_i for various conditions.

3.1 Steady state

In the steady state g_i, g_o and $\dot{m}$ are constant, and equation 3 reduces to:

$$\dot{m}(t) + \rho V N g_o(t) = \rho V N g_i(t)$$

so that

$$g_i = g_o + \dot{m}/\rho V N \qquad (4)$$

This case was examined in some detail by Loudon[7], in conjunction with the steady state heat balance equation, to predict the relative humidity of the internal air under various combinations of heat and moisture generation and ventilation rate. It will not be considered in more detail here, except to say that, in ordinary naturally ventilated rooms with N generally between about 0.5 and 1.5 ac h^{-1}, a simple rule of thumb would be

$$g_i - g_o \simeq \dot{m}/V$$

so that the moisture content excess in kg kg^{-1} is approximately equal to the rate of vapour generation in kg h^{-1} divided by the volume of the space in m^3.

In equation 3 it is assumed that the ventilation is by all fresh air at the volume flowrate NV m^3 h^{-1}. If there is partial recirculation, such as occurs in some ducted air heating systems, the moisture balance of equation 3 is modified:

$$\dot{m}(t) + (1 - R)\rho V N g_o(t) = (1 - R)\rho V N g_i(t) + \rho V \, \mathrm{d}g_i(t)/\mathrm{d}t$$

where R is the ratio of the mass flow rate of recirculated air to the mass flow rate of supply air to the room (the recirculation ratio).

This assumes that the volume of air contained in the ducting system and the time delay in the system are small compared with V and $1/N$ respectively.

In the steady state this gives:

$$g_i = g_o + \dot{m}/[(1 - R)\rho VN] \quad (5)$$

When R is zero there is no recirculation (all fresh air), and equation 5 reduces to the same case as equation 4. When R is unity there is full recirculation (no fresh air), and the case becomes that of the sealed room; equation 5 becomes invalid.

3.2 Dynamic state

In the dynamic state a number of cases and combinations of cases are worth examining. There are three independent variables of interest: $\dot{m}$, g_o and N; the effects of each of these on g_i as the dependent variable will be considered in turn. Since the system is assumed to be linear, the principle of superposition applies and the net result of simultaneous changes in the independent variables can be obtained by summation of the effects of the individual changes. Where the form of the time function is simple, a solution can be found by using the Laplace transform method[9]. For complicated or arbitrary patterns of variation, numerical solutions can be found, for example by the Runge–Kutta method[9].

3.2.1 Changes in the moisture generation rate $\dot{m}(t)$

One case of interest here is the effect of a step function change in $\dot{m}(t)$. Often this will be from zero to some steady value, but for generality we first consider a step change from one steady value $\dot{m}_1$ to a new steady value $\dot{m}_2$. Applying this to equation 3 and solving by Laplace transform gives the following solution for $g_i(t)$, assuming N and g_o are constant in time:

$$g_i(t) = g_o + \frac{\dot{m}_2}{\rho VN} - \frac{\dot{m}_2 - \dot{m}_1}{\rho VN}\exp(-Nt) \quad \text{for } t \geq 0 \quad (6)$$

This is illustrated in Figure 2. If the initial value of the moisture generation rate, $\dot{m}_1$, is zero then the equation simplifies to:

$$g_i(t) = g_o + (\dot{m}_2/\rho VN)(1 - \exp(-Nt)) \quad (7)$$

For example, in the case of the room described in section 2 above, with the outside air moisture content g_o steady at 0.006 kg kg^{-1} and an infiltration rate N of 1.5 ac h^{-1}, the inside air moisture content $g_i(t)$ (for $\dot{m}_1$ = zero and $\dot{m}_2$ = 0.35 kg h^{-1}) will be:

$$g_i(t) = 0.006 + 0.0032(1 - \exp(-1.5t))$$

The value of g_i will rise to 0.0085 kg kg^{-1} after one hour, and will experience 95% of its final change within about two hours. Generally, the dynamic part of the response will be complete within a time interval of about $3/N$ hours.

Rearranging equation 6 for t as the independent variable gives:

$$t = (1/N)\log_e\left(\frac{\dot{m}_2 - \dot{m}_1}{\dot{m}_2 - \rho VN\,\Delta g(t)}\right) \quad (8)$$

where $\Delta g(t)$ denotes $g_i(t) - g_o$, the rise of g_i above g_o at time t. Equation 8 can be used to find the time taken for $\Delta g(t)$ to reach some specified value, less than its maximum value $\dot{m}_2/\rho VN$.

The change in $\dot{m}$ may be produced by the entrance or exit of occupants or by their activities. Each person generates water vapour in the breath at a rate of about 50 g h^{-1}, and activities such as cooking, washing and drying clothes can release very large quantities of vapour[1]. In domestic situations, a free water surface will release vapour at a rate of about 30 kg h^{-1} per m^2 of area when at a slow simmer, and about 60 kg h^{-1} m^{-2} at a fast boil. A lid on a saucepan will reduce these figures by a factor of between 3 and 6, depending on the quality of fit. A boiling electric kettle will produce vapour at a rate of about 1.5 kg h^{-1} per kW of electric power.

The response of g_i to short-term inputs of water vapour is obviously relevant. If a step rise is produced in $\dot{m}$ then the value of g_i at time t is given by equation 6 above. If at some time t_1 the vapour generation is reduced to $\dot{m}_1$ or stopped, there will then be an exponential decay from that value at a rate depending on the ventilation:

$$g_i(t) = g_o + \frac{\dot{m}_1}{\rho VN} + \left[g_i(t_1) - \left(g_o + \frac{\dot{m}_1}{\rho VN}\right)\right] \times \exp[-N(t - t_1)] \quad (9)$$

for $t_1 \leq t \leq \infty$.

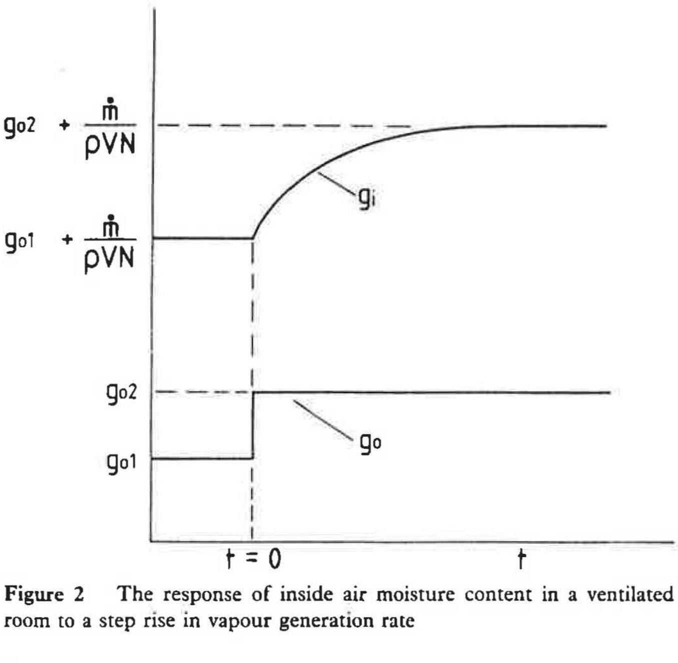

Figure 2 The response of inside air moisture content in a ventilated room to a step rise in vapour generation rate

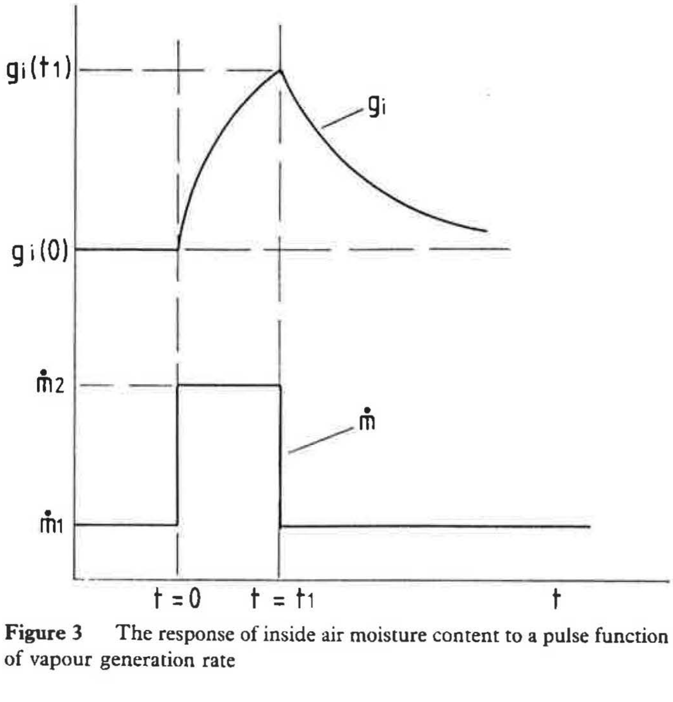

Figure 3 The response of inside air moisture content to a pulse function of vapour generation rate

The response to this pulse input is illustrated in Figure 3. Equation 9 relates to the method of measuring the ventilation rate in an enclosure by releasing a pulse of tracer gas and observing the decay in concentration with time.

3.2.2 *Changes in the outside air moisture content g_o*

The moisture content of the outside air is determined by the local meteorological conditions. It may undergo a stepwise increase when rain starts, or may rise or fall with an approximately constant slope as wind effects bring damper or dryer air to the site. As before, we assume a constant ventilation rate N and moisture generation rate $\dot{m}$.

For a stepwise change in g_o, the outside air moisture content, from one steady value g_{o1} to another steady value g_{o2}, the solution of equation 3 for $g_i(t)$ is

$$g_i(t) = g_{o2} - (g_{o2} - g_{o1})\exp(-Nt) + \dot{m}/\rho VN \quad \text{for } t > 0 \qquad (10)$$

The behaviour is similar in form to that of the previous case, as shown in Figure 4. In theory the inside air moisture content will never exactly reach the steady value of g_{o2}, but in practice the dynamic part of the response will be complete within about three time constants, i.e. an interval of $3/N$ hours.

For a ramp change in g_o, starting at an initial steady value $g_o(0)$ and changing at a steady slope $\dot{g}_o$ kg kg^{-1} h^{-1}, the response of g_i is given by:

$$g_i(t) = \dot{g}_o t - (\dot{g}_o/N)(1 - \exp(-Nt)) + g_i(0) \quad \text{for } t \geqslant 0 \qquad (11)$$

where $g_i(0)$ is the initial steady value of g_i at the start of the ramp, which is determined by the steady-state relationship of equation 4, so that

$$g_i(t) = \dot{g}_o t - (\dot{g}_o/N)(1 - \exp(-Nt)) + g_o(0) + \dot{m}/\rho VN \qquad (12)$$

The behaviour of $g_i(t)$ as predicted by this equation is quite interesting. At zero time the value of g_i is $g_o(0) + \dot{m}/\rho VN$ as would be expected intuitively. For values of time much greater than $1/N$, equation 12 reduces to the form

$$g_i(t) = \dot{g}_o t - \dot{g}_o/N + g_o(0) + \dot{m}/\rho VN \quad (t \gg 1/N)$$

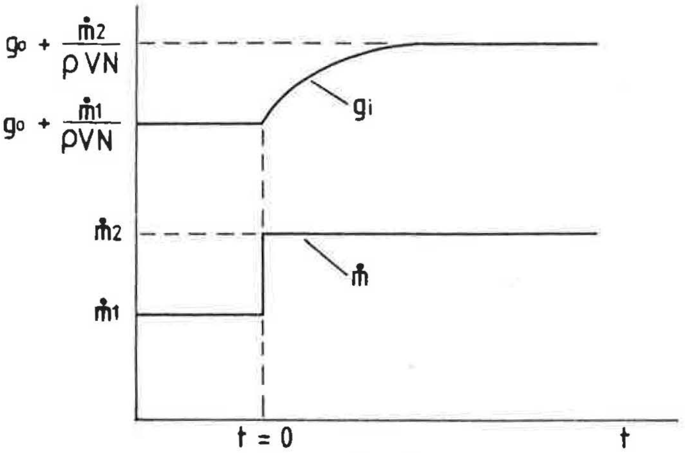

Figure 4 The response of inside air moisture content in a ventilated room to a step rise in outside air moisture content

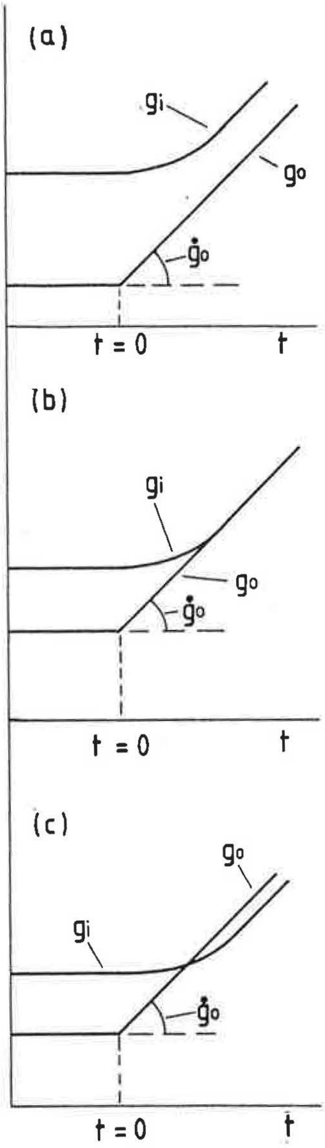

Figure 5 The response of inside air moisture content to a ramp change in outside air moisture content (a) with $\dot{m}/\rho V > \dot{g}_o$ (b) with $\dot{m}/\rho V = \dot{g}_o$ (c) with $\dot{m}/\rho V < \dot{g}_o$

which indicates a rise in g_i at the same rate as g_o but with a steady difference given by

$$\Delta g_o = \dot{m}/\rho VN - \dot{g}_o/N$$

Evidently, if $\dot{m}/\rho V > \dot{g}_o$ then g_i will remain greater than g_o, and *vice versa*. For the special case where

$$\dot{m}/\rho V = \dot{g}_o$$

then $g_i(t) = g_o(t)$ when $t \gg 1/N$. These cases are illustrated in Figure 5.

3.2.3 *Changes in the ventilation rate N*

In naturally ventilated buildings infiltration air is driven by a combination of the stack effect and wind-induced pressures which are essentially unpredictable except in the statistical sense. Some guidance is available from data published by BRE[8]. In mechanically ventilated buildings, however, the ventilation rate is controllable. It is of interest to examine the response of equation 3 to changes in ventilation rate N. A step change in ventilation rate would be produced by the switching on or off of an extractor fan or by a change of fan speed in a variable-air-volume system.

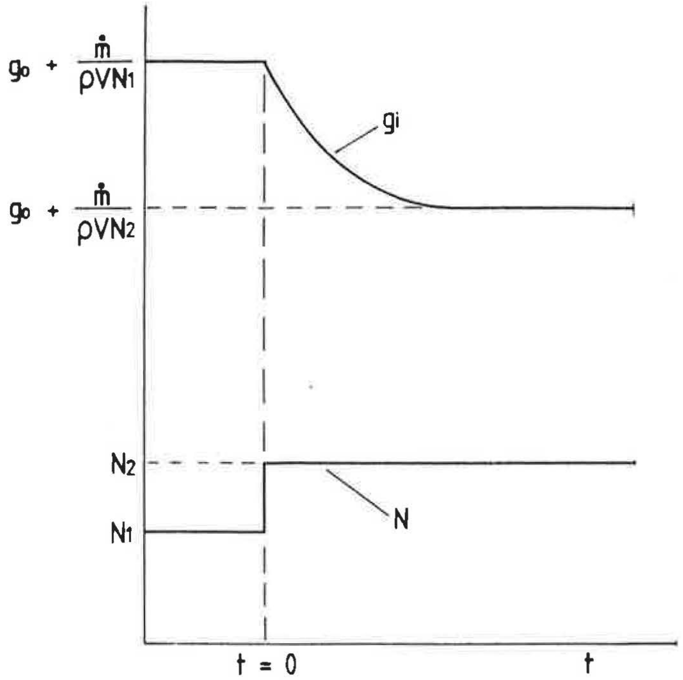

Figure 6 The response of inside air moisture content to a step rise in ventilation rate

Equation 3 may conveniently be solved for a step change in N by the use of the integrating factor method[9]. Choosing $\exp(N_2t)$ as the integrating factor and allowing N to change from one steady value N_1 to another steady value N_2 (neither of which is zero) gives the following equation for the response of $g_i(t)$, with steady values of $\dot{m}$ and g_o:

$$\begin{aligned} g_i(t) &= (g_o + \dot{m}/\rho V N_1)\exp(-N_2 t) \\ &\quad + (g_o + \dot{m}/\rho V N_2)(1 - \exp(N_2 t)) \\ &= g_o + \dot{m}/\rho V N_2 + (\dot{m}/\rho V) \\ &\quad \times (1/N_1 - 1/N_2)\exp(-N_2 t) \quad \text{for } t \geq 0 \end{aligned} \tag{13}$$

The internal air moisture content changes exponentially from its original steady value to its new steady value at a rate determined by the new ventilation rate (Figure 6).

For the case which was assumed previously, with $V = 60$ m^3, $\dot{m} = 0.35$ kg h^{-1}, and with a step change in N from $N_1 = 0.5$ to $N_2 = 1.5$ ac h^{-1}, we obtain from equation 13:

$$g_i(t) = 0.00924 + 0.00648\exp(-1.5t)$$

and the fall to the new steady value of g_i, 0.00924 kg kg^{-1}, is practically complete within about 2 h, i.e. an interval of $3/N_2$ hours. Note that if $N_1 > N_2$ the $g_i(t)$ will rise to a new value, higher than the initial state, and *vice versa*.

4 Conclusions

The mass flow balances on the moisture content of the air in a room have been presented, both for a sealed, non-ventilated room, and for a room with fresh air ventilation.

The differential equations of the water vapour flow balance have been solved for a number of simple cases, involving step or ramp changes in the vapour generation rate, the outside air moisture content and the ventilation rate.

The dynamic response of the inside air moisture content in these cases has been described and illustrated.

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