

A Procedure for Calculating Concentration Histories in Dwellings

K. E. SIRÉN*

The multi-chamber procedure is used to calculate the tracer gas concentration histories in a test house containing five rooms. For this calculation the air flows between adjacent rooms are evaluated using a simple analytical theory describing the air exchange through the doorways. The measured concentration histories during five runs in the test house are analyzed and compared with the calculated concentrations. Combination of the two theories seems to produce reasonable results. This method can for the present, however, be applied only to conditions usually existing in dwellings and to systems in which all the chambers are horizontally adjacent, due to the restricted validity range of the parameters used.

NOMENCLATURE

a	factor
C_i	concentration in chamber i
$C(t)$	concentration vector
$C(0)$	initial concentration vector
C_s	supply concentration
C	coefficient of discharge
g	acceleration due to gravity
H	height of opening
I	unit matrix
m_i	flow rate of contaminant or tracer gas into cell i
$m(t)$	contaminant flow rate vector
N	number of cells in the system
$\Delta p(z)$	pressure difference
p_v	pressure difference due to ventilation
p_w	pressure difference due to turbulence
Q_s	supply flow rate to chamber i
Q_{ej}	exhaust flow rate from chamber j
Q_{ji}	total flow rate from chamber j to chamber i
Q_{oi}	overall volumetric flow rate leaving chamber i
Q_i	inflow
Q_i^+	inflow, positive turbulence pressure
Q_i^-	inflow, negative turbulence pressure
Q	flow matrix
Q_s	diagonal supply flow matrix
Q_x	net flow rate
T	temperature
t	time
t'	dummy variable of integration
V_i	volume of chamber i
V	diagonal volume matrix
v^+	velocity of air flow, positive turbulence pressure
v^-	velocity of air flow, negative turbulence pressure
W	width of opening
z	vertical coordinate
z_0^+	neutral level, positive turbulence pressure
z_0^-	neutral level, negative turbulence pressure
Θ	temperature difference
ρ	density
τ	tau-matrix

INTRODUCTION

THE QUALITY of air is a concept commonly used to describe the quantity and type of contaminants in the air. The more unhealthy gases, particles and organic compounds the air contains, the lower its quality is, and *vice versa*. The health effects of many individual contaminants have already been determined, but very little is known about the health effects of different contaminant combinations. The consistence of the outdoor air, emissions from building materials, smoking, the number of occupants, etc., all have an effect on the indoor air quality. If the outdoor air is not too contaminated, the indoor air quality can usually be raised by increasing the ventilation rates. This, however, raises the energy consumption in the ventilated building; the colder the local climate, the greater the increase. Thus the rate of ventilation must be a compromise between the energy consumption and the indoor air quality.

The effect of the ventilation air flows on the energy consumption of the building can be evaluated using relatively simple calculation procedures. Obtaining any quantitative result describing the influence of the air flows on the health of the occupants is far more difficult. The concentrations of single contaminants affecting the indoor air quality can be calculated. The well-known multi-chamber theory provides a suitable basis for calculating the concentrations, which are assumed to be uniform in each room. Sufficient information on the system must naturally be available. Most of the input data concerning a real or an imagined system can be measured, estimated or chosen. However, the air flow rates between the rooms are difficult to measure and choosing is out of the question. The best alternative in this context seems to be to calculate these flow rates. The volume of work, the assumptions made and the information included in the results of such a procedure should be in harmony with the multi-chamber method. This usually eliminates the numerical computation methods because of their complexity. Analytical calculation of the air exchange

*Helsinki University of Technology, Laboratory of Heating, Ventilation and Air-Conditioning, Sähkömiehentie 4, 02150 Espoo, Finland.

between adjacent rooms was therefore chosen in this context.

THE MULTI-CHAMBER THEORY

The widely applied multi-chamber theory can also be used to calculate concentration histories in buildings [1]. Internal mixing in rooms is usually more effective than mixing between adjacent rooms. Thus a building, a dwelling or some other system, Fig. 1, containing several rooms can be divided, as a first approximation, so that each room constitutes one chamber. The chambers are connected through the doorways and the air flows between adjacent rooms in only one or two directions. As a rule the system is open, which means that it is connected with the environment through the supply and exhaust flows. Some approximations are made in deriving the equations. In each chamber complete and instantaneous mixing is assumed. Further, the release rate of contaminants is assumed to be much smaller than the air flows in the system. The conservation of mass for chamber i yields

$$V_i \frac{dC_i}{dt} = - \sum_{j \neq i}^N Q_{ji} C_i + \sum_{j \neq i}^N Q_{ij} C_j - Q_{ei} C_i + Q_{is} C_s + \dot{m}_i \quad (1)$$

In the above equation V_i is the volume of chamber i , C_i is the mass concentration of the contaminant in chamber i , t is the time, Q_{ji} is the volume flow rate of air from chamber i to chamber j , Q_{ei} is the exhaust flow rate from chamber i to outside the system, Q_{is} is the supply flow rate from outside the system to chamber i , C_s is the concentration of the supply air and \dot{m}_i is the mass flow rate of a contaminant released in chamber i . Using the notation

$$Q_{ii} C_i = \sum_{j \neq i}^N Q_{ji} C_i + Q_{ei} C_i \quad (2)$$

for the total contaminant mass flow rate leaving chamber i , equation (1) becomes as follows

$$V_i \frac{dC_i}{dt} = - Q_{ii} C_i + \sum_{j \neq i}^N Q_{ij} C_j + Q_{is} C_s + \dot{m}_i \quad (3)$$

Assuming that only the concentrations and the mass flow rates of the sources are time dependent, the equations for the conservation of mass for the whole system can be represented by a linear time-invariant vector matrix

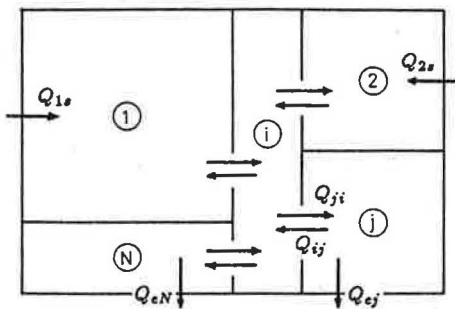


Fig. 1. A multi-chamber system.

differential equation

$$V \frac{dC(t)}{dt} = -QC(t) + Q_s C_s + \dot{m}(t), \quad (4)$$

where V is a $N \times N$ diagonal matrix containing the volumes of the chambers, Q is a square matrix called flow matrix [1] and Q_s is a diagonal matrix containing the supply flow rates. The concentrations in the chambers C , the concentration of the supply air C_s and the contaminant sources \dot{m} are vectors of length N . The solution of equation (4) is

$$C(t) = e^{-\tau^{-1}t} C(0) + \tau (I - e^{-\tau^{-1}t}) V^{-1} Q_s C_s + \int_0^t e^{-\tau^{-1}(t-t')} V^{-1} \dot{m}(t') dt' \quad (5)$$

The vector $C(0)$ contains the initial concentrations of the chambers at time $t = 0$ and the matrix τ is defined

$$\tau = Q^{-1}V. \quad (6)$$

The calculation of the concentration histories using the vector matrix equation (5) implies that all the quantities needed to describe the system are measured, calculated or chosen. The volumes of the chambers are often easiest to determine. The release rates of the contaminant sources can in most cases be estimated with such reliability that they are not critical as regards the inaccuracy of the computed results. When calculating e.g. CO_2 concentrations, the concentration of the supply air must also be known. This, however, usually causes no problem. The supply air flow into a chamber is in principle composed of the ventilation flow and the infiltration flow. When a building is planned the ventilation flows are chosen; in an existing building they can reasonably be measured. Evaluating the infiltration rates is somewhat more difficult. In a balanced ventilation system the infiltration flow rates are usually small compared with the ventilation flow rates and are thus not of any great importance. The situation is, however, different in an extract ventilation system or natural ventilation. Then the distribution of the infiltration air between the rooms is an important detail to determine. The solution to this problem is a tracer gas measurement procedure, e.g. the constant concentration method [2]. Finally the air flows between adjacent rooms must be known for the construction of the flow matrix. It is not enough to know only the net flow rates from one room to another. Generally an inflow, in the opposite direction from the net flow, arises between the rooms. This is caused by temperature differences between the rooms and the internal movement of air in the rooms. Measurement of the flows between adjacent rooms is also possible with multiple tracer gas techniques. With the same amount of work, however, the concentration histories in the system can be measured directly without any need for further calculations. Computing these flows therefore seems to be the best solution in this context.

AIR EXCHANGE BETWEEN ADJACENT ROOMS

Using numerical approaches the air flows between adjacent rooms could well be computed. However, the

amount of work and the level of information included in the results of such a method are usually not of the same magnitude as in the multi-chamber method. Simple analytical treatment of the problem here offers a better alternative. The theory for calculating analytically the air flows between horizontally adjacent rooms has been developed in steps by several researchers [3-5]. The following short review presents the basic ideas of this theory.

The system under consideration contains two rooms, Fig. 2. The doorway between the rooms has a height H and width W . The air temperature in each room is assumed to be uniform. If the air densities in the rooms are ρ_1 and ρ_2 , the pressure difference on level z between the rooms is given by

$$\Delta p(z) = g(\rho_2 - \rho_1)z + p_x \pm p_u \quad (7)$$

The pressure difference p_x is due to the ventilation system or some other external source and produces a net flow rate Q_x from room 1 to room 2. The turbulence pressure p_u operates in opposite directions over both half areas of the doorway. This pressure is a residue term, which is used to explain the air movement due to all those factors not included among the previous terms. Further assuming the flow through the doorway to be ideal and the air to be a perfect gas, the velocities of the air are obtained as

$$v^\pm(z) = \left(2g \frac{\Theta}{T} z + \frac{2(p_x \pm p_u)}{\rho} \right)^{1/2} \quad (8)$$

where Θ is the difference in air temperatures between the rooms, T is the mean air temperature in the system and ρ is the corresponding mean density. The superscript \pm refers to the turbulence pressure with two directions. On the basis of equation (8) it can be seen that the velocities are equal to zero on the levels

$$z_0^+ = - \frac{p_x + p_u}{g(\rho_2 - \rho_1)} \quad (9)$$

$$z_0^- = - \frac{p_x - p_u}{g(\rho_2 - \rho_1)} \quad (10)$$

which are called the neutral levels. The volume flow rates through the doorway are calculated by integrating the velocities over the appropriate area. The nature of the turbulence pressure divides the integration into two parts and the results must be added together. The upper and lower limits of integration are determined in accordance with the height of the doorway and the neutral levels. Thus, for example, in the case of Fig. 2 the inflow, which

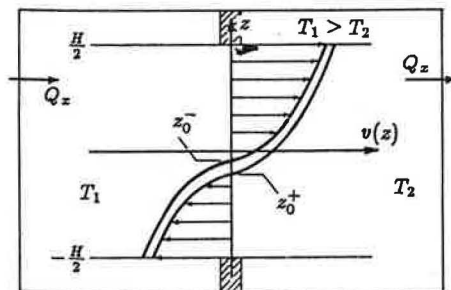


Fig. 2. The air velocities in a doorway.

acts in the opposite direction from the net flow, turns out to be

$$Q_+ = \frac{1}{2} CW \int_{-H/2}^{z_0^+} v^+(z) dz \quad (11)$$

$$Q_- = \frac{1}{2} CW \int_{-H/2}^{z_0^-} v^-(z) dz \quad (12)$$

The superscript $+$ refers to the positive value of the turbulence pressure and the superscript $-$ refers to the negative value of the turbulence pressure. The coefficient of discharge C takes into account the effects of the real frictional flow. Integrating equations (11) and (12) yields

$$Q_+ = \frac{1}{6} CWH \left(\frac{aH}{2} \right)^{1/2} \left(1 - \frac{4(p_x + p_u)}{\rho aH} \right)^{3/2} \quad (13)$$

$$Q_- = \frac{1}{6} CWH \left(\frac{aH}{2} \right)^{1/2} \left(1 - \frac{4(p_x - p_u)}{\rho aH} \right)^{3/2} \quad (14)$$

Here the factor $a = 2g\Delta\rho/\rho$, where $\Delta\rho$ is the difference between the air densities in the rooms. The total inflow is the sum of the positive and negative components

$$Q_1 = Q_+ + Q_- \quad (15)$$

The outflow, having the same flow direction as the net flow, is calculated in an analogous way. Depending on the position of the neutral levels, the temperature difference between the rooms and the mutual values of the pressure differences, the outflow and inflow are calculated using equations of different kinds. Eighteen different situations exist in all. In practice the flow rate is calculated by selecting for the pressure p_x a value such that the difference between the outflow and the inflow is equal to the net flow rate through the doorway.

PARAMETERS

The method explained above implies that the values both for the discharge coefficient C and the turbulence pressure p_u are known. The values presented earlier [4, 5] for these parameters are based on velocity measurements in the doorway. The temperature difference between the rooms and the mean velocity corresponding to the net flow rate through the doorway varied widely during these measurements. Under the circumstances usually existing in dwellings the variation in these quantities is, however, much smaller. Evaluation of the parameters was thus considered necessary. This was done according to the following procedure, called the stationary two-chamber method [6].

In a system containing two rooms, Fig. 3, the temperatures in the rooms are T_1 and T_2 . The direction of the net flow rate Q_x is from room 1 to room 2. A mass flow rate \dot{m}_2 of tracer gas is injected into room 2. The conservation of mass for chamber 1 then yields

$$V_1 \frac{dC_1}{dt} = -Q_2 C_1 + Q_1 C_2 \quad (16)$$

Here the assumptions included in the multi-chamber method have already been made. In a stationary situation the time derivative in equation (16) is equal to zero and

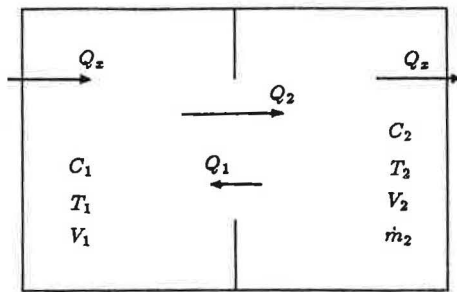


Fig. 3. A system for measuring the parameters.

we may write

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = f(C, p_u). \quad (17)$$

Using the theory for air exchange through the doorway, values for the parameters C and p_u can be computed from the concentrations measured in a stationary situation. Because there are two unknowns, at least two values must be available for each concentration.

Measurements according to this method were carried out in the test house of The National Swedish Institute for Building Research, Fig. 4. The bedroom and the kitchen were separated from the other rooms to form a system of two chambers. The width of the doorway between these two rooms was varied between 0.1–0.7 m using different door positions. The air was supplied to the bedroom and extracted from the kitchen. A mixture of N_2O and He was used as tracer gas. The gas was injected into the kitchen through sixty injection points to achieve as homogeneous a gas distribution as possible without using any mechanical mixing. The concentrations were measured on three levels in the centre of both rooms. The temperatures were measured at the centre point of each room and the net flow rate was found using the constant concentration method [2]. Several values for the concentrations C_1 and C_2 for each temperature and flow combination were achieved using different release rates for the tracer gas. At least three concentration levels were used for each pair of parameters calculated. The values of the parameters were

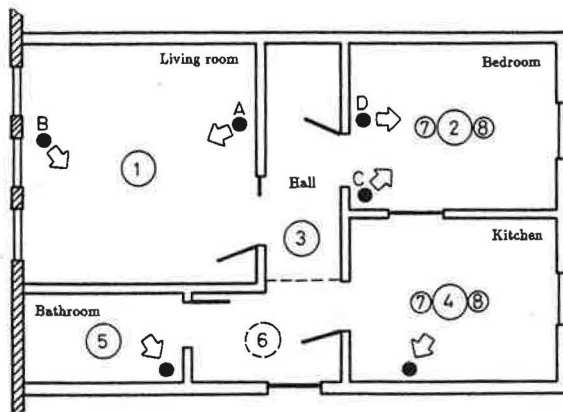


Fig. 4. The test house of The National Swedish Institute for Building Research.

selected to produce the minimum difference between the measured and calculated concentration values. As a criterion in this context the method of least squares was used. The values of the parameters obtained using the method described can be written as follows

$$C = 3.7v_x + 6.4v_x e^{-\Theta} - 0.90e^{-\Theta} + 0.96 \quad (18)$$

$$p_u = 0.003 Pa, \quad (19)$$

where v_x is the mean velocity [$m s^{-1}$] corresponding to the net flow rate through the doorway, Θ is the air temperature difference [K] between the centre points of the rooms. The allowable ranges for the arguments are $\Theta = 0-3$ K and $v_x = 0-0.05$ $m s^{-1}$. As can be seen, the turbulence pressure has a constant value but the coefficient of discharge strongly depends both on the temperature difference and the mean velocity. According to equation (18) some combinations of the arguments Θ and v_x lead to values greater than unity for the coefficient of discharge. This may give an odd impression in physical terms. The reason for this obviously lies in the approximations made in deriving the equations used and the effect produced by combining the two theories.

THE CALCULATION AND MEASURING PROCEDURES

To calculate the concentration histories in dwellings a MULTIC computer program was developed in Fortran programming language [7]. The computation is based on the combination of the multi-chamber theory and the theory of air exchange through doorways. The air flows between adjacent rooms are calculated using the values of equations (18) and (19) for the parameters. The input data are the temperatures, volumes, exhaust flow rates and initial concentrations of the chambers. The net flow rates and the dimensions of the doorways between the chambers are also included. A description of the internal contaminant sources must also be placed in the input data file. The output data contain the concentration histories of the chambers and some air quality related quantities. Among these are the purging flow rate, the mean age of the air and the transition probabilities [1]. The program can be applied only to conditions usually existing in dwellings. Further, all the chambers in the system must be horizontally adjacent, because the calculation of vertical flows is not included in the program. The present version is able to handle a system containing not more than ten chambers.

In the test house mentioned above, Fig. 4, a series of measurements was done to verify the results calculated with the MULTIC program. This time the system contained all five rooms in the test house. The house has a floor area of $70.2 m^2$ and a total volume of $175.7 m^3$. The living room, with three windows, is part of the south wall of the laboratory hall. As a consequence weather conditions outside the laboratory hall affect the conditions in the test house. The ventilation system used was either an extract system or a balanced system. The air was extracted from the kitchen and the bathroom. The supply air came through the roof into the living room and the bedroom. There were two alternative locations for the air inlets in both rooms. The supply air jet was

Table 1. Room air temperatures and other information related to the five runs analyzed

Run	Room air temperatures °C					T ₆	Ventilation system	Supply air grille	
	T ₁	T ₂	T ₃	T ₄	T ₅			Living room	Bedroom
FC03	23.7	24.3	24.0	24.3	23.9	24.0	extract	A	C
FC11	23.2	24.2	23.9	25.2	23.6	23.8	balanced	B	D
FC12	24.5	24.7	24.7	25.9	24.6	24.6	balanced	B	D
FC16	23.8	24.4	24.2	24.4	24.1	24.2	balanced	B	D
FC20	23.0	23.7	23.4	23.6	23.2	23.3	extract	B	D

directed to flow along the ceiling towards the centre of the room. The specific flow rate was 0.5 (m³ h⁻¹)/m³ during all tests. 68% of the exhaust flow was extracted from the kitchen. The infiltration flow rates were about 10% of the total supply flow in the extract system and 3% when the balanced system was in operation. The distribution of the supply air coming into the test house was measured separately using the constant concentration method. The air temperatures and concentrations were measured at the centre point of each room, points 1-5 in Fig. 4. Further, in the room where the tracer gas was released, the concentrations near the ceiling, point 8, and near the floor, point 7, were measured. The concentration and the temperature in the hall between the kitchen and the bathroom, point 6, were also monitored. A constant release of tracer gas mixture was injected through rotameters either into the kitchen or the bedroom. The injection technique was the same as that used in the parameter measurements. Mechanical mixing of the air was used only during the constant concentration measurements.

ANALYSIS

This section analyzes five runs made in the test house. The concentration histories measured were also compared with those calculated using the MULTIC program. The temperatures and other information related to the runs is presented in Table 1.

Run FC03, Fig. 5, was a step-up test with injection of tracer gas into the kitchen. All doors except the one between the bedroom and the kitchen were fully open. It can be seen from the concentration curves in Fig. 5 that the mixing in the kitchen was quite weak and that although there were sixty injection points, the fluctuations in the concentrations with respect to time and the differences with respect to place were quite large. Because the temperature in the kitchen was higher than in the hall, the inflow from the kitchen to the hall was in the upper part of the doorway and the outflow from the hall to the kitchen in the lower part of the doorway. As a consequence the concentration near the kitchen floor at measuring point 7 was distinctly lower than near the ceiling at point 8. Further, because the temperature in the hall was a little higher than in the bathroom, the air flowed from the kitchen along the ceiling straight to the bathroom and back again somewhat lower. This is indicated by the values of the concentrations at points 1, 2 and 3. This also indicates a weak connection between the two ends of the hall, which seems to be due to the small temperature difference and the elongated shape of the hall. The corresponding calculated concentration histories for a system containing five chambers are presented in Fig. 6. It can be seen that the calculated concentrations in the kitchen are probably a bit too high. However, comparison is difficult because of the large fluctuations in the measured values. The calculated and measured concentrations in the bathroom have nearly

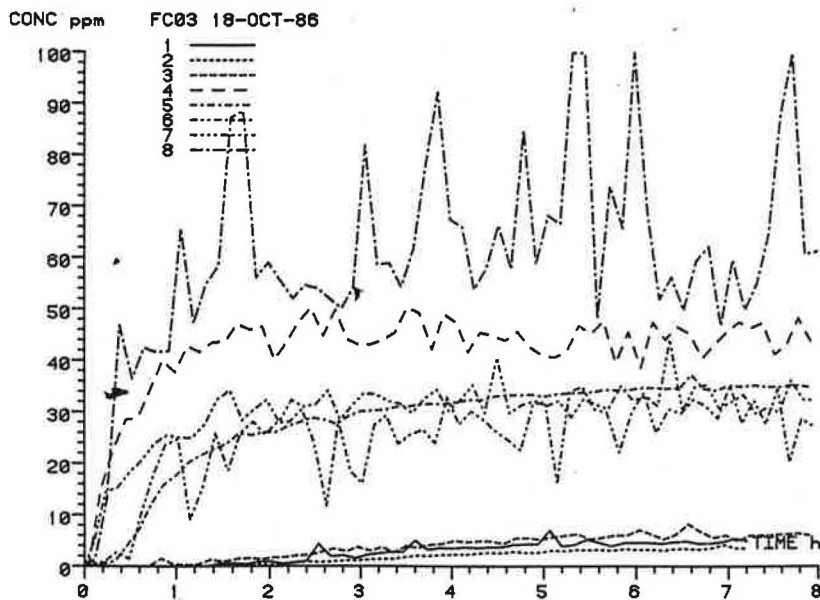


Fig. 5. Measured concentration histories during run FC03. Injection of tracer gas into the kitchen.

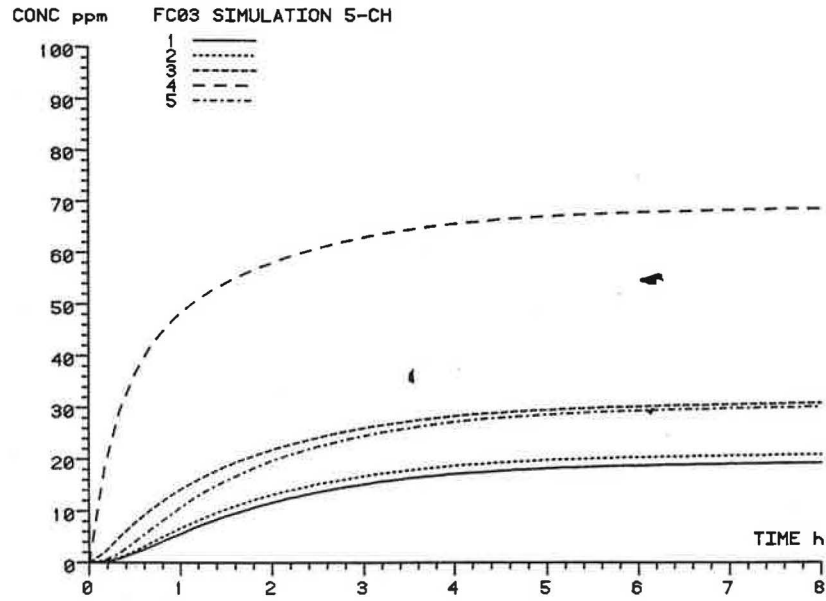


Fig. 6. Calculated concentration histories corresponding to run FC03. A system of five chambers.

the same values. On the other hand the calculated concentrations in the hall, living room and bedroom are too high. The calculation procedure overestimates the mixing in the longitudinal direction of the hall because it treats the hall as one chamber. Dividing the hall by the broken line in Fig. 4 transforms the system of five chambers into a system of six chambers. The dimensions of the opening between the different parts of the hall can be defined as the height and width of the hall in the plane of separation. The calculated concentration histories in the system containing six chambers are presented in Fig. 7. Now the concentrations at points 1, 2 and 3 are on the right level. The concentrations at points 5 and 6 also rise more steeply at the beginning, corresponding better to the measured values. Thus it can be concluded that the

test house in this case behaves more like a system of six chambers than a system of five chambers.

Run FC11, Fig. 8, was also a step-up test with injection of tracer gas into the kitchen. The door between the bedroom and the kitchen was closed. The door between the kitchen and the hall was only partly open so that the width of the doorway was 0.25 m. All the other doors were wide open. To achieve a slightly larger temperature difference between the hall and the kitchen, a heating power of 300 W was generated using a radiator located beneath the window in the kitchen. From Fig. 8 it can immediately be seen that the circulation due to the heating mixed the air and the tracer gas quite efficiently. The differences in the concentrations at points 4, 7 and 8 were smaller than in the previous run. The fluctuations with

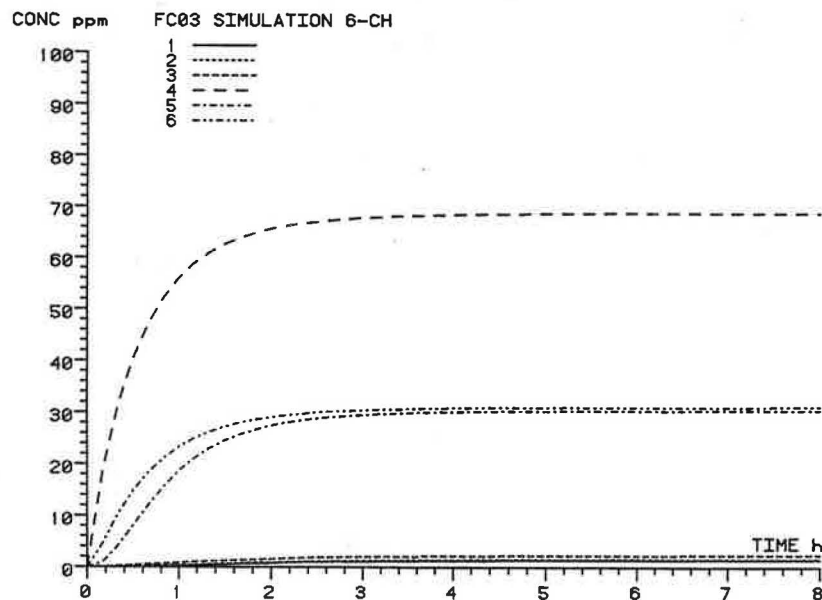


Fig. 7. Calculated concentration histories corresponding to run FC03. A system of six chambers.

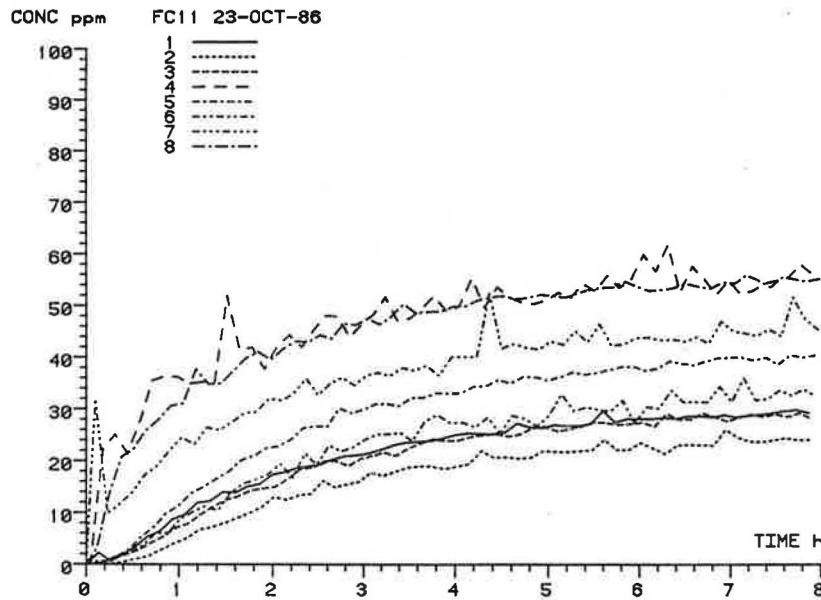


Fig. 8. Measured concentration histories during run FC11. Injection of tracer gas into the kitchen.

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respect to time were also noticeably smaller. Concentration 5 in the bathroom was higher than concentration 6 in the hall. This was due to the strong air flow along the ceiling from the kitchen to the bathroom. On the other hand concentrations 3 and 6 in the hall were quite close; this was because the kitchen door was ajar. The inflow from the kitchen to the hall was guided by the door towards the living room and the bedroom and raised concentration 3 in the hall. The rather large temperature difference between the living room and the hall caused concentrations 1 and 3 to have almost identical values. The concentrations calculated using a system of five chambers turn out to have values of the correct magnitude, Fig. 9. Only the concentrations in the living room and the bedroom are slightly lower than the mea-

sured values. On the other hand a system of six chambers produces too small values in the living room, hall and bedroom, Fig. 10. This is due to the guiding effect of the kitchen door. The calculation procedure cannot take such an effect into account. For the same reason concentrations 5 and 6 rise more steeply than the corresponding measured values, even though in a stationary situation the values are quite close. Hence the concentration histories calculated using a system of five chambers better describe the real phenomenon in this case.

Run FC12, Fig. 11, was a combined step-up and decay test conducted under practically the same circumstances as run FC11. Only the temperatures had slightly different values, Table 1. The injection of the tracer gas began

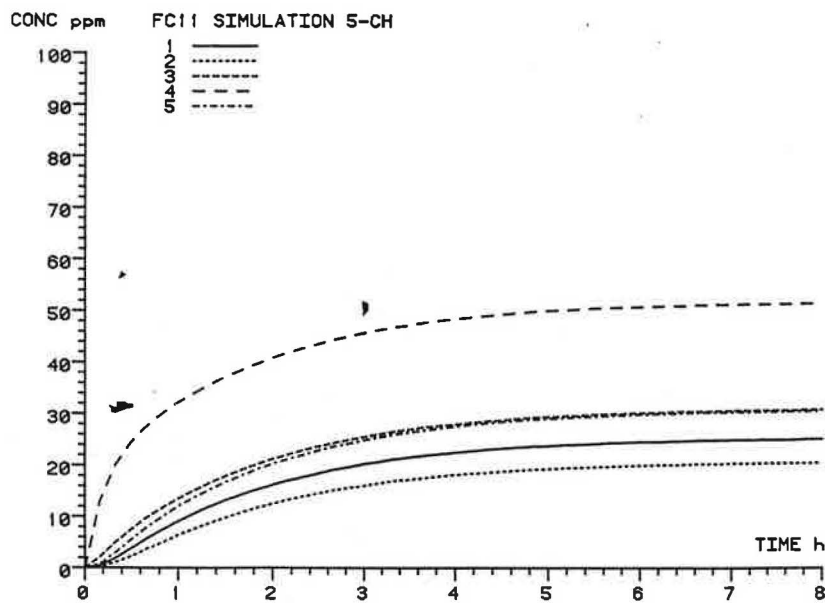


Fig. 9. Calculated concentration histories corresponding to run FC11. A system of five chambers.