Building and Environment, Vol. 23, No. 2, pp. 123–135, 1988.
Printed in Great Britain.

0360–1323/88 $3.00+0.00

3351

The Moisture Performance of Framed Structures—A Mathematical Model

M. J. CUNNINGHAM*

A mathematical model is developed for the moisture performance of a framed structure (e.g., a flat roof or a wall), containing a hygroscopic framing material and a cavity filled with air or insulation. A formula is developed that connects the enclosed and unenclosed drying time constants for the framing material. The enclosed drying time constant alone describes the longer term moisture behaviour of the structure (much greater than one day) under any driving forces given the linearity assumptions used. The model allows for anisotropic framing materials with initial moisture contents above or below fibre saturation.

NOMENCLATURE

- a half-width of framing material (m)
- A area (m^2)
- b half-height of framing material or half-depth of cavity (m)
- c half-width of cavity (m)
- c_i concentration of moisture in the air in region i (kg m^{-3})
- D_m diffusion coefficient under moisture concentration driving force (m^2 s^{-1})
- D_p diffusion coefficient under vapour pressure driving force (s)
- D_{pa} diffusion coefficient of the framing material in the direction framing to cavity (s)
- D_{pb} diffusion coefficient of the framing material in the direction framing to lining (s)
- D_{po} diffusion coefficient of the cavity material in the direction parallel to the linings (s)
- F air change rate (s^{-1})
- h mass transfer coefficient under vapour pressure driving force (s m^{-1})
- k proportionality constant linearising the sorption curve of a hygroscopic material (m^2 s^{-2})
- $\mathcal{K}$ ratio of k's (dimensionless)
- L dimensionless ratio of volume to surface mass transfer rates
- m moisture concentration (kg m^{-3})
- M initial moisture concentration (kg m^{-3})
- p vapour pressure (Pa)
- $\underline{p}$ final vapour pressure (Pa)
- $\bar{\underline{p}}$ weighted mean of vapour pressures (Pa)
- P initial vapour pressure (Pa)
- r total surface vapour resistance (N s kg^{-1} m^{-2})
- r_{ij} vapour resistance between region i and j (N s kg^{-1})
- r_{sa} total surface vapour resistance at surface to face cavity (N s kg^{-1} m^{-2})
- r_{sb} total surface vapour resistance at surface to face linings (N s kg^{-1} m^{-2})
- R total vapour resistance (N s kg^{-1} m^{-2})
- R universal gas constant (8310 J K^{-1} kmole^{-1})
- R_{ae} total vapour resistance between framing material and external regions via the cavity (N s kg^{-1} m^{-2})
- R_{ao} total vapour resistance between framing material and cavity (N s kg^{-1} m^{-2})
- R_{be} total vapour resistance between framing material and external regions via the linings (N s kg^{-1} m^{-2})
- R_{oe} total vapour resistance between the cavity and the external regions (N s kg^{-1} m^{-2})
- R_{wa} total vapour resistance of a half-width of framing material facing the cavity (N s kg^{-1} m^{-2})
- R_{wb} total vapour resistance of framing material facing the linings (N s kg^{-1} m^{-2})
- t time (s)
- t_a unenclosed drying time constant through the face of the framing material which will face cavity (s)
- t_b unenclosed drying time constant through the face of the framing material which will face the linings (s)
- t_o time constant associated with cavity performance (s)
- t_w drying time constant for unenclosed framing material (s)
- t_1 short term structure time constant, "equilibration time constant" (s)
- t_2 long term structure time constant, "drying time constant" (s)
- T Kelvin temperature (K)
- T_a unenclosed time for above fibre saturation drying through the face of the framing material which will face cavity (s)
- T_b unenclosed time for above fibre saturation drying through the face of the framing material which will face linings (s)
- V volume (m^3)
- W molecular weight of water (18 kg kmole^{-1})
- α surface mass transfer coefficient under mass concentration driving force (m s^{-1})
- γ dimensionless resistance ratio
- δ dimensionless resistance ratio
- ζ dimensionless resistance ratio
- η dimensionless resistance ratio
- ν ratio of the volume of the framing material to the volume of the cavity
- ξ dimensionless resistance ratio
- φ phase lag (radians)
- ω angular frequency (radians s^{-1})
- $\parallel$ parallel combination

Subscripts

- a edge of the framing material which will face the cavity
- b edge of the framing material which will face the linings
- i, j external regions
- o cavity
- w framing material

* BRANZ, Private Bag, Porirua, New Zealand.

BACKGROUND

THE MOISTURE performance of framed structures containing hygroscopic structural and insulant materials is complex and not necessarily well described by traditional means. The complexity arises out of the hygroscopic properties of the materials, the two dimensional nature of the flow, and the time dependency of the moisture performance of the structure. None of these issues is addressed in full by conventional design tools [1, 2]. This paper examines the particular case of the moisture performance of a flat roof or wall with a hygroscopic framing material and a cavity filled with air or insulation, and, by the nature of the model used, addresses the issues mentioned.

To elucidate the moisture behaviour of a structure, a logical approach would be to model the structure in detail with a numerical model and validate this modelling with experimental and field studies. Numerical modelling however does not necessarily lead to insights to system-wide performance nor to parameters characterising this performance. The philosophy adopted here and in earlier work [3–5] is to make appropriate simplifications to the differential equations describing the moisture performance of a structure, to enable these equations to be solved analytically. This approach leads to a small number of parameters, each with a clear physical meaning, which provide insights into the moisture performance of the structure and give an appropriate means for the researcher to communicate to the practitioner, whether he be engineer, architect or builder.

The key result derived is a formula for the drying time constant of the enclosed framing material, defined as the time constant for drying of the framing material once it is enclosed within the structure. Once this drying time constant is known then the long term behaviour of the structure under any driving force can be derived approximately. Enclosed framing material will dry more slowly than the unenclosed material and the result derived here expresses this quantitatively. Although the concept of an enclosed drying time constant is developed using steady state driving conditions, because of the assumptions of linearity which are made the moisture performance of the structure under any driving force variation (in particular periodic seasonal variations) can be given.

In a series of papers [3–5] the author has developed a more general analytical model describing the moisture behaviour of a building cavity containing hygroscopic material. An important approximation used in the development of the earlier model was to assume that the hygroscopic framing material dried exponentially below fibre saturation point. This work examines this approximation showing that it is soundly based in theory.

In New Zealand, timber framing is often enclosed with moisture contents much higher than fibre saturation despite recommendations to the contrary [6]. The implications of this in terms of the moisture performance of structures do not seem to have been addressed. The model developed here predicts that enclosed framing material with an initial moisture content above fibre saturation dries linearly, after an initial transient. An expression is given that compares the enclosed to the unenclosed linear drying rates.

OUTLINE

This work examines the case of a flat roof or wall which has the features that the framing material is joined to the inside and outside lining and the associated cavity may be filled with anything, in particular insulation or air.

The new features of this model are:

1. The model is two dimensional in the sense that moisture can flow from all four faces of the hygroscopic framing material (usually timber), both into the adjacent cavity as in previous work [3–5] and into the surrounding linings as for more traditional models [1, 2].
2. The adjacent cavity can be filled with any material, not just air as in previous work [3–5].
3. The framing material can have an initial moisture content that is above or below fibre saturation point.
4. The materials can be anisotropic, in particular they can have different moisture diffusion coefficients in the horizontal and vertical directions.
5. The assumption of exponential drying of the framing material below fibre saturation, is shown to be an approximation which is soundly based in theory. An expression is given for the size of the associated time constant in terms of the fundamental physical parameters (diffusion coefficients, size, etc.) describing the framing material.

The model consists of two coupled first order differential equations constructed by conserving moisture in the framing material and in the cavity. This means that, after linearising, the solution exhibits two time constants below fibre saturation. The shorter time constant can be interpreted as the time constant for the framing material to come into moisture equilibrium with the cavity material (the "equilibration time constant") while the longer time constant can be interpreted as the time constant for the drying of the initial construction moisture in the framing (the "drying time constant"). However, since the equations have been linearised, the time constants have a wider utility than this. By the theory of linear systems, see for example [7], they become the parameters that completely describe the moisture performance of the structure under any driving forces.

An expression is derived for the longer time constant that shows the increase in drying time of the enclosed framing material compared to the drying time of the unenclosed material. This expression generalises that derived in earlier work, reflecting the fact that moisture movement can now take place out of all four faces of the framing material. Whether the result is significantly different from that obtained earlier clearly depends on whether the flow of moisture through the alternative path provided (from the framing material through the linings) is of the same order or larger than flow through the path to the cavity. Examples are given to illustrate this.

It is shown that the effect of the insulation is twofold. Firstly, it lengthens the shorter time constant by an amount depending upon the hygroscopic nature of the cavity material. In the case of air, which cannot hold much water vapour, the equilibration time constant is very small, in the order of hours; in the case of insulation the time constant will be longer depending upon the exact hygroscopic nature of the material, that is the amount of water it can hold for a given vapour pressure.

Secondly, the insulation contributes to the longer (drying) time constant through its vapour resistance. Whether this is significant depends upon the relative size of the vapour resistance of the insulation compared to the vapour and air resistances of the cavity and the vapour resistance of the framing material.

If the framing material has an initial moisture content above fibre saturation then its vapour pressure is independent of moisture content and equal to the saturation vapour pressure at the temperature of the material. It is shown in this paper that this results in constant moisture concentration in the cavity and a linear fall in moisture concentration in the framing material after an initial transient in which the cavity and framing come into moisture equilibrium. An expression for this drying rate is derived.

In the first section of this paper the approximation which forms a basic assumption in this and previous work, namely of exponential drying of the hygroscopic materials, is put on a sound theoretical basis and an expression is given for the size of the associated time constant in terms of fundamental physical parameters (diffusion coefficients, size, etc.). In the next section the model is described, the differential equations for the model constructed and these equations are solved for the cases of initial framing material moisture content below and above fibre saturation. The behaviour of the solutions with the longer time constant are examined, being the solutions of most practical interest. An expression is given for the increase of the longer time constant for the enclosed framing material compared to the time constant when the framing material is unenclosed. A similar ratio is shown to hold if the initial moisture content of the framing material is above fibre saturation. This is the key result of this work.

The paper concludes with some examples deriving the value of the longer time constant which determines the drying rates of initial construction moisture. These results are compared to results derived in earlier work. The fact that, under the assumption of linearity, all other moisture performance characteristics of the structure can be determined once the enclosed drying time constant is known, is illustrated with examples showing how structures will behave under seasonal periodic driving forces.

THE EXPONENTIAL DRYING APPROXIMATION

It is assumed for this model, as in previous work [3–5], that the hygroscopic material dry, approximately exponentially, that is

$$\frac{\mathrm{d}m}{\mathrm{d}t} \propto -(m-\underline{m}), \tag{1}$$

where m is the moisture concentration in the hygroscopic material (kg m^{-3}) and $\underline{m}$ is its final equilibrium value. This is a lumped approximation in which the entire moisture content of the material is visualised as being concentrated at one point, and transfers in and out of the material through an effective vapour resistance. In an electrical analogue a hygroscopic material would be modelled by an RC circuit which has a time constant

$$t = RC. \tag{2}$$

In what follows it is shown that this exponential drying approximation is mathematically valid under the following conditions: (a) constant diffusion coefficient (concentration independent); (b) times greater than one time constant for the exponential solution. The physical validity or otherwise of these conditions is discussed at the end of this section.

Consider the case of drying (desorption) of a two sided plane sheet. Crank [8] shows that for this case

$$\frac{m-\underline{m}}{M_i-\underline{m}} = \sum_{n=1}^{\infty} \frac{2L^2 \exp\left(-\frac{\beta_n^2 D_m t}{a^2}\right)}{\beta_n^2(\beta_n^2+L^2+L)}, \tag{3}$$

where D_m is the diffusion coefficient for mass transfer under moisture concentration driving force, i.e.

$$\text{mass flux} = -D_m \frac{\partial m}{\partial x} = -D_m \frac{\mathrm{d}m}{\mathrm{d}p}\frac{\partial p}{\partial x},$$

i.e.

$$D_p = \frac{\mathrm{d}m}{\mathrm{d}p} D_m \tag{4}$$

also

$$L = \frac{a\alpha}{D_m}$$

and β_n are the positive roots of the equation

$$\beta_n \tan \beta_n = L,$$

α here is the surface mass transfer coefficient under mass concentration driving force, i.e.

$$\text{mass flux across the surface} = \alpha(m_w - m_o) \tag{5}$$

where m_w is the moisture concentration at the surface of the material and m_0 is the moisture concentration external to the material.

Consider in particular now the case of framing timber which will have parameters in the following range for the structures that are being considered, see for example Cunningham [5] and Siau [9]

$$k \sim 20\,\mathrm{m^2\,s^{-2}},$$
$$a \sim 2\text{–}5 \times 10^{-2}\,\mathrm{m},$$
$$\alpha \sim 10^{-7}\text{–}10^{-5}\,\mathrm{m\,s^{-1}},$$
$$D_m \sim 10^{-11}\text{–}10^{-9}\,\mathrm{m^2\,s^{-1}},$$

which in turn implies that L will be in the range of 2 to very large. For the moment take

$$t > \frac{a^2}{\beta_1^2 D_m},$$

which is t greater than one time constant for the first exponential term in formula (3).

Inspection of formula (3) for the range of values for L above shows that the series is very rapidly convergent for this range of t and can be well approximated by the first term, viz

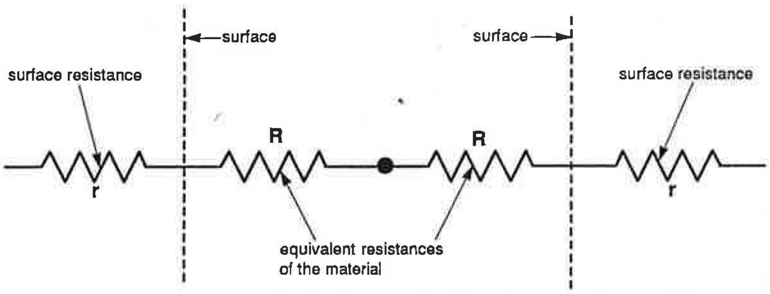

Fig. 1. Lumped vapour flow resistances for plane hygroscopic materials.

$$\frac{m-\underline{m}}{M_i-\underline{m}} \simeq \frac{2L^2 \exp\left(-\frac{\beta_1^2 D_m t}{a^2}\right)}{\beta_1^2(\beta_1^2+L^2+L)}. \tag{6}$$

For materials in intimate contact, for example insulation in contact with wood framing, it may be possible to ignore the surface resistance—this is the case of L infinite. Since $\beta_n = (2n-1)\pi/2$ for L infinite then in this case formula (3) reduces to [8]

$$\frac{m-\underline{m}}{M_i-\underline{m}} = \sum_{n=0}^{\infty} \frac{8 \exp\left(-\frac{D_m(2n+1)^2\pi^2 t}{4a^2}\right)}{(2n+1)^2\pi^2}, \tag{7}$$

which is also very rapidly convergent for t greater than one time constant for the first exponential term, i.e. for

$$t > \frac{4a^2}{\pi^2 D_m}.$$

In this case the following approximation can be written

$$\frac{m-\underline{m}}{M_i-\underline{m}} \simeq \frac{8}{\pi^2}\exp\left(-\frac{\pi^2 D_m t}{4a^2}\right). \tag{8}$$

Formula (6) and its special case formula (8) demonstrate that for times greater than one time constant the moisture desorption or absorption of the materials in our structure is exponential. In fact, for all but the smallest values of L considered here, it will be found the exponential approximation holds good for times considerably less than one time constant.

In so far then as the diffusion can be assumed constant, for times greater than one time constant, drying is exponential, i.e.

$$\frac{dm}{dt} \propto -(m-\underline{m}).$$

As explained above, this equation describes a lumped model. It remains to be shown what size the lumped parameters should be given. This is established as follows.

Let R be the equivalent total resistance to moisture movement from the middle of the sheet to one surface and r be the total surface mass transfer resistance. Total resistance is defined as

$$\text{total resistance} = \text{vapour resistance}/A,$$

where A is the area through which the moisture transfers.

Figure 1 shows that, as the moisture transfers through an area A on each of the two faces, the total resistance to moisture transfer out of the hygroscopic material R_t is given by the parallel combination of two resistances each equal to $R+r$, that is

$$R_t = \tfrac{1}{2}(R+r). \tag{9}$$

It is widely accepted that, below fibre saturation, moisture transfer in timber is driven by vapour pressure gradients [10, 11]. Vapour pressures can be determined from the moisture content using the sorption curve of the material. As in earlier work [3–5], to make progress, the simplification is made of linearising the sorption curve, i.e. it is assumed that

$$p = km, \tag{10}$$

where k is assumed constant.

In what follows we are concerned primarily with the long term behaviour of the structure and so can assume that the framing and cavity material are at a fixed temperature, being the mean temperature of the structure over the time period considered. This assumption is important to the usefulness of the linearity approximation implied by equation (10). When temperature variation becomes important the temperature dependency of k must be taken into account, see [5]. This issue is discussed further when unsteady driving forces are evoked, see below.

Note that from equation (4) above this implies that

$$D_m = kD_p. \tag{11}$$

Formula (5) can also be rewritten as

$$\text{mass flux across surface} = h(p_w - p_o), \tag{12}$$

implying that

$$\alpha = hk, \tag{13}$$

where h is the surface mass transfer coefficient under vapour pressure driving force.

If R_t is the effective vapour resistance for the hygroscopic framing material in the lumped model then equation (1) can be written as

$$V\frac{dm}{dt} = \frac{\underline{p}-p}{R_t},$$

where V is the volume of hygroscopic material associated with the area A across which the moisture flows, i.e.

$$V = 2Aa.$$

Hence using equation (10)

$$\frac{dp}{dt} = \frac{k}{V}\left(\frac{\underline{p}}{R_t} - \frac{p}{R_t}\right).$$

The solution to this equation is an exponential with a time constant t_a of

$$t_a = \frac{R_t V}{k}. \tag{14}$$

This is the time constant of the exponential drying solution to the lumped approximation being used.

On the other hand the (approximate) solution taken from Crank [8], namely equation (6), can be rewritten using equation (10) as

$$\frac{p - \underline{p}}{P_i - \underline{p}} \simeq \frac{2L^2 \exp\left(-\frac{\beta_1^2 D_m t}{a^2}\right)}{\beta_1^2(\beta_1^2 + L^2 + L)}$$

which has a time constant

$$t_a = \frac{a^2}{\beta_1^2 D_m} = \frac{a^2}{\beta_1^2 k D_p}, \tag{15}$$

by comparing (14) and (15) it follows that

$$R_t = \frac{ka}{2\beta_1^2 A D_m} = \frac{a}{2\beta_1^2 A D_p}. \tag{16}$$

When L is infinite $\beta_1 = \pi/2$, and by inspection of the tables of the values of β_n [8], it can be verified that within 2%

$$\frac{1}{\beta_1^2} = \frac{4}{\pi^2} + \frac{1}{L}.$$

Hence

$$R_t = \frac{a}{2AD_p}\left(\frac{4}{\pi^2} + \frac{1}{L}\right)$$

$$= \frac{2a}{\pi^2 A D_p} + \frac{1}{2Ah}. \tag{17}$$

Note that the moisture transfer is diffusion limited (diffusion very slow compared to surface mass transfer) if

$$\frac{1}{L} \ll \frac{4}{\pi^2}$$

in which case

$$R_t \to \frac{2a}{\pi^2 A D_p}. \tag{18}$$

Note also that if the moisture performance is diffusion limited then $R \gg r$ so that in equation (9)

$$R_t \to \tfrac{1}{2}R, \tag{19}$$

by comparing (18) and (19) it can be seen that

$$R = \frac{4a}{\pi^2 A D_p}, \tag{20}$$

and hence from equations (9) and (17)

$$r = \frac{a}{AD_p L} = \frac{1}{Ah}. \tag{21}$$

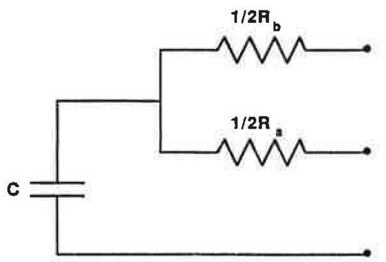

Fig. 2. Electrical circuit analogue for hygroscopic materials.

Consider now the case of material of rectangular cross section. Drying and wetting in this case can take place out of four faces. As the governing equation is linear the solution in two dimensions is just the product of two one dimensional solution equations derived above [equations (3), (6–8)], see for example Carslaw and Jaeger [12]. It follows that the time constant for drying from four faces in the exponential approximation is given by

$$\frac{1}{t_w} = \frac{1}{t_a} + \frac{1}{t_b},$$

or

$$t_w = t_a \parallel t_b. \tag{22}$$

Here $\parallel$ is the operation of determining the parallel combination of the operands, i.e. if

$$c = a \parallel b,$$

then

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{b},$$

or

$$c = \frac{ab}{a+b},$$

t_a and t_b are of the form given by equation (15), viz

$$t_a = \frac{a^2}{\beta_1^2 k_w D_{ap}} \tag{23}$$

and

$$t_b = \frac{b^2}{\beta_1^2 k_w D_{bp}}, \tag{24}$$

with D_{ap} and D_{bp} being the diffusion coefficients in the direction of the edge that will face the cavity or the linings respectively, while the resistances are of the form given by equation (20), viz

$$R_a = \frac{4a}{\pi^2 A_a D_{ap}}, \tag{25}$$

and

$$R_b = \frac{4b}{\pi^2 A_b D_{bp}}. \tag{26}$$

In the above analysis, the hygroscopic materials have been characterised in a lumped fashion with the parameters R_a, R_b, t_a and t_b. Alternatively an electrical circuit analogue can be used, see Fig. 2. This requires values to be given to the resistors and the capacitor illustrated.

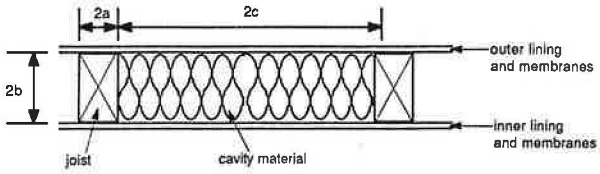

Fig. 3. Diagram of the structure type modelled.

Since the resistances and time constants are known the capacitance can be deduced by noting that for a simple RC circuit from formula (2)

$$c = \frac{t}{R}.$$

In Fig. 2 it can be seen that the total resistance is given by R_a and R_b in parallel, i.e.

$$\frac{1}{R} = \frac{1}{\frac{1}{2}R_a} + \frac{1}{\frac{1}{2}R_b}, \tag{27a}$$

or

$$R = \tfrac{1}{2}R_a \parallel \tfrac{1}{2}R_b, \tag{27b}$$

while the overall time constant t_w for the circuit is given by formula (22). It follows that

$$c = \frac{V}{k_w}. \tag{28}$$

Formulae (27) and (28) give the lumped parameters in the alternative electrical circuit analogue.

It has been shown in this section that provided the diffusion coefficient is constant and time periods greater than the time constant for drying of the hygroscopic materials (a few weeks for framing timber) are being considered then the drying is exponential. In reality the diffusion coefficient D_m is a function of moisture content. Furthermore, materials such as wood are not homogenous and initial moisture distributions are not uniform. Nevertheless the exponential drying indicated by equation (6) or (8) is a useful approximation and one for which there is some experimental evidence, see for example [13].

MODEL DEVELOPMENT AND ANALYSIS

The structure to be analysed is a flat roof or wall with a framing material which is joined to the inside and outside linings and an associated cavity filled with insulation or air, see Fig. 3. Both framing and cavity material can store moisture but the hygroscopic properties of the linings are ignored. Any membranes such as vapour barriers, building paper, sarking, etc. are lumped in with the linings.

By symmetry, for the analysis that follows only one half of the structure is considered, namely from the middle of the framing material to the middle of the insulation. The half-volume of the cavity V_o and the framing V_w are therefore given by

$$V_o = 2bc \times \text{depth},$$

$$V_w = 2ab \times \text{depth},$$

where a is half the width of the framing material; b is half the height of the cavity; c is the half-width of the cavity.

Moisture in the cavity and in the framing is conserved as follows:

> Increase in cavity moisture per unit time = flow of moisture from external regions by diffusion + flow of moisture from external regions by air leakage − flow of moisture to the framing material

which results in the equation

$$V_o \frac{\mathrm{d}m_o}{\mathrm{d}t} = \frac{p_w - p_o}{R_{wo}} + \sum_{i=1}^{2} \left(\frac{A_{io}(p_i - p_o)}{r_{io}} + V_o(F_{io}c_i - F_{oi}c_o) \right), \tag{29a}$$

and

> Increase in framing material moisture per unit time = flow of moisture from cavity + flow in moisture from exterior regions through the linings

giving

$$V_w \frac{\mathrm{d}m_w}{\mathrm{d}t} = \frac{p_o - p_w}{R_{wo}} + \sum_{j=1}^{2} \frac{A_{jw}(p_j - p_w)}{r_{jw}}, \tag{29b}$$

where the subscripts are defined as follows:

> i,j—external regions, $i, j = 1$ or 2; o—cavity; w—framing material

and

> c_p is the concentration of moisture (kg m^{-3}) in the air in region p; m_i is the moisture concentration (kg m^{-3}) in the material in region i; r_{pq} is the series sum of all vapour resistances between region p and q (N s kg^{-1}); R_{wo} is the total vapour resistance including area, between the centre of the framing material and the centre of the insulation (N s kg^{-1} m^{-2}); A_{pq} is the area (m^2) between region p and q; F_{pq} is the air change rate (s^{-1}) between region p and q.

Also the net air flow into the cavity is zero, i.e.

$$\sum_{i=1}^{2} (F_{io} - F_{oi}) = 0.$$

Note that the term $F_{io}c_i - F_{oi}c_o$ in equation (29a) assumes perfect mixing of the air flows in the cavity.

It is straightforward to include internal moisture sources, other moisture source regions [5], or leaks into the model at this stage but these are excluded here for simplicity.

Water vapour concentrations c_i are now converted to vapour pressure by assuming water vapour to be an ideal gas, i.e.

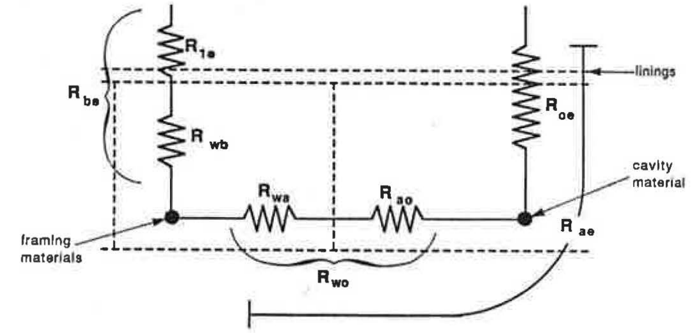

Fig. 4. Definitions of the lumped total vapour flow resistances.

$$p_i = \frac{c_i RT}{W}, \tag{30}$$

where R is the universal gas constant, T is the Kelvin temperature of the air and W is the molecular weight of water.

A number of lumping definitions are now made to simplify these conservation equations, see Fig. 4, i.e.

$$\frac{1}{R_{oe}} = \sum_{i=1}^{2}\left(\frac{A_{io}}{r_{io}} + \frac{V_o F_{oi} W}{RT}\right),$$

$$\frac{1}{R_{be}} = \sum_{j=1}^{2}\frac{A_{jw}}{r_{jw}},$$

$$R_{wa} = \frac{4a}{\pi^2 A_a D_{ap}},$$

$$R_{wb} = \frac{4b}{\pi^2 A_b D_{bp}},$$

$$R_{ao} = \frac{4c}{\pi^2 A_a D_{op}},$$

$$R_{ae} = R_{wo} + R_{oe},$$

$$R_{1e} = R_{be} - R_{wb}$$

$$\bar{p}_o = R_{oe}\sum_{i=1}^{2}\left(\frac{A_{io}}{r_{io}} + \frac{V_o F_{io} W}{RT}\right)p_i,$$

$$\bar{p}_w = R_{be}\sum_{j=1}^{2}\frac{A_{jw}p_j}{r_{jw}}, \tag{31}$$

where

$$p_o = k_o m_o,$$

and

$$p_w = k_w m_w,$$

and D_{op} is the diffusion coefficient of the cavity material in the direction parallel to the linings.

k_o and k_w are temperature dependent but are taken here as constant at their mean value over the range of temperatures under consideration, see discussion in the previous section.

With these definitions equations (29) become

$$\frac{V_o}{k_o}\frac{dp_o}{dt} = \frac{p_w - p_o}{R_{wo}} + \frac{\bar{p}_o - p_o}{R_{oe}} \tag{32a}$$

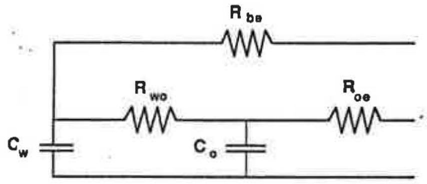

Fig. 5. Electric circuit analogue of the structure.

$$\frac{V_w}{k_w}\frac{dp_w}{dt} = \frac{p_o - p_w}{R_{wo}} + \frac{\bar{p}_w - p_w}{R_{be}}. \tag{32b}$$

Figure 5 shows an electrical equivalent of this structure as represented by equations (32).

The steady state solution to these equations is found by putting the time derivatives to zero and is

$$\underline{p}_o = \frac{R_{oe}\bar{p}_w + (R_{be} + R_{wo})\bar{p}_o}{R_{ae} + R_{be}},$$

$$\underline{p}_w = \frac{R_{ae}\bar{p}_w + R_{be}\bar{p}_o}{R_{ae} + R_{be}}, \tag{33}$$

where underlining is used to denote steady state values.

The steady state cavity and framing vapour pressures and moisture contents are seen to be a weighted mean of the driving (external) vapour pressures, the weights depending upon the relative ease with which each driving vapour pressure can influence the volume under consideration.

If R_{be} is infinite, that is, flow through the framing to the linings is negligible, then

$$\underline{p}_o = \underline{p}_w = \bar{p}_o,$$

as was found earlier [3–5].

The time dependent case with initial moisture contents below fibre saturation is solved first. The initial conditions are taken at $t = 0$ as

$$\begin{cases} p_o = P_o \quad \text{or} \quad m_o = M_o \\ p_w = P_w \quad \text{or} \quad m_w = M_w \end{cases} \tag{34}$$

with steady driving forces, i.e. the step function case. The following definitions are now made

$$t_a' = \frac{V_w R_{wo}}{k_w}$$

$$t_o = \frac{V_o R_{oe}}{k_o}$$

$$\frac{1}{t_p} = \frac{k_o}{V_o}\left(\frac{1}{R_{oe}} + \frac{1}{R_{wo}}\right)$$

$$v = \frac{V_w}{V_o}$$

$$\eta = \frac{R_{wo}}{R_{be}}$$

$$\lambda = \frac{R_{ae}}{R_{be}}$$

$$\mathscr{K} = \frac{k_w}{k_o}. \quad (35)$$

Note that in the case of the cavity containing air, under the assumption of an ideal gas, equation (30), we have

$$k_o = \frac{RT}{W},$$

and therefore

$$\mathscr{K} = \frac{k_w W}{RT},$$

as was defined earlier [3–5].

Since the driving forces are steady the temperature dependence of k_o and k_w is irrelevant so that the full solution of equations (32) under initial conditions (34) is found to be

$$m_w = A\,e^{-t/t_1} + B\,e^{-t/t_2} + \underline{m}_w$$

$$m_o = C\,e^{-t/t_1} + D\,e^{-t/t_2} + \underline{m}_o \quad (36)$$

where

$$\underline{m}_o = \frac{\underline{p}_o}{k_o}, \quad \underline{m}_w = \frac{\underline{p}_w}{k_w},$$

and the time constants t_1 and t_2 are

$$\frac{1}{t_1}, \frac{1}{t_2} = \frac{1}{2}\left(\frac{1}{t_o} + \frac{1}{t'_a} + \frac{v}{\mathscr{K} t'_a} + \frac{\eta}{t'_a} \pm \sqrt{\left(\frac{1}{t_o} + \frac{1}{t'_a} + \frac{v}{\mathscr{K} t'_a} + \frac{\eta}{t'_a}\right)^2 - \frac{4}{t_o t'_a}(1+\lambda)}\right) \quad (37)$$

Using the initial conditions (34) A, B, C and D can be evaluated as

$$A = \left\{\frac{1}{t'_a}\left(\frac{M_o}{\mathscr{K}} - (1+\eta)M_w\right) - \frac{1}{t_2}(\underline{m}_w - M_w) + \frac{\eta \bar{m}_w}{t'_a}\right\} \Big/ \left(\frac{1}{t_2} - \frac{1}{t_1}\right)$$

$$B = M_w - \underline{m}_w - A, \quad C = \mathscr{K}\left(1 + \eta - \frac{t'_a}{t_1}\right)A,$$

$$D = \mathscr{K}\left(1 + \eta - \frac{t'_a}{t_2}\right)B.$$

The significance of these solutions is examined in the next section.

Equations (32) are now solved in the case where the initial moisture content of the framing material is above fibre saturation. Other conditions are as before, i.e. the initial cavity moisture content is below fibre saturation and the driving forces are steady.

This case gives rise to qualitatively different solutions, because as long as the moisture content of the framing material is above fibre saturation its vapour pressure remains constant at the saturation vapour pressure appropriate to the temperature of the material. It is assumed here that this vapour pressure remains as the driving force for drying at the surface for as long as the mean moisture content of the framing material is above fibre saturation. If this statement were strictly true then moisture transfer out of the framing material would be controlled by surface resistance only. However the true situation is more complex than this and no attempt is made here to derive an equivalent resistance formula comparable to equation (17) for the case of moisture contents above fibre saturation. Note that dp_w/dt must be replaced by $k_w\, dm_w/dt$ in equation (32b), since p_w is now constant.

With these assumptions equation (32a) has the solution

$$p_o = (P_o - \underline{p}_o)\,e^{-t/t_p} + \underline{p}_o \quad (38)$$

or

$$m_o = (M_o - \underline{m}_o)\,e^{-t/t_p} + \underline{m}_o$$

where

$$\underline{p}_o = \frac{R_{oe} p_w + R_{wo} \bar{p}_o}{R_{ae}}$$

and after substituting this result into the modified form of equation (32b) the expression for m_w is

$$m_w = M_w + \frac{t_p}{V_w}\left(\frac{P_o - \underline{p}_o}{R_{wo}}\right)(1 - e^{-t/t_p}) - \frac{t}{V_w}\left(\frac{p_w - \bar{p}_o}{R_{ae}} + \frac{p_m - \bar{p}_w}{R_{be}}\right).$$

These results show that, after an initial transient, as long as the moisture content of the framing material remains above fibre saturation the vapour pressure and moisture content in the cavity material remains constant at $\underline{p}_o$ and $\underline{m}_o$ respectively while the framing material dries out at a constant rate given by

$$\frac{dm_w}{dt} = -\frac{1}{V_w}\left(\frac{p_w - \bar{p}_o}{R_{ae}} + \frac{p_w - \bar{p}_w}{R_{be}}\right). \quad (39)$$

The transient has a time constant of t_p which is found to have a value of less than an hour for loose cavities containing air to values in the order of a day for tight cavities filled with insulation. This transient can be interpreted as the time taken for the cavity to come into equilibrium with the framing.

PROPERTIES OF THE LONGER TIME CONSTANT SOLUTIONS

The solution below fibre saturation exhibits two time constants while the solution above fibre saturation exhibits a short time constant and a rate term. This section concentrates on the properties of the longer time constant or the rate part of the solution since they are of more practical interest. The short term behaviour of a structure will be more complex than this model allows because of the influence over these time periods of the possible hygroscopic nature of the linings and initial non-uniform moisture concentrations.

It must first be shown that the short time constant transient is rapid enough to be ignored over the longer time periods of interest here, namely weeks and months.

An expression for the initial transient time constant t_p, equation (35), for the case of initial framing moisture content above fibre saturation has already been derived. Using values of parameters likely to be encountered in practice this will have values in the range of an hour to a day, as mentioned before.

Similar considerations for the below fibre saturation case show that the shorter time constant t_1 is approximately

$$t_1 = \frac{t_o}{1+\dfrac{vt_o}{\mathscr{K}t_a'}+\dfrac{t_o}{t_a'}(1+\eta)}, \tag{40}$$

which also will have values in the order of an hour to a day.

In the below fibre saturation case once the initial t_1 transient has passed the solutions (36) of equations (32) can be rewritten in a simpler form. Since now

$$\frac{dp_o}{dt} \simeq 0$$

then from equation (32a)

$$p_o = \left(\frac{p_w}{R_{wo}}+\frac{\bar{p}_o}{R_{oe}}\right)\Big/\left(\frac{1}{R_{wo}}+\frac{1}{R_{oe}}\right)$$

which upon substituting into equation (32b) and solving the resulting differential equation gives

$$p_w = (P_w - \underline{p}_w)\,e^{-t/t_2} + \underline{p}_w$$

where

$$\underline{p}_w = \frac{R_{ae}\bar{p}_w + R_{be}\bar{p}_o}{R_{ae}+R_{be}}$$

as before (equation (33)) and where

$$t_2 = \frac{V_w}{k_w} R_{ae} \parallel R_{be}$$

or

$$t_2 \doteq \frac{V_w}{k_w}\,\frac{R_{ae}R_{be}}{R_{ae}+R_{be}}. \tag{41}$$

Equations (40) and (41) represent the simplified form of the two time constants t_1 and t_2 first derived in equations (37). Recall that k_w and k_o are temperature dependent so that t_1 and t_2 will take on different values for different mean temperatures.

In the equivalent circuit analogue, Fig. 5, once the capacitor representing the cavity is fully charged, it no longer contributes significantly to the circuit performance. Hence the circuit becomes a simple RC combination and can be redrawn as in Fig. 6.

Intuitively one would expect that once the framing material has been enclosed its drying time t_2 would be longer than for the unenclosed material. This can be placed on a quantitative basis by comparing the time constant t_2 in formula (41) above to the time constant t_w, equation (22) for unenclosed drying for the framing material.

From equation (41)

$$\frac{1}{t_2} = \frac{k_w}{V_w R_{ae} \parallel R_{be}}$$

$$= \frac{k_w}{V_w}\left(\frac{1}{R_{wo}+R_{ao}+R_{oe}}+\frac{1}{R_{wb}+R_{le}}\right)$$

$$= \frac{k_w}{V_w}\left(\frac{1}{R_{wo}(1+\gamma)}+\frac{1}{R_{wb}(1+\delta)}\right), \tag{42}$$

where

$$\gamma = \frac{R_{ao}+R_{oe}}{R_{wa}}, \tag{43}$$

and

$$\delta = \frac{R_{le}}{R_{wb}}. \tag{44}$$

For comparison, take the case of unenclosed drying being diffusion limited, i.e. diffusion very much slower than surface mass transfer. Unenclosed timber will probably be in this regime because the appropriate parameter L will be 2 or larger as shown above. In this case from equation (14)

$$t_a = \frac{V_w}{k_w}(R_{wa}+r_{sa}) \simeq \frac{V_w R_{wa}}{k_w}, \tag{45}$$

where r_{sa} is the surface resistance at the edge to face the cavity, and

$$t_b \simeq \frac{V_w R_{wb}}{k_w}.$$

Expression (42) now becomes

$$\frac{1}{t_2} = \frac{1}{t_a(1+\gamma)}+\frac{1}{t_b(1+\delta)} \tag{46a}$$

or

$$t_2 = t_a(1+\gamma) \parallel t_b(1+\delta). \tag{46b}$$

This is the key result of this work in that it shows how the long term time constant—physically, the drying time of the framing material once enclosed in the structure—is increased over the unenclosed value, according to the air and vapour tightness construction details of the structure and the driving forces upon it.

However the value of this result goes much further than this: the value of this time constant allows the moisture behaviour of the structure to be predicted for any driving forces in so far as linearity can be assumed; in particular, provided that the temperature dependence of k_w and k_o can be allowed for at least approximately, the seasonal behaviour of the cavity can now be derived, see [5]. In the electric circuit analogue, knowing the longer

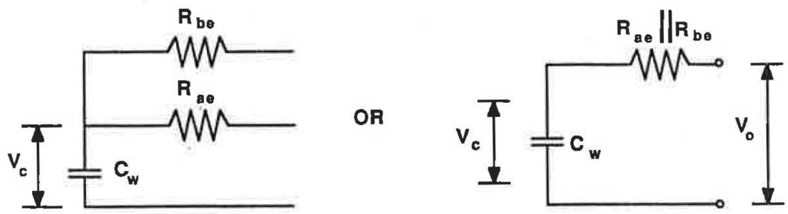

Fig. 6. Simplified equivalent circuit for the structure.

time constant t_2 is equivalent to knowing the size of the capacitor and resistor in the equivalent circuit shown in Fig. 6 and once this is known the performance of the circuit under any driving voltage can be predicted. It will be appreciated that what is predictable knowing only t_2 is the longer term behaviour only of the structure. The shorter term behaviour with time periods of less than say, one day, is governed by t_1 in this model and by even more complex considerations in reality.

The formula derived in earlier work can be recovered by considering the case of the cavity material containing air and noting that δ is infinite if moisture flow to the framing material is insignificant. Formula (46) for the drying time constant is found to reduce under these circumstances to

$$\frac{t_2}{t_a} = 1 + \frac{\nu t_o}{\mathscr{K} t_a} \tag{47}$$

which is identical to the expression derived in earlier work [3–5].

A similar analysis can be carried out for the constant rate of drying term, equation (39), in the above fibre saturation case as follows.

In this case it is necessary to examine the time T_2 taken for the framing to dry from a moisture content m_1 above fibre saturation down to a moisture content m_2 also above fibre saturation. Let T_a be the time taken for the framing material to dry through this range in air through the faces that will face the cavity and T_b be the time taken to dry through the other two faces to air. At steady state if p_e is the external vapour pressure then by conservation of moisture in the framing material it follows that

$$V_w \frac{\mathrm{d}m_w}{\mathrm{d}t} = \frac{p_w - p_e}{R_{wa} + r_{sa}},$$

for drying through the edges to face the cavity, with p_w constant in this case, then

$$T_a = \frac{V_w(m_2 - m_1)(R_{wa} + r_{sa})}{p_w - p_e}.$$

Similarly

$$T_b = \frac{V_w(m_2 - m_1)(R_{wa} + r_{sb})}{p_w - p_e}.$$

Once the framing material is enclosed and steady state has been reached the governing equation (39) becomes

$$\frac{\mathrm{d}m_w}{\mathrm{d}t} = -\frac{1}{V_w}(p_w - \bar{p}_e)\left(\frac{1}{R_{ae}} + \frac{1}{R_{be}}\right),$$

where $\bar{p}_e$ is a weighted driving vapour pressure defined by

$$\bar{p}_e = \left(\frac{\bar{p}_o}{R_{ae}} + \frac{\bar{p}_w}{R_{be}}\right) \Big/ \left(\frac{1}{R_{ae}} + \frac{1}{R_{be}}\right).$$

Hence the time T_2 taken for the enclosed framing to dry from a moisture content m_1 down to a moisture content m_2, both above fibre saturation, is given by

$$\frac{1}{T_2} = \frac{(p_w - \bar{p}_e)}{V_w(m_2 - m_1)}\left(\frac{1}{R_{ae}} + \frac{1}{R_{be}}\right). \tag{48}$$

In terms of the time taken for the unenclosed framing material to dry equation (48) becomes

$$\frac{1}{T_2} = \frac{1}{\xi T_a} + \frac{1}{\zeta T_b}, \tag{49}$$

taking $p_e = \bar{p}_e$ for comparative purposes and where

$$\xi = \frac{R_{ae}}{R_{wa} + r_{sa}}, \quad \zeta = \frac{R_{be}}{R_{wb} + r_{sb}}.$$

Only if $r_{sa} \ll R_{wa}$ and $r_{sb} \ll R_{wb}$ could $\xi = 1 + \gamma$ and $\zeta = 1 + \delta$ in which case equation (49) for the above fibre saturation would take the same form as equation (46) for below fibre saturation. If anything, for the above fibre saturation case, the surface resistances are likely to dominate so that equations (46) and (49) must remain in distinctly different forms.

Whether the expressions derived here for the enclosed drying time compared to the unenclosed drying time (equation (46)) give rise to results that are significantly different from those that would be obtained using the expression derived in earlier work, equation (47), clearly depends upon the size of the parameters involved—if the alternative path for moisture transfer from the framing material through the linings is a relatively important one then the results may differ significantly, otherwise not. This is pursued further in the next section in which some specific examples are given.

In earlier work [3], as a result of the formula for the increase in drying time for enclosed framing material, three types of structures were identified according to the tightness. They were: "hygroscopically controlled", "intermediate", and "construction controlled". These concepts remain useful here. If γ and δ are small, much less than one, then the structure can be described as "hygroscopically controlled", i.e. the moisture behaviour of the framing material does not differ significantly from the unenclosed material and hence is dominated only by the hygroscopic nature of the material. If γ and δ are large, or more precisely if t_2 is significantly greater than t_w, then the moisture behaviour can be described as "construction controlled" because the rate of drying is determined by the air and vapour tightness of the structure. The intermediate case lies between these extremes.

One of the new features in this work is to allow for the possible presence of insulation in the cavity. The effect of the insulation is to increase both the short and long term constants t_1 and t_2.

The shorter time constant t_1 is lengthened depending upon the hygroscopic nature of the cavity material. This can be seen by rewriting t_1, equation (40), in terms of resistances. This gives

$$t_1 = \frac{V_o R_{oe}}{k_o\left(1 + \frac{R_{oe}}{R_{wo}} + \frac{\mathscr{K}}{v}\left(\frac{R_{oe}}{R_{wo}} + \frac{R_{oe}}{R_{be}}\right)\right)},$$

k_o becomes smaller for more hygroscopic materials and R_{oe} becomes larger for higher vapour resistance materials—both terms will increase the smaller time constant t_1. If the cavity material is air which cannot hold much water then t_1 is very small, in the order of hours; otherwise the time constant will be longer depending upon the exact hygroscopic nature of the material and its vapour resistance, perhaps up to the order of days.

The insulation increases the longer (drying) time constant through its vapour resistance. This can be seen by examining equation (46)

$$t_2 = t_a(1+\gamma) \parallel t_b(1+\delta)$$

$$= t_a\left(1 + \frac{R_{ao} + R_{oe}}{R_{wa}}\right) \Bigg\| \, t_b(1+\delta).$$

As the cavity material vapour resistance increases so does R_{ao} and R_{oe} which in turn increases γ and hence the long term drying constant t_2. Even for relatively low vapour resistance materials such as fibreglass this increase in γ can be quite significant, see the examples below.

EXAMPLES

In this section some examples are presented to illustrate the application of formula (46) to find the drying time constant for a building structure. Further examples show how knowledge of this time constant implies complete knowledge of the moisture behaviour of the structure [5] (within the limitations of the model assumptions).

Two structures are considered and two subcases are considered for each structure. Structure 1 has 50 × 50 mm timber joists spaced at 500 mm centres with the cavity in between filled with fibreglass. Structure 2 is similar except that the joist is 50 × 100 mm and is orientated so that the cavity between the joists is 100 mm across and also filled with insulation. For definiteness the structure is considered to be 1 m deep in the third dimension but of course its exact value is irrelevant in finding time constants.

For each structure two subcases are considered. Subcase (a) has the internal linings plus membranes with a vapour resistance of 2 GNs kg^{-1} and the external linings with a vapour resistance of 1 GNs kg^{-1}. Subcase (b) has the internal linings with a vapour resistance of 20 GNs kg^{-1} and the external linings with a vapour resistance of 10 GNs kg^{-1}.

In all cases the driving air pressures and air permeabilities are such that the air change in the cavity is 0.5 air changes per hour. (To calculate the long term time constant the direction of air movement is not needed.) The mean temperature of the structure is taken as 11°C. The diffusion coefficient (vapour pressure driven) for the wood is taken as 7.32×10^{-12} s and for the insulation as 1.67×10^{-10} s. k_w is taken as 20 m^2 s^{-2}.

Formulae (46) are used to calculate the time constants t_a and t_b for open air drying of the joists. This comes to 20 days for drying through faces 50 mm apart and 80 days through faces 100 mm apart. Hence from formula (22) the overall drying time in air will be 10 days for a 50 × 50 mm joist and 16 days for a 50 × 100 mm joist.

To find how these drying times increase when the structure is enclosed we proceed as follows. R_{oe} is calculated from formula (31) and γ calculated from its definition formula (43). Similarly R_{be} and R_{wb} are calculated from their definitions, formulae (31). R_{1e} is then found by subtracting R_{wb} from R_{be} as indicated in formulae (31) and δ calculated from its definition, formula (44). This allows the drying time constant t_2 to be calculated from formula (46).

Table 1 shows the results of these calculations and the values that would be obtained from earlier work, formula (47), assuming drying through the joist edges facing the insulation only.

Two opposing effects have been included in this model in refining earlier work. Firstly the framing material enclosed drying time has been decreased because an alternative path from the joist through the linings has been included. This effect is most significant in case 1(a) where the alternative path presents the lowest resistance of all cases considered, causing the enclosed drying time constant to be reduced to 17.2 days. Secondly the drying time has been increased because moisture passing through the cavity must pass through insulation rather than air as assumed in the earlier work. These opposing effects have nearly cancelled in all cases except 1(a). However it is possible that this model overestimates the contribution from the cavity insulation vapour resistance because the effect of lumping is to cause all the moisture to flow to the centre of the cavity and then turn at right angles to flow to the linings; in reality flow from the joist to the linings via the cavity is fully two-dimensional, causing the effective vapour resistance to be less than that given by this model.

To highlight the fact that knowledge of the drying constant t_2 implies complete knowledge of the longer term moisture performance of the structure (assuming linearity), the case of periodic driving forces will now be considered. It was shown in [5] that the chief factor contributing to non-linearity is the temperature dependence of k_w and k_o. This problem was partially overcome by using as a value for t_2 its mean value over the range of temperatures that are of concern, written as $\bar{t}_2$.

Consider now for example the important case of seasonal variation in the moisture content of the structure. In this case the driving forces can be approximated as [5]

$$\bar{p} = \tilde{p} + \Delta p \sin \omega t$$

where $\tilde{p}$ is the mean value of $\bar{p}$ and Δp is the maximum deviation of the driving force from this mean and ω is the angular frequency of the driving forces.

Table 1. Drying time constants for various structures

Quantity	Formula number	1. 50 × 50 mm Joist: 1(a) Low resistance linings	1(b) High resistance linings	2. 100 × 50 mm Joist: 2(a) Low resistance linings	2(b) High resistance linings
γ	(43)	0.50	1.19	0.61	1.66
δ	(44)	1.02	9.75	0.52	4.92
Drying time constant for unenclosed framing, t_w	(22)	10.0 days	10.0 days	16.0 days	16.0 days
Drying time constant for enclosed framing, t_2	(46)	17.2 days	36.4 days	25.5 days	47.8 days
Drying time constant for enclosed framing from earlier work	(47)	22.1 days	35.9 days	24.0 days	45.0 days

Table 2. Seasonal moisture behaviour for various structures

Quantity	1. 50 × 50 mm Joist: 1(a) Low resistance linings	1(b) High resistance linings	2. 100 × 50 mm Joist: 2(a) Low resistance linings	2(b) High resistance linings
Drying time constant for enclosed framing, t_2	17.3 days	36.8 days	25.8 days	48.5 days
Phase lag	0.56 months	1.10 months	0.81 months	1.35 months
Amplitude response	0.96	0.84	0.91	0.77

The moisture content of the framing material can be simply determined by examining the *RC* circuit analogue in Fig. 6. From circuit analysis the voltage V_c across the capacitor for this circuit is

$$V_c = \frac{V_o}{\sqrt{1+(\omega\bar{t}_2)^2}},$$

where V_o is the driving voltage and $\bar{t}_2$ is the meaned time constant of the circuit.

The phase φ of the voltage across the capacitor lags the phase of the driving voltage by

$$\varphi = \tan^{-1}(\omega\bar{t}_2).$$

Taking the driving period as 1 year, Table 2 contains the amplitude response and phase lag of the framing material vapour pressure and hence moisture content compared to the driving forces for each of the four cases analysed in the examples above. For example in the case of the structure with a 50 × 100 mm joist and high vapour resistance linings, the maximum seasonal moisture content in the joist occurs 1.35 months later than the peak driving forces, and the value of the deviation of the moisture content from the yearly mean value is only 77% of that which would be predicted from assuming the timber was in moisture equilibrium with the driving forces. In fact as pointed out earlier [5], the droop in amplitude and phase lag is only significant for tight structures, that is, structures in which the enclosed drying time constant is significantly longer than the unenclosed time constant.

CONCLUSIONS

This work is an attempt to find through analytical modelling, parameters that characterise the moisture performance of the structure as a unit. These system-wide parameters are not easy to find if numerical modelling is used. The system-wide parameter discovered here is the time constant t_2 which describes the drying of the enclosed timber frame. The existence of this parameter and an understanding of its meaning should provide insight into the moisture performance of a structure and provide a means for the researcher to communicate to the practitioner, be he engineer, architect or builder.

However this simplicity has not been obtained without cost. Quite strong simplifications have been made to enable the differential equations to be solved, the most significant of which is to assume that over long time periods (say 1 year) one can satisfactorily take account of the temperature dependence of the parameter being used to describe the sorption curves, k, by using its mean value over the time period under consideration. Other simplifications are less significant, but still important. It has been shown for example that the assumption of exponential drying is only true if the diffusion coefficient is concentration independent. In fact, experimental evidence exists, see [10], to show that the exponential drying approximation is a valid one.

Three ways forward are indicated which still preserve the essential philosophy underlying this work, viz. to provide system-wide parameters with simple physical meaning in order to allow a better intuitive understanding of the moisture performance of the structure. Ways forward are: 1, experimental validation; 2, numerical modelling to test the limits of the analytical solutions; and 3, refinement of the analytical solutions addressing some of the simplifications mentioned above.

Until the conclusions of this model and others like it are tested experimentally it remains uncertain to what degree the concepts derived here will be reflected in the actual moisture performance of structures. Programmes are in place to provide the necessary experimental validation, see for example [14].

The model developed here has two time constants. The longer term time constant is interpreted as the time constant for the drying of the enclosed framing material and is given by

$$\frac{1}{t_2} = \frac{1}{t_a(1+\gamma)} + \frac{1}{t_b(1+\delta)}$$

or

$$t_2 = t_a(1+\gamma) \parallel t_b(1+\delta). \quad (50)$$

This is the key result of this work. The formula describes quantitatively how the drying time of the framing material enclosed in a structure is increased over the unenclosed drying time. Once this longer term time constant is known then under the assumption of linearity used here the moisture performance of the model under any (longer term) driving force is known. In particular, the seasonal moisture performance of the structure can be ascertained.

The shorter time constant can be interpreted as the time constant for the process of the framing material and the cavity material to come into equilibrium. It is not claimed here that the short term moisture performance of a real structure has been accurately modelled. Other factors not considered will have an important influence in the shorter term such as the hygroscopic nature of the linings, nonuniform initial moisture distributions, non-linearities, etc.

The formula for the drying time constant generalises the formula obtained in earlier work because more detail has been included in the model described in this paper. In particular a flow path connecting the framing material directly to the linings has been included and the cavity material does not now have to be air only.

This paper has also shown that, provided the diffusion coefficient of the framing material can be taken as approximately constant, then it can be safely assumed that drying of the framing material below fibre saturation is exponential. It has been shown that if the moisture content of the framing material is above fibre saturation then the framing material dries linearly at a slower rate than if it were unenclosed.

The chief value of an analytical model such as that developed here is to condense into one formula a single parameter, the long term drying time constant, which will explain and predict the moisture behaviour of a wide number of different structures under a very wide range of driving forces.

REFERENCES

1. W. P. Marsh, *Thermal Insulation and Condensation*. Construction Press, Lancaster (1979).
2. G. Keiper, W. Cammerer and A. Wagner, A new diagram to evaluate the performance of building constructions with a view to water vapour diffusion. C.I.B. W40 Working Commission (1976).
3. M. J. Cunningham, A new analytical approach to the long term behaviour of moisture concentrations in building cavities—I. Non-condensing cavities. *Bldg Envir.* **18**, 109–116 (1983).
4. M. J. Cunningham, A new analytical approach to the long term behaviour of moisture concentrations in building cavities—II. Condensing cavity. *Bldg Envir.* **18**, 117–124 (1983).
5. M. J. Cunningham, Further analytical studies of building cavity moisture concentrations. *Bldg Envir.* **19**, 21–29 (1984).
6. Code of practice for specifying timber and wood based products for use in building. NZS 3602 : 1975. New Zealand Standard Association, Wellington (1975).
7. Y. Takahashi, M. Rabins and D. M. Auslander, *Control and Dynamic Systems*. Addison-Wesley, Mass. (1972).
8. J. Crank, *The Mathematics of Diffusion*. Oxford University Press, London (1970).
9. J. F. Siau, *Transport Processes in Wood*. Springer-Verlag, Berlin (1984).
10. G. Bramhall, Sorption diffusion in wood. *Wood Sci.* **12**, 3–13 (1979).
11. M. Bomberg, Moisture flow through porous building materials. Report 52, Division of Building Technology, Sweden (1974).
12. H. S. Carslaw and J. C. Jaeger, Conduction of heat in solids. Oxford University Press, London (1959).
13. M. J. Cunningham, Drying of construction moisture in timber framed flat roofs. I. Low air leakage. *ASHRAE Trans.* **93**, Part 2, 153–170 (1987).
14. Research programme of work 1986–87. Building Research Association of New Zealand, Judgeford (1986).