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Temperature Under a House With Variable Insulation

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A calculation method of high accuracy is presented for the two-dimensional steady-state ground temperature under a long house. The thermal insulation thickness is arbitrarily variable along the ground surface. Explicit, iterative formulas with rapid convergency are given. The heat loss from the house and the ground surface temperature are given for three types of insulations. The method is available as a PC-program.

NOMENCLATURE

- a_n Fourier coefficient for the temperature under the thermal insulation
- b_n Fourier coefficient for the prescribed temperature above the thermal insulation
- B width of the house
- C_{mn} matrix coefficient
- d equivalent thermal insulation thickness
- d_i thermal insulation thickness
- $\tilde{d}$ equivalent insulation thickness for the extra thermal insulation at the edges of the house
- D width of the extra thermal insulation
- f prescribed temperature above the insulation
- f' non-dimensional temperature above the insulation
- h_s non-dimensional steady-state heat loss factor
- J_{mn} matrix coefficient
- L total length of the thermal insulation
- N number of Fourier coefficients
- q_s steady-state heat loss for the house
- T temperature in the ground
- T_0 annual mean temperature at the ground surface
- T_i temperature inside the house
- U non-dimensional temperature in the ground
- w complex plane $w = \xi + i \cdot \eta$
- x horizontal coordinate
- x' non-dimensional horizontal coordinate
- y vertical coordinate
- y' non-dimensional vertical coordinate
- z' complex plane $z' = x' + i \cdot y'$
- α angle giving the position of the extra thermal insulation
- γ angle giving the position of the house
- η Im(w)
- λ thermal conductivity of the ground
- λ_i thermal conductivity of the thermal insulation
- ξ Re(w)

INTRODUCTION

THIS PAPER presents a semi-analytical method for an accurate calculation of the steady-state temperature in the ground under a house, and of the heat loss. The heat flow process in the ground is assumed to be two-dimensional. This will give a good approximation for a vertical cross-section of a long house. Figure 1 shows the vertical cross-section for such a house. The total heat loss from the house to the ground is denoted by q_s.

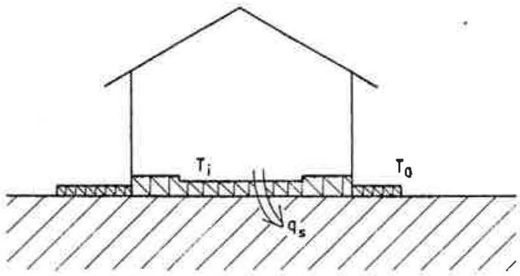

Fig. 1. Steady-state heat flow in the ground under a house with variable thermal insulation thickness along the ground surface.

The ground is assumed to be homogeneous and semi-infinite. The house has no cellar. The insulation is placed directly on the ground surface. The insulation thickness under the house and at the ground surface outside the house may vary from point to point. The indoor temperature T_i is constant for the case shown in Fig. 1. The temperature T_0 outside the house is also constant. In the general case, the given temperature above the insulation may vary from point to point. The presented semi-analytical method gives the unknown temperature in the ground. The ground surface temperature under the building foundation is of great interest, since it is an important factor for condensation and ensuing moisture damages.

PROBLEM

The heat conduction problem

The heat flow region is shown in Fig. 2 together with the boundary conditions. (The coordinate-axis y points downwards in the ground.) The steady-state temperature is denoted by $T(x, y)$. The soil $y > 0$, is homogeneous with thermal conductivity λ. The width of the house is B. The ground surface is covered with insulation for

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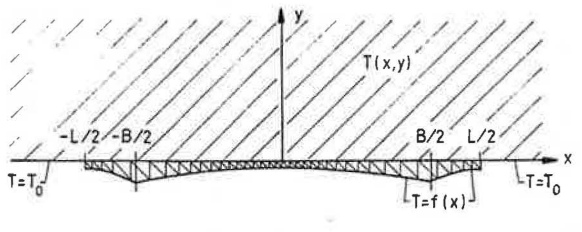

Fig. 2. Boundary condition at the ground surface with a variable insulation thickness $d_i(x)$.

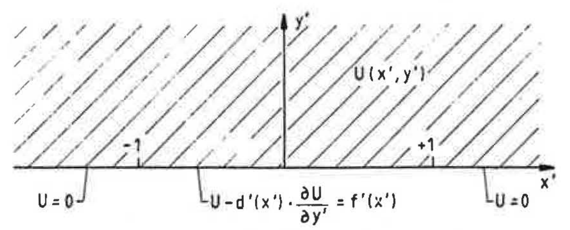

Fig. 3. Heat flow problem in non-dimensional form.

$-L/2 < x < L/2$, where L is equal to or greater than B. The insulation is of thickness $d_i(x)$, $-L/2 < x < L/2$, and thermal conductivity λ_i. The insulation thickness is greater than or equal to zero. The prescribed temperature above the insulation is denoted by $f(x)$. The temperature $T(x, y)$ satisfies the heat conduction equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad -\infty < x < \infty, \quad y > 0. \tag{1}$$

The boundary conditions at the ground surface are:

$$T = T_0 \quad |x| > L/2, \quad y = 0,$$

$$T - d(x)\frac{\partial T}{\partial y} = f(x) \quad |x| < L/2, \quad y = 0. \tag{2}$$

The equivalent insulation thickness d is introduced. It is defined by:

$$d(x) = \frac{\lambda \cdot d_i(x)}{\lambda_i}. \tag{3}$$

A soil layer of thickness d has the same thermal resistance as the insulation layer of thickness d_i since $d/\lambda = d_i/\lambda_i$.

Scaling

The heat conduction problem can be expressed in a non-dimensional form. The coordinates are scaled with the length $L/2$:

$$x' = \frac{x}{L/2}, \tag{4}$$

$$y' = \frac{y}{L/2}. \tag{5}$$

The non-dimensional temperature U is defined as:

$$U = \frac{T - T_0}{T_i - T_0}. \tag{6}$$

The temperature T_i is the indoor temperature or, in the general case when the temperature varies along the surface, for instance the maximum value of $f(x)$. The non-dimensional boundary temperature f' is defined as:

$$f' = \frac{f - T_0}{T_i - T_0}. \tag{7}$$

The scaled insulation thickness is:

$$d' = \frac{d}{L/2}. \tag{8}$$

The heat conduction equation in non-dimensional form is then:

$$\frac{\partial^2 U}{\partial (x')^2} + \frac{\partial^2 U}{\partial (y')^2} = 0 \quad -\infty < x' < \infty, \quad y' > 0. \tag{9}$$

The boundary conditions become:

$$U = 0 \quad |x'| > 1, \quad y' = 0$$

$$U - d'(x')\frac{\partial U}{\partial y'} = f'(x') \quad |x'| < 1, \quad y' = 0. \tag{10}$$

Figure 3 shows the heat flow problem in the non-dimensional form.

Heat loss

The total heat loss from the house to the ground. q_s (W/m), is obtained by integration over the house width B:

$$q_s = \int_{-B/2}^{B/2} -\lambda \left.\frac{\partial T}{\partial y}\right|_{y=0} dx. \tag{11}$$

With the non-dimensional coordinates and temperature this becomes:

$$q_s = \lambda(T_i - T_0) \int_{-B/L}^{B/L} -\left.\frac{\partial U}{\partial y'}\right|_{y'=0} dx'. \tag{12}$$

The integral in (12) is non-dimensional. It is a steady-state heat loss factor, which is denoted by h_s:

$$h_s = \int_{-B/L}^{B/L} -\left.\frac{\partial U}{\partial y'}\right|_{y'=0} dx'. \tag{13}$$

Thus, the heat loss may be written:

$$q_s = \lambda(T_i - T_0) \cdot h_s. \tag{14}$$

The heat loss factor depends on a number of non-dimensional parameters. For a given temperature f, it depends on the width of the slab and the insulation distribution. The simplest case is when the insulation thickness is constant and there is no insulation outside the house. The heat loss factor then depends only on d/B.

METHOD

Conformal mapping

To solve the heat conduction problem the heat flow region in the plane $z' = x' + i \cdot y'$ is mapped into a semi-infinite strip in the plane $w = \xi + i \cdot \eta$:

$$z' = x' + i \cdot y' = \cos(iw) \quad \text{or} \quad \begin{aligned} x' &= \cos(\eta) \cdot \cos h(\xi) \\ y' &= \sin(\eta) \cdot \sin h(\xi) \end{aligned}. \tag{15}$$

The mapping is shown in Fig. 4.

The boundary consists of three parts A, B and C. The temperature U is determined in the w-plane as a function

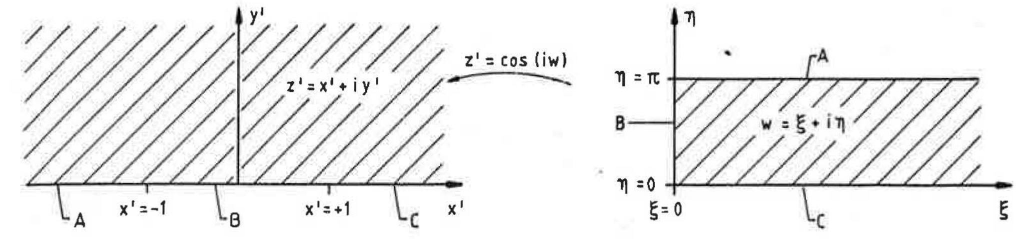

Fig. 4. The conformal mapping.

of ξ and η. The heat conduction equation in the new coordinates is according to [1]:

$$\frac{\partial^2 U}{\partial \xi^2} + \frac{\partial^2 U}{\partial \eta^2} = 0. \qquad (16)$$

For the boundary condition at B we have to write down the normal derivative with respect of y' for $U(\xi(x', y'), \eta(x', y'))$. This becomes: $\partial U/\partial y' = \partial U/\partial \xi \cdot \partial \xi/\partial y' + \partial U/\partial \eta \cdot \partial \eta/\partial y'$. From the Cauchy–Riemann condition we have: $\partial \eta/\partial y' = \partial \xi/\partial x'$. The ξ-coordinate is constant along the boundary segment B. Thus with (15) we have:

$$\frac{\partial U}{\partial y'} = \frac{\partial U}{\partial \xi} \frac{1}{\sin(\eta)} \quad 0 < \eta < \pi, \quad \xi = 0. \qquad (17)$$

The boundary conditions are then:

$$U = 0 \quad \eta = 0, \quad \xi > 0 \quad \eta = \pi, \quad \xi > 0, \qquad (18)$$

$$U - \frac{d'(\eta)}{\sin(\eta)} \cdot \frac{\partial U}{\partial \xi} = f'(\eta) \quad 0 < \eta < \pi, \quad \xi = 0. \qquad (19)$$

Numerical solution

For the sake of simplicity, only symmetric insulation distributions and boundary temperatures will be discussed. The insulation thickness d_i and the temperature f are then even functions of x. In the w-plane this corresponds to functions symmetric about $\eta = \pi/2$. Let us express f' in a Fourier series. The even terms of the series vanish due to the symmetry about $\eta = \pi/2$:

$$f'(\eta) = \sum_{n=0}^{\infty} b_n \cdot \sin[(2n+1)\eta]. \qquad (20)$$

A general solution U is given by:

$$U(\xi, \eta) = \sum_{n=0}^{\infty} a_n \cdot \sin[(2n+1)\eta] \cdot e^{-(2n+1)\xi}. \qquad (21)$$

This expression satisfies (16) and (18). With (21) inserted in (19), the following equation is obtained:

$$\frac{d'(\eta)}{\sin(\eta)} \sum_{n=0}^{\infty} a_n(2n+1) \cdot \sin[(2n+1)\eta]$$

$$= \sum_{n=0}^{\infty} (b_n - a_n) \cdot \sin[(2n+1)\eta] \quad 0 < \eta < \pi. \qquad (22)$$

Equation (22) is multiplied by the factor $\sin(\eta) \cdot \sin[(2m+1)\eta]$, and integrated over $0 < \eta < \pi/2$. For every integer value of m, following equations, which determine the unknown a_n, are obtained:

$$\sum_{n=0}^{\infty} a_n(2n+1) J_{mn} = \sum_{n=0}^{\infty} (b_n - a_n) \cdot C_{mn} \quad m = 0, 1, \ldots, \infty, \qquad (23)$$

where

$$J_{mn} = \int_0^{\pi/2} d'(\eta) \cdot \sin[(2n+1)\eta] \cdot \sin((2m+1)\eta)\, d\eta \qquad (24)$$

and

$$C_{mn} = \int_0^{\pi/2} \sin[(2n+1)\eta] \cdot \sin(\eta) \cdot \sin((2m+1)\eta)\, d\eta$$

$$= \frac{1}{4} \left\{ \frac{1}{2m+2n+1} + \frac{1}{2n-2m+1} + \frac{1}{2m-2n+1} - \frac{1}{2m+2n+3} \right\}. \qquad (25)$$

A finite number of terms in (23) is used in the numerical calculation. Let N denote the number of equations and unknown coefficients. For large equation systems, it is convenient to use an iterative method of solution. Let a_m^k be the values for iteration step k. The new value is obtained from (23) with the use of the latest values of the other coefficients ($m = 0, 1, 2, \ldots, N-1$, $m \neq n$). The following iterative formulas are used:

$$a_m^0 = b_n$$

$$a_m^{k+1} = \frac{b_m C_{mm} + \sum_{n=0}^{m-1} \{(b_n - a_n^{k+1}) C_{mn} - a_n^{k+1}(2n+1) J_{mn}\}}{(2m+1) J_{mm} + C_{mm}}$$

$$+ \frac{\sum_{n=m+1}^{N-1} \{(b_n - a_n^k) C_{mn} - a_n^k (2n+1) J_{mn}\}}{(2m+1) J_{mm} + C_{mm}}$$

$$m = 0, 1, \ldots, N-1. \qquad (26)$$

When the differences $a_m^{k+1} - a_m^k$ all are less than a given small value, an approximate solution is found.

The heat loss factor is, according to (13), (15), (17) and (21):

$$h_s = 2 \cdot \sum_{n=0}^{\infty} a_n \cdot \cos((2n+1)\gamma)$$

$$\gamma = \arccos(B/L) \qquad (27)$$

In the numerical calculations, where N coefficients are used, the sum (27) is approximated with the first N terms.

Table 1. Heat loss factor $h_s(0.1)$ for different numbers of coefficients N

N	h_s	Error (%)
1	2.061	12
5	2.375	2
10	2.347	1
25	2.334	0.2
50	2.331	0.04
100	2.33019	0.01
200	2.32996	0.003
400	2.32989	<0.001

Accuracy

The iterative formula is used in the examples below, and it has been working without any problem in all cases that have been tested. There has been no need to try other methods for the solution of equation system (23).

The accuracy of the method is tested for a house with constant insulation thickness. There is no insulation outside the house. The heat loss factor then depends on d/B only. For the case studied the parameter d/B is equal to 0.1. In the calculation the temperature is approximated with the first N terms in (21). The number of coefficients is varied between 1 and 400. The heat loss factor h_s (0.1) is given in Table 1.

The iterative formula (26) is used. The iterations are stopped when the change in h_s is less than 10^{-5} in two consecutive iterations. The maximum number of iterations is 10.

For use in practice it should be sufficient to use $N \approx 25$, which gives an error of about 1% and a number of iterations less than 20, for the most common cases. See the examples below. Generally, the accuracy decreases and the number of iterations increases, when the insulation profile and the boundary temperature become more complex. In particular, this is the case when a segment has zero insulation thickness.

PC-model

The general case with piece-wise constant insulation thickness has been adapted to a small PC-program [2], which runs under MS-DOS on IBM-PC compatible computers. The cases treated below are special cases handled by this program.

EXAMPLES

The heat loss to the ground, and the ground surface temperature for a house with constant indoor temperature is of special interest. It is given below for some frequently occurring cases. For the basic case the insulation thickness is constant. For the other two cases there is an extra insulation at the edges, either inside or outside the house.

The Fourier coefficients a_n for the temperature under the insulation are calculated according to the iterative formula (26). The coefficients b_n and J_{mn} that are used in the formula are given explicitly for three cases below.

Constant insulation thickness

The insulation thickness of the ground slab is constant, and there is no insulation outside the house ($L = B$). According to (4) and (15):

Table 2. Heat loss factor $h_s(d/B)$ for a ground slab with constant insulation thickness and no outside insulation

d/B	h_s	d/B	h_s
0.05	2.827	0.55	1.083
0.10	2.330	0.60	1.026
0.15	2.030	0.65	0.974
0.20	1.814	0.70	0.928
0.25	1.647	0.75	0.886
0.30	1.511	0.80	0.848
0.35	1.398	0.85	0.813
0.40	1.302	0.90	0.781
0.45	1.219	0.95	0.751
0.50	1.147	1.00	0.724

$$\frac{x}{B/2} = cos(\eta) \quad \xi = 0. \tag{28}$$

There is an insulation for $-B/2 < x < B/2$. In the coordinate η this corresponds to $0 < \eta < \pi$. The insulation thickness and the non-dimensional scaled temperatures above the insulation are:

$$d'(\eta) = \frac{d}{B/2} \quad 0 < \eta < \pi \quad \xi = 0,$$

$$f'(\eta) = 1.0 \quad 0 < \eta < \pi \quad \xi = 0. \tag{29}$$

The temperature f' is given by a Fourier series according to (20) and (29). The Fourier coefficients become:

$$b_n = \frac{4}{\pi} \frac{1}{2n+1}. \tag{30}$$

According to (24) and (29) the coefficient J_{mn} becomes:

$$J_{mn} = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} \cdot d/B & m = n \end{cases} \tag{31}$$

The heat loss factor is given in Table 2 for a few cases. Figure 5 shows the temperature under the insulation, $U(x', 0)$, for a few values of d/B. The number of coefficients N is 200. The estimated error is less than 0.1%.

Extra insulation at the edges inside the house

Figure 6 shows a house with constant insulation thickness except for an extra insulation at the edges. The extra insulation has the equivalent insulation thickness $\tilde{d}$ and the width D. There is no insulation outside the house

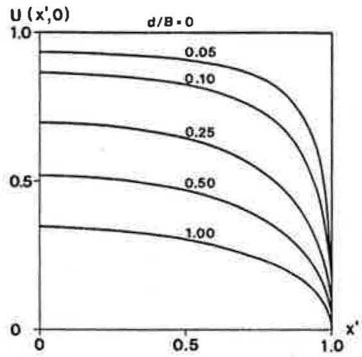

Fig. 5. Temperature under the insulation, $U(x', 0)$, for different d/B.

Table 3. Heat loss factor $h_s(d/B, \tilde{d}/d, D/B)$ for a ground slab of the type shown in Fig. 6

	$d/B = 0.2$			$d/B = 0.4$			$d/B = 0.6$	
D/B	$\tilde{d}/d$	h_s	D/B	$\tilde{d}/d$	h_s	D/B	$\tilde{d}/d$	h_s
0.05	−0.5	2.033	0.05	−0.5	1.450	0.05	−0.5	1.138
0.05	0.5	1.730	0.05	0.5	1.249	0.05	0.5	0.986
0.05	1.0	1.685	0.05	1.0	1.221	0.05	1.0	0.965
0.05	1.5	1.657	0.05	1.5	1.204	0.05	1.5	0.953
0.05	2.0	1.638	0.05	2.0	1.193	0.05	2.0	0.945
0.10	−0.5	2.125	0.10	−0.5	1.537	0.10	−0.5	1.215
0.10	0.5	1.683	0.10	0.5	1.211	0.10	0.5	0.955
0.10	1.0	1.610	0.10	1.0	1.163	0.10	1.0	0.919
0.10	1.5	1.564	0.10	1.5	1.133	0.10	1.5	0.896
0.10	2.0	1.532	0.10	2.0	1.113	0.10	2.0	0.881
0.20	−0.5	2.218	0.20	−0.5	1.647	0.20	−0.5	1.323
0.20	0.5	1.623	0.20	0.5	1.155	0.20	0.5	0.905
0.20	1.0	1.509	0.20	1.0	1.072	0.20	1.0	0.846
0.20	1.5	1.454	0.20	1.5	1.020	0.20	1.5	0.799
0.20	2.0	1.381	0.20	2.0	0.984	0.20	2.0	0.771
0.30	−0.5	2.269	0.30	−0.5	1.719	0.30	−0.5	1.401
0.30	0.5	1.581	0.30	0.5	1.109	0.30	0.5	0.863
0.30	1.0	1.436	0.30	1.0	0.997	0.30	1.0	0.771
0.30	1.5	1.337	0.30	0.5	0.924	0.30	1.5	0.712
0.30	2.0	1.265	0.30	2.0	0.872	0.30	2.0	0.671

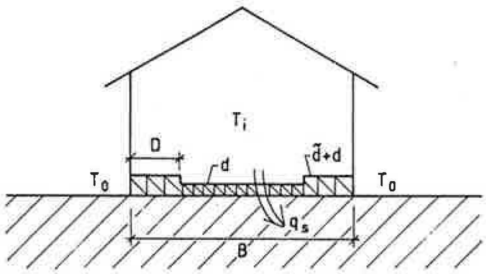

Fig. 6. Extra insulation at the edges inside the house.

$(L = B)$. The boundary temperature is constant over $-1 < x' < 1$. This gives:

$$f'(\eta) = 1.0 \quad 0 < \eta < \pi. \tag{32}$$

The Fourier coefficient b_n is given by (30).

There is an extra insulation for $B/2 - D < x < B/2$ and $-B/2 < x < -B/2 + D$. In the coordinate η this corresponds to $0 < \eta < \alpha$ and $\pi - \alpha < \eta < \pi$. The scaled insulation thickness is:

$$d'(\eta) = \frac{d}{B/2} \quad \alpha < \eta < \pi - \alpha$$

$$d'(\eta) = \frac{d + \tilde{d}}{B/2} \quad 0 < \eta < \alpha \quad \pi - \alpha < \eta < \pi. \tag{33}$$

$$\alpha = \arccos\left(1 - \frac{D}{B/2}\right).$$

According to (24) and (33) the coefficients J_{mn} become:

$$J_{mn} = \begin{cases} \dfrac{\tilde{d}}{B}\dfrac{1}{2}\left\{\dfrac{\sin(2\alpha(n-m))}{n-m} - \dfrac{\sin(2\alpha(n+m+1))}{n+m+1}\right\} & n \neq m \\ -\dfrac{\tilde{d}}{B}\dfrac{1}{2}\dfrac{\sin(2\alpha(2m+1))}{2m+1} + \dfrac{\tilde{d}}{B}\alpha + \dfrac{d}{B}\dfrac{\pi}{2} & n = m \end{cases} \tag{34}$$

Table 4. Heat loss factor h_s for the case $d = 0$ and $\tilde{d}/B = 10$

D/B	0.03	0.05	0.075	0.10	0.20	0.30
h_s	3.05	2.73	2.46	2.26	1.75	1.40

The heat loss factor is a function of d/B, $\tilde{d}/d$ and D/B. It is given in Table 3. Figure 7 shows the temperature at the ground surface, $U(x', 0)$, for a few cases. The number of coefficients N is 200. The number of iterations is about 14. The estimated error is less than 0.1%.

Table 4 gives the heat loss factor for a special case where $d = 0$ and $\tilde{d}/B = 10$. The number of coefficients N is 200. The number of iterations is a few hundred up to a thousand. The estimated error is about 1%. The heat loss factors for an uninsulated floor are given in Table 4. The width D of the extra insulation then corresponds to the thickness of the wall. We assume that there is approximately no heat flow up into the wall. This case can be compared with the approximation according to Macey, see [3]. He approximated the ground temperature under the wall with the indoor temperature. The heat loss is obtained by an integration of the heat flow between the walls of the building. For our case the ground temperature under the wall will vary between the outdoor

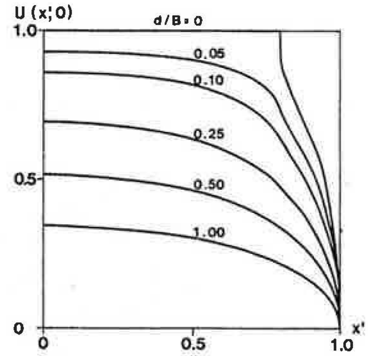

Fig. 7. Temperature under the insulation, $U(x', 0)$, for the case shown in Fig. 6 with $D/B = 0.1$, $\tilde{d}/D = 1.0$ and different values of d/B.

Table 5. Heat loss factor $h_s(d/B, \tilde{d}/d, D/B)$ for a ground slab of the type shown in Fig. 8

$d/B = 0.2$			$d/B = 0.4$			$d/B = 0.6$		
D/B	$\tilde{d}/d$	h_s	D/B	$\tilde{d}/d$	h_s	D/B	$\tilde{d}/d$	h_s
0.05	0.5	1.651	0.05	0.5	1.214	0.05	0.5	0.970
0.05	1.0	1.633	0.05	1.0	1.209	0.05	1.0	0.968
0.05	1.5	1.626	0.05	1.5	1.207	0.05	1.5	0.967
0.05	2.0	1.622	0.05	2.0	1.206	0.05	2.0	0.966
0.10	0.5	1.596	0.10	0.5	1.174	0.10	0.5	0.941
0.10	1.0	1.558	0.10	1.0	1.160	0.10	1.0	0.934
0.10	1.5	1.541	0.10	1.5	1.154	0.10	1.5	0.932
0.10	2.0	1.532	0.10	2.0	1.151	0.10	2.0	0.930
0.20	0.5	1.553	0.20	0.5	1.135	0.20	0.5	0.910
0.20	1.0	1.488	0.20	1.0	1.105	0.20	1.0	0.894
0.20	1.5	1.456	0.20	1.5	1.093	0.20	1.5	0.888
0.20	2.0	1.436	0.20	2.0	1.086	0.20	2.0	0.884
0.30	0.5	1.536	0.30	0.5	1.116	0.30	0.5	0.893
0.30	1.0	1.456	0.30	1.0	1.076	0.30	1.0	0.870
0.30	1.5	1.413	0.30	1.5	1.057	0.30	1.5	0.866
0.30	2.0	1.387	0.30	2.0	1.046	0.30	2.0	0.854
0.50	0.5	1.522	0.50	0.5	1.100	0.50	0.5	0.877
0.50	1.0	1.428	0.50	1.0	1.046	0.50	1.0	0.844
0.50	1.5	1.374	0.50	1.5	1.019	0.50	1.5	0.828
0.50	2.0	1.338	0.50	2.0	1.001	0.50	2.0	0.818

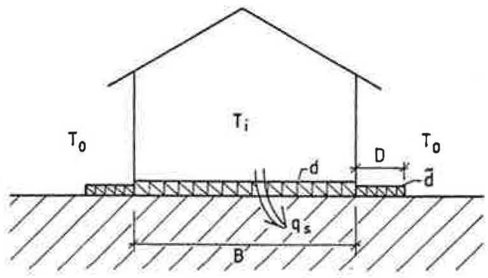

Fig. 8. Extra insulation at the edges outside the house.

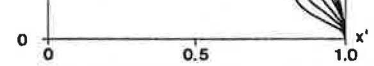

Fig. 9. Temperature under the insulation, $U(x', 0)$, for the case shown in Fig. 8 with $D/B = 0.1$, $\tilde{d}/D = 1$ and different values of d/B.

and indoor temperature. This gives a colder region under the floor and an ensuing greater heat loss. The heat loss according to Macey is given in the CIBS guide [4]. For a case with $B = 10$ m and $D = 0.3$ m ($D/B = 0.03$) the heat loss factor becomes equal to 2.2. This should be compared with 3.1 from our calculation. It is an difference of about 30%. For the case $D/B = 0.075$ we get the numbers 1.8 and 2.4.

Extra insulation at the edges outside the house

Figure 8 shows a house with constant insulation thickness and an extra insulation at the edges outside the house. The extra insulation has the equivalent insulation thickness $\tilde{d}$ and the width D. The total width of the insulation L becomes $B+2D$.

The edges of the house are given by $x = B/2$ and $x = -B/2$. In the coordinate η this corresponds to $\eta = \alpha$ and $\eta = \pi-\alpha$. The boundary temperature is:

$$f'(\eta) = 1.0 \quad \alpha < \eta < \pi-\alpha$$

$$f'(\eta) = 0 \quad 0 < \eta < \alpha \quad \pi-\alpha < \eta < \pi. \tag{35}$$

$$\alpha = \arccos\left(\frac{B/2}{B/2+D}\right).$$

The Fourier coefficient b_n becomes:

$$b_n = \frac{4}{\pi}\frac{1}{2n+1}\cos\left[(2n+1)\alpha\right]. \tag{36}$$

The scaled insulation thickness is:

$$d'(\eta) = \frac{d}{B/2+D} \quad \alpha < \eta < \pi-\alpha$$

$$d'(\eta) = \frac{\tilde{d}}{B/2+D} \quad 0 < \eta < \alpha \quad \pi-\alpha < \eta < \pi. \tag{37}$$

$$\alpha = \arccos\left(\frac{B/2}{B/2+D}\right).$$

According to (24) and (36), the coefficient J_{mn} becomes:

$$J_{mn} =$$

$$\frac{\tilde{d}-d}{B+2D}\frac{1}{2}\left\{\frac{\sin(2\alpha(n-m))}{n-m} - \frac{\sin(2\alpha(n+m+1))}{n+m+1}\right\} \quad n \neq m$$

$$-\frac{\tilde{d}-d}{B+2D}\frac{1}{2}\frac{\sin(2\alpha(2m+1))}{2m+1}$$

$$+\alpha\frac{\tilde{d}-d}{B+2D}+\frac{d}{B+2D}\frac{\pi}{2} \quad n = m. \tag{38}$$

The heat loss factor is a function of d/B, $\tilde{d}/d$ and D/B. It is given in Table 5. Figure 9 shows the temperature at the ground surface $U(x', 0)$ for a few cases. The number of coefficients $\bar{N}$ is 200. The number of iterations is about 15. The estimated error is less than 0.1%.

EXTENSION OF THE MODEL

The method, which here is presented for the symmetric cases, can directly be extended to asymmetric cases. It may also be applied to insulated cellars and other thermal problems, where a finite part of the boundary is thermally insulated, and the rest has a constant temperature. The actual heat conduction region must first be mapped into the half-plane $\mathrm{Re}(z') > 0$. This results in a changed condition at the insulated part of the boundary. This corresponds to a variable insulation thickness. A factor which depends on the derivative of the transformation function will be added to the insulation thickness. The method accepts this, and thus it can solve the new problem. The coefficients J_{mn} must however in most cases be calculated numerically.

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