

Uncertainty in Air Flow Calculations Using Tracer Gas Measurements

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Tracer gas techniques are becoming widely used to measure the ventilation rates in buildings. As more detailed information is required for both energy and indoor air quality purposes, researchers are turning to complex, multizone tracer strategies. Both single gas and multiple gas techniques are being utilized, but only multigas techniques are capable of uniquely determining the entire matrix of air flows. In any of these measurement techniques, the determination of the precision of the result is critical for understanding its significance. This report derives expressions for determining the uncertainties in the air flows from the measured data. Examples indicate that real-time techniques are more precise than integrated techniques and that multigas techniques are more precise than single-gas techniques.

NOMENCLATURE

C	Instantaneous tracer gas concentration
C	Multizone tracer gas concentration matrix
C_T	Target concentration (for constant concentration)
E	Exfiltration from a zone to outside, $m^3 h^{-1}$
N	Number of zones
N'	Number of tracer gasses
I	Infiltration to a zone from outside, $m^3 h^{-1}$
Q	Ventilation, $m^3 h^{-1}$
Q	Ventilation matrix, $m^3 h^{-1}$
S	Instantaneous source strength of tracer gas, $m^3 h^{-1}$
S	Multizone tracer source strength matrix, $m^3 h^{-1}$
t	Time, h
V	Volume, m^3
V	Zone volume matrix, m^3
δ_{ij}	Kronecker delta function (equals unity if $i = j$, otherwise is zero)
\bar{X}	Overbar: The time average of the instantaneous quantity X
e_{xy}	Correlation coefficient between variables x and y
σ_x^2	Variance of variable x (x^2)
$\sigma_{x,y}$	Covariance of variables x and y (xy)
$\sigma_{i,l,n}$	Covariance of zonal data errors for gas n , $m^6 h^{-2}$
i, j, l, m	Indices indicating zone ($1 \dots N$)
k, m	Indices indicated tracer gas ($1 \dots N'$)

INTRODUCTION

TRACER GASES are used for a wide range of diagnostic techniques including leak detection [1, 2] and atmospheric tracing [3]. One application which has had a resurgence in the last decade is the use of tracer gasses to measure ventilation (i.e. air flow) in buildings [4]. Ventilation is an important process in buildings because of its impact on both energy requirements and indoor air quality—both of which are topics of concern to society. Measurement of the tracer gas combined with conservation laws allows a quantitative determination of the

tracer transport mechanism (i.e. a measurement of the air flow).

The vast majority of the ventilation measurements made to date have involved a single tracer gas deployed in a single zone. This technique has proven very useful for building which may be treated as a single zone (e.g. houses) and for more complex buildings in which there are isolatable sub-sections. However, as the need to understand more complex buildings has grown, tracer techniques that are able to treat multiple zones have been developed [5]. Multizone techniques recognize that not only does air flow between the outside and the test space, but there are air flows between different parts (i.e. zones) of the test space and, in the complete case, they are able to measure these flows.

As in any experimental techniques, there are uncertainties associated with the fundamental measurements and these errors propagate to become uncertainties in the determination of air flows. Some work on the error analysis of the single zone problem has been done. For example, Heidt [6] has demonstrated that optimal precision in tracer decay measurement is on the order of the inverse air-change rate (i.e. the turn-over time); and D'Ottavio [7] has shown a decrease in precision when a two-zone building is treated as a single zone.

Because of the highly coupled nature of multizone air flows, the uncertainties of the calculated air flows are, in general, correlated. Little work on the multizone error analysis exists. This report derives the error propagation expressions and presents the results for the common types of measurement techniques.

BACKGROUND

To describe multizone air flows, a matrix form of the continuity equation can be used. For every zone of the system there will be a row in both the concentration and source-strength matrices. For every unique tracer there

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will be a column in those matrices. If there are N zones, the volume* and air flow matrices will be square matrices of order N . If there are as many tracer species as there are zones, the problem is called *complete* and there will be an exact answer; we shall focus our attention to the complete problem and therefore assume that all of the matrices are square. Thus, in matrix notation the continuity equation can be seen as follows:

$$\mathbf{V} \cdot \dot{\mathbf{C}} + \mathbf{Q} \cdot \mathbf{C} = \mathbf{S}, \quad (1a)$$

or, using explicit indices

$$\sum_{j=1}^N (V_{ij} \dot{C}_{jk} + Q_{ij} C_{jk}) = S_{ik}. \quad (1b)$$

To reiterate, \dot{C}_{ij} , C_{ij} and S_{ij} all represent the respective value of the j^{th} tracer gas in the i^{th} zone.

The interpretation of the air flow matrix requires a bit more explanation. The diagonal elements, Q_{ii} represent the total flow into or out of that zone from all sources (including outside) and should have positive sign. The off-diagonal elements represent the flows between zones and should have a negative sign; specifically, $-Q_{ij}$ is flow from the j^{th} zone to the i^{th} zone. Since the flow from the j^{th} zone to the i^{th} zone can be different from the flow from the i^{th} zone to the j^{th} zone, this matrix need not be symmetric.

It is often useful to calculate the air flow into or out of a particular zone to the environment (i.e. outside), which must be positive. From the previous definitions these flows can be calculated by summing the appropriate column or row respectively:

$$E_j = \sum_{i=1}^N Q_{ij} \geq 0, \quad (2a)$$

$$I_i = \sum_{j=1}^N Q_{ij} \geq 0. \quad (2b)$$

The total infiltration for the structure is the sum of all the individual elements:

$$Q_0 = \sum_{i=1}^N \sum_{j=1}^N Q_{ij} = \sum_{i=1}^N I_i = \sum_{j=1}^N E_j. \quad (3)$$

Given all of the physicality constraints the air flow matrix must be positive definite and well conditioned in any non-trivial case.

CALCULATION OF UNCERTAINTY

Depending on the experimental conditions, there are a variety of ways of solving the complete problem. Virtually all of them are subsets of the general case of inverting the continuity equation:

$$\mathbf{Q} = (\mathbf{S} - \mathbf{V} \cdot \dot{\mathbf{C}}) \cdot \mathbf{C}^{-1}, \quad (4a)$$

or, equivalently, using the matrix indices explicitly:

$$Q_{ij} = \sum_{k=1}^{N'} \left(S_{ik} - \sum_{l=1}^N V_{il} \dot{C}_{lk} \right) C_{kj}^{-1}. \quad (4b)$$

In all of the following analysis it is assumed that the concentration matrix is generally invertible (i.e. has non-zero determinant). Physically, this requires that the information of any one of the N tracer gases be independent from all of the other tracer-gas information.

To the extent that there is uncertainty in the measured data, there will be an uncertainty in the calculated air flows. Since each element of the air flow matrix is calculated from some combination of the same measured data elements, the errors air flows will, in general, be correlated—indicating that care must be taken in the calculation of the uncertainties.

If variations in the measured data are small, then all of the error information is contained in the *covariance* matrix. The covariance matrix of a calculated quantity is the sum of the covariance matrix of the data weighted by the dependency of the calculated quantity on the data. If, for example, we have a set of quantities, Y_i , which depend on a set of measured data, x_m , the covariance of calculated values is defined as follows:

$$\sigma_{Y_i, Y_j} = \sum_m \sum_l \frac{\partial Y_i}{\partial x_m} \frac{\partial Y_j}{\partial x_l} \sigma_{x_m, x_l}. \quad (5)$$

The simple error in any one term in the square root of the appropriate diagonal element of the matrix (This definition will be generally applied.):

$$\sigma_{Y_i}^2 \equiv \sigma_{Y_i, Y_i}. \quad (6)$$

If the data elements are themselves independent all of the cross terms drop out and the covariance simplifies:

$$\sigma_{Y_i, Y_j} = \sum_m \frac{\partial Y_i}{\partial x_m} \frac{\partial Y_j}{\partial x_m} \sigma_{x_m}^2. \quad (7)$$

In our case the calculated quantities are in themselves matrices which depend on the tracer source strength, the tracer concentration, and the time rate of change of that concentration. The source strength and concentration are physically independent and, therefore, their errors can be assumed to be uncorrelated. As long as the errors in the concentration do not have an explicit time dependence, the errors in the time rate of change of the concentration are also uncorrelated. The expression for the covariance of the air flows can be written as follows:

$$\sigma_{Q_{ij}, Q_{i'j'}} = \sum_{m=1}^N \sum_{n=1}^{N'} \left(\frac{\partial Q_{ij}}{\partial S_{mn}} \frac{\partial Q_{i'j'}}{\partial S_{mn}} \sigma_{S_{mn}}^2 + \frac{\partial Q_{ij}}{\partial \dot{C}_{mn}} \frac{\partial Q_{i'j'}}{\partial \dot{C}_{mn}} \sigma_{\dot{C}_{mn}}^2 + \frac{\partial Q_{ij}}{\partial C_{mn}} \frac{\partial Q_{i'j'}}{\partial C_{mn}} \sigma_{C_{mn}}^2 \right). \quad (8)$$

The variances of the data will of course depend on the instrumentation and analysis technique used and hence cannot be simplified further. The partial derivatives depend only on the physics and therefore can be simplified using the defining relation for the air flows. The dependence of air flow on the source strength is as follows:

$$\frac{\partial Q_{ij}}{\partial S_{mn}} = \sum_{k=1}^{N'} \delta_{im} \delta_{nk} C_{kj}^{-1}, \quad (9a)$$

* For most practical purposes the volume matrix can be assumed diagonal with the individual zone volumes as the entries. If, however, there is *short circuiting* of the tracer source from one zone to another, this process can manifest itself as an off-diagonal volume element. We shall not, therefore, assume diagonality. Note also that the sum of each column must be equal to the physical volume of the zone.

$$\frac{\partial Q_{ij}}{\partial S_{mn}} = \delta_{im} C_{nj}^{-1}. \quad (9b)$$

Similarly the dependence of air flow on the time change of concentration can be calculated:

$$\frac{\partial Q_{ij}}{\partial C_{mn}} = -V_{im} C_{nj}^{-1}. \quad (10)$$

The partial derivative with respect to the concentration presents a slightly more difficult problem because it includes the derivative of the inverse concentration matrix:

$$\frac{\partial Q_{ij}}{\partial C_{mn}} = \sum_{k=1}^{N'} \left(S_{ik} - \sum_{l=1}^N V_{il} C_{lk} \right) \frac{\partial C_{kj}^{-1}}{\partial C_{mn}}. \quad (11)$$

We can use the fact that (for any non-singular concentration matrix):

$$\sum_{i=1}^N \left[\frac{\partial}{\partial C_{mn}} \left(\sum_{k=1}^N C_{ki}^{-1} C_{li} \right) \right] C_{ij}^{-1} = 0, \quad (12)$$

to derive:

$$\frac{\partial C^{-1}}{\partial C_{mn}} = -C_{km}^{-1} C_{nj}^{-1}. \quad (13)$$

Using this expression and our defining relation for the air flows we get the following expression for the derivative.

$$\frac{\partial Q_{ij}}{\partial C_{mn}} = -Q_{im} C_{nj}^{-1}. \quad (14)$$

Putting all of these expressions together, the formula for the covariance becomes:

$$\sigma_{Q_{ij}, Q_{i'j'}} = \sum_{n=1}^{N'} \left(C_{nj}^{-1} C_{nj'}^{-1} \right) \sigma_{i,i';n}, \quad (15)$$

where we have defined the covariance of the data of zone i with zone i' for tracer gas n as follows:

$$\sigma_{i,i';n} = \sum_{m=1}^N \delta_{im} \delta_{i'm} \sigma_{S_{mn}}^2 + V_{im} V_{i'm} \sigma_{C_{mn}}^2 + Q_{im} Q_{i'm} \sigma_{C_{mn}}^2. \quad (16)$$

It should be noted that this variance of the data represents the uncertainty of the continuity of tracer gas n in zone i caused by the uncertainty in the measured quantities. If a statistical fitting process is used to find the solution to the continuity equation, this variance can be used to weight the deviations. A more detailed discussion is beyond the scope of this report and will be the topic of a future report.

The errors in the individual matrix elements of the air flow are the diagonal elements of the covariance matrix:

$$\sigma_{Q_{ij}}^2 = \sum_{n=1}^{N'} (C_{nj}^{-1})^2 \sigma_{i,i;n}^2. \quad (17)$$

When looking for correlations between the errors of different quantities, it is often more useful to deal with the correlation rather than covariance matrix. The correlation matrix can be calculated from the covariance matrix as follows:

$$r_{Q_{ij}, Q_{i'j'}} = \frac{\sigma_{Q_{ij}, Q_{i'j'}}}{\sigma_{Q_{ij}} \sigma_{Q_{i'j'}}}. \quad (18)$$

Uncertainty of infiltration and exfiltration

The elements of the air flow matrix are the flow between zones and the total flow to all zones including outside. Since a sum of these elements must be taken in order to get the flow between a zone and outside, the covariances must be considered in calculating the uncertainty associated with the infiltration and exfiltration:

$$\sigma_{E_j, E_j} = \sum_{i=1}^N \sum_{i'=1}^N \sigma_{Q_{ij}, Q_{i'j}}, \quad (19a)$$

$$\sigma_{E_j, E_j} = \sum_{n=1}^{N'} (C_{nj}^{-1} C_{nj}^{-1}) \sum_{i=1}^N \sum_{i'=1}^N \sigma_{i,i';n}, \quad (19b)$$

$$\sigma_{E_j, E_j} = \sum_{n=1}^{N'} (C_{nj}^{-1} C_{nj}^{-1}) \sum_{m=1}^N (\sigma_{S_{mn}}^2 + V_m^2 \sigma_{C_{mn}}^2 + E_m^2 \sigma_{C_{mn}}^2). \quad (19c)$$

Similarly, we can calculate the infiltration uncertainty:

$$\sigma_{I_i, I_i} = \sum_{j=1}^N \sum_{j'=1}^N \sigma_{Q_{ij}, Q_{i'j}}, \quad (20a)$$

$$\sigma_{I_i, I_i} = \sum_{n=1}^{N'} \left(\sum_{j=1}^N C_{nj}^{-1} \right)^2 \sigma_{i,i;n}. \quad (20b)$$

To the extent that the sums over i and j in the above expressions cause cancellation of errors, the variances above will be reduced. Unless the zones are completely uncoupled (i.e. have no air flow to other zones), there will be some reduction. However, as the exfiltration expressions contain some errors which are already independent, there will be more reduction for the infiltration variances.

Similarly, the variance in total infiltration is the sum of the elements of the covariance matrix:

$$\sigma_{Q_0}^2 = \sum_{i=1}^N \sum_{i'=1}^N \sum_{j=1}^N \sum_{j'=1}^N \sigma_{Q_{ij}, Q_{i'j'}}, \quad (21a)$$

$$\sigma_{Q_0}^2 = \sum_{n=1}^{N'} \left(\sum_{j=1}^N C_{nj}^{-1} \right)^2 \sum_{m=1}^N (\sigma_{S_{mn}}^2 + V_m^2 \sigma_{C_{mn}}^2 + E_m^2 \sigma_{C_{mn}}^2). \quad (21b)$$

SIMPLIFICATIONS

The preceding equations are quite general and depend only on the fact that the errors physical measurements are uncorrelated. However, in many real experiments more is known about the physical conditions and the size of certain terms. (Computationally, use of these simplifications is really only justified if the number of zones is sufficiently large that the cost of inversion of an $N \times N'$ matrix is prohibitive.) In this section we treat two of the more common special cases which lead to some simplification in the error analysis and allow more insight into the uncertainties.

Independent tracer errors

In many experimental situations, the uncertainty in the data will not strongly depend on the explicit type of tracer gas; in such a case the continuity errors are independent of tracer gas:

$$\sigma_{i,i';n} \rightarrow \sigma_{i,i'} \equiv \frac{1}{N'} \sum_{n=1}^{N'} \sigma_{i,i;n}. \quad (22)$$

Thus the sum over n and m in equation (16) can be separated, and the correlation matrix can be expressed as follows:

$$r_{Q_{ij}, Q_{if}} = r_{I_i, I_f} r_{E_j, E_f} \quad (23)$$

where the infiltration and exfiltration correlation matrices are given by the following expressions for the exfiltration:

$$r_{E_j, E_f} = \frac{\sum_{n=1}^{N'} (C_{nj}^{-1} C_{nf}^{-1})}{\left(\sum_{n=1}^{N'} (C_{nj}^{-1})^2 \sum_{n=1}^{N'} (C_{nf}^{-1})^2 \right)^{1/2}}, \quad (24)$$

and the infiltration correlation:

$$r_{I_i, I_f} = \frac{\sigma_{i,i'}}{\sigma_i \sigma_{i'}}. \quad (25)$$

As can be verified by direct substitution, these quantities are the correlation matrices of the infiltration and exfiltration, in the limit of tracer-invariant errors:

$$\sigma_{I_i, I_f} = r_{I_i, I_f} \sum_{j=1}^N \sum_{j'=1}^N r_{E_j, E_{j'}} \sigma_{Q_{ij}} \sigma_{Q_{if}}, \quad (26a)$$

$$\sigma_{E_j, E_{j'}} = r_{E_j, E_{j'}} \sum_{i=1}^N \sum_{i'=1}^N r_{I_i, I_{i'}} \sigma_{Q_{ij}} \sigma_{Q_{if}}. \quad (26b)$$

This separation has the interesting property that the instrumental factors (i.e. the data errors) are separated from the conditioning of the concentration matrix.

Weak error coupling

In many experimental strategies currently in use, the bulk of the uncertainty comes from the diagonal elements of equation (17) (i.e. when $i = i'$). This *weak error coupling* is only violated if the errors in the concentration (cf. the source strength and concentration time change) are the dominant uncertainty and the air flow matrix has significant off-diagonal elements.

In the weak coupling limit the infiltration correlations become the identity matrix:

$$r_{I_i, I_f} \rightarrow \delta_{if} \quad (27)$$

the data errors become diagonal:

$$\sigma_{i,i';n} \rightarrow \delta_{ii'} \sigma_i^2, \quad (28)$$

and the covariance matrix becomes block diagonal:

$$\sigma_{Q_{ij}, Q_{if}} \approx \delta_{if} \sigma_i^2 \sum_{n=1}^{N'} (C_{nj}^{-1} C_{nf}^{-1}). \quad (29)$$

In this limit the errors on the exfiltration become uncorrelated and the uncertainty in these flows simplifies to the simple result one would have predicted assuming independence of errors:

$$\sigma_{E_j}^2 = \sum_{i=1}^N \sigma_{Q_{ij}}^2. \quad (30)$$

* The issue of mixing is really an issue of accuracy not precision and thus a detailed discussion is beyond the scope of this report. For our current purposes we will treat the mixing as though it were a random error—part of our measurement uncertainty.

Note that no such simplification occurs for the *infiltration* air flows and that the correlations must be maintained to get proper estimates of uncertainty:

$$\sigma_{I_i, I_f} = \delta_{if} \sum_{j=1}^N \sum_{j'=1}^N r_{E_j, E_{j'}} \sigma_{Q_{ij}} \sigma_{Q_{if}}, \quad (31)$$

but (unlike the exfiltration flows) the infiltration flows are uncorrelated and thus may be treated as independent.

EXPERIMENTAL TECHNIQUES

Most of the experimental techniques currently in use have some common features. For example, short-circuiting is usually ignored and the volume matrix is assumed to be known and diagonal. Also, for practical purposes, the source-strength matrix is usually diagonal as well. The techniques, further, tend to fall into one of two categories which we will designate by *real-time measurement* and *integrated measurement*.

Real-time measurement

In the real-time techniques, typified by our Multi-Tracer Measurement System (MTMS) currently under development, and Princeton University's Constant Concentration Tracer Gas (CCTG) system [8], the concentration of tracer gas in each zone is measured in a period short compared to its time rate of change and the source strength is actively controlled by some real-time control algorithm. Thus, the errors in the estimation of the source strength and concentration time change will be dominated by mixing,* and the independent-tracer-error assumption is likely to be valid. Furthermore, the analysis is usually done over a period of time containing many individual concentration measurements and the contribution to the errors from the uncertainty in the concentration will be small. Thus, the weak-error-coupling assumption is also likely to be valid.

Integrated techniques

In the integrated techniques, typified by the Brookhaven PerFluorocarbon Tracer (PFT) technique [9], a constant emission source is used over a period of time long compared to the time rate of change of the concentration. A continuous sampling method is used to generate an average concentration which is then used to calculate the air flows. (The \dot{C} term is ignored.) If the time period is sufficiently long, the \dot{C} error will be small and mixing will not be a significant factor in estimation of the random error. The other errors will be determined by the precision of the source strength and concentration. Thus, in general, neither of the simplifications need to apply, but the covariance of the data should be calculated explicitly to determine it.

EXAMPLE EXPERIMENTS

The most common techniques currently in use are typified by the MTMS, CCTG, and PFT systems. We have selected datasets from each to demonstrate the error analysis technique. The examples selected are all three-zone, continuous injection tests. Only detail necessary to understand the example is repeated herein.

As an example of a real-time complete multizone

Table 1. Example uncertainties from MTMS dataset

$Q_{ij} \pm \sigma_{Q_{ij}}$	1	2	3	Outside
1	50 ± 11	-0 ± 17	-41 ± 25	9 ± 9
2	-1 ± 1	83 ± 5	-72 ± 5	9 ± 1
3	-2 ± 3	-83 ± 56	159 ± 53	74 ± 15
Outside	46 ± 11	0 ± 58	46 ± 59	92 ± 18

system, we have extracted data from our MTMS development system used on a zone-heated single-family house. The errors in the individual concentration measurements are in the 2–5% range and the errors in the individual flows are in the 1–4% range. This dataset was analysed using a half-hour time constant and represents one (half-hour) period from out of a larger dataset. Table 1 contains the results of the analysis for both air flow and uncertainty.

Physically, the off-diagonal elements must be non-positive, but this fact is not built into the error analysis and therefore, some uncertainties appear to allow values in an unphysical range.

To see how the various flows couple and to see if any of our simplifications would be justified, we calculate the (symmetric) correlation matrix as shown in Table 2.

The correlation matrix clearly shows the block diagonal behavior expected for the weak coupling limit. *A posteriori* we can conclude that this dataset meets both simplifications. Furthermore, it is also apparent that zones two and three are strongly (anti)correlated.

Incomplete systems: single tracer gas; constant concentration

The analysis herein has considered the complete problem in which there are exactly as many tracers as zones. In many circumstances, it is possible to extract useful information when there are more zones than tracers.

We will consider the one such system currently in use, the multizone constant concentration system using a single tracer gas [8]. If a single tracer gas is so controlled as to have the same concentration in every zone, the infiltration (from outside) to that zone can be calculated even though the other air flow elements cannot:

$$I_i = \frac{1}{C_T} S_i \quad (32)$$

Our error analysis equations for the infiltration can be used to estimate the uncertainty of these infiltrations:

$$\sigma_{I_i}^2 = \frac{1}{C_T^2} \left(\sigma_{S_i}^2 + V_i^2 \sigma_{C_T}^2 + \sum_{m=1}^N Q_{im}^2 \sigma_{C_m}^2 \right). \quad (33)$$

Since the technique assumes constant concentration, we can break up the data uncertainties into the mean deviation from the target concentration and the variance around it:

$$\sigma_{I_i}^2 = \frac{1}{C_T^2} (\sigma_{S_i}^2 + I_i^2 \sigma_{C_T}^2 + V_i^2 \sigma_{C_i}^2 + \delta^2 I_i \sigma_C^2). \quad (34)$$

where the second term represents the error from not keeping the average concentration at target, and the last term is the error due to cross flows in which:

$$\delta^2 I_i \equiv \left(\sum_{m=1}^N Q_{im}^2 \right) - \left(\sum_{m=1}^N Q_{im} \right)^2, \quad (35)$$

represents the variance of the individual flow elements around the infiltration and we have assumed that variance to be independent of zone. The error in the time rate of change of the concentration can be approximated by the variance in the concentration and the time used to analyse the data:

$$\sigma_{C_i} \approx \frac{\sigma_C}{\Delta t_{\text{analysis}}}. \quad (36)$$

This uncertainty expression still contains the (unknown) values of the air flows. Thus to make any kind of error analysis, some reasonable bounds on these values must be made based on *a priori* knowledge of the experimental conditions. Clearly this technique will work best when the zone-to-zone air flows are small or when the concentration can be tightly controlled around its target value.

Bohac [8, p. 178] and, independently, Kvisgaard [10] report that the error for their constant concentration systems is approximately 5%. However, their calculations were based only on the first two terms in equation (34), and thus will always tend to underestimate the uncertainty of the calculated air flows. While the third term may be reduced by increasing the analysis time, the last term may still represent significant error.

Since this error equation cannot be resolved without independently knowing the interzonal flows, we will use

Table 2. Correlation matrix for MTMS dataset

		$[r_{Q_{ij}, Q_{ij}}]$								
i	j	1	1	1	2	2	2	3	3	3
		1	2	3	1	2	3	1	2	3
1	1	1.00	0.23	-0.52	0.00	-0.00	-0.00	-0.00	0.00	0.00
1	2	0.23	1.00	-0.90	-0.00	0.00	-0.00	0.00	-0.00	0.00
1	3	-0.52	-0.90	1.00	-0.00	-0.00	0.00	0.00	0.00	-0.00
2	1	0.00	-0.00	-0.00	1.00	-0.07	-0.11	-0.00	0.00	0.00
2	2	-0.00	0.00	-0.00	-0.07	1.00	-0.94	0.00	-0.00	0.00
2	3	-0.00	-0.00	0.00	-0.11	-0.94	1.00	0.00	0.00	-0.00
3	1	-0.00	0.00	0.00	-0.00	0.00	0.00	1.00	-0.09	-0.07
3	2	0.00	-0.00	0.00	0.00	-0.00	0.00	-0.09	1.00	-0.94
3	3	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.07	-0.94	1.00

Table 3. Single gas; multizone error ($\text{m}^3 \text{h}^{-1}$)

Zone	Infiltration	CCTG 5%	Uncertainty CCTG ∞	Actual CCTG	MTMS
1	9	<1	2	12	9
2	9	<1	5	7	1
3	74	4	8	15	15

the air flows from the previous dataset and the measurement uncertainties from the description of the CCTG to estimate the uncertainties in the flow. The reported standard deviation of the concentration ranges from 0–15% of target; we will use 5% as a representative value. The comparison follows in Table 3. The "CCTG 5%" column represents the value error estimate using only that reported in [8]. The "CCTG ∞ " column represents the error including the δI term, but not the \dot{C} term (as would be appropriate for a long analysis time) and the "Actual CCTG" column contains the error analysis that should be compared with the previous MTMS analysis (repeated in last column).

The actual errors for the two methods come out on the same order of magnitude, which is not surprising, since the data errors in the two methods are similar. It should be noted, however, that these errors are an order of magnitude larger than would be calculated by the methods in [8] because those methods do not take into account the errors associated with the variance of the concentrations.

PFT method

An error analysis of the PFT method was done by D'Ottavio [11] in which the matrices were augmented with an addition row and column to account for the outside and then a matrix error propagation method that assumed small, normally distributed errors was used to find the uncertainties in the flows.

The D'Ottavio method left one issue undiscussed. The equation used in the analysis assumed steady state and thus ignored the contribution of the \dot{C} term. This assumption is reasonable for long periods of time, but can cause some additional uncertainty in the result. The error analysis should contain the \dot{C} error term, having a value on the order of the average concentration divided by the length of the experiment. Only if the associated error term is small compared to the other error terms is its neglect warranted. For sufficiently long experiments, this

will undoubtedly be the case; but a careful error analysis should include the term. Thus, the D'Ottavio uncertainties will tend to be low.

We have used the method presented herein to compute the uncertainties from the values in the D'Ottavio paper, and display them in Table 4.

In this example Q_{13} is physically disallowed—although there is overlap in the allowable range. Such results are an artifact of the analysis method used and represent a potential bias, which is not accounted for in our uncertainty analysis.

We have used our formulae to calculate the total correlation matrix for this dataset. Contained in Table 5 it again allows us to determine the correlations between errors and the validity of any simplifying assumptions.

This matrix tends towards a block diagonal matrix. The sum of the off-diagonal elements (e.g. 2, 1; 1, 1) are too large for the weak coupling limit to apply in this circumstance. Nevertheless, between these two tables, the value and error associated with any combination of air flows can be calculated.

Intercomparison

We can use the last dataset to make a theoretical intercomparison of the uncertainties of the three techniques. We use the three infiltration flows: the PFT data is copied from the table above; for the other two it is assumed that there was a system of the kind described above running

Table 4. Example air flows and uncertainties for PFT dataset ($\text{m}^3 \text{h}^{-1}$)

$Q_{ij} \pm \sigma_{Q_{ij}}$	1	2	3	Outside
1	667 ± 107	-314 ± 64	15 ± 25	368 ± 61
2	-132 ± 43	454 ± 52	-212 ± 33	110 ± 33
3	-17 ± 5	-23 ± 6	293 ± 43	254 ± 37
Outside	518 ± 92	118 ± 69	97 ± 42	733 ± 59

Table 5. Correlation matrix for PFT dataset [$r_{Q_{ij}, Q_{ij}}$]

i	j	1	1	1	2	2	2	3	3	3
		1	2	3	1	2	3	1	2	3
1	1	1.00	-0.84	0.17	-0.48	0.21	-0.03	-0.21	0.10	-0.00
1	2	-0.84	1.00	-0.55	0.42	-0.28	0.11	0.19	-0.15	0.00
1	3	0.17	-0.55	1.00	-0.09	0.18	-0.28	-0.07	0.08	0.04
2	1	-0.48	0.42	-0.09	1.00	-0.65	0.23	-0.24	0.10	0.02
2	2	0.21	-0.28	0.18	-0.55	1.00	-0.67	0.09	-0.10	0.03
2	3	-0.03	0.11	-0.28	0.23	-0.67	1.00	0.23	0.33	-0.49
3	1	-0.21	0.19	-0.07	-0.24	0.09	0.23	1.00	-0.17	-0.48
3	2	0.10	-0.15	0.08	0.10	-0.10	0.33	-0.17	1.00	-0.63
3	3	-0.00	0.00	0.04	0.02	0.03	-0.49	-0.48	-0.63	1.00

Table 6. Comparison of uncertainties for three techniques ($\text{m}^3 \text{h}^{-1}$)

Zone	Infiltration	PFT	Uncertainty CCTG	MTMS
1	368	61	33	4
2	110	33	25	1
3	254	37	8	3
Total		8%	5%	1%

during the month and that an average infiltration was calculated for that period (i.e. all three methods have been corrected to the time period used in the PFT example). Thus for the two real time cases the purely random error will be greatly reduced by the large number of measured data points. The results are shown in Table 6.

The "Total" row displays the percentage error of the total infiltration for the building.

DISCUSSION

We have presented a detailed error analysis for use with multizone tracer gas analysis techniques, which yields estimates for the uncertainty of the calculated air flows from errors in the measurements of tracer gas data. Our analysis assumes that the underlying model used in the analysis is simply that of the multizone continuity equation.

There are several potential biases possible in such tracer techniques. Most of these biases are associated with mixing problems and the adequacy of the zonal model. To some extent poor mixing can be compensated for by increasing the uncertainties in the data to account for the facts that the injected flow is not instantaneously mixed that that the measured concentration may be noisier than the actual zonal average. Such techniques, however, can only be used on the margin; very poor mixing can easily result in the breakdown of the assumptions of the zonal model. In such cases a more complex model must be developed for both the interpretation of the data and its associated error analysis.

The examples used herein all use continuous injection. Many researchers use decay techniques for multizone buildings using both single gas [12] and multigas [13-15] variants. Because of the special kinds of assumptions often made, and because of the unique biases which can affect this analysis technique, a comparison of uncertainties is beyond the scope of this report.

There are other types of bias that occur which are not included in this analysis. For example, using long-term averaged data can cause a bias in the estimation of the average air flows [16]. Such bias is not included in this error analysis. Other types of errors may be associated with different measurement strategies [17]. Bias and linearity in the measurement equipment is also not accounted for explicitly in our derivation, nor is the effect of time lags [18]. Errors associated with exogenous variables (e.g. source strength dependence on temperature) are also not considered. Similarly, the analysis herein has assumed that the uncertainty of the input data is independent of its value; thus any such non-linearities that exists will not be reflected in our uncertainties.

The examples we have used serve to show that for a given environment the real-time systems (e.g. MTMS) will probably be more precise than the integrated systems (e.g. PFT). (The incomplete, real-time systems such as the CCTG fall in between for those quantities that they calculate.) Such a result is not surprising, because the real-time systems take orders of magnitude more data (in the example used) than do the integrated systems.

Precision alone should not, however, be used to indict a particular type of system. Each kind of system may have different practical advantages—as well as different biases and precision—and hence, different applications. The trade off between different accuracies, precisions, and practical advantages will be up to the user to determine. This report has derived the relationships necessary to calculate the uncertainties of the air flows from the data.

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REFERENCES

1. P. H. Singer, Helium leak detection in vacuum systems and IC packages. *Semicond. Int.* **10**, 51-55 (1987).
2. R. McCullough, Leak testing. *ASTM Stand. News* **10**, 32-33 (1982).
3. G. J. Ferber, K. Telegadas, J. L. Heffler, C. R. Dickson, R. Dietz and P. Krey, Demonstrations of a long-range atmospheric tracer system using perfluorocarbons. BNL-33846, Brookhaven National Laboratory (1983).
4. P. Lagus and A. K. Persily, A review of tracer gas techniques for measuring airflows in buildings. *ASHRAE Trans.* **91**, 1075-1087 (1985).
5. D. T. Harje, G. S. Dutt and D. L. Bohac, Documenting air movements and infiltration in multicell buildings using various tracer-gas techniques. *ASHRAE Trans.* **91-2B**, 2012-2027 (1985).
6. F. D. Heidt and H. Werner, Microcomputer-aided measurement of air change rates. *Energy Bldgs* **9**, 313-320 (1986).
7. T. W. D'Ottavio and R. N. Dietz, Errors resulting from the use of single zone ventilation models on multi-zone buildings: implications for energy conservation and indoor air quality studies. *ASHRAE Trans.* **91-2B**, 1777-1789 (1985).
8. D. J. Bohac, The use of a constant concentration tracer gas system to measure ventilation in buildings. PU/CEES Rpt No. 205, Princeton University (1986).
9. R. N. Dietz, R. W. Goodrich, E. A. Cote and R. F. Wieser, Detailed description and performance of a passive perfluorocarbon tracer system for building ventilation and air exchange measurements. ASTM Special Tech. Publ. No. 904, p. 203, American Society of Testing and Materials, Philadelphia, PA (1986).

10. B. Kvisgaard, P. F. Collet and J. Kur, Research on fresh-air change rate: 1, Rpt, Tech. Inst. of Copenhagen, ISBN 87-5711-460-7 (1985).
11. T. W. D'Ottavio, G. I. Senum and R. N. Dietz, Error analysis techniques for perfluorocarbon tracer derived multizone ventilation rates, *Bldg Envir.* **23**, 187-194 (1988).
12. R. R. Walker, M. D. Perera, O. D. Oglesby and M. B. Hathaway, Evaluation of a simple technique for measuring infiltration rates in large and multicelled buildings using a single tracer gas. *ASHRAE Trans.* **91-2B**, 2028-2043 (1985).
13. J. R. Waters and M. W. Simons (1987) The evaluation of contaminant concentration and air flows in a multizone model of a building. *Bldg Envir.* **22**, 305-315 (1987).
14. C. Irwin, R. E. Edward and A. T. Howarth, The measurement of airflows using a rapid response tracer gas technique. *Bldg Serv. Engng Res. Techn.* **6**, 146-152 (1985).
15. J. J. Prior, C. J. Martin and J. G. F. Littler, An automatic multi-tracer-gas method for following interzonal air movement. *ASHRAE Trans.* **91-2B**, 1997-2011 (1985).
16. M. H. Sherman, Analysis of errors associated with the passive ventilation measurement technique. *Bldg Envir.* **24**, 131-139 (1989).
17. M. Sandberg and C. Blomqvist, Quantitative estimate of the accuracy of tracer gas methods for the determination of the ventilation flow rate in buildings. *Bldg Envir.* **20**, 139-150 (1985).
18. J. R. Waters, Time lags in a two-zone air movement model. *Bldg Serv. Res. Tech.* **18**, 43-46 (1987).