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A Predictive Stochastic Model For Indoor Air Quality

F. HAGHIGHAT*
P. FAZIO*
T. E. UNNY†

A method for estimating the uncertainty in the prediction of indoor air quality is described. The pollutant concentration is sensitive to meteorological data, source, etc. which are stochastic in nature. This method is based on Itô stochastical differential equations which provides the statistical characteristic of variables of interest. This model can simultaneously consider randomness in the inputs and the coefficients. Randomness in this case is modelled as a Guassian white noise process. The moment equations are developed which provide the mean and variance of pollutant concentration and required fresh air. This information is useful in design of the ventilation systems. Illustrative examples are presented.

INTRODUCTION

RECENTLY, the issue of indoor air quality has gained impetus due to energy conservation measures which have resulted in tighter house construction and reduced air exchange rate. A considerable effort is now being devoted to estimate the indoor air pollutant concentrations and to assess the risk from exposure to pollutants. Many current ventilation standards are based on the removal requirement of various pollutants in buildings. To meet the requirement, the amounts of various pollutants that may be present in the room air must be below safe concentration levels as specified in the ASHRAE standard 62-1981 [1]. The knowledge of the maximum concentration level of pollutants is particularly important if the health effects of the specific pollutant are not known, or are considered harmful, i.e. to chemically hypersensitive people [2].

Deterministic models are usually based on a pollutant mass balance for a particular indoor volume [3, 4]. There are several limitations to these approaches. Specifically, modelling the system deterministically implies that the spread of input parameter values is zero, and that we are 100% certain of the corresponding output values. The deterministic approach is valid if the effect of fluctuations in the forcing functions (windspeed, outdoor air temperature and pollutant concentration, etc.) is negligible when compared to the mean values. When random fluctuations are significant, the variance of response can no longer be neglected.

The empirical approaches are obtained by a statistical least-square fit to a set of data [5]. The drawback of this method is that the effects of all potential fluctuations are smoothed out, and the user is restricted to a reference room (i.e. cannot take into account the temporal and spatial variations in mixing coefficients). Therefore the essential physics of the problem is lost.

The approaches noted above predict only the expected behaviour of the state of the system. That is, these approaches yield the single point probability information (single statistic, the expected value of pollutant concentration). However, they are limited by the fact that the parameters defining the system have been considered to be deterministic and not all of the types of uncertainties describing the system are taken into account. Therefore, the design value is multiplied by a safety factor to account for the uncertainty in the design parameter and input variables. The safety factor is based on engineering judgement.

An alternative approach to using safety factors to account for uncertainties in design is to use a stochastic model of the process at the outset. This approach has been successfully applied to many physical systems [6, 7]. In this approach, the pollutant mass balance equation reduces to a stochastic differential equation, the solution of which represents the relationship between the statistics of the random input variables and the parameters of the model. The results of the analysis developed herein can be considered to provide complementary information to those which can be derived from traditional deterministic methods.

The primary objective of this study is to develop stochastic modelling procedures for predicting the quality of indoor air under random variations of external and internal load and system parameters. The motivation for such study arises from the fact that all the forcing functions in the mass balance equation such as infiltration (wind speed and temperature), source, etc. are random in nature.

* Centre for Building Studies, Concordia University, Montreal, Quebec, H3G 1M8, Canada.
† Department of Systems Design Engineering, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada.

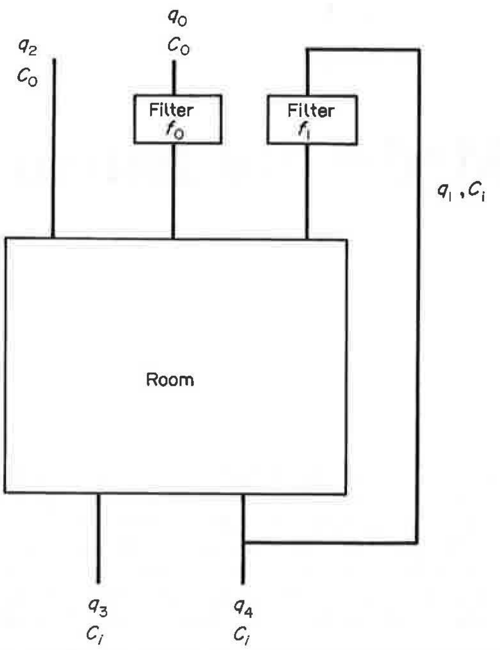

Fig. 1. Ventilation system for time varying indoor–outdoor model [8].

MODEL DEVELOPMENT

A technique often used to estimate expected indoor pollutant concentrations is based on pollutant mass balance. The general model of pollutant mass balance for an interior space can be written as

$$\frac{\mathrm{d}C}{\mathrm{d}t} = f(C, t), \tag{1}$$

where C is the n-vector of pollutant concentration and $f(C, t)$ is a vector of functions which quantitatively represents the rate of change in concentration with respect to time. The solution to this differential equation for appropriate boundary conditions can be found by ordinary calculus. The method may be illustrated with an example. Consider a room for which the mass balance for pollutant flow into and out of it can be written as (Fig. 1), [8]

$$v\frac{\mathrm{d}C_i}{\mathrm{d}t} = -k(q_0+f_1q_1+q_2)C_i + k(q_0(1-f_0)+q_2)C_o+S-R, \tag{2}$$

where C_i is the indoor concentration and C_o is outdoor concentration; q is the volumetric flow rate for make-up air (q_0), recirculation (q_1), infiltration (q_2), exfiltration (q_3), and exhaust (q_4); f is the filter efficiency for make-up (f_0) and recirculation air (f_1); v is the volume of the room; S is the indoor source emission rate; R is the indoor sink removal rate; and k is a factor which accounts for the inefficiency of mixing. The solution for Eq. (1), considering all the parameters are constant and with an initial value of $C_i = C_s$, is:

$$C_i = \frac{[k(q_0(1-f_0)+q_2)C_o+S-R]}{k(q_0+q_1f_1+q_2)} \times [1-\mathrm{e}^{\frac{-k}{v}(q_0+q_1f_1+q_2)t}]+C_s\mathrm{e}^{\frac{-k}{v}(q_0+q_1f_1+q_2)t}. \tag{3}$$

This is a deterministic solution which assumes that there is no uncertainty in either the parameter values or the inputs and hence the output can be predicted with certainty. That is not the case for indoor air pollution modelling, since most of the parameters cannot be predicted with certainty. As an example, infiltration rate, q_2, is a function of the temperature and pressure differences between indoor and outdoor air. Temperature differences cause the stack effect, which is a result of warm air rising. A study showed that the outdoor air temperature is stochastic in nature [9]. Haghighat *et al.* [6, 7], showed that the indoor room air temperature is also stochastic in nature because of fluctuations in outdoor condition and internal heat gains. Mustacchi *et al.* [9] showed that the wind speed is stochastic in nature, which is another factor influencing the infiltration rate.

Studies by Tamura [10], and Sherman and Grimsrud [11] showed that the variation in the infiltration rate can be as high as 60% of the mean value. Similar observation about the uncertainty of the values of other parameters such as S, C_o, k, q can be found in the literature [8, 12, 13]. The randomness can be included in the parameters as:

$$\begin{aligned} S &= \bar{S}+S' \\ C_o &= \bar{C}_o+C'_o \\ q_0 &= \bar{q}_0+q'_0 \\ q_2 &= \bar{q}_2+q'_2 \end{aligned} \tag{4}$$

where the quantities with an overbar are the deterministic parts, and the primed parts are random noise. Equation (1) is not adequate to accommodate this uncertainty, because the parameters are known only in terms of their distributions. Therefore, for realistic modelling, Eq. (1) is redefined as a vector of stochastic differential equation of the form,

$$\frac{\mathrm{d}C}{\mathrm{d}t} = f(C,t)+G(C,t)n(t), \tag{5}$$

where $n(t)$ is a noise term and $G(C, t)$ is a function denoting the sensitivity of the system to the noise term. Substituting Eq. (4) in Eq. (2), the $f(C, t)$ and $G(C, t)$ becomes:

$$f(C,t) = \left[\frac{-kC_i}{v}(\bar{q}_0+f_1q_1+\bar{q}_2) + \frac{\bar{C}_ok}{v}(\bar{q}_0(1-f_0)+\bar{q}_2)+\frac{\bar{S}}{v}-\frac{R}{v}\right]$$

$$G(C,t) = \left[\frac{k(\bar{C}_o(1-f_0)-C_i)}{v} \quad \frac{k(\bar{C}_o-C_i)}{v} \times \frac{k(\bar{q}_0(1-f_0)+\bar{q}_2)}{v} \quad \frac{1}{v}\right]$$

$$n(t) = [q'_0 q'_2 C'_o S']^T.$$

The product term C_oq_0 has been linearized as follows:

$$C_oq_0 = (\bar{C}_o+C'_o)(\bar{q}_0+q'_0) = \bar{C}_o\bar{q}_0+\bar{C}_oq'_0+C'_o\bar{q}_0.$$

The term $C'_oq'_0$ is neglected because it is second order, the product is small compared to the mean valves, and the resulting equation is then linear and solvable. The random noise term should have two features. First it should be sufficiently comprehensive to provide an adequate description of the random disturbance and second, it should provide for the existance and uniqueness of the solution. In general, white noise $[W(t)]$ is not only able to describe the random fluctuations in the environment in a physically meaningful way, but it also provides a unique and satisfactory solution. In order to make the set of Eq. (5) readily solvable, it is both useful and reasonable to consider that the stochastic noise belongs to wide band processes with a flat frequency spectrum up to very high frequencies. These wide band processes are almost delta-correlated so that they can be approximated by white noise processes. Now Eq. (5) becomes:

$$dC = f(C,t)\,dt + G(C,t)\,dB(t) \tag{6}$$

where $f(C,t)$ and $G(C,t)$ are given continuous functions having continuous second-order derivatives with regard to all variables for $C \in R$, and $B(t)$ is an independent Wiener processes $[dB(t) = W(t)\,dt]$. Equation (6) is called a Stochastic Differential Equation (SDE). A more complete discussion of the subject can be found in articles [6, 7, 14].

There is a lemma in stochastic Itô calculus that states that if ϕ is a scalar valued real function of the solution process to the SDE (6) such that it has continuous first and second derivatives, then with reference to SDE (6) the stochastic differential equation satisfied by ϕ is [14]:

$$\frac{d\theta}{dt} = \sum_{j=1}^{n} f_j(C,t)\frac{\partial\theta}{\partial T_j} + \frac{1}{2}\sum_{i,j=1}^{n}(GDG^T)_{ij}\frac{\partial^2\theta}{\partial T_i\,\partial T_j} + \frac{\partial\theta}{\partial t}, \tag{7}$$

where R is the diagonal covariance parameter matrix [which shows how $W(t)$ is correlated with itself],

$$\langle W(t)W(t-s)^T\rangle = D(t)\delta(t), \tag{8}$$

where $\delta(t)$ is the Dirac–Delta function, and $\langle\cdot\rangle$ denotes expectation. The expectation of Eq. (7) yields,

$$\frac{d\langle\theta\rangle}{dt} = \sum_{j=1}^{n}\left\langle f_j(C,t)\frac{\partial\theta}{\partial T_j}\right\rangle + \frac{1}{2} \times \sum_{i,j=1}^{n}\left\langle (GDG^T)_{ij}\frac{\partial^2\theta}{\partial T_i\,\partial T_j}\right\rangle + \left\langle\frac{\partial\theta}{\partial t}\right\rangle \tag{9}$$

let

$$\theta = \prod_{i=1}^{n} C_i^{k_i}, \tag{10}$$

where the k_i are positive integers satisfying

$$\sum_{i=1}^{n} k_i = m,$$

where m is the order of the moment and n is the number of variables.

For example, in the case of the previous problem, if the pollutant of interest is CO_2 then, $n = 1$, $\theta = C_i^{k_i}$, $m = 1,2$ (i.e. first and second moment). The moment equations are generated by substituting appropriate value for k_i in Eq. (10). For the first moment (mean value), the equation can be generated by setting

Table 1. Assumptions for one compartment model

Term		Mean	Standard deviation
Room size	v	250 m^3	0.0
Outdoor concentration	C_o	0.2 g m^{-3}	0.2 g m^{-3}
Source	S	410 mg h^{-1}	40 mg h^{-1}
Sink	R	0.0	0.0
Factor for inefficient mixing	k	0.3	0.0
Ventilation rate			
make up air	q_0	125 m^3 h^{-1}	25 m^3 h^{-1}
recirculation	q_1	520 m^3 h^{-1}	0
infiltration	q_2	50 m^3 h^{-1}	20 m^3 h^{-1}
Filter efficiency for			
recirculated air	f_1	0.0	0.0
make up air	f_0	0.0	0.0

$$k_1 = 1;\quad \frac{d\bar{C}_i}{dt} = \frac{-k}{v}(\bar{q}_0 + f_1 q_1 + \bar{q}_2)\bar{C}_i + \frac{k}{v}(\bar{q}_0(1-f_0)+\bar{q}_2)\bar{C}_o + \frac{\bar{S}-R}{v}, \tag{11}$$

which is the same as the deterministic equations given in Eq. (2). Similarly, the second moment equation is obtained as given below:

$$k_1 = 2;\quad \frac{d\bar{C}_i^2}{dt} = \left[\frac{-2k}{v}(\bar{q}_0+f_1q_1+\bar{q}_2) + \frac{k^2}{v^2}(\sigma_{q_0}^2+\sigma_{q_2}^2)\right]\overline{C_i^2} + 2\left[\left(\frac{k}{v}(\bar{q}_0(1-f_0)+\bar{q}_2) - \frac{k^2}{v^2}(1-f_0)\sigma_{q_0}^2 - \frac{k^2}{v^2}\sigma_{q_2}^2\right)\bar{C}_o + \left(\frac{\bar{S}-R}{v}\right)\right]\bar{C}_i + \frac{k^2}{v^2}\left[\overline{C_o^2}((1-f_0)^2\sigma_{q_0}^2+\sigma_{q_2}^2) + (\bar{q}_0(1-f_0)+q_2)\sigma_{c_o}^2 + \frac{\sigma_s^2}{k^2}\right]. \tag{12}$$

The third moment is obtained in a similar manner:

$$k_1 = 3;\quad \frac{d\bar{C}_i^3}{dt} = \left[\frac{-3k}{v}(\bar{q}_0+f_1q_1+\bar{q}_2) + \frac{3k^2}{2v^2}(\sigma_{q_0}^2+\sigma_{q_2}^2)\right]\overline{C_i^3} + 3\left[\left(\frac{k}{v}(\bar{q}_0(1-f_0)+\bar{q}_2) - \frac{k^2}{v^2}(\sigma_{q_0}^2(1-f_0)+\sigma_{q_2}^2)\right)\bar{C}_o + \frac{\bar{S}-R}{v}\right]\overline{C_i^2} + \frac{3k^2}{2v^2}\left[\left((1-f_0)^2\sigma_{q_0}^2+\sigma_{q_2}^2)\overline{C_o^2} + (\bar{q}_0(1-f_0)+\bar{q}_2)^2\sigma_{c_o}^2 + \frac{\sigma_s^2}{k^2}\right)\right]\bar{C}_i. \tag{13}$$

As mentioned earlier, the air infiltration, the make up air, the outdoor concentration, and the source are considered to be independent white noise stochastic processes, distributed as, $N_{q_0}(\bar{q}_0, \sigma_{q_0})$, $N_{q_2}(\bar{q}_2, \sigma_{q_2})$, $N_{c_o}(\bar{S}, \sigma_s)$ and $N_{c_o}(\bar{C}_o, \sigma_{Co_2})$ respectively. The mean values and standard deviations are considered time independent and are given in Table 1. A numerical method is applied

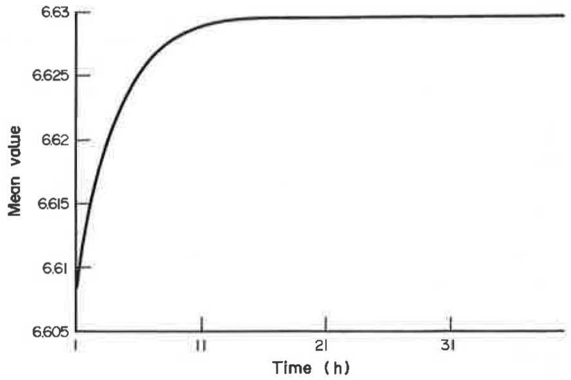

Fig. 2. Mean hourly pollutant concentration vs. time.

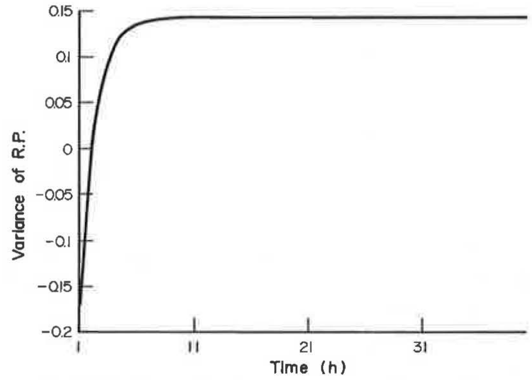

Fig. 3. Variance of pollutant concentration vs. time.

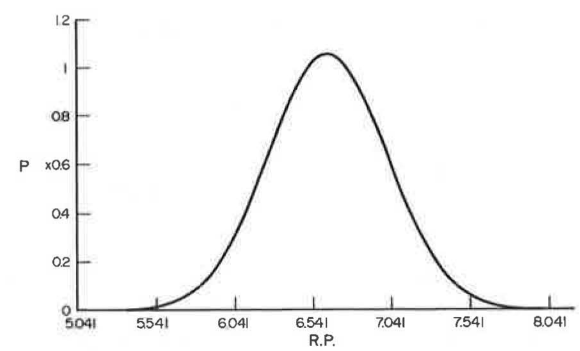

Fig. 4. The probability distribution of pollutant concentration.

to the set of Eqs (11) and (12). Figures 2 and 3 show the mean value and variance of CO_2 concentration in indoor air respectively as functions of time. The steady-state values for mean and variance are reached for times greater than 10 h respectively. The probability density of the CO_2 concentration at steady-state is also shown in Fig. 4.

The third central moment furnishes a measure of skewness, or departure from symmetry about the mean of the distribution. Skewness is obtained through the solution of Eq. (13). It was found that the probability distribution of CO_2 is negatively skewed. Skewness of distribution implies the need for considering asymmetric bound for allowable deviation from mean values.

Two-compartment models

Reible and Shair [15] proposed a two-compartment model, in order to study the operation of a ventilation system. The assumption in this model is that the air

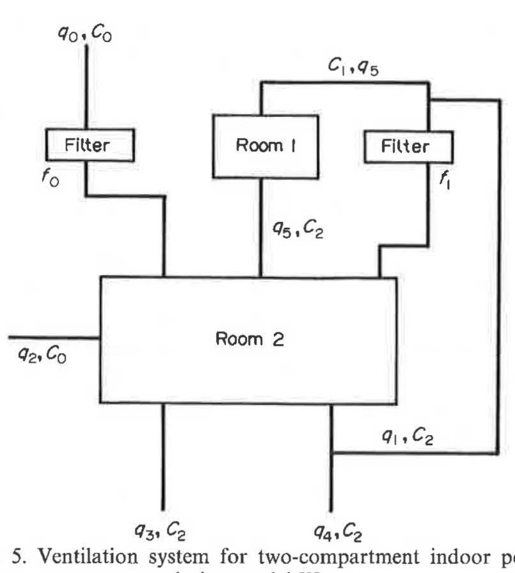

Fig. 5. Ventilation system for two-compartment indoor pollution model [8].

supplied to compartment 1 can only be transferred through ventilation ducts or from hallways through doors and other openings (see Fig. 5). The mass balance equations for each of the compartments can be written as:

$$\frac{dC_1}{dt} = -\left(\frac{k_1q_5+k_1v_1}{v_1}\right)C_1+\frac{k_1q_5}{v_1}C_2+\frac{S_1-R_1}{v_1},$$

$$\frac{dC_2}{dt} = -\frac{k_2(q_1f_1+k_2v_2+q_0+q_2)}{v_2}C_2+\frac{k_2q_5(1-f_1)}{v_2}C_1 +\frac{S_2+R_2+k_2q_2C_0+k_2(1-f_0)q_0C_0}{v_2}, \tag{14}$$

where the nomenclature is as before. Now, considering that q_0, C_0, S_1, q_2 and S_2 are stochastic in nature, we can write,

$$\begin{aligned} q_0 &= \bar{q}_0+q'_0 \\ q_2 &= \bar{q}_2+q'_2 \\ C_0 &= \bar{C}_0+C'_0 \\ S_1 &= \bar{S}_1+S'_1 \\ S_2 &= \bar{S}_2+S'_2. \end{aligned} \tag{15}$$

Then the Eq. (14) becomes:

$$\frac{dC_1}{dt} = -\left(\frac{k_1q_5+k_1v_1}{v_1}\right)C_1+\frac{k_1q_5}{v_1}C_2+\frac{\bar{S}_1-R_1}{v_1}+\frac{S'_1}{v_1}$$

$$\frac{dC_2}{dt} = -\frac{k_2(q_1f_1+k_2v_2+\bar{q}_0+\bar{q}_2)}{v_2}C_2+\frac{k_2q_5(1-f_1)}{v_2}C_1 +\frac{\bar{S}_2+R_2+k_2\bar{q}_2\bar{C}_0+k_2(1-f_0)\bar{q}_0\bar{C}_0}{v_2} +\frac{S'_2+k_2(q'_2\bar{C}_0+\bar{q}_2C'_0+k_2(1-f_0)(\bar{q}_0C'_0+q'_0\bar{C}_0)}{v_2}. \tag{16}$$

Now, the set of mass balance can be written as given in

Eq. (6), where the terms $G(C, t)$ and $\mathrm{d}B_t$ are given by:

$G(T, t) =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & \dfrac{1}{v_1} \\ \dfrac{k_2}{v_2}\bar{C}_0 & \dfrac{k_2(\bar{q}_2+\bar{q}_0(1-f_0))}{v_2} & \dfrac{k_2(1-f_0)\bar{C}_0}{v_2} & \dfrac{1}{v_2} & 0 \end{bmatrix}$$

$$\mathrm{d}B_1^T = [q'_2\, C'_0\, q'_0\, S'_2\, S'_1]\,\mathrm{d}t \tag{17}$$

and

$$R(t) = \begin{bmatrix} \sigma_{q_2}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{C_0}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{q_0}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{S_2}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{S_1}^2 \end{bmatrix}.$$

The set of the first moment equations can be derived using Eqs (9) and (10), for $n = 2$ (i.e. C_1 and C_2), $\theta = C_1^{k_1}C_2^{k_2}$.

$$k_1 = 1,\quad k_2 = 0;\quad \frac{\mathrm{d}\bar{C}_1}{\mathrm{d}t} = -\left(\frac{k_1q_5+k_1v_1}{v_1}\right)\bar{C}_1+\frac{k_1q_5}{v_1}\bar{C}_2+\frac{\bar{S}_1-R_1}{v_1}$$

$$k_1 = 0,\quad k_2 = 1;\quad \frac{\mathrm{d}\bar{C}_2}{\mathrm{d}t} = -\frac{k_2(q_1f_1+k_2v_2+\bar{q}_0+\bar{q}_2)}{v_2}\bar{C}_2+\frac{k_2q_5(1-f_1)}{v_2}\bar{C}_1 + \frac{\bar{S}_2+R_2+k_2\bar{q}_2\bar{C}_0+k_2(1-f_0)\bar{q}_0\bar{C}_0}{v_2}, \tag{18}$$

the equation for the second moment is obtained as follows:

$$k_1 = 2,\quad k_2 = 0;\quad \frac{\mathrm{d}\overline{C_1^2}}{\mathrm{d}t} = -\frac{2(k_1q_5+k_1v_1)}{v_1}\overline{C_1^2}+\frac{2k_1q_5}{v_1}\overline{C_1C_2}+\frac{2(\bar{S}_1+R_1)}{v_1}\bar{C}_1+\frac{\sigma_{S_1}^2}{v^2}$$

$$k_1 = 1,\quad k_2 = 1;\quad \frac{\mathrm{d}\overline{C_1C_2}}{\mathrm{d}t} = -\frac{(k_1q_5+k_1v_1)}{v_1}\overline{C_1C_2}+\frac{k_1q_5}{v_1}\overline{C_2^2}+\frac{\bar{S}_1-R_1}{v_1}\overline{C_2} - \frac{k_2(q_1f_1+k_2v_2+\bar{q}_0+\bar{q}_2)}{v_2}\overline{C_1C_2} + \frac{k_2q_5(1-f_1)}{v_2}\overline{C_1^2} + \frac{\bar{S}_2+R_2+k_2\bar{q}_2\bar{C}_0+k_2(1-f_0)\bar{q}_0\bar{C}_0}{v_2}\bar{C}_1,$$

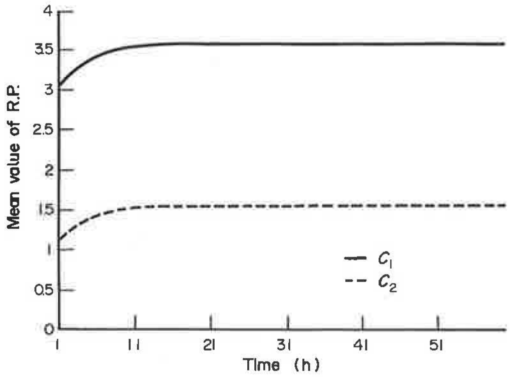

Fig. 6. Mean hourly concentration of C_1 and C_2 vs. time.

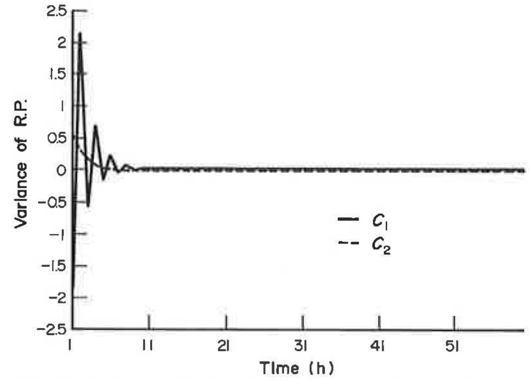

Fig. 7. Variance of C_1 and C_2 concentration vs. time.

$$k_1 = 0,\quad k_2 = 2;\quad \frac{\mathrm{d}\overline{C_2^2}}{\mathrm{d}t} = -\frac{2k_2(q_1f_1+k_2v_2+\bar{q}_0+\bar{q}_2)}{v_2}\overline{C_2^2} + \frac{2k_2q_5(1-f_1)}{v_2}\overline{C_1C_2} + \frac{2(\bar{S}_2+R_2+k_2\bar{q}_2\bar{C}_0+k_2(1-f_0)\bar{q}_0\bar{C}_0)}{v_2}\overline{C_2} + \frac{k_2^2}{v^2}\overline{C_0^2}\sigma_{q_2}^2 + \frac{k_2^2(\bar{q}_2+\bar{q}_0(1-f_0))^2}{v_2^2}\sigma_{C_0}^2 + \frac{k_2^2(1-f_0)^2}{v_2^2}\overline{C_0^2}\sigma_{q_0}^2 + \frac{1}{v_2^2}\sigma_{S_2}^2. \tag{19}$$

The mean values and variances of C_1 and C_2 are obtained through the solution of Eqs (18) and (19) numerically. The solution of these equations provides the statistical properties of the indoor pollutant concentration as function of the statistics of the random input variables and parameters of the model. The mean value and the variance of the pollutant are given in Figs 6 and 7. Based on this information, the designer is able to design the ventilation system with different levels of confidence. The probability density functions of C_1 and C_2 are shown in Figs 8 and 9. Such complimentary information, characteristic of the stochastic nature of the system, would greatly assist a designer in selecting appropriate components to match the desired level of confidence in meeting performance indices such as the indoor pollutant concentration. Such knowledge is particularly important if the health effects of the specific pollutant are

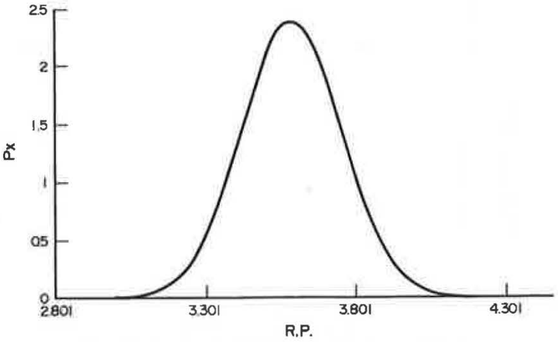

Fig. 8. The probability distribution of C_1.

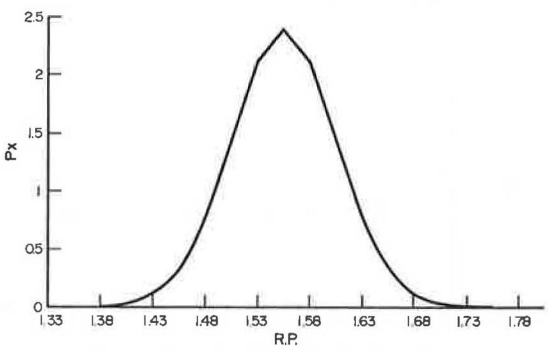

Fig. 9. The probability distribution of C_2.

Table 2. Assumptions for two-compartment model

Term		Mean	Standard deviation
Room size			
room 1	v_1	250 m^3	0
room 2	v_2	5000 m^3	0
Outdoor concentration	C_o	0.5 g m^{-3}	0.0004 g m^{-3}
Source			
room 1	S_1	410 mg h^{-1}	50 mg h^{-1}
room 2	S_2	1640 mg h^{-1}	164 mg h^{-1}
Sink			
room 1	R_1	0.0	0.0
room 2	R_2	0.0	0.0
Factor for inefficient mixing			
room 1	k_1	0.2	0.0
room 2	k_2	0.3	0.0
Ventilation rate			
room 2 make up air	q_0	1700 m^3 h^{-1}	50 m^3 h^{-1}
room 2 recirculation air	q_1	3800 m^3 h^{-1}	0
infiltration rate	q_2	50 m^3 h^{-1}	10 m^3 h^{-1}
room 1 air	q_5	600 m^3 h^{-1}	0
Particle deposition	k	0.05 h	0.0

not known, or if any exposure to a specific contaminant is unsafe, i.e. for chemically hypersensitive people.

Manual derivation of moment equation of pollutant mass balance in a multi-compartment model is not an efficient method, since the number of equations generated increases very rapidly as the number of variables increases. Automatic formulation procedures based on network theoretical technique have been developed [16, 17]. These techniques are highly suited for computer implementation. Such procedures require only simple interconnection information, nominal component values such as infiltration rate, room volume, and source or sink. The resulting equations are ordinary differential equation with constant coefficients and can be solved using efficient numerical techniques.

CONCLUSION

In this paper a stochastic differential equation has been introduced to describe the pollutant mass balance in a room. This approach enables one to incorporate the observed variability in the input variables, and the parameters of the model to characterize the variability in the output variable concentration. The behaviour of the variables can be derived in terms of the mean values and the variance associated with these quantities. This method provides complementary information to that derivable from the deterministic method. This information can help in determining the confidence level, which can then be used to calculate the exceedance probability of variables. The information thus desired can, assist designers in selecting appropriate components to match the desired level of confidence in meeting performance indices such as pollutant concentrations. The Standard deviation of the parameters given in Tables 1 and 2 are hypothetical value. Further research is required to study the statistical properties of these parameters. In most cases in indoor air quality problems, variations in source terms, outdoor air concentrations and air flow rates are almost highly autocorrelated, often cross-correlated. This information can be included in this model easily.

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