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SUMMARY OF

"State Equation of General Diffusion System Using Network Concepts and Theory of System Parameter Identification"

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written by

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Introduction

A general diffusion system consisting of conduction, transmission and advection can be systematically modelled in the form of a state-equation using this Network Concepts. In the previous paper,[1] analytical time integration scheme by projective representation giving exact solutions for simulations was proposed, and it was proved that the eigenvalues of the system have negative real parts. In the present paper, the inverse problem or the system parameter identification problem are solved generally. The system parameters are estimated from observational data of state and input variables.

State equation

Extended conductance is defined, including not only conduction and transmission but also advections.

Then between nodes i and j, extended conductance Cij = Cji, and Cij by advection has a property of asymmetry Cij = g, Cji = O, when flow rate from node j to i is g. Capacity of node i is mii, xi, xj are states of nodes i and j respectively representing temperature or concentration in cade of jth generator among ng and it represents solar radiation component or gas generator, and γij, is free input ratio from gj to node i, then the ith node equilibrium equation can be written as egn(1). In case of obtaining lumped parameter system by finite difference method or control volume method, this equation should be replaced by

$$\sum_{j=i}^{n} mij \cdot \dot{\chi} j$$

Anyway, detailed explanation of unitying or standardizing theory of all lumped parametrization methods is described in another paper[2].

$$Mii\cdot \dot{\chi} i\cdot = \sum^{n+no} Cij\cdot \chi j - \sum^{n+no} Cji\cdot\chi i + \sum_{j=1}^{ng}\gamma ij\cdot gj \qquad (1)$$

Above equation has somewhat unfamiliar form, but this can be rewritten into comprehensible form egn(2), using the law of mass balance egn (3).

$$Mii\cdot\chi i = \sum_{j=1}^{n+no} Cij\cdot(\chi j-\chi i) + \sum_{j=1}^{ng}\gamma ij\cdot gj \qquad (2)$$

$$\sum_{j=1}^{n+no} Cij = \sum_{j=1}^{n+no} Cji \qquad (3)$$

In which n is the order of the system and equals to number of state dependent nodes, no is exogenous state independent of the nodes number. The state equation can be constituted by egn(1) automatically, defining state vector $\chi = {}^{t}(g_1, \ldots, g_{ng})$.

$$M\cdot \chi = \dot{C}\cdot \chi + Co\cdot \chi o+R\cdot g \qquad (4)$$

Consequently, system parameters to be identified are above Mij, Cij, γij, and the observational states and inputs are χ, χo and g, where mij denotes the i + h row and j + h column element of capacity matrix of M.

Measured equation for system parameters

Vector m is defined that containing parameters Mij in arbitrary element order, vector C for Cij, and vector γ for γij in a similar manner. The law of mass conservation (3), or constraints from the symmetry of conduction Cij = Cji exists among the elements of vector C, therefore linear dependence

$$C = \lfloor \cdot Cm \qquad (5)$$

can be made by partial vector Cm in C. Matrix $\lfloor$ will be called reducing matrix of C. Elements of matrix $\lfloor$ are -1, o, or 1, and this can be mathematically proved. Note that this constraint plays very important role, because characteristic of eigenvalue's negative real part is brought from it. Furthermore another reducing matrix can be defined to obtain more primal parameters such as conductivity, and derived from known system parameters to the left side, egn(6) can be obtained.

$$y(t) = -\tilde{M}\cdot\dot{\chi} + [\tilde{C},\tilde{C}o] \begin{matrix}\chi \\ \chi o\end{matrix} + \tilde{R}\cdot g \qquad (6)$$

The equation can be transformed into the explicit form for M, Cm γ as egn(7).

$$y(t) = D(\chi i)\cdot m + X(\chi i)\cdot \underline{|}\cdot Cm + G(gj)\cdot \gamma \qquad (7)$$

Above transformation can be implemented automatically by general algoritm on the basis of network formulation. Where ~ represents remaining matrix after known parameters leaving for left side. Thus some of all parameters are needed to be given for the identification. In the actual situation, there are many cases of given parameters, such as multi-cell's volume in measuring air flows[3].

Egn.(7) can be made simply reduced to egn(8).

$$Y(t) = Z(t)\cdot a \qquad (8)$$

Where definitions Z(t)=[D(χi), X(χi)·L, G(gj], and ^{t}a = (^{t}M, ^{t}Cm, $^{t}\sim$) are used. Now y(t) will be called measurement vector, X(t) will be called measurement matrix, and egn.(8) will be called measurement equation for system parameters. Vector a will be called system parameters vector.

System parameters estimation

Error vector e(t) of measurement equation is represented by egn.(9). Then the evaluating function of e(t) can be defined by a time integration of quadratic form for the period [o,T] such as egn.(10).

$$e(t)=Y(t)-Z8t)\cdot a \qquad (9)$$

$$JJs(a)= \int_{o}^{t} +e(t)\cdot W(t)\cdot e(t)dt = \sum_{j=1} \int_{(j-1)\Delta t}^{j\Delta t} +e(t)\cdot W(t)\cdot e(t)dt \qquad (10)$$

Where right side is egn.(10), the integration for [o,T] is divided into short periods of Δt, an W(t) is providing inbiased estiamte of Markov-Estimate, and the calculation of W(t) will be described later. Finite differentiating the egn.(10) for the purpose of digital computing availability, the approximated evaluating function becomes egn.(11).

$$J(a)=\sum_{j=1}^{p}\frac{1}{\Delta t^2}\,{}^te_j\cdot W_j\cdot e_j=\sum_{j=1}^{p}\frac{1}{\Delta t^2}\cdot{}^t(y_j-Z_j\cdot a)\cdot W_j\cdot(y_j-Z_j\cdot a) \qquad (11)$$

Minimizing condition of J(a) by a is egn.(12), and after little manipulation, a the estimation of a, can be solved as egn.(12).

$$\frac{\partial J}{\partial a}=0 \qquad (12)$$

$$a=\left(\sum_{j=1}^{p}{}^tZ_j\cdot W_j\cdot Z_j\right)^{-1}\cdot\left(\sum_{j=1}^{p}{}^tZ_j\cdot W_j\cdot y_j\right) \qquad (13)$$

Λo is co-variance matrix of measurement equation's error caused by observartional Gaussian Noise and is expressed by egn.(14). Weighting matrix W_j is therefore given by egn.(15).

$$\Lambda o = M\cdot \mathrm{diag}(\tau Xn\cdot{}^t\tau\chi n)\cdot{}^tM + [C,Co]\cdot \mathrm{diag}(\tau\chi n+n_o)^t\cdot[C,Co] + R\cdot \mathrm{diag}(\tau g\cdot\tau g)\cdot{}^tR \qquad (14)$$

$$W_j=\left[\int_{(j-1)\Delta t}^{j\,\Delta t}\Lambda o\,dt\right]^{-1} \qquad (15)$$

Where defining $\tau\chi i$ as standard deviation of observational noise in χi, and τgi as in gi, so the vectors $\tau\chi n$, $\tau\chi$ n+no and $\tau g={}^t(\tau g1,\ldots,\tau g_{ng})$. Error transmission structure, described by egn.(14), is using egn.(13) requires iteration process to covergence, when Markov-Estimate should be implemented. But in may actual situations, those standard deviations of noise are unknown, best fit estimation could be used for practical use, substituting Wj as a mere unit matrix. In this case, no iterations is needed, and calculation can be completed in one step.

Recursive identification

Now matrix Ak is defined by egn.(16), emplpying Woodbury's matrix inversion lemma[4], egn.(17) is obtained.

$$Ak^{-1} = \sum_{j=1}^{k} {}^{t}Zj \cdot Wj \cdot Zj = Ak^{-1}_{-1} + {}^{t}Zk \cdot Wk \cdot Zk \qquad (16)$$

$$Ak = Ak-1 - Ak-1 \cdot {}^{t}Zk \cdot (Wk^{-1} + {}^{t}Zk \cdot Ak-1 \cdot Zk)^{-1} \cdot Zk \cdot Ak-1 \qquad (17)$$

The egn.(13) is transformed using egn.(17), the time discrete system for a can be deducted to egn.(18). This is a time varying system. Transition matrix Φ and driving matrix β of the system are represented by egn.(19) and egn.(20) respectively.

$$\alpha k = \Phi^{k}{}_{-1} + \beta_{k} \cdot Y_{k} \qquad (18)$$

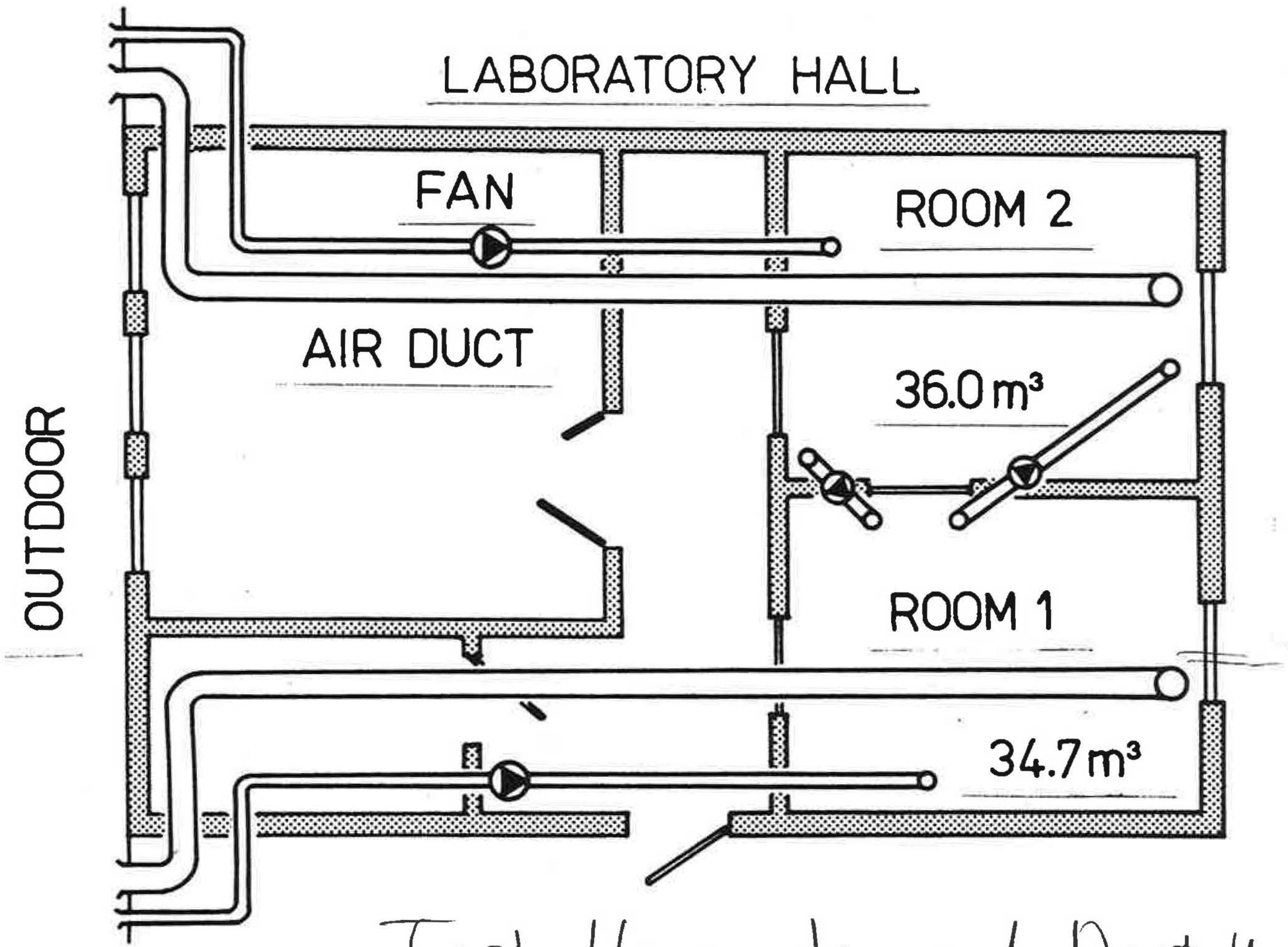
$$\Phi_{k} = Ea - Ak-1 \cdot {}^{t}Zk \cdot (Wk^{-1} + Zk \cdot Ak-1 \cdot {}^{t}Zk) \cdot Zk^{-1} \qquad (19)$$

$$\beta\ k = Ak \cdot {}^{t}Zk \cdot Wk \qquad (20)$$

Where matrix Ea is a unit matrix sized a, and initial Ao must be set to Ea. Matrix Wk always should be set to unit matrix En in the "Recursive identification" implementing process using egns.(18), (17), (19) and (20).

AIR DUCT AI ROOM 1 ROOM 2 AIR DUCT

OUT DOOR

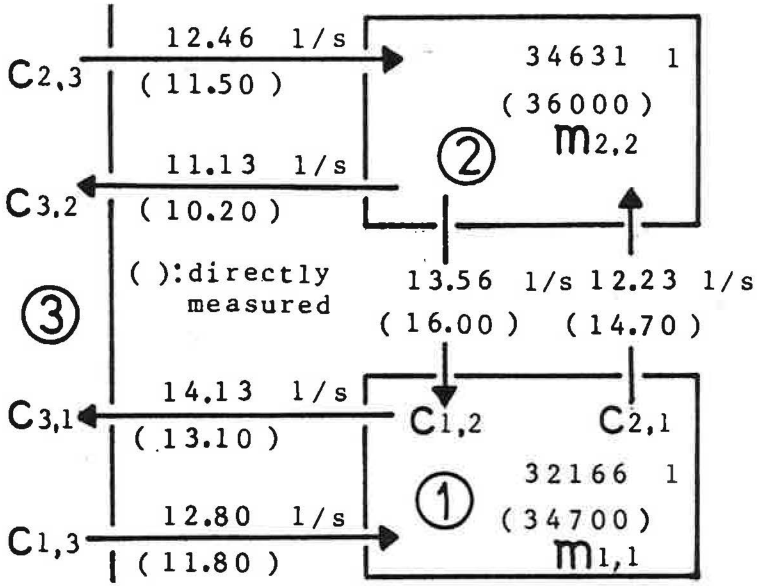


Test House plan and Duct Works

45%

ROOM 1 ROOM 2 FAN FAN FAN FAN

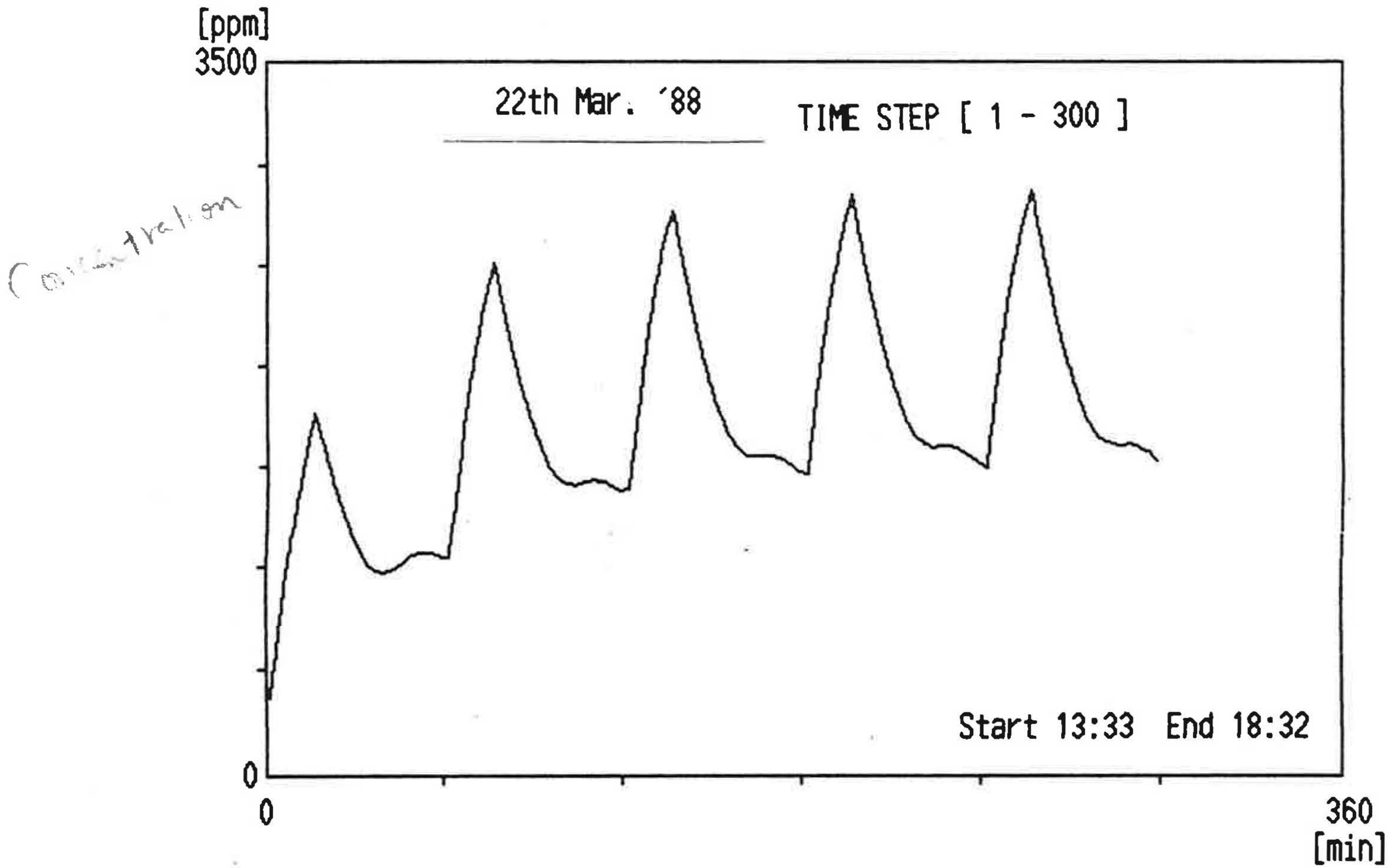
AIR DUCT ROOM 1 ROOM 2 34.7 m³ 36.0 m³



Batch Identification Results

effective volumes are also estimated.

75%


[ppm]
3500
22th Mar. '88
TIME STEP [1 - 300]
Start 13:33 End 18:32
0
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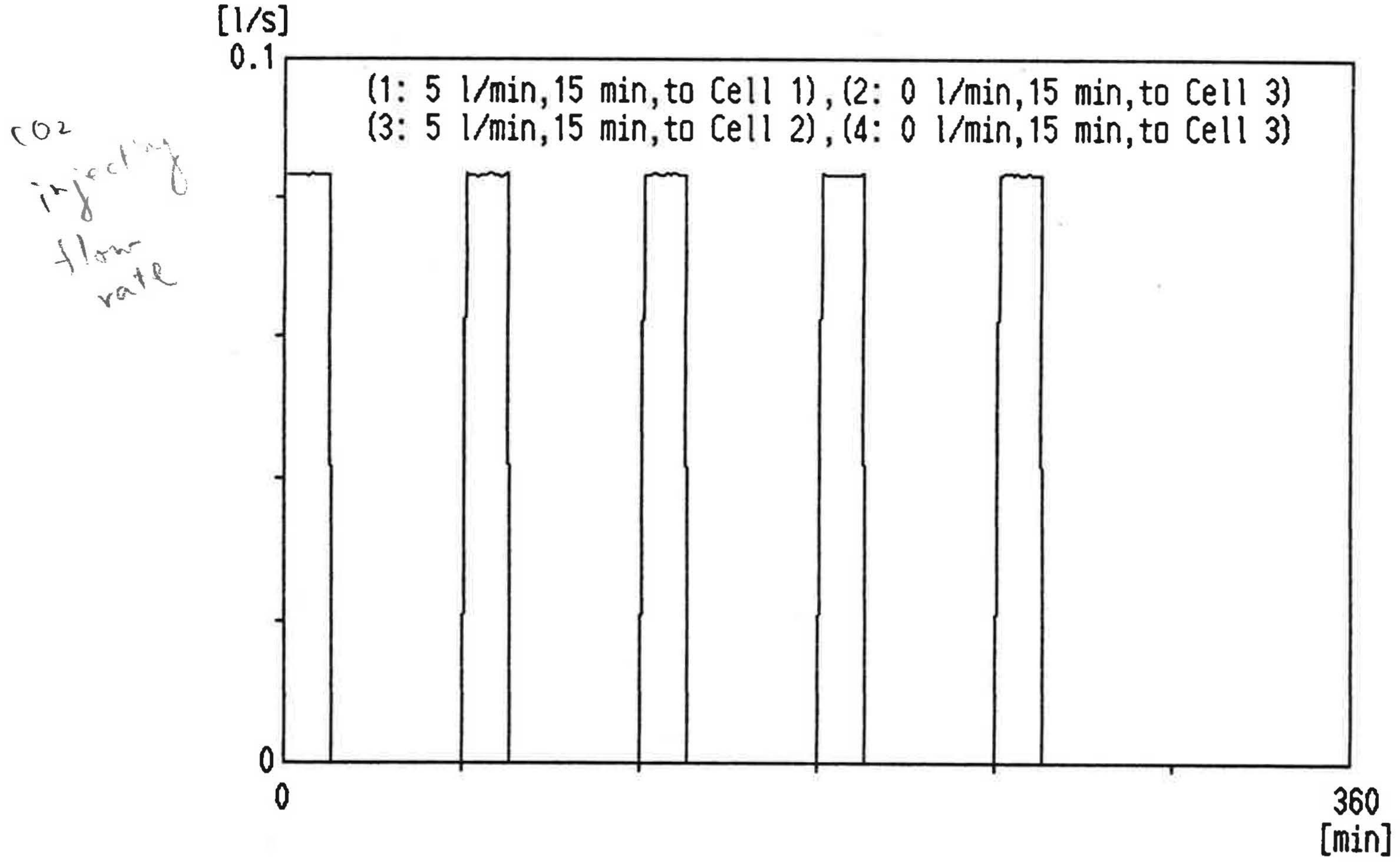
33%

room 1

ROOM NUMBER [1]   TIME STEP [1 - 300]



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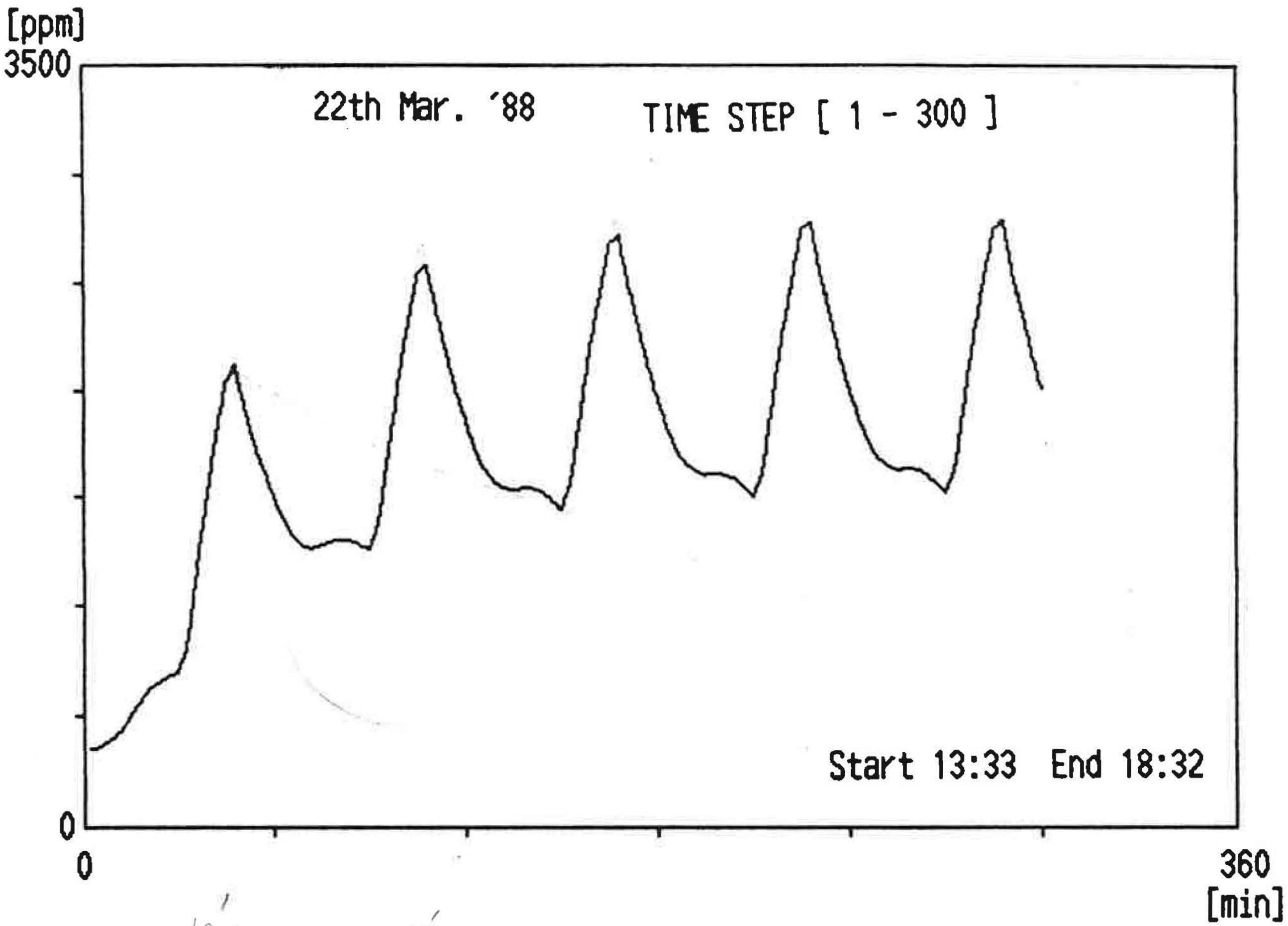

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TIME STEP [1 - 300]
Start 13:33 End 18:32
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33 %

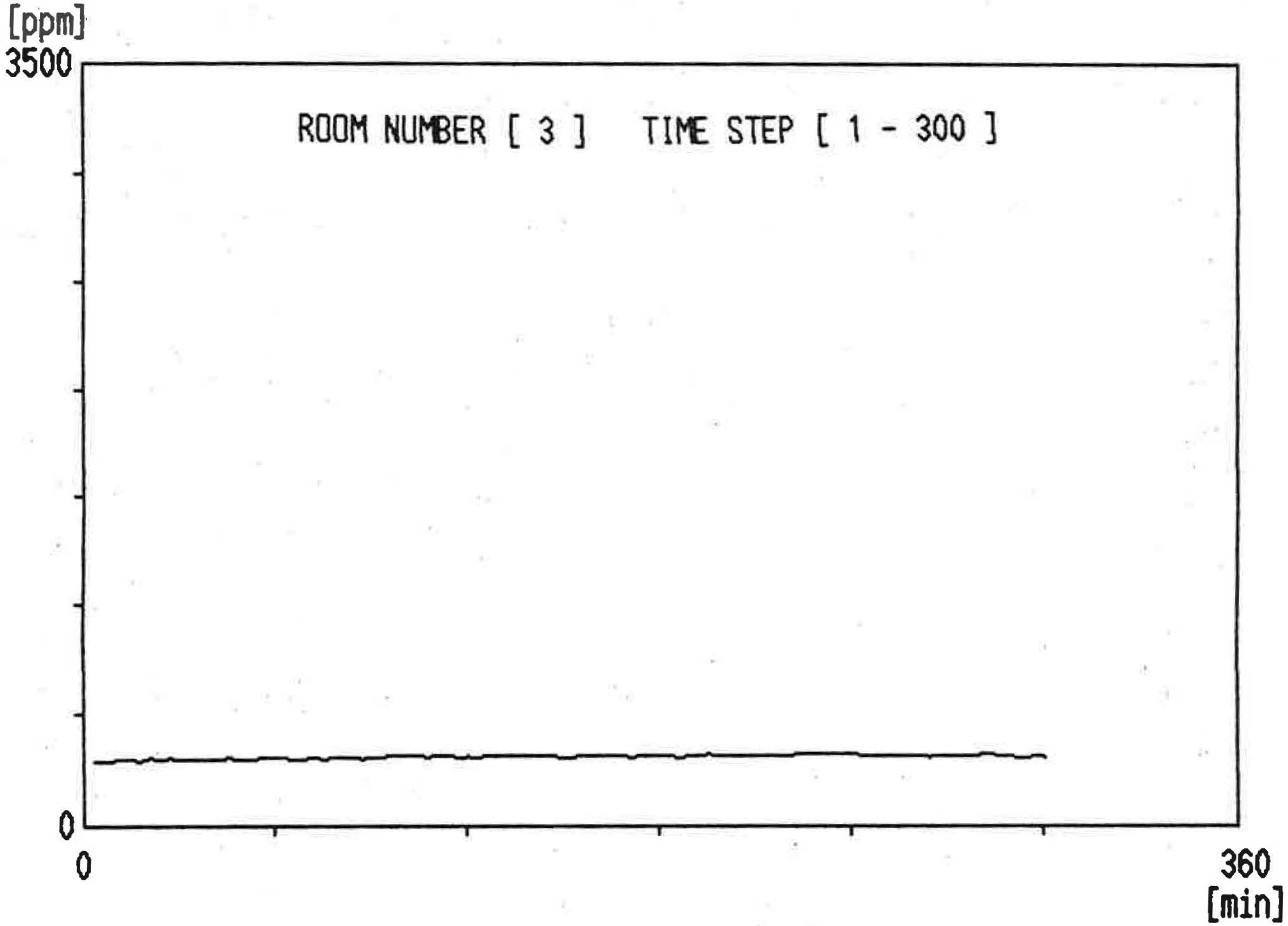
room. 2

Identified parm.

estimated error standard deviation (σ)

同定パラメータ	誤差推定標準偏差
m 1,1	1371.8 l
m 2,2	2432.1 l
c 3,2	3.464 l/s
c 3,1	3.489 l/s
c 2,3	1.494 l/s
c 2,1	2.939 l/s
c 1,3	0.905 l/s
c 1,2	1.773 l/s

80%


[ppm]
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ROOM NUMBER [3] TIME STEP [1 - 300]
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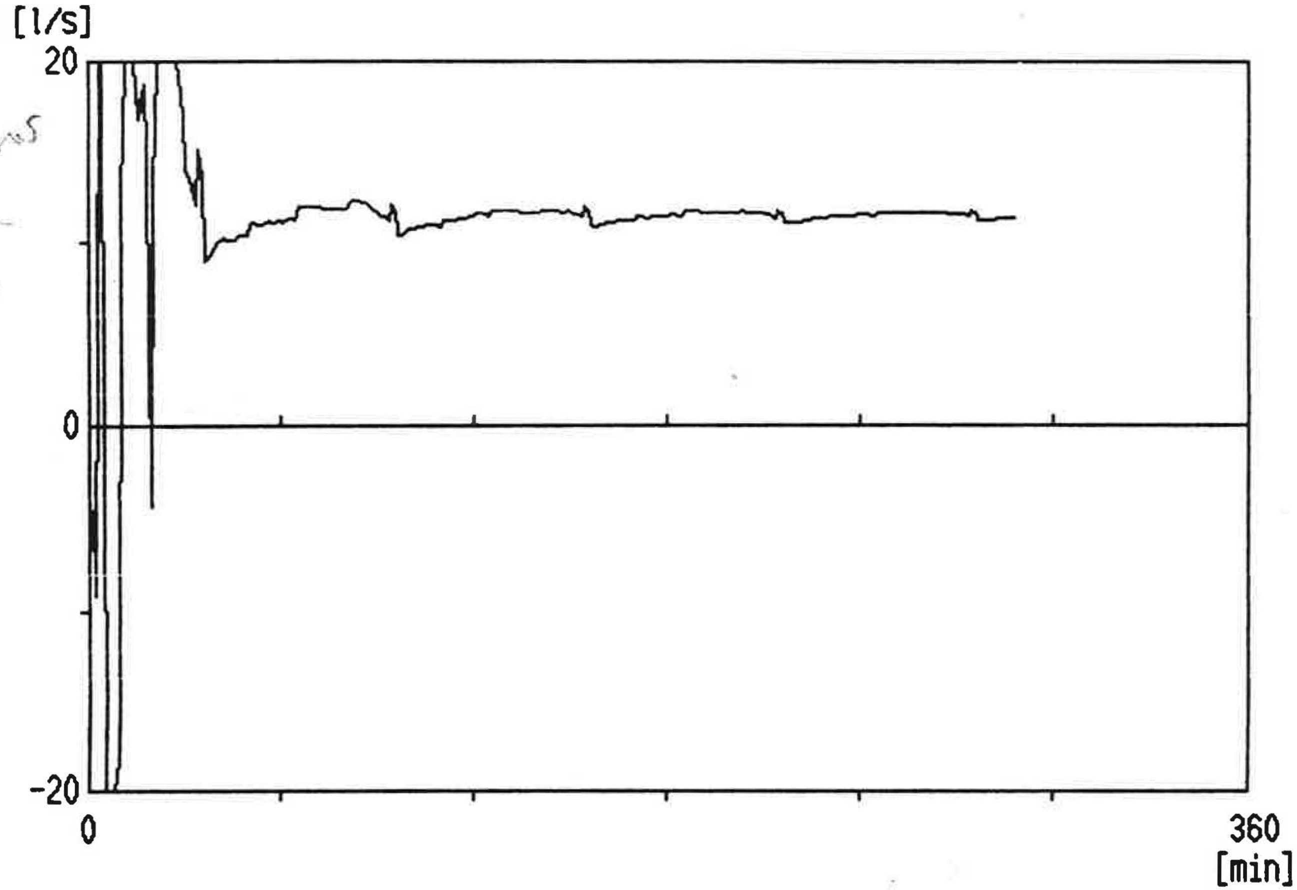
360
[min]

C(2,3) TIME STEP [1 - 288]

Airflow rate



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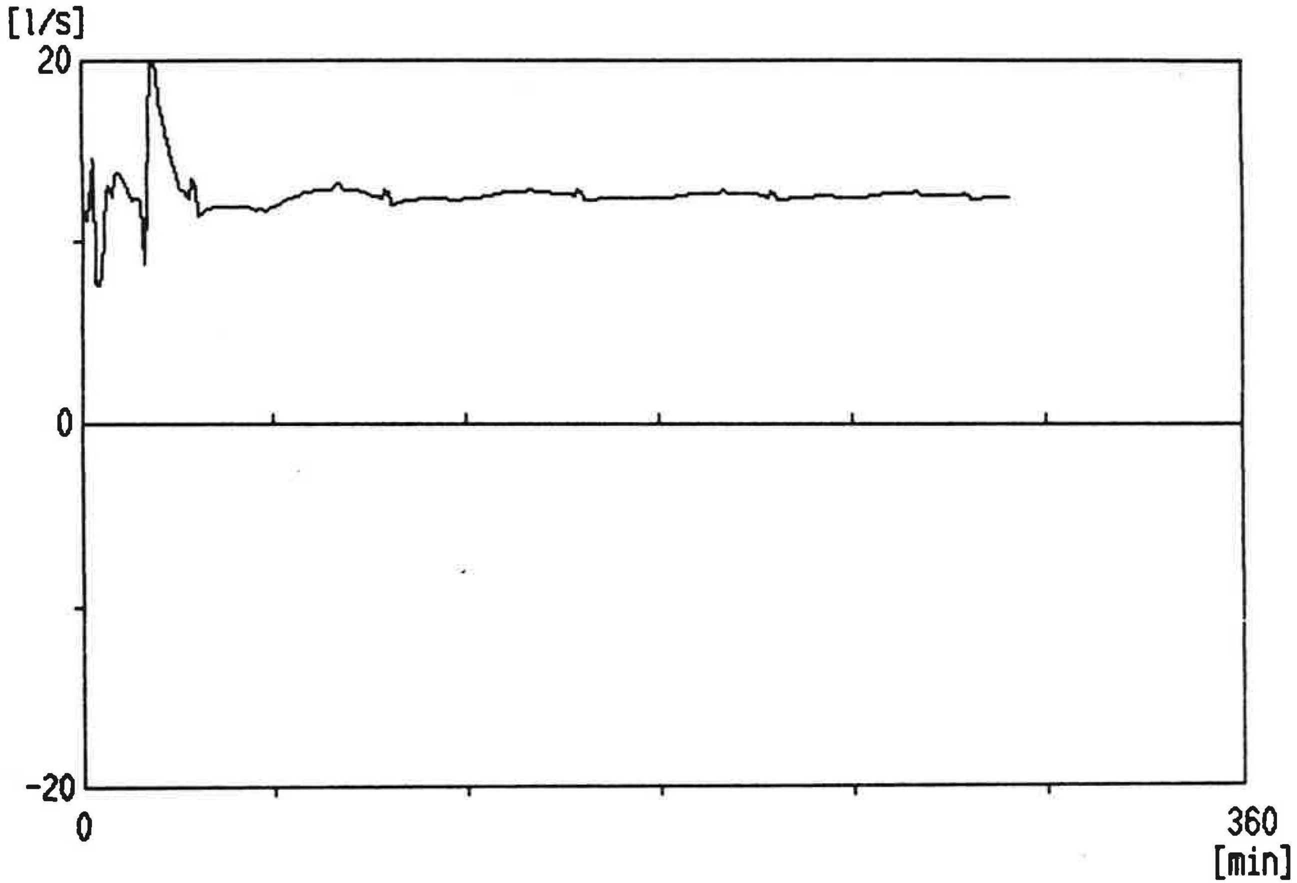
C(1,2)   TIME STEP [1 - 288]



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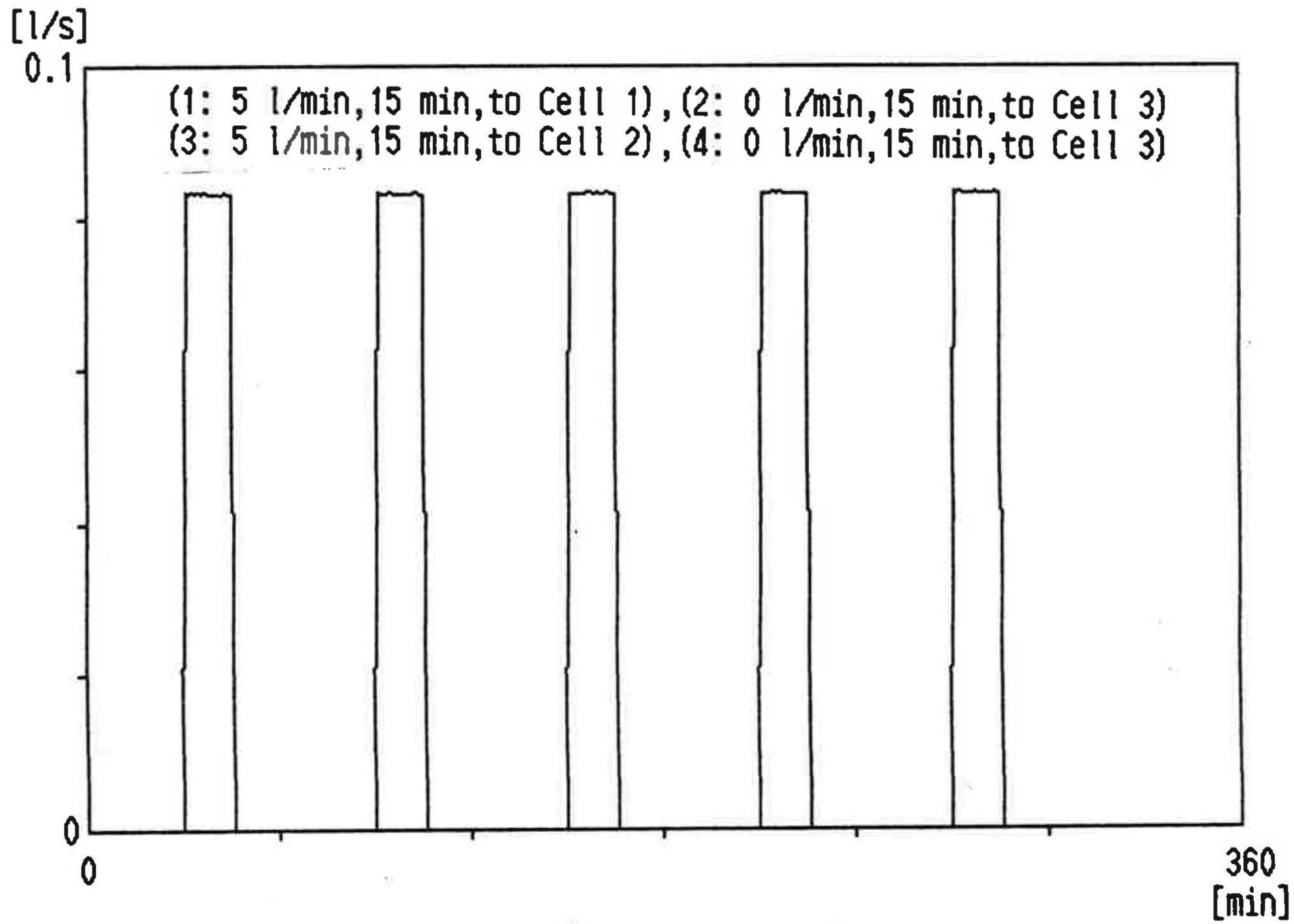
ROOM NUMBER [2] TIME STEP [1 - 300]



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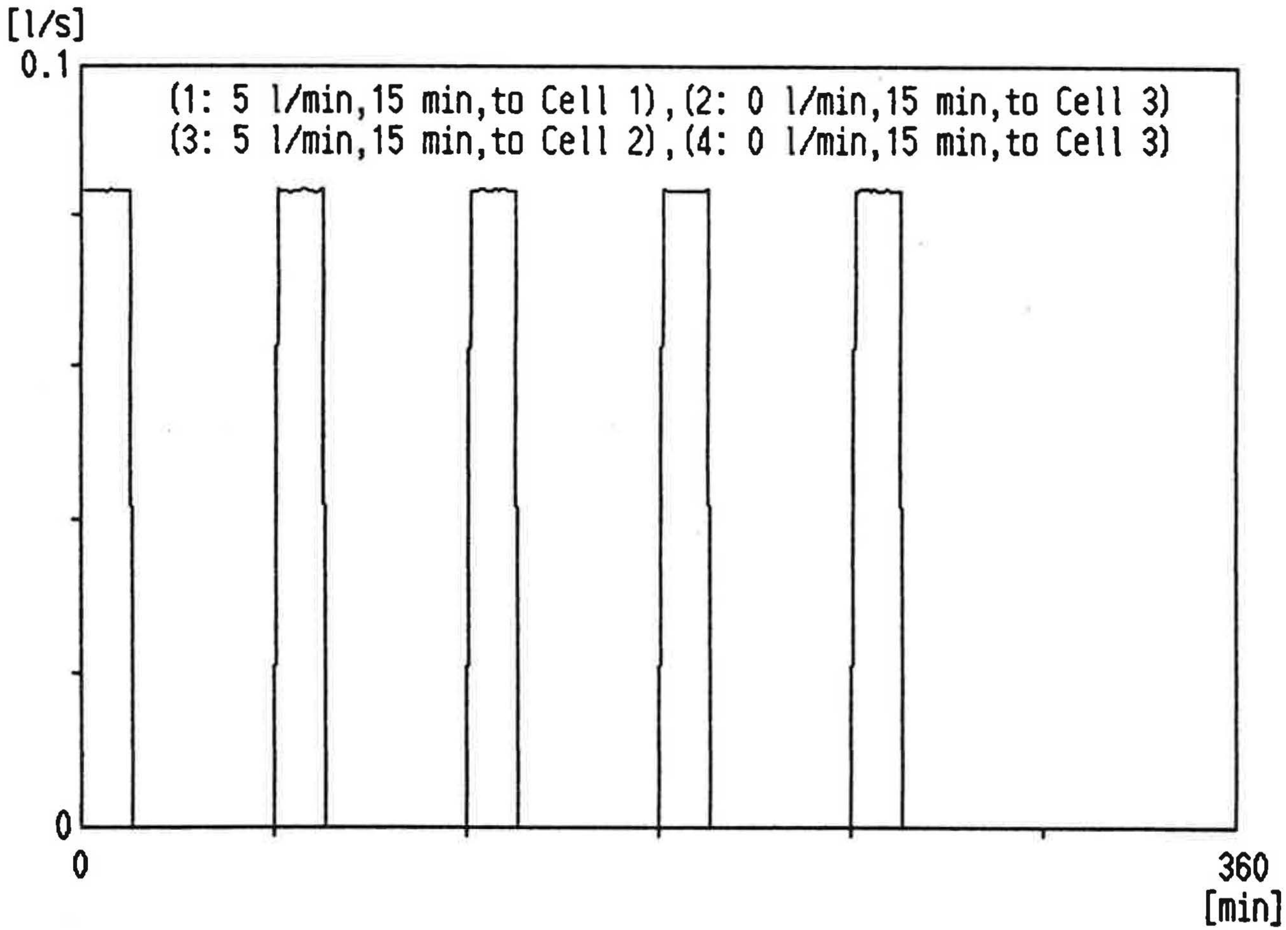
ROOM NUMBER [1]   TIME STEP [1 - 300]

CO2 injection flow rate



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