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No. 3157

DESIGN MODELS TO HANDLE RADIATIVE AND CONVECTIVE EXCHANGE IN A ROOM

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ABSTRACT

The traditional method of handling heat exchange in a room for design purposes was to suppose that all heat from the plant, radiative as well as convective, was input at a so called "air temperature". This is an evident misnomer, since air temperature as such cannot drive longwave radiation as the model actually assumed. The "environmental temperature" (t_{ei}) concept has been introduced in the UK to get round the difficulty. This paper presents an analysis of when an approach along these lines may be logically acceptable. The surface-to-surface system of radiant exchange is first reduced to a surface-to-star point (T_r) exchange by a least squares fit. It is then shown that the space-averaged *observable* radiant temperature T_{avr} can be approximated by the value of the temperature generated at T_r when the radiant output from heating appliances and casual gains is taken to act at T_r. (Strictly speaking, this is not possible, since T_r is only a convenient fiction.) Now it is normally assumed that convective gains can be treated as though input at the space averaged air temperature, T_a. Thus we can set up a "binary star" model, formed from the radiant star pattern centred on T_r and the convective star pattern centred on T_a, can be set up. It provides an attractive model for the internal exchange of heat in a room. An *equivalence theorem* can be demonstrated, and it serves to show that the binary star system, based on T_r and T_a, can, in certain well-defined conditions, be replaced by a single star system, centered on an index temperature, T_{ra}, (the "rad-air" temperature). T_{ra} in fact serves the same function as did as "air temperature" in the old fashioned sense. The model based on T_{ra} is workable but it is physically unattractive, and a model that handles convection and radiation separately may provide a better design procedure. Environmental temperature (t_{ei}) is a form of T_{ra}, but the logic of setting it up is seriously flawed.

INTRODUCTION

The four walls of a room together with its floor and ceiling can be regarded as a control volume. Outside it lies the fabric of the building which provides resistance to and storage for the flow of heat, the ventilation process and the external environment. Within it, convective and longwave heat transfer mechanisms serve to move heat between the various sources - radiators, lighting, equipment, and occupants - and the air and walls of the room. This article is concerned with the interplay of mechanisms within the control volume. Further, the treatment is presented at a level appropriate to design methods for sizing heating and cooling equipment in a room; these processes can be modeled with any degree of detail using a computer model, but such treatment is unnecessarily complicated in a design context.

The traditional standpoint is set out in for example the 1963 ASHRAE Guide and Data Handbook, Fundamentals and Equipment, and also the 1965 IHVE Guide. Calculations centered around a global room temperature, T_i say, termed "air temperature". To express the heat flow to the inner surface of an outer wall, E was taken to denote an emissivity value, seemingly not properly defined, h_r was the usual linearized radiant heat transfer coefficient

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$(4\sigma T_{mean}^3)$, and h_c the convective coefficient. . T_i was taken to drive a flow of heat through a transmittance Eh_r+h_c (about 8 W/m^2K) to the inner surface of an outer wall at T_w, (Figure 1) so that T_i must be taken to have radiative properties as well as convective properties, despite its name. T_i also served to drive the ventilation loss of heat, and its value was further taken to estimate the comfort temperature in the room. Both radiative and convective components of all internal sources of heat were taken to be input at T_i. The model provides a rather coarse account of room heat transfer, but it remains the basis of many plant design calculations today.

In the UK in the 1960s, a move was made to provide a better index temperature than T_i, which would take more explicit account of the longwave radiant exchange in a room. The concept of "environmental temperature", t_{ei}, was developed. It was a linear mix of mean air temperature t_{ai} and of the mean surface temperature, t_m:

$$t_{ei} = (1/3).t_{ai} + (2/3).t_m$$

This model will be termed the "environmental temperature model" (ETM, Figure 2). (In discussing the ETM, temperature will be denoted by t, following the IHVE Guide notation. The author's thesis will be presented using T.) According to the model, t_{ei} drives a heat flow from the room as a whole to a surface (internal as well as external) through a transmittance of $(6/5)Eh_r+h_c$. The ventilation loss is driven by the air temperature, t_{ai}, and there is a conductance of $4.8\Sigma A$ (W/K) between t_{ei} and t_{ai}. ΣA is the total internal area of the room. (The conductance was later notated as $h_a\Sigma A$.)

In the ETM, convectively input heat was taken to act at t_{ai}, as expected, but the longwave radiation, Q_r, from all sources was taken to act as the value $1½.Q_r$ at t_{ei}, and at the same time a quantity $½.Q_r$ was taken to be *extracted* from t_{ai}. The comfort temperature - dry resultant temperature, t_c - was taken to be given by a node on the $h_a\Sigma A$ conductance with a value

$$t_c = (1/2).t_{ai} + (1/2).t_m$$

$$= (1/4).t_{ai} + (3/4).t_{ei}$$

This system was adopted by the UK Institution of Heating and Ventilating Engineers (now the Chartered Institution of Building Services Engineers, CIBSE) as its recommended procedure for conducting heating design calculations in the 1970 revision of its Guide, Section A5 (IHVE 1971). It was revised in 1979 (CIBS 1979). (The factor of (6/5) in the quantity $(6/5)Eh_r+h_c$ was given in the papers supporting the IHVE Guide but not in the Guide itself.)

The present author suspected from the outset that the reasoning which had been used to arrive at t_{ei} was flawed and that the model's handling of radiant exchange was oversimplified. It was not immediately apparent, however, exactly where the reasoning broke down, nor what degree of simplification of radiant exchange was acceptable in a design context. These problems have now been solved. The present paper is intended to give an overview of how radiant exchange can be so simplified, how a model based on a global room temperature such as T_i can be arrived at logically, and where the shortcomings in the ETM lie.

THE BINARY STAR MODEL

In this section, it is to be shown how, for design purposes, the convective and radiant exchange mechanisms can be modeled with good accuracy using two superposed star-based thermal networks, one for convective and one for radiant exchange.

The Convective Network

The convective network is based on a central mean air temperature, T_a. The value of T_a is arrived at conceptually by making measurements of air temperature at uniformly spaced locations in the room, and averaging. There may be steadily sustained temperature differences between, say, floor and ceiling, but these are disregarded in the design model as far as global behavior is concerned. We assume that we know the h_c values at the mean surface temperatures T_1, T_2, ... of the room, although we recognize that there is uncertainty in what values to select. The conductance linking T_a to the *j*th surface is then A_jh_{cj}. Heat input convectively to the room is taken to be input at the T_a node. A measuring device such as a thermometer is linked to T_a through a conductance A_gh_{cg}, where the subscript p denotes the thermometer or probe properties. The $A_p h_{cp}$ conductance is of course very, very much smaller that of a typical room conductance, $A_j h_{cj}$.

This is the conventional model for convective exchange in a room. It is to be argued that radiation can be so manipulated that it can be handled in virtually the same way. This is not obvious, nor is it exact, but it is adequate for design purposes.

Radiation as a Surface-Surface Exchange

Consider an empty room with six surfaces at mean temperatures T_1 to T_6. If they are blackbody, the direct radiant exchange between surfaces j and k is proportional to $\sigma(T_j^4-T_k^4)$. As mentioned earlier, this can be written on linearization as $4\sigma T_m^3.(T_j-T_k)$ or $h_r.(T_j-T_k)$. h_r is about 5.7 W/m²K. The radiant conductance between these surfaces is given as $A_j F_{jk} h_r$, where F_{jk} is the viewfactor between them and is related to room geometry. There are 15 such conductances if the six surface temperatures are specified. Figure 3 shows the pattern for a four-sided enclosure.

Radiation as a Surface-Star Point Exchange

The above system is too complicated for design use, and the traditional and environmental temperature models both tacitly assume that radiation can be exchanged via a star point of some sort. The present author has provided a logical foundation for this assumption (Davies 1983) and it is sketched in Appendix 1.

If the surface J radiates to an enclosing blackbody surface at a uniform temperature, the radiant conductance is simply $A_j h_r$. If the surface "radiates" to an intermediate node, the radiant star node T_r (Figure 4), its conductance will be greater than $A_j h_r$ and will be written as $A_j h_r/\beta_j$, where $\beta_j < 1$. We have to have some sort of logical procedure to find values of β such that, seen from the outside, the behavior of the equivalent star circuit (Figure 4) is as like that of the parent (or delta) circuit (Figure 3) as is possible.

To do this, we write down an expression for the net resistance, R_{jk}^{Δ}, between nodes j and k in the parent network when the other four nodes are taken to be adiabatic. (The direct resistance is $1/A_j F_{jk} h_r$. The net resistance can be found as the ratio of two determinants.) The resistance of the star network is simply

$$R_{jk}^{*} = \beta_j/A_j h_r + \beta_k/A_k h_r$$

If the difference $R_{jk}^{\Delta}-R_{jk}^{*}$ were zero, the two circuits would be identical in their external effects. This cannot be achieved simultaneously for all 15 pairs of nodes, so we form the sum of the squares of the non-dimensionalized differences,

$$S = \Sigma\Sigma(1 - R_{jk}^{*}/R_{jk}^{\Delta})^2, \qquad j = 1 \ldots 5, \qquad k = j+1 \ldots 6,$$

and by simultaneous adjustment of the values of β_j, S can be minimized.

By examining a range of rectangular enclosures of all shapes, we find that β_j is largely determined by the ratio

$$f_j = A_j/(\text{total surface area})$$

Then $$\beta_j \simeq 1-f_j-3.53(f_j^2-½f_j)+5.04(f_j^3-¼f_j)$$

with a standard deviation of 0.0067. The root mean square difference between the delta and star circuits is given as $(S/15)^{½}$ and this is for the most part less than 0.02. (Details are given in Davies (1983) and in Appendix 1).

Thus we conclude that the surface to surface or delta pattern which provides an exact description of the geometrical aspects of radiant exchange in a rectangular enclosure, can, as far as its *external* effects are concerned, be replaced with good accuracy by a suitably designed surface to star pattern.

The qualitative idea of this transform is of long standing, but this may be the first attempt to design the star network in an optimal manner and to examine the accuracy it provides.

<u>The Emissivity Conductance</u>

If a surface is not blackbody but is grey with an emissivity of ε_j, we have to include two further features, shown in Figure 5:

1. A black body equivalent node T_j' which replaces T_j itself as the termination of the geometrically based conductances, either in the exact form with the $A_jF_{jk}h_r$ conductances, or in the approximately equivalent form with conductances of type A_jh_r/β_j.
2. An emissivity based conductance $A_j\varepsilon_jh_r/(1-\varepsilon_j)$ is to be located between T_j, the thermodynamic temperature of surface A_j, and its T_j' node.

T_j' is the linearized equivalent of radiosity in conventional radiant exchange theory. It has the property that all longwave radiation from an internal source of radiation that falls on the surface A_j is to be completely absorbed at T_j', not partly absorbed and partly reflected at T_j itself.

In the star-based system, the emittance conductance is in series with the geometrical conductance and they can be combined as

$$[(A_j\varepsilon_jh_r/(1-\varepsilon_j))^{-1} + (A_jh_r/\beta_j)^{-1}]^{-1} \qquad (= A_jE_j{}^*h_r, \quad \text{say})$$

<u>The Average Radiant Temperature</u>

The temperature T_r is a fictitious temperature with no physical significance. It simply serves as a convenient device to model the *external* effect of the real radiant exchange. The radiant inputs should be taken to act at the several T_j' nodes, but suppose for the moment that their total, Q_r, were to act at T_r. This is physically meaningless, but it is very easy to calculate T_r as an increment above wall temperature, which can be conveniently taken to be zero.

Let us set aside such fictions for the moment and consider what radiant effect can actually be observed in an enclosure (taken as air free so as to avoid consideration of convection) when a source of radiant heat is present within the enclosure.

Suppose we have an enclosure with blackbody surfaces all at a reference temperature of zero, (e.g 0°C). A pure radiant source of strength, Q_r, is placed at its center and the temperature is sensed by a probe of some kind at points over a uniform array of points within the room, (exactly as air temperature was supposed to be sensed). The average radiant temperature, T_{avr}, can be found from the local values. In relation to the labor of performing design calculations, evaluation of T_{avr} for a room of given dimensions is a laborious task. Details are given in Appendix 2.

<u>The Two Global Radiant Temperatures</u>

We thus have two global measures of the radiant temperature in the room relative to its walls, due to the presence of an internal radiant source:

1. The radiant star temperature, T_r, network based, fictitious but very easily evaluated,

2. The average radiant temperature, T_{avr}, physically based and on the same footing as T_a, but laborious to evaluate (and indeed, specific to situation).

The computations based on a wide variety of enclosure shapes show that T_{avr} tends to be a little larger - some 14% on average - than T_r although it varies less with shape. If the radiant source is placed at the wall of the enclosure - a more realistic position for a radiator - T_{avr} is reduced somewhat, and it turns out that, for practical design purposes, T_r provides a satisfactory estimate of the physical parameter, T_{avr}.

<u>The Radiant Star Model</u>

The star model for radiant exchange thus consists of a series of conductances of type $A_jE_j{}^*h_r$ linking the surface nodes, T_j, to the fictitious radiant star node, T_r. This is in itself a useful simplification, but it has the further advantage that the longwave radiant heat input can be treated as though input at T_r, and if so done, T_r provides an estimate of T_{mvr}, the physically significant quantity. (If the exchange is treated as a surface to surface exchange, T_{mvr} requires separate calculation).

The Binary Star Model

It was noted at the outset that the model for convective exchange consists of a series of conductances of type $A_j h_{cj}$ linking the surface nodes, T_j, to the physically based mean air temperature node, T_a. Since the radiant and convective processes procede quite independently of each other within the enclosure and only interact at solid surfaces - those of the room itself, or of furnishings or sensors - a physically based model of the enclosure must consist of the two networks superposed so as to form a binary star pattern.

T_a and T_r denote, respectively, the average perceptible air temperature and an estimate of the average observable radiant temperature in the enclosure. Large heat flows - of the order of kilowatts - can be input at T_a and T_r, and they result in only modest rises of temperature. The resultant perceived temperature, whether by a sensor or an occupant, must be a linear mix of the two. Dry resultant temperature is formed as

$$T_c = ½.T_a + ½.T_r$$

The T_c node is of course linked to these nodes by very small conductances, because of the very small dimensions of, say, a thermometer bulb. T_c is an estimate of T_p. If a thin pencil of solar radiation falls on a thermometer bulb, it will lead to a marked increase in T_p but a negligible increase in T_a or T_r. Thus T_c and T_p are not on the same footing as T_a and T_r.

Figure 6 shows the binary star model links for a four-surface enclosure. The most important quantities from the designer's point of view are the comfort temperature and the convective and radiative heat inputs needed to maintain it.

THE RAD-AIR MODEL

It is the purpose of this section to show that the binary star model of the last section, in which convective and radiative processes are handled separately, can be transformed in restricted circumstances to a single star model centered on a node to be called the "rad-air" node, T_{ra}, since it will be found to be a linear combination of T_r and T_a. In a later section, it will be shown that the environmental temperature model is in most respects the same as the rad-air model.

It is convenient first to state an equivalence theorem.

The Equivalence Theorem

Consider a very simple thermal (or electrical) circuit (circuit A in Figure 7a) comprised of three nodes, T_a, T_1, and T_r. A conductance C links T_a with T_1 and a conductance R links T_1 with T_r. (T_a and T_r will be given the meanings they had in the previous section, and C and R are to denote convective and radiant conductances, but this interpretation is not needed for the moment.) A heat flow, Q_r, is supposed input at T_r and there are heat losses of Q_f from T_1 and Q_v from T_a. If T_a is fixed in some way, it is an elementary calculation to find T_1 and T_r.

Consider now another circuit - circuit B (Figure 7b) - which consists of T_a, T_1, and C as in circuit A, but which lacks T_r, R, and the heat input. Suppose that a node T_{ra} is located on C, so as to define conductances (C+R).C/R to T_a and (C+R) to T_1. Suppose that a heat input of Q_r.(1+C/R) is input at T_{ra} and at the same time a flow of Q_r.C/R is *extracted* from T_a.

It is easily shown that the temperature established at T_1 and the heat flows from T_1 and T_a are identical to those in circuit A. T_{ra} too can, of course, be found. T_r does not appear explicitly in circuit B but its value can be constructed as

$$T_r = T_{ra}.(1+C/R) - T_a.C/R$$

Thus circuit B provides exactly the same information - values of T_1, T_r, Q_r, and Q_v - as did the parent circuit A.

If there are further heat inputs in circuit A at T_a and T_1, they are simply included in circuit B without change.

This theorem can be used in connection with an elementary building model.

The Model for a Basic Enclosure

We consider the most elementary building enclosure possible. It consists of an internal surface, all of whose area, A, is at a single uniform temperature, T_1; the fabric provides a conductance F_1 between T_1 and the ambient temperature of T_o. The air temperature, T_a, is linked to ambient by the ventilation conductance, V, and to T_1 by the convective conductance, C_1, equal to Ah_c. A pure radiant source, Q_r, is present within the enclosure, and its output is taken to act at T_r, linked to T_1 through the radiant conductance R_1, equal to AE^*h_r. The thermal circuit for the enclosure (circuit A' in Figure 8a) is that of circuit A, together with the two loss mechanisms F_1 and V.

If the equivalence theorem is applied to circuit A', it transforms to circuit B' (Figure 8b) which consists of the following sequence of nodes and conductances:

T_a, $(C_1+R_1).C_1/R_1$, T_{ra}, (C_1+R_1), T_1, F_1, T_o, V, back to T_a.

In this configuration, we input $Q_r.(1+C_1/R_1)$ at T_{ra} and withdraw $Q_r.C_1/R_1$ from T_a. According to the theorem, the real temperatures T_a and T_1 have the same values as they did in circuit A'. and of course T_r can be constructed from information provided by circuit B'.

The overall conductance from T_{ra} to T_o via T_1 is a U value-like conductance. If λ denotes the conductivity and d the thickness of the outer wall material, then

$$\frac{1}{C_1+R_1} + \frac{1}{F_1} = \frac{1}{Ah_c+AE^*h_r} + \frac{d}{A\lambda} + \frac{1}{Ah_o} = \frac{1}{A}\,\frac{1}{h_c+E^*h_r} + \frac{d}{\lambda} + \frac{1}{h_o} = \frac{1}{A\,U}$$

Restrictions to the Rad-Air Model

An enclosure consisting of a single isothermal surface is too idealized to be of any value. Suppose instead that we have an enclosure consisting of an outer wall of inside temperature T_1 and five internal surfaces at another uniform temperature T_2. T_a and T_r exist as before but we now have additional convective and radiant links, C_2 and R_2 (see Figure 9a). There will be a heat loss mechanism, F_1, from T_1. There may or may not be a loss mechanism from T_2.

It can be shown - after some algebra - that if

$$C_2/R_2 = C_1/R_1 \quad (= \alpha,\ \text{say})$$

the equivalence idea holds: We can replace T_r by a node T_{ra}, which has links

C_1+R_1 to node T_1,

C_2+R_2 to node T_2

and $x = (C_1+C_2+R_1+R_2).(C_1+C_2)/(R_1+R_2)$ - or $\Sigma(C+R).\alpha$ - to node T_a.

An input of Q_r at T_r in the physical circuit can be replaced by an input of $Q_r.(1+\alpha)$ at T_{ra} together with an extract of $Q_r.\alpha$ from T_a in the equivalent circuit. T_a, T_1, and T_2 in circuit B' (Figure 9b) then have their circuit A' values and the value of T_r in circuit A' can be constructed from the values of T_a and T_{ra}.

We assume that the same result holds when the enclosure surface consists of three or more portions at different temperatures. If $C_2/R_2 \neq C_1/R_1$, the equivalence does not hold exactly.

Discussion of the Rad-Air Model

The rad-air model is a single star model: it is centered on T_{ra}, which is linked to the surface J through a conductance C_j+R_j which lumps the convective and radiative mechanisms. T_j may be linked to the exterior through some simple or complicated thermal path. T_{ra} is linked to T_a through the conductance (C+R).α, (C denotes ΣC_j and R denotes ΣR_j), and T_a is linked to the exterior by the ventilation conductance. As a working tool, the model has a number of points both to commend and to deprecate it.

In its favor we may note:

1. The model retains the familiar U-value concept for conduction losses.

2. The conductance (C+R).α prevents a radiant input being too readily 'lost' by the ventilation process. (This will be explained more fully later.) This feature is useful in performing calculations on overheating due to solar gains. These are very quick and rough estimates, and it is sufficient to assume that all the solar gain is input at T_{ra}.

The model however contains a number of unattractive features:

1. It is only exact if $C_1/R_1 = C_2/R_2$, etc. There are several reasons why this will not be true. h_c varies from surface to surface. ε may vary from surface to surface, (though only in special cases), and radiative conductances are not simply proportional to surface area.

2. The conductances of type C_j+R_j lump together totally unlike physical processes. In no real sense can convective and radiative energy fluxes be said to "flow together" to a surface.

3. The (C+R).α conductance defies any kind of physical interpretation.

4. It is physically meaningless to input an augmented energy flux into some node and to extract part of it from a neighboring node.

5. The rad-air temperature can be given an interpretation of a kind: it a weighted mean of the average air and radiant star temperatures.

$$T_{ra} = \frac{C.T_a}{C+R} + \frac{R.T_r}{C+R}$$

but this is of formal rather than of substantive value.

6. T_{ra} is not a generally accepted measure of comfort temperature. Comfort or dry resultant temperature is usually taken as

$$T_c = ½.T_a + ½.T_r.$$

T_c can be represented by a node on the (C+R).α conductance. This, however, is conceptually wrong. T_c is a measured or perceived temperature at a thermometer bulb or the human body; it is a local value, and the conductances linking T_c to the room are orders of magnitude smaller than the room conductances themselves. Thus T_c should be linked to T_a and T_r by very small conductances. The rad-air model does not include T_r explicitly and so is unable to provide a low conductance link to T_c. If T_c is placed on (C+R).α, as it must be in the rad-air model, it is linked to the room by high conductances.

If a thermometer bulb is placed in a narrow beam of strong sunshine, it will register a high value, although the heating effect in the enclosure as a whole may be negligible. The binary star model can handle this situation but the rad-air model cannot.

7. It is easy in a design context to calculate the effect of a heat input at T_{ra} since all the paths from it are in parallel. If heat is input elsewhere, at T_a or T_1 say, some thermal circuit analysis is needed. To the research worker this complication is trivial, but to a thermal services engineer, for whom the intricacies of enclosure heat transfer are quite peripheral, the complication is a hindrance. A procedure has been developed in connection with the environmental temperature model - which is a form of

the rad-air model - which involves the use of a so-called "surface factor", F' say. F' is less than unity. Using this device, the real heat input, Q_1 at T_1 say, can be scaled down to the value, $F'.Q_1$. If the factor F' is chosen appropriately, the reduced heat input, $F'.Q_1$, if applied at T_{rm} (i.e., not at T_1 itself), has the same effect everywhere in the thermal circuit as does the application of Q_1 at T_1, but with the exception of that part of the circuit that contains T_1. The temperature at T_1 is underestimated, and a post-scaling adjustment has to be applied to restore T_1 to its proper value.

This procedure works, but it is an unsatisfactory device since contradicts the basic principle of Conservation of Heat Flow.

8. Finally, it may be remarked that in stating the equivalence theorem, T_r was replaced by T_{rm} and T_a was left intact. This represents an unsymmetrical handling of the elementary circuit. (We could have left T_r and replaced T_m.) This choice of transformation has the effect that, when it is applied to the elementary enclosure, we can accommodate an external loss of heat from T_a - the ventilation loss in fact - but we can no longer handle an external loss from T_r, the radiative loss that would occur from an open window, for example. This does not matter in a cold climate where ventilation losses are important but where the effect of open windows can be ignored, but it limits the use of a rad-air like model to situations where the enclosure concerned has no open apertures.

EXISTING SINGLE STAR MODELS

The section on the Binary Star Model showed how enclosure heat transfer could be handled in a design context by keeping separate the convective and radiative heat transfer mechanisms and expressing them in the form of two independent star circuits. The section following it showed how these mechanisms could be combined in a formal manner to form a one star model, the rad-air model. In the present section, the status of the existing-one star models, mentioned in the first section, is to be examined.

The Traditional Model

The traditional model, it will be recalled, simply spoke of a "room temperature", T_i, at which all heat was input and from which all heat was lost by conduction through the fabric and by ventilation. This situation can be derived from the binary model simply by superposing the T_a and T_r nodes. It is obvious that this is only possible if T_a and T_r happened to be equal, and in general this will not be so. The physical inappropriateness may be seen by noting that longwave radiation is handled as though input at T_r. The ventilation loss is driven from T_a. If T_a and T_r are superposed, the circuit will allow radiantly input heat to be "lost" directly by ventilation, without the necessary intervention of a solid surface. This is absurd. Thus the traditional model can only be a rather crude method of handling room heat exchange. Whether it is adequate in a design context is another matter. The author believes that it is adequate for many design purposes.

The radiant conductances of the traditional model were not set up correctly, but that is better discussed in connection with the environmental temperature model.

The Environmental Temperature Model

The environmental temperature model (ETM) is closely similar to the rad-air model. Unfortunately, the ETM was set up on the basis of an oversimplification of radiant exchange, and the concept of environmental temperature, as it is defined, is inadmissible. The defects are fully discussed in Davies (1986). Appendix 3 of the present paper shows that it is illegal to attempt to form an index temperature from surface and air temperatures, and environmental temperature is such a quantity.

The ETM was based on considerations of a cubic enclosure with an outer surface, area A, emissivity E, at a temperature t_m, five internal surfaces (5A) at t_ℓ, (subscript ℓ, not 1), and we are forced to assume that they have an emissivity of unity - they are blackbody surfaces - although this was probably not intended. The radiant conductance between t_m and t_ℓ was given as AEh_r, without distinguishing between the emittance and geometrical components of this quantity.

Environmental temperature t_{ei} itself was arrived at by finding that temperature which would drive the same heat flux to t_m as was physically driven by t_a and the air temperature t_{ai}. The argument included mention, however, of *mean surface temperature* which proves to be an irrelevant concept, either as far as the radiant exchange or the convective exchange in the room is concerned. Furthermore, constancy of heat flux proves to be an inappropriate principle with which to establish t_{ei}. It turns out that t_{ei}, as it is defined, is an absurd quantity, and the model centering on environmental temperature, if one adhers rigidly to the logic by which it was set up, leads to some ridiculous conclusions. It can be shown that t_{ei} may "depend" on the value of the emittance of a non-existent surface; the model further asserts that there will be no radiant exchange between floor and ceiling if the outer wall has zero emittance. These matters are discussed more fully in Appendix 3.

The radiant conductance between the surface at t_m and the t_{ei} node as expressed in the ETM is

$$R_{etm} = (6/5)AEh_r$$

The correct value, as given in the binary star model, is

$$R_{bsm} = [(A\varepsilon h_r/(1-\varepsilon))^{-1} + (Ah_r/\beta)^{-1}]^{-1} = AE^*h_r$$

In the ETM, E is taken as 0.9. In the binary star model, we can chose ε arbitrarily, and we will take the same value as that for E - 0.9. For a cube, $\beta = 5/6$. Then

$$R_{etm} = 1.08\ Ah_r$$
$$R_{bsm} = 1.06\ Ah_r$$

and so the ETM value is near enough correct numerically, even if not in principle. ($E^* = 1.06$ for one surface of a cube.)

With values of $h_c = 3\ W/m^2K$ and h_r of $5.7\ W/m^2K$, we should have for a cubic enclosure,

$$C/R = \Sigma C_j/\Sigma R_j = 6Ah_c/(6AE^*h_r) = 3/(1.06x5.7) = 0.497 \simeq ½$$

Thus α or C/R is here equal to about ½, and a radiant input at T_r in the binary star model has to be replaced by an input of $Q_r.(1+\alpha) = 1½.Q_r$ at T_{rm}, together with the withdrawal of $½.Q_r$ at T_a.

But these are precisely the values acting at the t_{ei} and t_{ai} nodes of the ETM. Thus *operationally speaking,* the ETM is a rad-air model. The value of t_{ei}, as arrived at by performing the operations recommended in the CIBSE 1979 Guide Section A (p A5-8), is to be identified with T_{rm}. The value of t_{ei}(operational), however, is greater than, and so conflicts with, the value of t_{ei}(defined).

Thus the environmental temperature model is fundamentally a rad-air model, whose derivation has been marred by a number of logical flaws. It will have the strengths and weaknesses of the rad-air model listed above.

SUMMARY AND DISCUSSION

The foregoing analyses have demonstrated a series of results:

1. The *external* effect of the physical surface to surface radiant exchange in an enclosure can be modeled with good accuracy if the network is replaced by a surface to star-point node, T_r, and the links between the blackbody surfaces at T_j and T_r are sized using the exact view factor relations between the surfaces together with a least squares technique. T_r has no physical significance. If the surface J is thermally grey, we have to introduce the concept of the "blackbody equivalent node", T_j'; T_j' is the linearised equivalent of radiosity. The emissivity conductance $A_j h_r \varepsilon_j/(1-\varepsilon_j)$ acts between T_j and T_j' and is thus in series with the surface to star node conductance $A_j h_r/\beta_j$ to form the conductance $R_j = A_j E_j^* h_r$. Longwave radiation which physically falls on surface J is to be taken to be completely absorbed at T_j', and not partly absorbed and partly reflected at T_j itself. (The same is true of the *diffusely* reflected component of shortwave radiation; shortwave absorptivity values replace the ε_j values, and of course h_r is not relevant.)

2. If an enclosure contains a pure radiant source Q_r, the temperature perceived by internal objects - furnishings, occupants, measuring devices - on which it falls is higher than that due to wall temperature alone. The perceived temperature has a spacial distribution and the space-averaged radiant temperature is defined as T_{avr}. Although T_r and T_{avr} are conceptually quite different quantities, it is found that for design purposes, T_{avr} can be estimated from the value at the star node T_r, if the radiant input traversing the space is taken to act at T_r. (Radiation from the back of a wall-mounted radiator and which does not traverse the space, but falls directly on the wall, is taken to be received at the corresponding T' node.) This leads to a very simple star-based model to handle radiant exchange.

3. Heat that is input convectively to an enclosure is routinely taken to be input at the average air temperature, T_a, with its convective C_j links to the room. Since convection and radiation procede independently of each other in a room, their joint effect can be modeled by superposing the two star-based networks so as to form the "binary star" model.

4. In the binary star model, the conductances linking the various nodes - T_j to T_m and T_j', and T_j' to T_r are all macro-conductances, of order of hundreds of W/K. Comfort temperature T_c may be estimated as dry resultant temperature, $\frac{1}{2}T_r + \frac{1}{2}T_a$, but the physical dimensions of a sensor, or even of a human occupant, are small in relation to room dimensions, with the result that the conductances linking T_c to T_r and T_a are small, of order mW/K or µW/K.

5. It was shown above, using the equivalence theorem, (a circuit theorem, similar to those of Thevenin or of Norton), that the binary star model of an enclosure can be reduced, exactly, to such a single star model - the 'rad-air' model, centered on T_{ra}. T_{ra} is a linear combination of T_r and of T_a but the model does not handle the two nodes in a symmetrical way: T_a is retained, but T_r is *replaced* by T_{ra}. The equivalence is only exact, however, if $C_1/R_1 = C_2/R_2$, etc., and the model is not a physically attractive one. In particular, since T_r no longer forms part of the model, comfort temperature T_c cannot be modeled using low conductance links to T_a and T_r; instead, it has to be modeled as a node on the very large - and artificial - link between T_a and T_{ra}.

6. Design models in current use are single star models, based on an air index temperature, or on environmental temperature. Such models are numerically very easy to evaluate. The environmental temperature model proves to be, operationally speaking, a rad-air model but the radiant conductances it incorporates are incorrectly evaluated and environmental temperature - *as it is defined* - is an absurd quantity.

We can thus list some conclusions regarding the value of these design models.

__The Traditional or Air-Index Model__ The model in effect superposes T_r and T_a and is thus not logical. This model of enclosure heat exchange, however, can be understood at one level with little effort. It provides a procedure of sufficient accuracy to cover most heating design exercises and appears to be widely used for routine sizing of plant.

__The Binary Star Model__ This model is physically based in that it keeps radiant and convective exchange separate. It is logically based in that it handles radiant effects in the manner noted in (1) and (2) above. It is flexible in that it allows the designer to take account of the shape of a rectangular room, and the emissivity and convective coefficient at each surface. It permits the comfort temperature to modeled in a conceptually correct mnner. It is computationally little more involved than the air-index model. At one level, it, too, can be understood with little effort by a design engineer.

__The Environmental Temperature Model (ETM)__ This model was advanced in the 1960s in the UK to obviate the illogical features of the traditional air-index model. Unfortunately, its own logic is flawed, although this may not matter when the model is used for a simple enclosure where surface emissivities are all large (around 0.9) and convective coefficients are moderate (around 3 W/m²K). The ETM incorporates the 4.8ΣA conductance between t_{ei} and t_{ai}, and this represents a significant improvement on the traditional model when checking the likelihood of overheating due to solar gains.

The Rad-Air Model . The rad-air model is a derivation of the binary star model, arrived at when the convective and radiative processes are merged. However, the way in which the model is set up invites the user to try to understand more deeply what is involved. For a proper understanding, the user must follow through the reasoning leading to the binary star model, (1 to 4 above), and then come to terms with the equivalence theorem (5), only to arrive at a conceptually recondite model. The ETM and the rad-air model prove to be structurally very similar, and, indeed, the rad-air model appears to achieve what the ETM originally set out to achieve.

In Davies (1987), Table 2, the author has provided a simple comparison of estimates of temperatures found using the binary star, air-index, and rad-air models. The same basic values of conductances and heat inputs are used for all three. In fact, the radiant conductances are based on the same radiant transmittance (the $E_j{*}h_r$ value) for each surface; only numerically different values for h_{cj} are input. This difference, however, leads to significant differences in temperature estimates.

A number of computer models are available to check designs for enclosures that cannot be reliably examined by these simple models of heat exchange.

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APPENDIX 1

The Star Network Conductances

This section presents the details of the analysis leading to the expression from which we can find the star network conductances.

We consider the exchange of radiation between two surfaces of a rectangular enclosure of dimensions l x d x h. The surfaces are supposed to be blackbody radiators, each at a uniform temperature. T_1 and T_4 denote the temperatures of the floor and ceiling surfaces, areas $A_1 = A_4 = l \times d$, T_2 and T_5 denote the temperatures of the east and west walls and T_3 and T_6 those of the north and south walls, with areas similarly expressed.

When the temperature difference is linearized, the conductance between nodes T_j and T_k is given as

$$G_{jk} = G_{kj} = A_j F_{jk} h_r = A_k F_{kj} h_r$$

The view factor F_{jk} from surface J of size b x unity to the *adjacent* surface K of dimension, c x unity, is given by

$$\begin{aligned}F_{jk} = (1/\pi b)\{&\tfrac{1}{4}[(b^2+c^2-1).\ln(b^2+c^2+1) - (b^2-1).\ln(b^2+1) - (c^2-1).\ln(c^2+1)]\\ &+ \tfrac{1}{4}[-(b^2+c^2).\ln(b^2+c^2) + b^2.\ln(b^2) + c^2.\ln(c^2)]\\ &+ [-(b^2+c^2)^{1/2}.\tan^{-1}(b^2+c^2)^{-1/2} + b.\tan^{-1}(1/b) + c.\tan^{-1}(1/c)]\}\end{aligned}$$

The view factor F_{jk} from surface J, of size b x c, to the *opposite* surface, separated by unit distance is

$$F_{jk} = (1/\pi)\{(1/bc).[\ \ln(b^2+1) + \ln(c^2+1) - \ln(b^2+c^2+1)]$$
$$+ 2[(1/b)(b^2+1)^{1/2}.\tan^{-1}(c/(b^2+1)^{1/2}) + (1/c)(c^2+1)^{1/2}.\tan^{-1}(b/(c^2+1)^{1/2})]$$
$$- 2[\ (1/b).\tan^{-1}(c) + (1/c).\tan^{-1}(b)\ ]\}$$

For a cubic enclosure, b = c = 1. In this case, F_{jk}(adjacent) = 0.200044 and F_{jk}(opposite) = 0.199825. Each of these rounds to the well-known value of 0.2, but they are not, in fact, identical.

The direct resistance between nodes J and K is simply $1/G_{jk}$. The net resistance, however, is less than this because of the multiple paths provided by the other 14 conductances. We wish to find the net resistance, R_{jk}^{Δ}. (The superscript Δ denotes the resistance provided by the surface-to-surface or delta network.) To do this, we note that if a heat flow, Q_1, is input at the node T_1, continuity requires that

$$\Sigma G_{1k}.(T_1 - T_k) = Q_1, \qquad k = 2, \ . \ . \ 6$$

Continuity at the set of six nodes leads to the matrix equation

$$\begin{bmatrix} +G_{11} & -G_{12} & -G_{13} & -G_{14} & -G_{15} & -G_{16} \\ -G_{21} & +G_{22} & -G_{23} & -G_{24} & -G_{25} & -G_{26} \\ -G_{31} & -G_{32} & +G_{33} & -G_{34} & -G_{35} & -G_{36} \\ -G_{41} & -G_{42} & -G_{43} & +G_{44} & -G_{45} & -G_{46} \\ -G_{51} & -G_{52} & -G_{53} & -G_{54} & +G_{55} & -G_{56} \\ -G_{61} & -G_{62} & -G_{63} & -G_{64} & -G_{65} & +G_{66} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix}$$

where $G_{11} = G_{12} + G_{13} + G_{14} + G_{15} + G_{16}$, etc.

Suppose that we wish to find R_{23}^{Δ}. We can set $T_3 = 0$ so that the number of equations is reduced to five. We make nodes 1, 4, 5 and 6 adiabatic. Then

$$Q_1 = Q_4 = Q_5 = Q_6 = 0$$

By definition, $R_{23}^{\Delta} = T_2/Q_2$ and so by Cramer's rule,

$$R_{23}^{\Delta} = \det[2,3]/\det[2]$$

where det[2,3] is the determinant of the above matrix of G_{jk} values, which is formed by omitting the row and column through G_{22} and the row and column through G_{33}. det[2] is the determinant formed by omitting the row and column through G_{22}, but in fact det[1] = det[2] = det[3].....

Thus if the room dimensions, l, d and h are specified, the set of 15 values of R_{jk}^{Δ} can be found. $R_{12} = R_{15} = R_{42} = R_{45}$, but the value of R_{14} is unique. Thus there are six distinct numerical values for R_{jk}^{Δ} if $l \neq d \neq h$.

The value of the resistance between node j and the radiant star node is β_j/A_jh_r. So the resistance between nodes j and k via T_r is

$$R_{jk}^{*} = R_j + R_k = \frac{\beta_j}{A_jh_r} + \frac{\beta_k}{A_kh_r}$$

If R_{23}^{*} were to equal R_{23}^{Δ}, the star and delta circuits would be identical in their external effects, as far as inputs at nodes 2 and 3 were concerned. In general, however, $R_{jk}^{*} \neq R_{jk}^{\Delta}$ and $R_{jk}^{*} - R_{jk}^{\Delta}$ determines the difference.

An overall measure of the difference of reponse of the star network from that of the parent delta network might be found by summing the 15 values of $R_{jk}^{*} - R_{jk}^{\Delta}$, but two changes are needed. First, the largest R values are associated with the smallest and so the least important surfaces. To avoid biasing the sum, we nondimensionalize the difference as

$(R_{jk}^* - R_{jk}^\Delta)/R_{jk}^\Delta$. Second, $R_{jk}^* - R_{jk}^\Delta$ may be positive or negative and to avoid the spurious canceling of such differences, we take the square of the difference. The overall difference in response of the star and delta circuits is thus expressed as the sum of the 15 terms:

$$S = \Sigma\Sigma((R_{jk}^* - R_{jk}^\Delta)/R_{jk}^\Delta)^2$$

The "optimal" star arrangement is found by minimizing S with respect to the six β values which have been hitherto arbitrary. So

$$\partial S/\partial\beta_1 = \partial S/\partial\beta_2 = \ldots\ldots = 0$$

This leads to a set of six simultaneous equations for R_j, but since $R_4 = R_1$, etc., the set reduces to three, the first of which is

$$R_1(1/G_{12}^2 + 1/G_{13}^2 + 1/G_{14}^2) + R_2/G_{12}^2 + R_3/G_{13}^2 = 1/G_{12} + 1/G_{13} + ½/G_{14}$$

From the solution of these equations, the β values are found as

$$\beta_j = A_j h_r . R_j$$

(In fact, the β values do not depend on the choice of h_r, and a value of unity can be assumed in performing the calculations.)

To evaluate β, a series of enclosures was examined with values of l/h from 0.1 up to 10 in 10 equal fractional steps, and the same values for d/h. The values l/h = d/h = 1 denote a cubic enclosure. This makes a total of 11^2 = 121 enclosures, each of which yields three β values, many of which are, of course, coincident.

One may enquire how well such optimally determined star links represent the parent network. The quantity $\delta = (S/15)^{1/2}$ denotes the root mean square difference between the set of resistances, expressed fractionally, across the nodes of the star and delta networks. Its value is shown for a selection of the above set of enclosures in Table 1.

TABLE 1
Values of the Root Mean Square Deviation δ between the Responses of the Star and Delta Networks

l/h d/h	0.40	0.63	1.00	1.58	2.51
0.40	.016	.015	.017	.018	.018
0.63	.015	.010	.011	.015	.019
1.00	.017	.011	.000	.010	.016
1.58	.018	.015	.010	.011	.015
2.51	.018	.019	.016	.015	.017

These values - deviations of 1% or 2% - show that a suitably designed delta circuit can closely approximate the characteristics of the parent delta network.

The value of δ for a cubic enclosure is not exactly zero. The links between T_r and each of the six surface nodes of a cubic enclosure are, of course, equal; (each is $(6/5)Ah_r$). The surface-surface links of the delta network, however, are not identically equal to each other, and for a cube, δ = 0.000073. A small deviation from cubic form leads to a rapid increase in δ, (see Davies [1983], Figure 5).

It turns out that the β values depend largely upon the fractional area f_j of the surface concerned:

$$f_j = A_j/(\text{total area of the enclosure})$$

Some indication of their distribution is shown in Figure A1. It can clearly be represented satisfactorily by a fitted curve. For theoretical reasons, for a vanishingly small surface (f_j = 0), β_j must be unity, and for a relatively very large surface (f_j nearly ½), β_j must be ½. The distribution deviates sufficiently from linear to justify fitting a cubic curve to it, which must be of the form

$$\beta_j \simeq 1 - f_j + A(f_j^2 - \tfrac{1}{2}f_j) + B(f_j^3 - \tfrac{1}{4}f_j)$$

A least squares fit gave values of $A = -3.53$ and $B = 5.04$ with a standard deviation of 0.0067. These considerations lead back to a simplified approximate expression for view factors in a rectangular room (Davies 1984).

The star node T_r is a fictitious node and heat cannot be "input" at it. If nevertheless we do input a radiant flow Q_r there, the temperature generated in relation to the walls at zero will be

$$T_r = Q_r / (\Sigma (A_j h_r / \beta_j))$$

We can define a non-dimensionalized temperature β_r as

$$\beta_r = T_r \cdot h_r \Sigma A_j / Q_r .$$

Then
$$\beta_r = \Sigma A_j / (\Sigma A_j / \beta_j)$$

β_r was determined for the above enclosures using the least squares expression for β_j. The values are given in the upper lines of Table 2.

TABLE 2

Values of the Radiant Star Temperature and Average Radiant Temperature (Expressed Non dimensionally as β_r and β_{avr}.)

d/h \ l/h	0.40	0.63	1.00	1.58	2.51
0.40	.810	.805	.777	.746	.719
	.909	.915	.921	.924	.925
0.63	.805	.832	.827	.805	.780
	.915	.914	.916	.915	.911
1.00	.777	.827	.843	.832	.810
	.921	.916	.915	.914	.909
1.58	.746	.805	.832	.827	.805
	.924	.915	.914	.916	.915
2.51	.719	.780	.810	.805	.778
	.925	.911	.909	.915	.921

Comment on the values of β_r is postponed until later.

APPENDIX 2
The Average Radiant Temperature in an Enclosure

We wish to determine the average observable temperature in an enclosure due to the presence of an internal pure radiant source, Q_r. Suppose for simplicity that Q_r is placed in the center of the enclosure and that it is of small dimensions. The flux across a spherical surface, radius R and centered on the source, is $Q_r/4\pi R^2$. Suppose that a small spherical sensor or probe, radius r, is placed a distance R from the source. It intercepts a flow of $(\pi r^2).Q_r/4\pi R^2$, which brings it to a temperature of T_p above the wall temperature of zero. The radiant flow from the probe to the walls is $T_p.4\pi r^2.h_r$. The probe temperature so established is then

$$T_p = Q_r / (16\pi R^2 h_r)$$

The space averaged value of T_r is found as

$$T_r = \frac{\iiint T_r . dx. dy. dz}{\iiint dx. dy. dz}$$

This too can be non-dimensionalised. Further we note that

$$R^2 = x^2 + y^2 + z^2$$

So

$$\beta_{avr} = T_p . \Sigma A_j h_r / Q_r$$

$$= \frac{\Sigma A_j}{16\pi} \int_{-l/2}^{l/2} \int_{-d/2}^{d/2} \int_{-h/2}^{h/2} \frac{dx. dy. dz}{x^2 + y^2 + z^2}$$

A full table of values of β_{avr} is provided in Davies (1983), Table 5. A selection is given as the second lines of Table 2 here.

It will be seen that β_{avr} varies comparatively little with enclosure shape. β_r is on average some 14% less and varies more with enclosure shape. If, however, the radiant source had been placed at the wall - a more realistic position for a hot water radiator - β_{avr} decreases. Further, we note that the local value of T_r must vary considerably over the volume of the enclosure. Also, T_r is itself sensitive to the probe shape; a flat plate probe place edge-on to the source would record a near zero temperature regardless of position.

In the interests of having a simple procedure to handle radiant exchange it appears sufficient to assume that β_r and β_{avr} are near enough equal, that is to say, the average observable radiant temperature, T_{avr}, the quantity upon which thermal comfort depends, can be estimated with sufficient accuracy for design purposes as the radiant star temperature, T_r, a fictitious construct, if the radiant flow to the enclosure is taken to be input at T_r.

APPENDIX 3

A Mean of Surface and of Air Temperatures?

It is shown here that an attempt to form a single index temperature from surface and air temperature components of an enclosure, using considerations of heat flow from the enclosure to one of its surfaces, leads to an absurd quantity.

Consider an enclosure containing air at temperature T_a and consisting of two surfaces. The smaller is plane, of area a, temperature T_1, and emissivity ε_1. The larger surface is of arbitrary shape, area A, temperature T_2, and emissivity ε_2. The conductance, aEh_r, between T_1 and T_2 is given as

$$\frac{1}{aEh_r} = \frac{1-\varepsilon_1}{a\varepsilon_1 h_r} + \frac{1}{ah_r} + \frac{1-\varepsilon_2}{A\varepsilon_2 h_r}$$

Suppose that the convective heat transfer coefficient between the air and the smaller surface is h_{c1}. The heat flow from the enclosure to the surface a is

$$Q = aEh_r . (T_2 - T_1) + ah_{c1} . (T_a - T_1)$$

If a tends to zero, both the radiant and convective components of this flow tend to zero. Suppose we express this flow in terms of an index temperature based on T_2 and T_a:

$$Q = a(Eh_r + h_{c1}) \frac{Eh_r . T_2 + h_{c1} . T_a}{Eh_r + h_{c1}} - T_1$$

The heat flow is now expressed in terms of the index temperature

$$T_e = \frac{Eh_r . T_2 + h_{c1} . T_a}{Eh_r + h_{c1}}$$

Again, if a becomes zero, Q too becomes zero as it must. Now if a = 0, the quantity Eh_r becomes $\varepsilon_1 h_r$, independently of ε_2, (provided that $\varepsilon_2 > 0$) and the index temperature becomes

$$T_{e,\, a=0} = \frac{\varepsilon_1 h_r . T_2 + h_{c1} . T_a}{\varepsilon_1 h_r + h_{c1}}$$

T_e has preserved its form for the situation when a = 0. Does it provide a valid index for this single surface enclosure? We note that T_e includes mention of T_2 and of T_a, now the only relevant enclosure temperatures, and to this extent, T_e provides a valid description of the enclosure. However, it includes too the values ε_1 and h_{c1} as weighting factors; these derive from the smaller and now non-existant surface, and so are now totally irrelevant to the enclosure. Furthermore, T_e does not include any mention at all of ε_2, nor of the convective coefficient h_{c2} for the larger surface. Thus T_e does not contain the information that is even qualitatively needed to form a valid enclosure index; T_e is a meaningless construct.

But T_e was arrived at as the index that drives the same heat flow to T_1 as do the real driving temperatures, T_2 and T_a. Thus we establish the principle: *considerations of heat flow from an enclosure to a bounding surface do not enable us to arrive at an index temperature for the enclosure.*

Environmental temperature t_{ei} was arrived at by exactly the argument used to reach T_e above. Thus t_{ei}, as it is defined in the CIBSE Guide, cannot provide a meaningful index temperature for the enclosure. t_{ei} does not include proper consideration of the emissivity of the larger surface which constitutes the model enclosure - a cube - on which it is based, or, indeed, any consideration at all of the convective coefficient (h_{c2}) at the larger surface. As an illustration of its absurdity, we may note three consequences.

1. Returning to the cubic enclosure, the CIBSE Guide defines t_{ei} as

$$t_{ei} = \frac{(6/5)Eh_r . t_m + h_{c1} . t_{ai}}{(6/5)Eh_r + h_{c1}} \approx (2/3) . t_m + (1/3) . t_{ai}$$

ε_1, the emissivity of the outer surface, may have any value between zero and unity. Suppose $\varepsilon_1 = 0$; then E = 0 and so

$$t_{ei} = t_{ai}.$$

Thus t_{ei} is independent of the temperature of the five possibly blackbody surfaces of the enclosure; it cannot provide a representative temperature for the enclosure. Further, the Guide defines comfort or dry resultant temperature as

$$t_c = ½t_m + ½t_{ai}$$

$$= ¾t_{ei} + ¼t_{ai}$$

and so in this case,

$$t_c = t_{ai}$$

Comfort temperature is now based on air temperature alone, and is *independent* of the temperature of the five internal surfaces - an evident absurdity.

2. According to the environmental temperature model, all radiant exchange between surfaces takes place via t_{ei}, and the conductances are proportional to $(6/5)Eh_r + h_{c1}$. But if E = 0, the conductances are simply proportional to h_c alone. Thus the model now has no means of describing any radiant exchange between, say, the floor and ceiling, if they happen to be at different temperatures.

3. Since the convective part of these conductances is based on h_{c1} alone, the model omits any mention of the convective coefficient h_{c2} between air and the five internal surfaces. Thus internal convective exchange is based on the wrong coefficient. The model fails again.

It is clear that environmental temperature is a badly malformed index. It was shown earlier that it is possible to construct a valid room index temperature - the rad-air temperature, T_{ra} - in order to combine convection and radiation. Since emissivities usually differ little from surface to surface, and convective coefficients do not vary grossly, we should not usually expect big numerical differences between t_{ei} and T_{ra} and their consequent conclusions, but that cannot be held to be a justification for using so structurally incorrect an index.

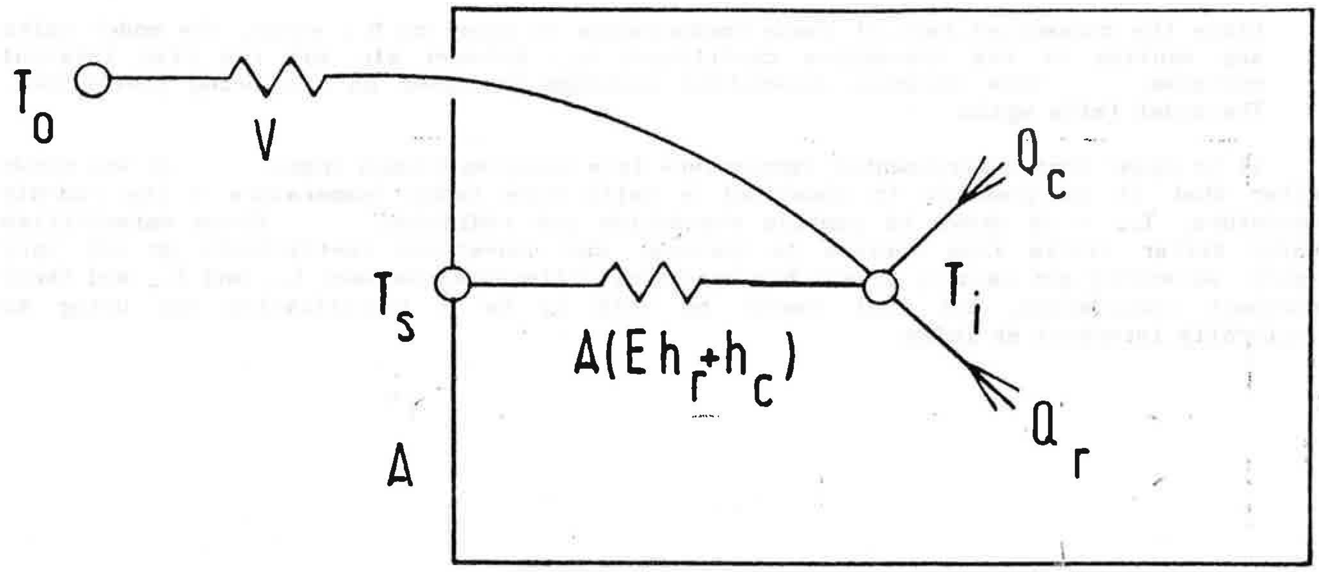

Figure 1 *The traditional model for handling heat transfer within a room, centered on 'air temperature' T_i. Convectively and radiantly input heat flows (Q_c and Q_r) are both taken to act at T_i, and T_i drives a heat flow proportional to Eh_r + h_c to the outer wall, inner surface at T_s. The ventilation conductance, V, acts between T_i and ambient temperature T_o.*

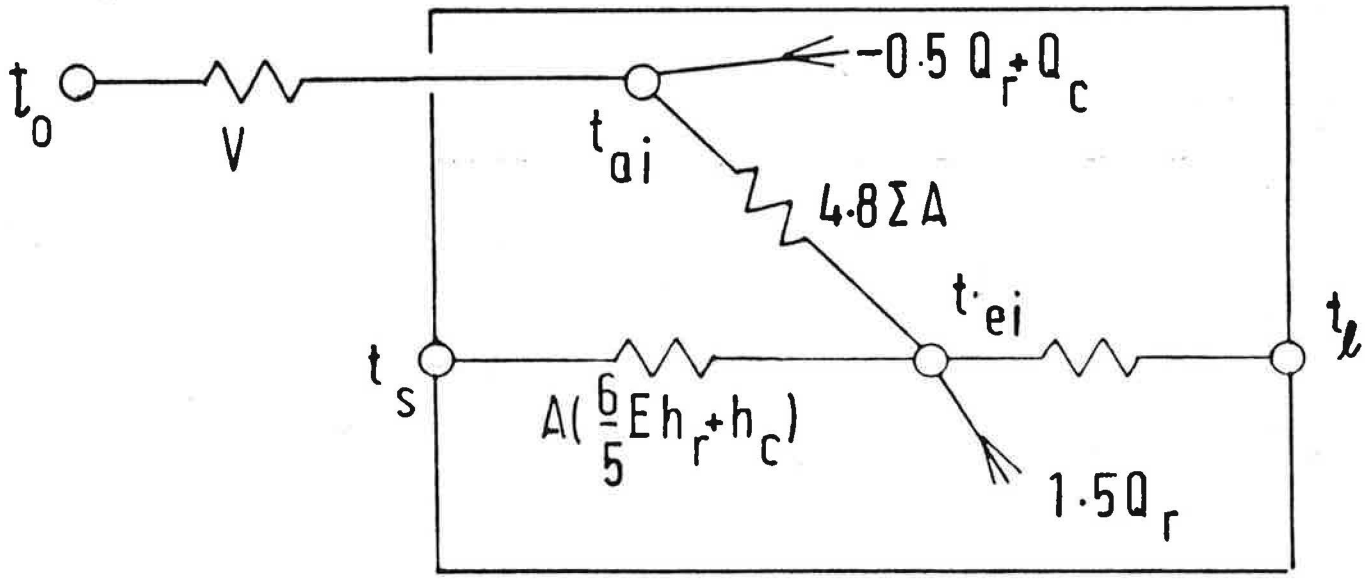

Figure 2 *The environmental temperature model, centered on environmental temperature t_{ei}. The augmented radiant input $1_1/2.Q_r$ acts at t_{ei}; the excess is withdrawn from the air temperature node, t_{ai}, so that the input there is - $1/2.Q_r$ + Q_c. There is a transmittance of $(6/5)Eh_r$ + h_c between t_{ei} and any room surface, internal or external. V acts between t_{ai} and t_o, and there is a conductance $4.5.\Sigma A$ between t_{ei} and t_{ai}. Comfort temperature in the form of dry resultant temperature t_c is a node on the $4.8.\Sigma A$ conductance and has a value $3/4.t_{ei}$ + $1/4.t_{ai}$.*

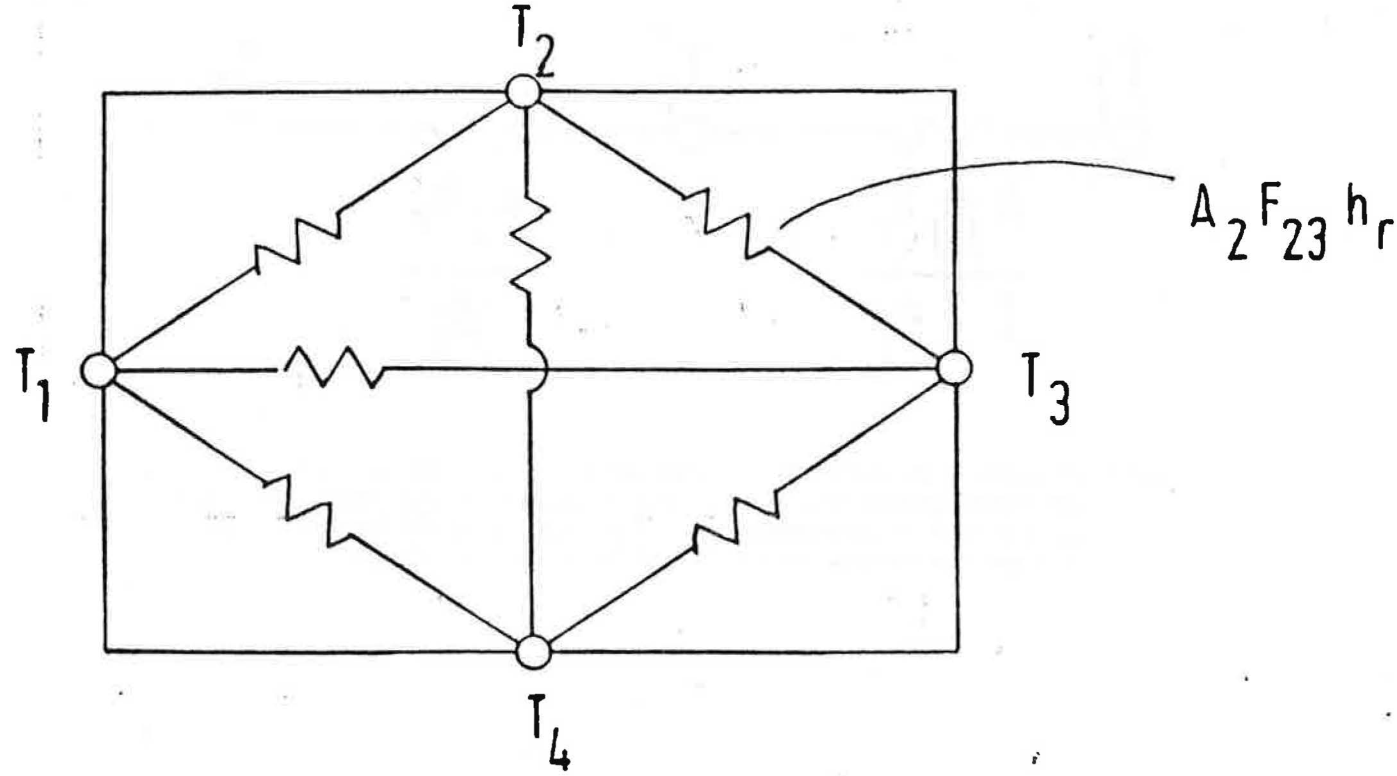

Figure 3 The surface-to-surface or 'delta' network for radiant exchange in an enclosure with four black body surfaces

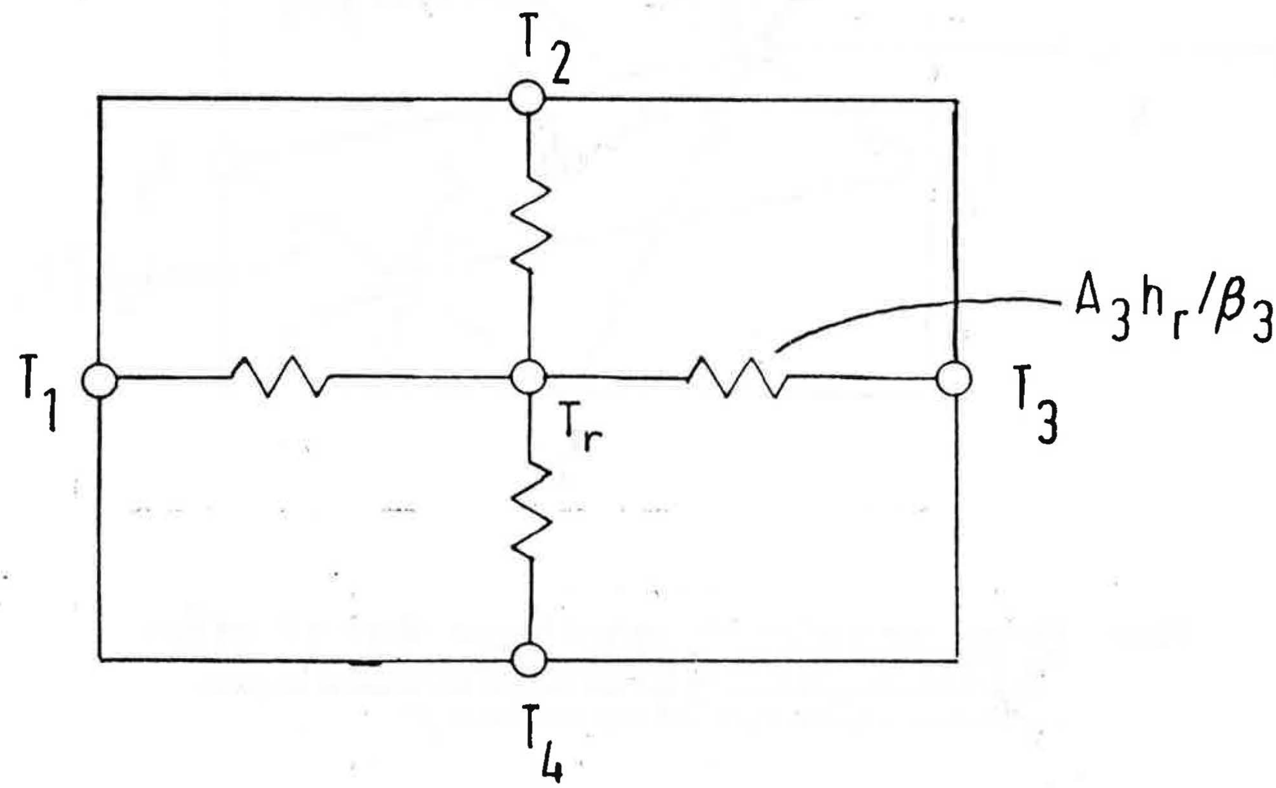

Figure 4 The surface-to-star node network, centered on T_r, which has approximately the same external effect as the parent delta network

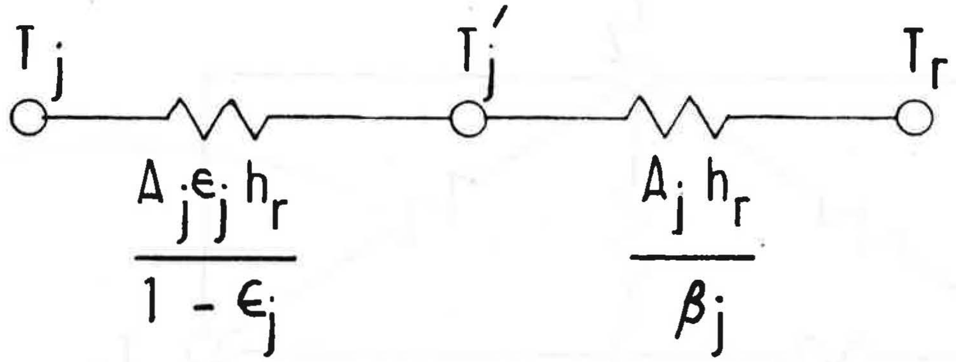

Figure 5 *Inclusion of the blackbody equivalent temperature of a surface and its emissivity conductance into the pattern of radiant exchange. This conductance, together with the geometrical conductance in star-centered form, constitute the combined conductance $A_jE_j{}^*h_r$, which acts between T_j and T_r*

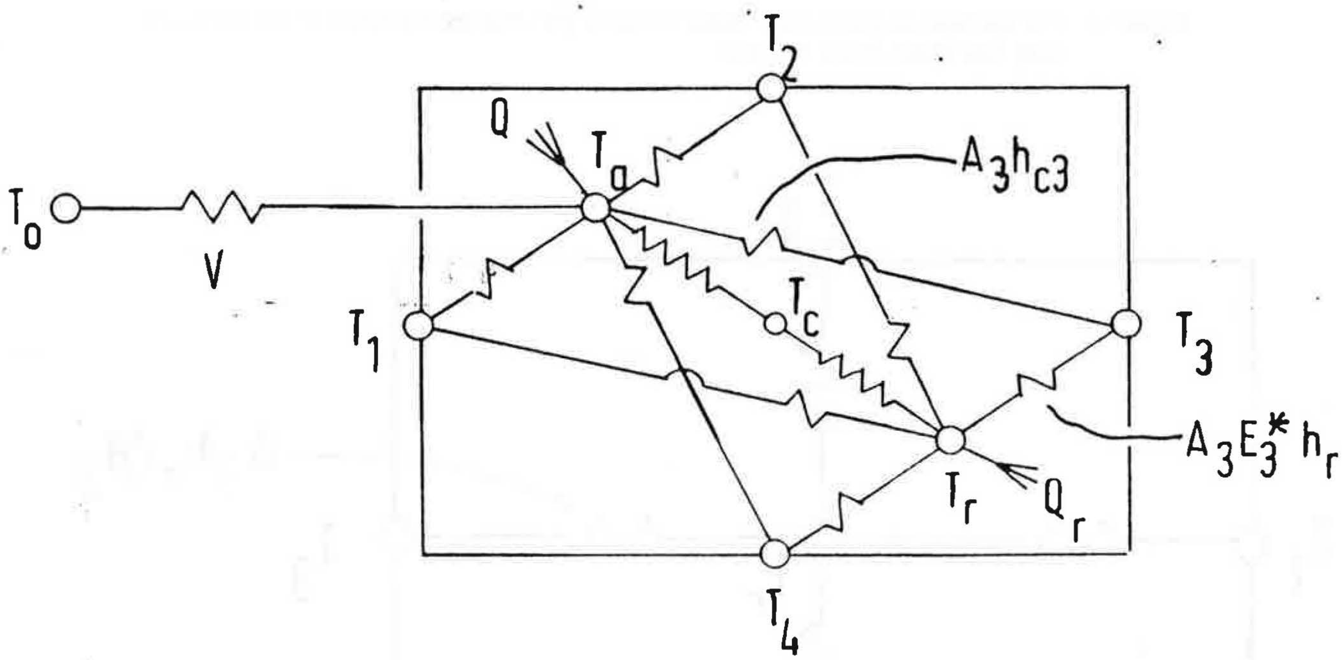

Figure 6 *The binary star model for a four surfaced enclosure. Q_r acts at T_r and Q_c at T_a. There is a transmittance of $E_j{}^*h_r$ between T_r and any surface, and a transmittance of h_{cj} between T_a and any surface. Dry resultant temperature, T_c, is linked to T_r and T_a through very high resistances*

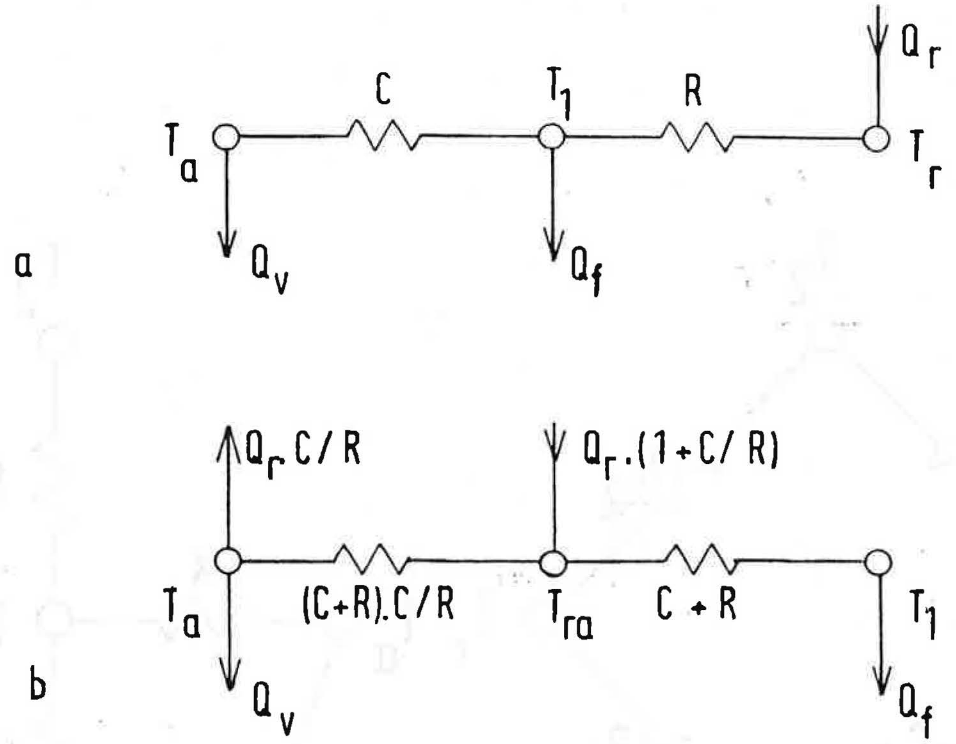

Figure 7 *This figure illustrates the equivalence theorem. The characteristics of Figure 7a can be obtained exactly from those Figure 7b. In Figure 7b, T_a continues to remain independent of radiant transfer, but T_{ra} is a linear combination of T_r and T_a, and replaces T_r*

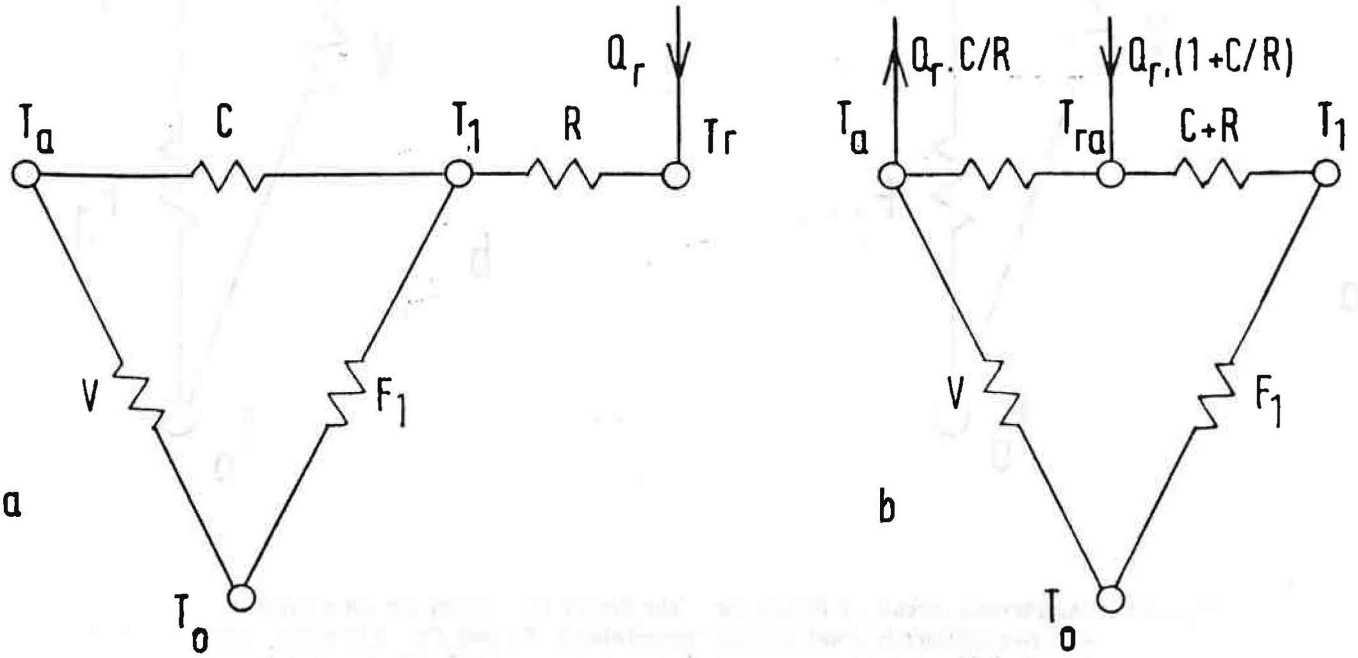

Figure 8 *The thermal circuit of Figure 8a illustrates the binary star model (in which air temperature T_a and radiant temperature T_r are expressed explicitly) for the simplest of all building enclosures - one in which there is a uniform internal surface temperature T_1. F_1 denotes a fabric loss conductance from the surface temperature T_1 to T_o, and V the ventilation loss from T_a to T_o. According to the equivalence theorem, the characteristics of the enclosure can be deduced exactly from the equivalent circuit of Figure 8b, which is the corresponding single star model, centered on the rad-air temperature T_{ra}.*

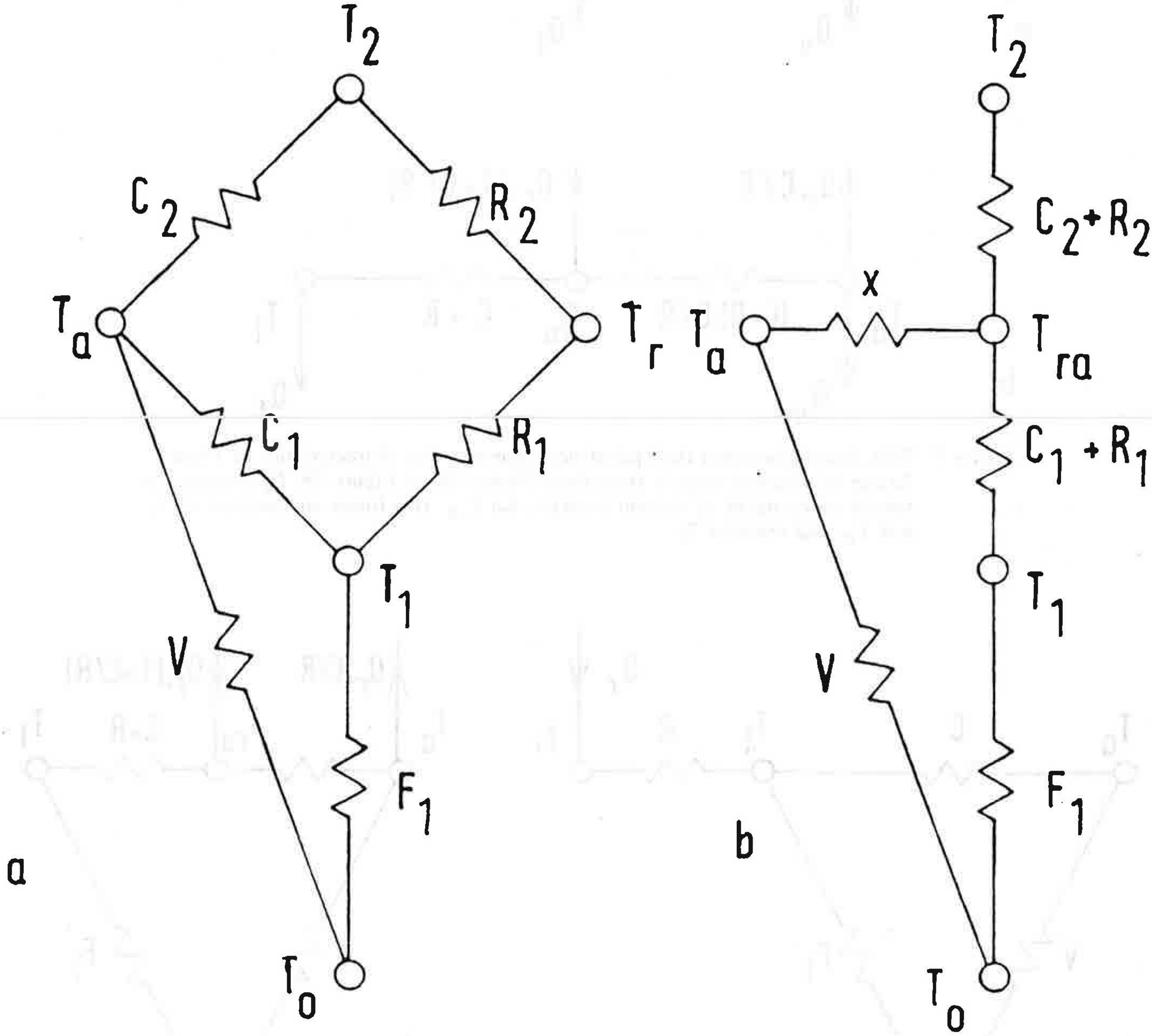

Figure 9 The thermal circuit of Figure 9a is the binary star model for an enclosure with two distinct internal surface temperatures, T_1 and T_2. The single star model of Figure 9b is exactly equivalent to it only if $C_1/R_1 = C_2/R_2$

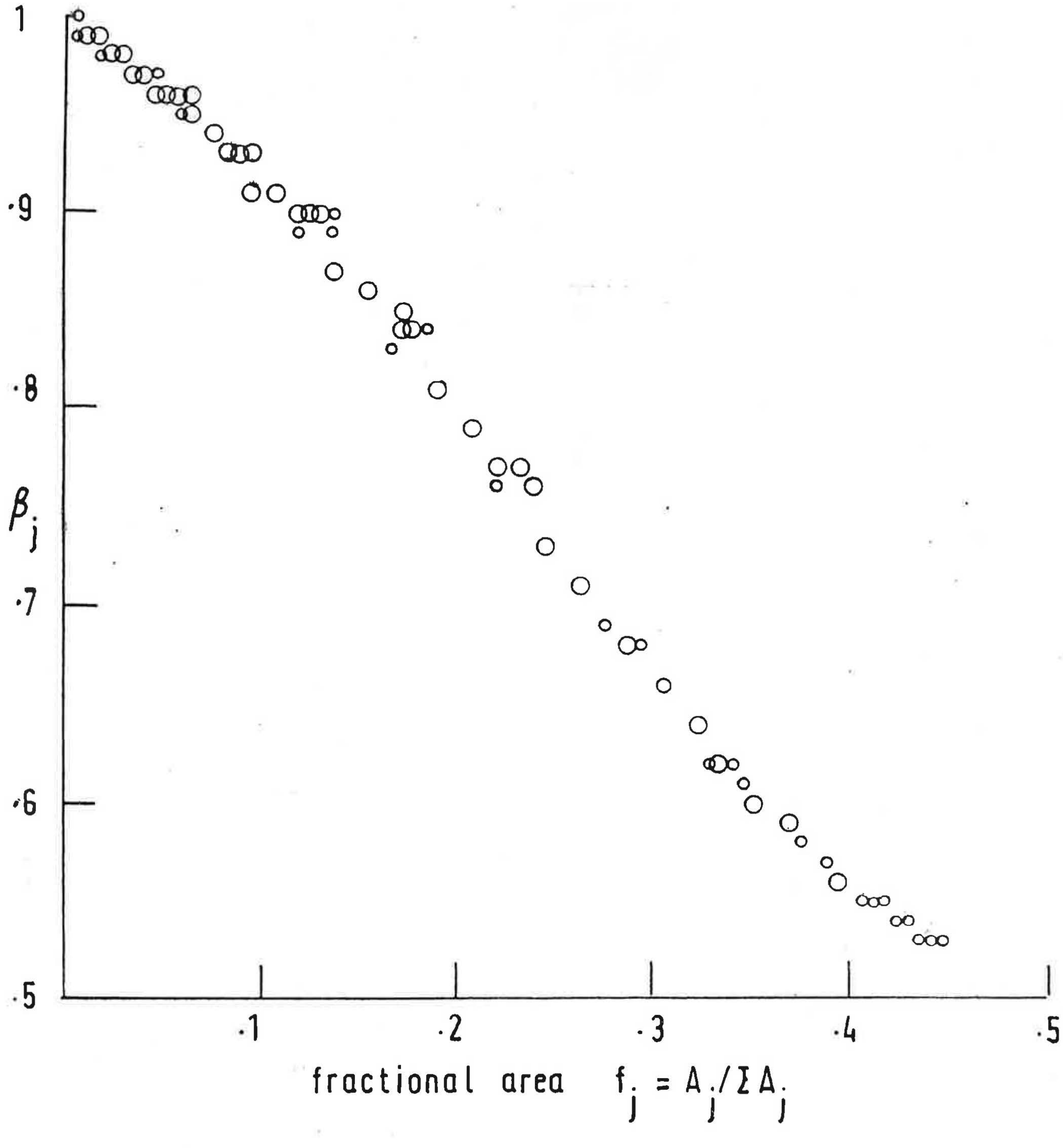

Figure A-1 *The figure shows the distribution of values of β obtained by the least squares procedure which was used to transform the surface-to-surface pattern of radiant exchange in a rectangular enclosure to a surface-to-star point pattern. They were found from a range of enclosures of different shapes. β values are presented as a function of the fractional area of the surface with which the β value is associated. (Small circles indicate the location of between 1 and 5 β values and large circles, 6 and more values. The total number of values is 3.11^2 or 363.*