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The Modelling of Stairwell Flows

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This paper extends an earlier investigation of scale effects on buoyancy-driven recirculating flows in stairwells of the kind adopted in domestic accommodation. Further consideration is given to the role of Reynolds number, which proves to have unexpected features, possibly because stairwell flows fall into the range of incipient instability. A technique is developed to introduce explicitly the fraction defining the way in which the energy loss from the system is divided between the regions above and below the stairway. Finally, it is shown that a single empirical constant suffices to complete relationships among key features of the processes of heat and mass transfer. The resulting formulae are suitable for incorporation within computer models of energy balances for complete buildings.

NOMENCLATURE

A	minimal cross-sectional area through which recirculating fluid must rise and return
C, C_1, C_2	dimensionless constants appearing in relationships between parameters characterising the system
c_p	specific heat at constant pressure
DT	difference in temperature between rising air and descending air $(T_H - T_C)$
Fr	Froude number, defined in equation (2)
h	measure of the height of the convection-driven system
K	dimensionless loss coefficient, dependent on Reynolds number
$\dot{Q}$	rate at which energy is supplied to the lower part of the system
$\dot{Q}_1$	rate at which energy is lost from the lower part of the system
$\dot{Q}_2$	rate at which energy is lost from the upper part of the system
Re	Reynolds number, defined in equation (5)
r	ratio of energy loss from upper part of the system to the total energy input $(\dot{Q}_2/\dot{Q})$
S	a dimensionless system characteristic (Re/Fr), defined in equation (29)
St	Stanton number based on total energy input
St_2	Stanton number based on energy lost from the upper part of the system, defined in equation (19)
T	absolute temperature
T_1, T_2	mean temperatures for regions below and above the stairwell, respectively
T_H, T_C	mean temperatures for heated upwards-flowing air and cooler downwards-flowing air, respectively
$\dot{V}$	recirculating volume flow rate
β	bulk modulus of air $(1/T)$
ΔT	difference in temperature between upper and lower parts of the system $(T_2 - T_1)$
υ	kinematic viscosity of the fluid (air)
ρ	mean density of the fluid (air)

1. INTRODUCTION

A RECENT paper by the first author [1] addressed the little studied problem of predicting the coupled flows of energy and mass up and down a stairwell. Dimensional arguments, supplemented by experimental results obtained in a one-half-scale model, displayed some essential features of this class of flows, which are of importance in understanding and ultimately in predicting the migration of energy through a building.

The emphasis in the paper referred to above was on the design of experiments on stairwell flows and on the interpretation of results obtained using scale models. The present paper returns to the question of the design and interpretation of experiments, by investigating the significance of the Reynolds-number régime into which stairwell flows fall.

An attempt is also made to determine the role of the division of energy losses (from a purely recirculating flow) between the regions below and above the stairway. In the experimental model studied by the authors it transpires that about two-thirds of the heat supplied to the apparatus is convected up the stairwell to be transferred to the surroundings through the walls of the upper parts of the model. Evidently this ratio will be quite different in other circumstances, and a useful empirical description of stairwell performance must include provision for this variability. Throughout, we consider the simplest case of purely recirculating flow. The important role of leakage between the environment and the building containing the stairwell will be discussed in a subsequent study.

2. THE ROLE OF REYNOLDS NUMBER REVISITED

The modelling of stairwell performance in the earlier paper [1] fell into two parts. In the first, the viscosity of the circulating fluid was neglected, and the relationship

$$\frac{\Delta T}{T} \propto \mathrm{Fr}^2, \qquad (1)$$

was found, by dimensional arguments, to relate ΔT, the driving temperature differential between lower and upper

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chambers, to Fr, the Froude number characterising the motion. The Froude number chosen was

$$\mathrm{Fr} = \frac{\dot{V}}{A(gh)^{1/2}}, \tag{2}$$

where $\dot{V}$ is the recirculating volume flow, A is the minimum cross-sectional area through which the buoyant fluid must both pass and return, and h is a scale for the height of the convection-driven system.

The result (1) can be interpreted as a balance between the buoyancy forces driving the motion and resistance proportional to the square of the velocities induced in the stairwell:

$$\Delta T \propto V^2, \tag{3}$$

being equivalent to

$$\Delta \rho g h \propto \tfrac{1}{2}\rho V^2 \tag{4}$$

which displays the balance of forces more explicitly, V being a typical velocity.

The second analysis of [1] generalised the results given above by introducing a loss coefficient, dependent upon Reynolds number

$$\mathrm{Re} = \dot{V}/\upsilon A^{1/2}. \tag{5}$$

Thus

$$\Delta \rho g h \propto K(\mathrm{Re})\tfrac{1}{2}\rho V^2, \tag{6}$$

where K is the loss coefficient.

In the flows most familiar to workers in the field of fluid mechanics it is found that $K(\mathrm{Re})$ is a decreasing function of Reynolds number, which may often be specified using a power law: $K \propto \mathrm{Re}^{-n}$, where n is a positive number. However, in the experiments reported in [1] it was found that the optimum power-law representation of the experiments was a different power law, namely

$$\Delta T \propto \mathrm{Fr}^3 \propto \mathrm{Re}^3 \propto \dot{V}^3. \tag{7}$$

Comparison with equation (4) suggests that this is equivalent to a loss coefficient displaying the singular behaviour

$$K \propto \mathrm{Re}. \tag{8}$$

A possible explanation of this unexpected behaviour can be determined by reference to the experiments which gave rise to it. For the model considered, the minimum flow area is $A = 0.76 \times 0.608$ m $\doteq 0.462$ m^2. The flow rates generated ranged from $\dot{V} = 0.040$ to 0.075 m^3 sec^{-1}. Taking the kinematic viscosity to have the typical value $\upsilon = 15.7$ mm^2 sec^{-1}, we find that

$$\mathrm{Re} = 3800\text{–}7000. \tag{9}$$

Note that this is the Reynolds number based on the gross parameters characterising the system, while the overall resistance to the fluid motion is the resultant of various smaller-scale processes adjacent to the walls and to the stairs themselves. The Reynolds numbers characteristic of these small-scale processes will be somewhat less than the gross-flow values found above. Thus the significant Reynolds numbers for the resistance-generating process could fall in the range, around Re = 1000, of transition between generally stable, viscosity-dominated flow and rather unstable, inertia-dominated flow. The terms 'laminar' and 'turbulent' are deliberately avoided, as possibly giving a misleading impression of the actual flow regimes.

Several conclusions follow from the realisation that stairwell flows fall inconveniently near the régime of transition:

(a) The resistance characteristic must be expected to display a somewhat complicated variation with Reynolds number, over the range of practical importance.
(b) A rising characteristic ($\mathrm{d}K/\mathrm{dRe} > 0$) need not be seen as implausible within this region, although the opposite tendency may well be encountered at values of Re beyond the range considered in the experiments considered here.
(c) Since the dependence on Reynolds number is rather large and of an uncertain character, it is necessary to match model and prototype values of Reynolds number quite closely, if reliable predictions of prototype behaviour are to be obtained.

3. THE ROLE OF THE ENERGY-LOSS DISTRIBUTION

The parameters used in the earlier study to characterise the buoyancy-driven flow in a stairwell included

$$\Delta T = T_2 - T_1 \tag{10}$$

where T_1 and T_2 are mean temperatures for the regions below and above the stairs. While these are the temperatures of greatest practical interest, certainly in respect of human comfort and possibly also in relation to energy losses from the system under consideration, the temperature difference of greater significance for the flow itself is the differential

$$DT = T_H - T_C \tag{11}$$

between the heated, rising stream and the cooler, descending stream. This latter differential is directly dependent on the ratio $\dot{Q}_2/\dot{Q}_1$, between the conductive heat flows from the above-stairs and below-stairs regions. This was acknowledged in the earlier paper, where the results of dimensional analysis were given in the form

$$\frac{\mathrm{Fr}^2}{\Delta T/T} = f(\dot{Q}_2/\dot{Q}_1, \mathrm{Re}). \tag{12}$$

A further advantage in using the temperatures T_H and T_C to characterise the process, rather than T_1 and T_2, is that the latter pair must be construed as averages over a considerable volume (where the variations are large) while the former pair are more localised and more readily measured.

We now argue that instead of adopting the starting point

$$\dot{V} \sim \Delta T, \beta g h, \dot{Q}_2/\dot{Q}_1, A, K, \tag{13}$$

which led to equation (8), we should take instead

$$\dot{V} \sim DT, \beta g h, A, K, \tag{14}$$

as more closely related to the physical processes driving the motion.

We note also that in the absence of a flow of heat from the upper chamber ($\dot{Q}_2 = 0$) no convection will occur ($DT = 0$). Hence we may replace another starting point of the earlier paper, namely

$$\dot{V} \sim \dot{Q}/\rho c_p, \dot{Q}_2/\dot{Q}_1, \beta gh, A, K, \tag{15}$$

by

$$\dot{V} \sim \dot{Q}_2/\rho c_p, \beta gh, A, K. \tag{16}$$

Expressing equations (9) and (10) in dimensionless form, we have

$$\frac{\mathrm{Fr}^2}{DT/T} = f_1(\mathrm{Re}), \tag{17}$$

$$\frac{\mathrm{Fr}^3}{\mathrm{St}_2} = f_2(\mathrm{Re}), \tag{18}$$

with

$$\mathrm{St}_2 = \frac{\dot{Q}_2}{\rho c_p T A (gh)^{1/2}} \tag{19}$$

the Stanton number based on the upper energy loss.

The experimental results considered in the earlier paper suggested that both

$$f_1(\mathrm{Re}), f_2(\mathrm{Re}) \propto 1/\mathrm{Re}. \tag{20}$$

Thus we obtain

$$\frac{DT}{T} = C_1 \mathrm{Re}\mathrm{Fr}^2 \tag{21}$$

$$\mathrm{St}_2 = C_2 \mathrm{Re}\mathrm{Fr}^3 \tag{22}$$

and

$$\mathrm{St} = C_2 \mathrm{Re}\mathrm{Fr}^3/r, \tag{23}$$

where C_1 and C_2 are constants, St is the Stanton number based on the rate at which energy is supplied, $\dot{Q}$, and $r = \dot{Q}_2/\dot{Q}$ is the fraction conducted from the upper part of the model.

To these results of dimensional analysis and experiment we may add the energy balance

$$\dot{Q}_2 = \rho c_p \dot{V}(T_H - T_C), \tag{24}$$

or

$$\mathrm{St}_2 = \mathrm{Fr}DT/T. \tag{25}$$

This is exact, if T_H and T_C are taken to be suitable averages over the ascending and descending flows.

On introducing the results (11, 12) into equation (18), we obtain

$$f_1(\mathrm{Re}) = f_2(\mathrm{Re}) = f(\mathrm{Re}). \tag{26}$$

In the same way the more specific results (15, 16) lead to

$$C_1 = C_2 = C. \tag{27}$$

Thus we see that only one function of Reynolds number (or, if the modelling of equation (14) is correct, a single empirical constant) is required to characterise a particular stairwell configuration. Here the word 'characterise' is used to mean 'define the relationships between the dimensionless quantities DT/T, Fr, Re, St and r.'

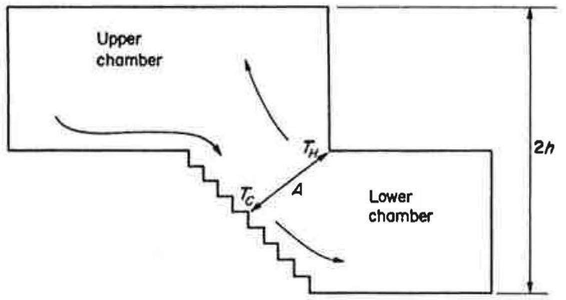

Fig. 1.

Table 1. Basic performance characteristics of a model stairwell

$\dot{Q}$ (W)	T (°C)	$\dot{V}$ (dm³/s)	1000Fr	1000St	Re	$C_2^{1/4}$*	DT (Cdeg)
100	29	40.7	25.5	0.1762	3810	0.208	1.2
300	34	53.0	33.2	0.529	4970	0.210	2.7
600	39	70.2	44.0	1.057	6580	0.188	6.4
900	44	74.6	46.7	1.586	6990	0.196	7.7

* Mean 0.200

4. EVALUATION FOR A PARTICULAR CASE

The half-scale model stairwell within which detailed measurements have been carried out by the writers' colleagues has the following geometrical characteristics:

$h = 1.218$ m (one half the height of a two-storey model)
$A = 0.462$ m² (area of the minimum area between stairway and ground-floor ceiling).

These critical dimensions are shown in Fig. 1.

Studies of the mass and energy flows through the minimum area revealed that the energy-loss ratio is sensibly constant:

$$\dot{Q}_2/\dot{Q} = 2/3, \tag{28}$$

throughout the range of conditions examined. The volume flows $\dot{V}$ generated by specified heat inputs $\dot{Q}$ are shown in Table 1.

The temperatures given in the second column are averages of nine readings distributed within the upper and lower chambers. Consideration is given to the fourth root of the constant C_2 since it is the relationship $\dot{V} \propto \dot{Q}^{1/4}$ which is the key output from this analysis. The near-constancy in the values of the characterising constant C_2 is not a coincidence: the Reynolds-number variations (20) were chosen to produce just this behaviour. The tabled values of the temperature difference $DT = T_H - T_C$ are found from equation (21), with $C_1 = C_2 = 0.0016$.

Introducing the system characteristic

$$S = \frac{\mathrm{Re}}{\mathrm{Fr}} = \frac{(Agh)^{1/2}}{v} \tag{29}$$

(independent of the flow condition) we can characterise the particular stairwell geometry by the set of equations

$$\frac{\mathrm{Fr}^4}{r\mathrm{St}} = \frac{\mathrm{Fr}^3}{DT/T} = \frac{1}{CS} = \frac{625}{S}. \tag{30}$$

5. CONCLUDING REMARKS

The results (30) provide a simple way of relating the rate of air circulation and the driving temperature difference to the heat input that gives rise to them. It is a matter for further experimental investigation to discover whether the constant C varies markedly from one stairwell type to the next, and possibly to develop a body of empirical data characterising the more common stairwell forms.

For some geometries and for some Reynolds-number ranges the particular power-law dependence upon Reynolds number which has been used here may not suffice. The description of such flows requires the more general forms

$$\frac{\mathrm{Fr}^3}{r\mathrm{St}} = \frac{\mathrm{Fr}^2}{DT/T} = f(\mathrm{Re}), \qquad (31)$$

derived from equations (17, 18, 26) or, more closely paralleling equation (30):

$$\frac{\mathrm{Fr}^4}{r\mathrm{St}} = \frac{\mathrm{Fr}^3}{DT/T} = \mathrm{Fr}\,f(\mathrm{Re}) = \frac{\mathrm{Re}\,f(\mathrm{Re})}{S}. \qquad (32)$$

Our experience suggests that the product $\mathrm{Re}\,f(\mathrm{Re})$ will vary only modestly.

Finally, a point of general physical interest should be noted. In the basic energy balance (25):

$$\mathrm{St}_2 = r\mathrm{St} = \mathrm{Fr}\,DT/T, \qquad (33)$$

the energy extracted at the upper level is displayed as the product of the rate of recirculation $\dot{V} \propto \mathrm{Fr}$ and the temperature differential $DT \propto \mathrm{Fr}^3$ (from equation (30) or elsewhere). Thus we see that an increase in the upper-level energy extraction (or for constant r an increase in energy input) is only weakly reflected in the rate of recirculation ($\dot{V} \propto \mathrm{St}^{1/4}$), the more significant response being that of the temperature differential ($DT \propto \mathrm{St}^{3/4}$).

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