

ASHRAE/DOE/BTECC
Thermal Performance of the Exterior
Envelopes of Buildings III
December 2-5, 1985
Clearwater Beach, Florida

THERMAL BRIDGES: A TWO-DIMENSIONAL AND THREE-DIMENSIONAL TRANSIENT THERMAL ANALYSIS

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ABSTRACT

Thermal bridges are parts of the building envelope where, due to the two-dimensional or three-dimensional character of the heat conduction, either the inside surface temperatures are rather low, which can cause condensation, or the heat losses are rather high. In this paper thermal bridges are analyzed by numerical methods, shortly described in the first section. They are based on energy balance techniques. Features of these models are their implementation on personal computers, the simple use and the graphical output by means of plots of isothermals and streamlines, permitting a direct evaluation of both aspects of thermal bridges. As shown in the next section, a two-dimensional steady-state analysis is sufficient to quantify both thermal bridge problems. The importance of thermal bridges in uninsulated or insulated cavity walls or massive walls is deduced. For example, for traditional insulated cavity wall constructions, the conductive heat losses exceed the onedimensionally calculated heat losses with 30% to 40%. The application of cavity insulation will not affect the condensation risk; it is also shown that with regard to this condensation risk, outer insulation of massive walls is much favorable than inner insulation. In a third section further study based on three-dimensional and transient calculations shows some particular points of interest: the symmetry between an observed mold growth pattern and the calculated isothermal field, the influence of inside surface conductances, transient surface condensation, and the relativity of one-dimensional heat loss calculations for complex-shaped buildings.

INTRODUCTION

The occurrence of surface condensation and its attendant mold growth on the inside wall surfaces of buildings induced the study (and the definition) of thermal bridges. Surface condensation appears when the surface temperature is lower than the dew point of the surrounding air. Therefore, both quantities, surface temperature and dew point, should be known in order to predict the phenomenon.

The inside surface temperature on a building element separating indoors and outdoors depends on the following parameters:

- the outdoor temperature; consequently surface condensation is seasonally determined.
- the indoor temperature; hence more damage appears in unheated rooms (as bedrooms).
- the configuration and the thermal properties of the different materials in the building element. For flat walls (in which the heat transfer is one-dimensional) it suffices to know the thermal resistance: condensation will appear sooner on walls with a low thermal resistance (e.g., single glazing). For complex elements (with two-dimensional or three-dimensional heat transfer) the surface temperature cannot be predicted so simply. Especially in the case of thermal bridges, low inside surface temperatures are caused by the use of well-conducting materials (metal, concrete, stone), by the shape of the inside and outside surfaces, and by the place of insulating materials.

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- the inside surface conductance. Again some typical thermal bridge configurations hinder the convective and radiative heat exchange with the inside environment, causing lower surface temperatures.

By means of numerical techniques, such as those described below, the temperature field in complex configurations can be predicted. The minimal inside surface temperature on the thermal bridge if outside and inside temperature are 0°C and 1°C respectively is called the dimensionless thermal bridge temperature (θ_{tb}).

The dew point of the air depends of the following parameters:

- the humidity of the outside air.
- the vapor production in the building (by people, plants, washing and cooking activities, etc.)
- the volume of the building : small houses are more accessible to condensation.
- the air ventilation rate : low ventilation rates cause high humidity - the energy crisis has certainly affected the occurrence of condensation and mold growth. The improvement of the airtightness of buildings to decrease heat losses has caused increased humidity ratios.
- the presence of airdrying elements : single glazing for example, can keep down the dew point by condensation. Therefore, replacing single by double glazing can result in higher humidity ratios and consequently in surface condensation on thermal bridges (on which surface temperatures are lower than on the double glazing).

(It's clear that if there is a humidity control unit, these parameters do not affect the dew point.)

Contrary to surface temperatures, the dew point of the air can hardly be predicted, as the vapor production and ventilation rate are normally unknown. The importance of the above-mentioned parameters mostly can be shown only in case studies. For this reason statistical data regarding the dew point in houses are used to predict the occurrence of surface condensation and to formulate requirements for the thermal performance of the building envelope. For Belgian houses, a simple requirement for the dimensionless surface temperature on thermal bridges is deduced : $\theta_{tb} > 0.7$ (Standaert 1982).

The second problem of thermal bridges, revealed by the energy crisis, is the relative importance of heat losses through them. It means that the classical one-dimensional heat transfer calculations underestimate the real conductive heat losses. Using the same numerical models as for the calculation of the temperature field, the two-dimensional or three-dimensional heat flow through complex building elements can be determined. The difference between the real heat losses through a thermal bridge and the onedimensionally calculated ones, in the case of an inside-outside temperature difference of 1 Kelvin, is defined as the linear coefficient of transmission of that thermal bridge: U_1 [W/mK]; (Règles Th 1974). This means that, once the U_1 -value of a thermal bridge is known (from a numerical calculation), the one-dimensional calculation of heat losses through a wall can be corrected by adding the U_1 -value multiplied by the length (i.e. perpendicular to the section) of the thermal bridge in this wall. The U_1 -value of a thermal bridge is mainly affected by the material configuration (possible short-circuit effect), but it must be emphasized that the conventions needed for the one-dimensional calculations induce a partially artificial thermal bridge effect. The U_1 -values given in this paper were obtained by taking into account the projected outside surface of the building elements as the heat loss surface in the one-dimensional calculations. Moreover, only the U-values of the outside visible major wall-parts (cavity wall, window, roof, visible concrete column) were integrated in the one-dimensional calculations.

NUMERICAL METHODS FOR TWO-DIMENSIONAL AND THREE-DIMENSIONAL HEAT TRANSFER

Review of Principles Used

The developed numerical methods are based on energy balance techniques. The energy conservation law and the law of Fourier are applied on finite volumes around nodal points (created by a chosen grid), by which a system of linear equations is obtained. While finite difference methods or finite element methods are techniques to numerically solve the governing differential equations, the principles used here are the same as those that make the differential equation, though the difference between these methods is rather artificial because identical linear equations are (or can be) obtained.

Computer programs were made to calculate two-dimensional and three-dimensional steady-state and transient heat transfer; further programs allow for the determination of local inside surface conductances. A description of the algorithms used is given in detail in Standaert (1984). The program to calculate two-dimensional steady-state problems, named KOBRU82, is

described in Standaert (1982). The most relevant characteristics of these programs are the following :

- The studied objects are put in a rectangular grid. This indeed is a restriction, but it facilitates the data input, and construction elements do mostly have a rectangular shape.
- A linear equation in the unknown temperatures in each node of the object and in the neighboring nodes is formed; the parameters in the equation can be deduced from the known geometry, the thermal properties of the materials, and the boundary conditions.
- The matrix of the obtained system of equations is a positive definite symmetric band matrix on which the Cholesky decomposition (Martin and Wilkinson 1971) can be applied, which is advantageous for calculation speed and memory saving.
- For transient problems the Crank-Nicolson method (Richtmeyer and Morton 1967) is used in the time domain. A system of equations is formed and solved for each time step. Nevertheless the Cholesky decomposition must be carried out completely only twice, so that the sequential solutions can be obtained by fast matrix calculations.
- By means of interpolation, the knowledge of the obtained node temperatures can be translated in a plot of isothermals. For two-dimensional steady-state problems, a supplementary calculation of the stream function values in the nodes leads to a plot of streamlines. By means of this dual graphical representation, both mentioned thermal bridge aspects can be analyzed. Results of three-dimensional calculations are clearly represented by isothermals on surfaces and sections in an axonometric view. For dynamic problems, node temperature and heat flows are plotted versus time. A clear visualization is produced by video pictures, which show a (usually accelerated) evolution of isothermal plots.
- In the procedure, the boundary conditions must be formulated either as known surface temperature, as a known heat flow, or as a known ambient temperature with known surface conductance. Because of the expected interaction between the inner surface conductances and the temperature field, which could be important in the case of thermal bridges, the computer programs were completed with a module to calculate these conductances. The construction element and the surfaces of the adjacent room are divided in elementary surfaces (coinciding with the grid partition of the element). For each surface element an energy balance equation is formed: the sum of incident radiation, convection and conduction equals zero. Starting from the calculated viewfactors between the surfaces, the radiative part can be formulated using fundamental laws (Sparrow and Cess 1970). Convective terms are deduced from empirical laws. The conductive part is obtained from simple one-dimensional heat transfer formulae (for room walls) or from the above described two-dimensional or three-dimensional calculations. Therefore and because of the non-linearity of the balance equations, an iterative procedure is used to solve the problem.
- The programs are written in an extended BASIC and run on desktop computers (memory requirements: 32 Kb for two-dimensional problems, 256 Kb for three-dimensional problems).

Compared with analytical solutions and experimental results, it is shown (Standaert 1984) that the accuracy of the numerical models is satisfactory for applications in building physics, as the next section illustrates.

Example

A horizontal section of a brickwork wall, insulated at the inside, is shown in Figure 1a. There is a discontinuity in the insulation at the connection of an inner wall with the outer wall. Because of symmetry only a half connection is considered. The wall (2.4 m width, 2 m height) was placed between two climatic rooms: one room (2.4 m x 2.4 m x 2 m) is heated by a convector, giving an air temperature of 21.0°C and a globe temperature of 20.6°C; in the other room a cold airstream with an air temperature of 0.6°C and an air velocity of 4 m/s passes the wall surface. For the wall the geometrical data, the thermal conductivities, and the assumed grid are shown in the same figure. A complete description of the test equipment is given in Standaert (1984). Using the procedure described above, the two-dimensional heat transfer was calculated in steady state. The results are shown in :

- Figure 1b : isothermals; the calculated thermal bridge temperature is 14.2°C, the measured value is 13.6°C. The calculated dimensionless temperature $\theta_{tb} = 0.67$.
- Figure 1c : streamlines; the total heat loss amounts to 13.9 W/m (for 1 m high part of half of the wall). Because the U-value of the (nondisturbed) wall is known ($U = 0.42 \text{ W/m}^2\text{K}$) the linear value U_1 of the thermal bridge can be calculated: $U_1 = 0.18 \text{ W/m}^2\text{K}$.

- Figure 2a : calculated versus measured temperature on the inner wall side.
- Figure 2b : calculated surface conductances on the inner wall side (black globe temperature as reference).

If a constant inner surface conductance of $8 \text{ W/m}^2\text{K}$ is assumed (Figure 3b), the results shown in Figure 3a are obtained : the calculated thermal bridge temperature amounts to 15.2°C , the calculated heat loss amounts to 14.4 W/m . Though the concurrence was better in the previous calculation, it illustrates that this easier calculation procedure is satisfactory for practical case studies.

TWO-DIMENSIONAL STEADY-STATE ANALYSIS

Thermal Bridges in Cavity Walls

Cavity walls have been commonly used in Belgian (and some other European) constructions since about 1950. The cavity (width $\pm 6 \text{ cm}$), separating the inner and the outer leaf (thickness, respectively, 14 cm and 9 cm) was introduced to avoid water penetration. To provide stability, connections were made between inner and outer leaf around windows and doors, in basements, at floor levels; some construction elements also pass through the cavity, e.g., terraces, concrete beams, and columns. From the energy crisis onward, the cavity was considered an obvious place to put the insulation without changing the building technology, and because of this the mentioned discontinuities became thermal bridges.

A typical example is given in Figure 4a, which shows a horizontal section of the connection between a window and a cavity wall. The calculated isothermals and streamlines, if the cavity is empty, are shown in Figure 4b and c. There is only a small thermal bridge effect: isothermals are nearly parallel and streamlines are nearly perpendicular to the mean surface of the wall. The dimensionless thermal bridge temperature amounts to 0.67; the linear U-value amounts 0.13 W/mK . The calculation results of a cavity filled with insulation material (thermal conductivity 0.04 W/mK) are shown in Figure 5a and b. The extending isothermals and contracting streamlines are typical. The dimensionless thermal bridge temperature amounts to 0.68, the linear U-value to 0.32 W/mK .

To determine the influence of thermal bridges on condensation and on heat losses, a lot of typical thermal bridges in cavity walls were studied numerically (Standaert 1983). The conclusions from this analysis are the following :

- If the cavity is not insulated, a mean linear U-value of 0.1 W/mK is found. For a typical one-family house, a supplement of only a small percentage must be added to the onedimensionally calculated conductive heat losses to obtain the real losses.
- If the cavity wall is filled with insulation material, a mean linear U-value of 0.4 to 0.5 W/mK is found. For the same (insulated) house, the supplement to the onedimensionally calculated heat losses amounts to 35% !
- The dimensionless thermal bridge temperature of typical thermal bridges in uninsulated cavity walls is 0.55 to 0.75 (11°C to 15°C if the outside and inside temperature are respectively 0°C and 20°C).
- If the cavity is filled, this temperature normally does not drop; it mostly remains, as in the above example. Therefore cavity filling itself will not cause condensation or mold growth, neither will it make them disappear (if the inside air humidity remains unchanged). In view of the fact that dew points with a value of 10°C to 12°C occur in statistically moist houses, thermal bridges will cause condensation and mold growth in these cases.
- The most important conclusion is that thermal bridges have to be avoided: this can be done by providing a continuity in the thermal insulation. Though the energy savings are less than what is expected from one-dimensional calculations, cavity insulation is mostly efficient to improve existing buildings if the air humidity is not too high.

Thermal Bridges in Massive Walls

Many European buildings constructed before 1940 consist of massive brickwork walls with a thickness of 20 cm to 30 cm . To insulate them, the insulation material can be placed either at the inside or at the outside. While there are no thermal bridges in the original construction, because of a general lack of insulation, they appear at the discontinuities in the thermal insulation.

| | | |
|---------------------|----------------------------|----------------------|
| -- no insulation | : $U_1 = 0.3 \text{ W/mK}$ | $\theta_{tb} = 0.65$ |
| -- inner insulation | : $U_1 = 0.8 \text{ W/mK}$ | $\theta_{tb} = 0.64$ |
| -- outer insulation | : $U_1 = 0.8 \text{ W/mK}$ | $\theta_{tb} = 0.78$ |

Specific Thermal Bridges

THREE-DIMENSIONAL AND TRANSIENT ANALYSIS

Figure 7 shows an axonometry of a three-dimensional corner in a flat: the reinforced concrete structure consists of width columns and beams that support the floor slab. The filling walls are in brickwork and pre-cast concrete panels are placed at the outside. The cavity between the brickwork and the panels is filled with mineral wool, while there is only an air cavity between the concrete structure and the panels. The floor slab between the room in question and the room above penetrates to the outside. In a bedroom of the flat, important mold growth occurs (Figure 8a). The three-dimensional temperature field was calculated with the following assumptions :

- ```
-- grid of 936 nodes
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### Influence of the Inner Surface Conductance

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-- thermal conductivities (,densities and specific heats) :
 outer leaf : k = 0.7 W/mK (ρ = 1600 kg/m3 c = 850 J/kgK)
 inner leaf : k = 0.4 W/mK (ρ = 1100 kg/m3 c = 850 J/kgK)
 insulation : k = .04 W/mK (ρ = 40 kg/m3 c = 840 J/kgK)
 concrete : k = 2.5 W/mK (ρ = 2300 kg/m3 c = 930 J/kgK)
-- steady-state boundary conditions : outside : θ = 0°C he = 23 W/m2K
 inside : θi = 20°C hi = 8 W/m2K
 adiabatic sections
```



The results are represented in Figure 9b. Minimal temperatures are 12.6°C on the column and 13.6°C in the three-dimensional corner. The assumed inner conductance had a value that is deducted for plane walls. Nevertheless, it is expected that the radiative and convective heat transfer will be lower. Using the iterative method mentioned above the surface conductances were calculated under the following assumptions :

- thermal conductivities and outside boundary conditions as above.
- temperature of the surrounding walls of a fictive room: 20°C
- emissivity of all surfaces: 0.9
- air temperature: 20°C
- convective conductance for all surfaces:  $h_c = 3 \text{ W/m}^2\text{K}$ ; so only the course of the radiative part is calculated.

The results of this second calculation are shown in Figure 9c. The temperature on the column and in the corner amount to 11.7°C and 12.6°C, respectively. The calculated surface conductances on these places are, respectively, 7 W/m<sup>2</sup>K and 4.5 W/m<sup>2</sup>K. An other example of this simulation procedure with an experimental verification is given in Knapen and Standaert (1985). It can be concluded that the supplementary decrease of surface temperatures obtained by this more complete calculation procedure is only small, which means that a calculation with a constant (eventually smaller) surface conductance is satisfactory in practice.

### Transient Surface Condensation

The same thermal bridge is considered as in the previous case study. The evolution of the temperature field was calculated under the following assumptions :

- thermal properties listed above
- surface conductances :  $h_i = 8 \text{ W/m}^2\text{K}$      $h_e = 23 \text{ W/m}^2\text{K}$
- ambient temperatures : outside  $\theta^e = 0^\circ\text{C}$  (constant)
- inside  $\theta_i^e = 10^\circ\text{C}$     for  $t \leq 0 \text{ s}$
- $= 10 + 10x(1 + \frac{t}{3600})^\circ\text{C}$     for  $0 < t \leq 3600 \text{ s}$
- $= 20^\circ\text{C}$     for  $t > 3600 \text{ s}$
- (simulation of the heating of a room)

- time step : 300 s

The results are represented in the following figures :

- Figure 10a and b : isothermals at  $t = 0 \text{ s}$  and  $t = 3 \text{ h}$
- (Figure 9a : isothermals at  $t = \infty$ )
- Figure 11a : temperatures versus time
- i : assumed inside temperature
- e : assumed outside temperature
- 1 : temperature in point 1 (cavity wall)
- 2 : temperature in point 2 (concrete column)
- 3 : temperature in point 3 (three-dimensional corner)
- 4 : temperature in point 4 (external corner)
- d : dew point in the room under the following assumptions :
- humidity of the outside air : 3 g/m<sup>3</sup>
- ventilation rate : 1 /h
- specific volume of the air : 0.84 m<sup>3</sup>/kg
- inside vapor production : 3 g/m<sup>3</sup> for  $t \leq 0 \text{ s}$
- 6 g/m<sup>3</sup> for  $t > 0 \text{ s}$
- no condensation or hygroscopicity considered

These results can be interpreted and concluded as following :

- The temperature in the three-dimensional corner (point 3) evolves slowly, not only compared with the one-dimensional surface temperature (point 1), but also compared with the column temperature (point 2), which is lower in steady-state but exceeds the corner temperature during an important period. This is so because of the small contact of the three-dimensional corner with the inner environment as compared with the surrounding cold and capacitive material. On the contrary, the temperature in point 4, e.g., evolves faster because of the relatively important contact with the inside. The difference in evolution rate is illustrated too by the different shape and the different gradients of the isothermals at  $t = 0 \text{ h}$  against at  $t = 3 \text{ h}$ .
- The dew point evolution exceeds the temperature evolutions in points 2 and 3 during a considerable period. Though there is no condensation in steady state, it will occur during that period, but only in a small zone near the corners. Small mold growth patterns were observed in case studies, as shown for example in Figure 11b. This calculation shows that, due to special transient conditions, surface condensation can occur in corners, though it is not expected from a steady-state analysis.

### Transient Heat Losses

The same case study as above is considered. The heat losses through the inner surfaces (a : cavity wall, b : concrete column, c: concrete floor, d : side of the beam, e : lower side of the beam) were calculated. The results are represented in Figure 12. It shows that the thermal bridge effect is important : though the floor slab is an inner wall, its heat loss (going outwards) is as large as the heat loss through the cavity wall. Nevertheless, there is no pronounced transient thermal bridge effect on the heat losses. On the other hand the results lead to the question as to how far one-dimensional heat loss calculations can be used for constructions with complex or differently shaped outside and inside surfaces.

### CONCLUSION

This paper shows the importance of thermal bridges and their effect on the condensation risk, on the one hand, and on the heat losses, on the other. If constructions are insulated, the heat loss through thermal bridges becomes rather important, while the inner surface temperatures will rise only in the case of outer insulation; cavity insulation and inner insulation normally result in the same surface temperatures as obtained without insulation. All this is shown by means of a two-dimensional steady-state analysis. Further, it is illustrated that especially three-dimensional corners are sensitive to condensation. By calculating the specific radiative and convective heat transfer, it is possible to obtain a better approximation of reality, though calculations with constant surface conductances suffice in practice. Though no condensation is expected from a steady-state analysis, it can occur due to special transient conditions. The principal conclusion is that thermal bridges must be avoided. For the improvement of existing buildings, control of surface temperatures and heat losses is recommended if there are critical points; mostly a two-dimensional steady-state calculation will suffice.

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### ACKNOWLEDGMENTS

This research was supported by the Belgian Ministry of Science in the framework of the National Program of Energy, and by the Catholic University of Leuven.

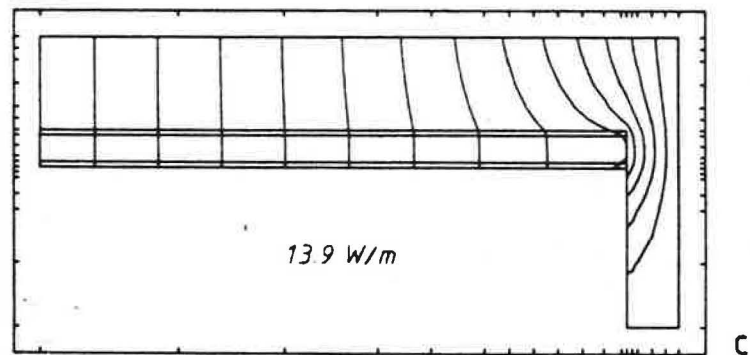
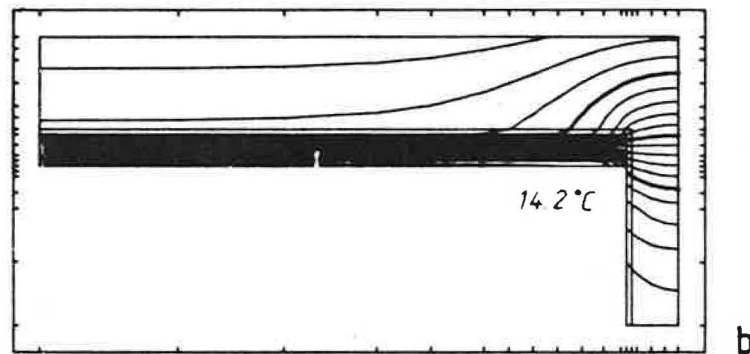
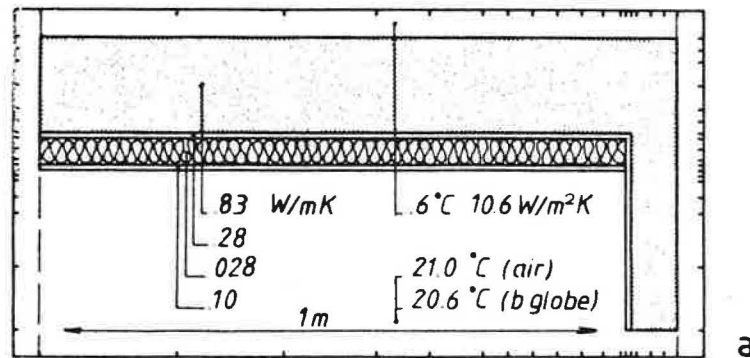


Figure 1. Horizontal section of brickwork wall, insulated inside.  
(a) Geometry, thermal conductivities and boundary conditions,  
(b) isotherms, (c) streamlines

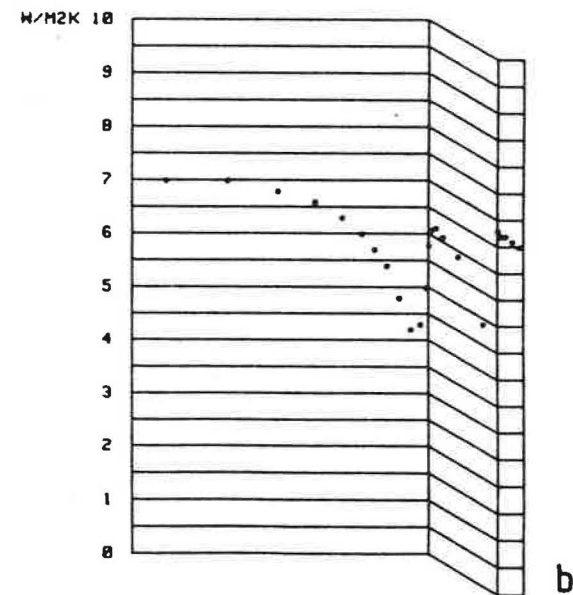
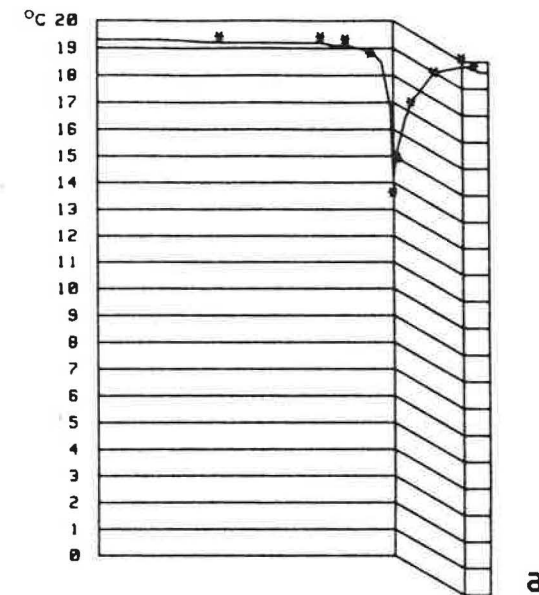


Figure 2. (a) Calculated vs. measured temperatures on the inner wall side; (b) calculated inner surface conductances



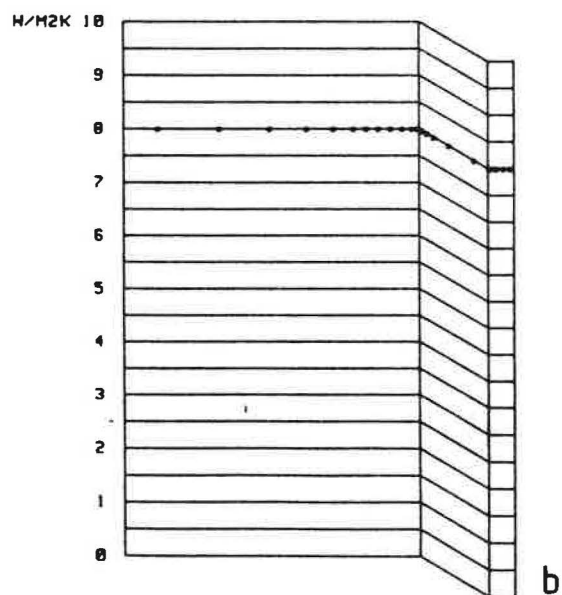
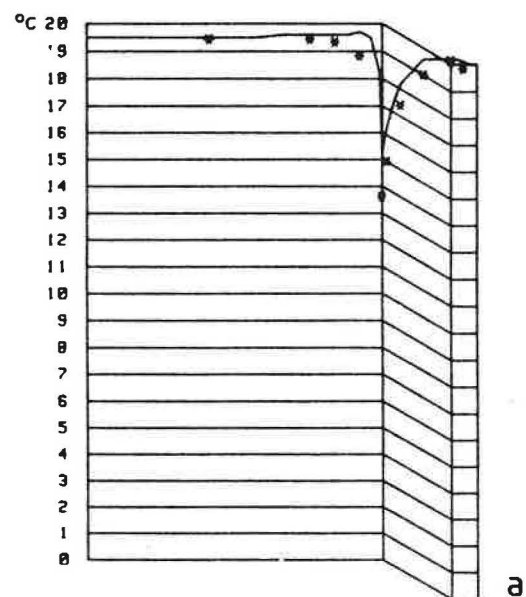


Figure 3. (a) Calculated vs. measured temperatures on the inner wall side; (b) assumed constant inner surface conductance

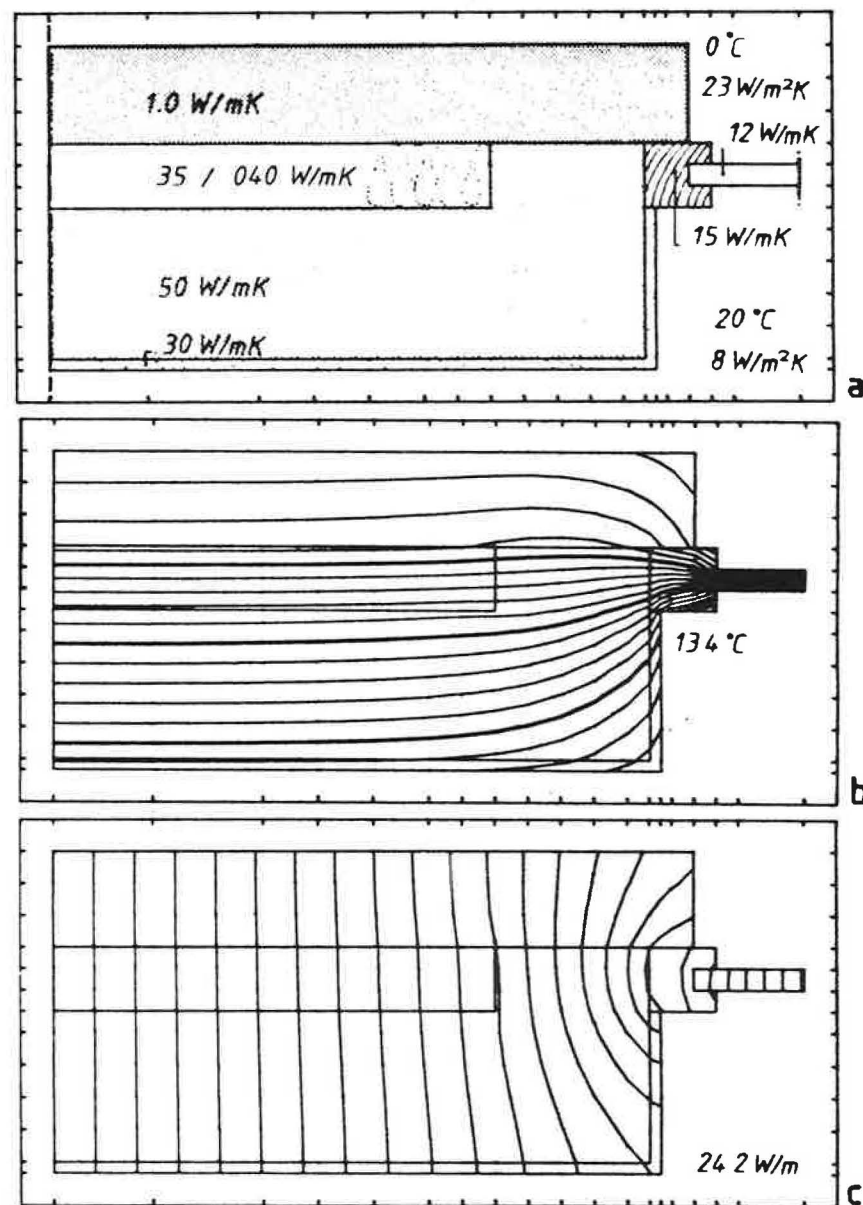


Figure 4. Horizontal section of connection between window and cavity wall. (a) Geometry, thermal conductivities and boundary conditions, (b) isotherms, (c) streamlines

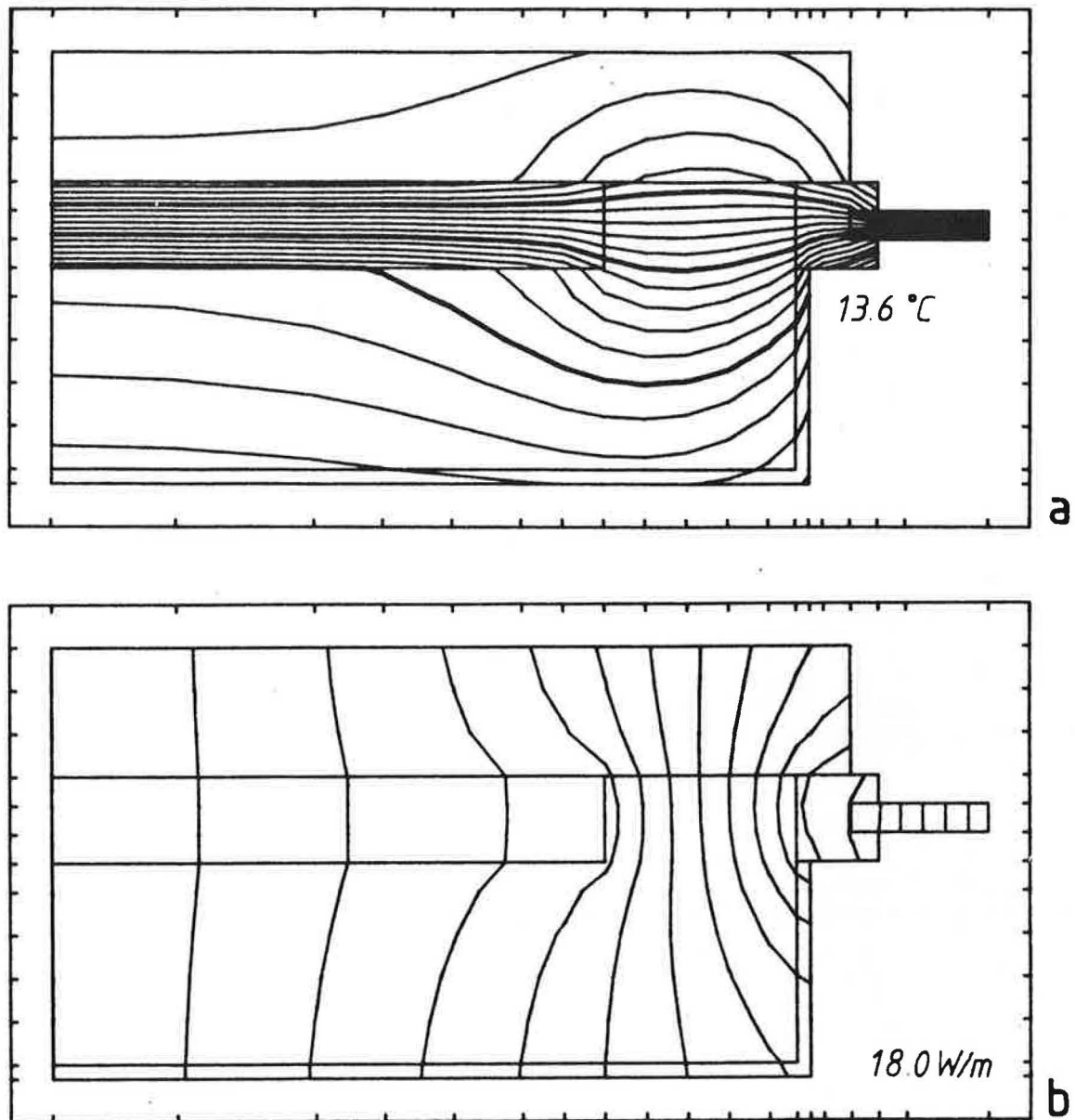


Figure 5. Horizontal section of connection between window and insulated cavity wall. (a) Isotherms, (b) streamlines

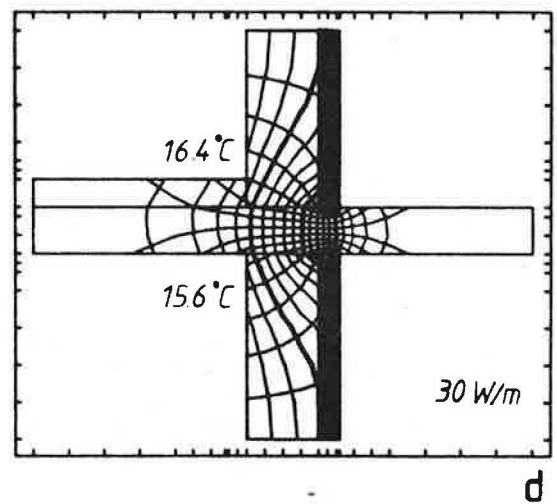
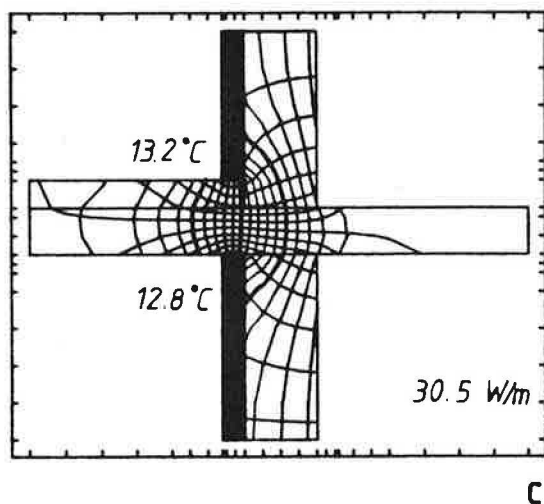
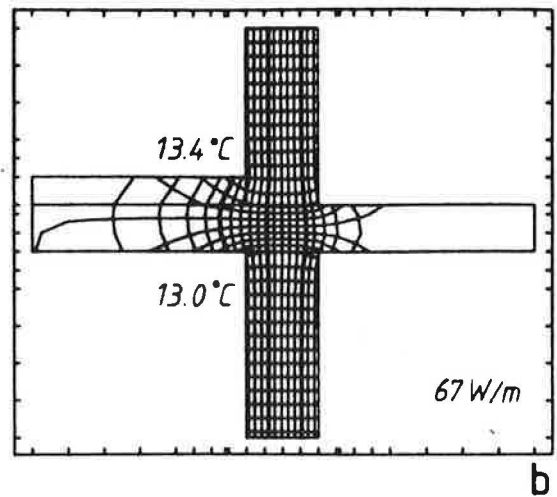
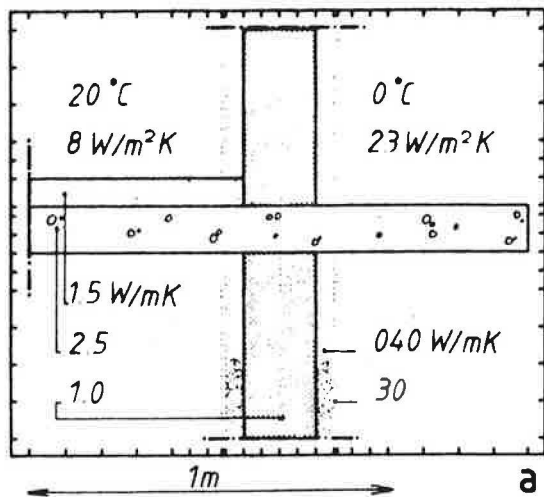


Figure 6. Vertical section of concrete slab penetrating outer wall. (a) Geometry, thermal conductivities and boundary conditions; and isotherms and streamlines with (b) no insulation, (c) inner insulation, and (d) outer insulation

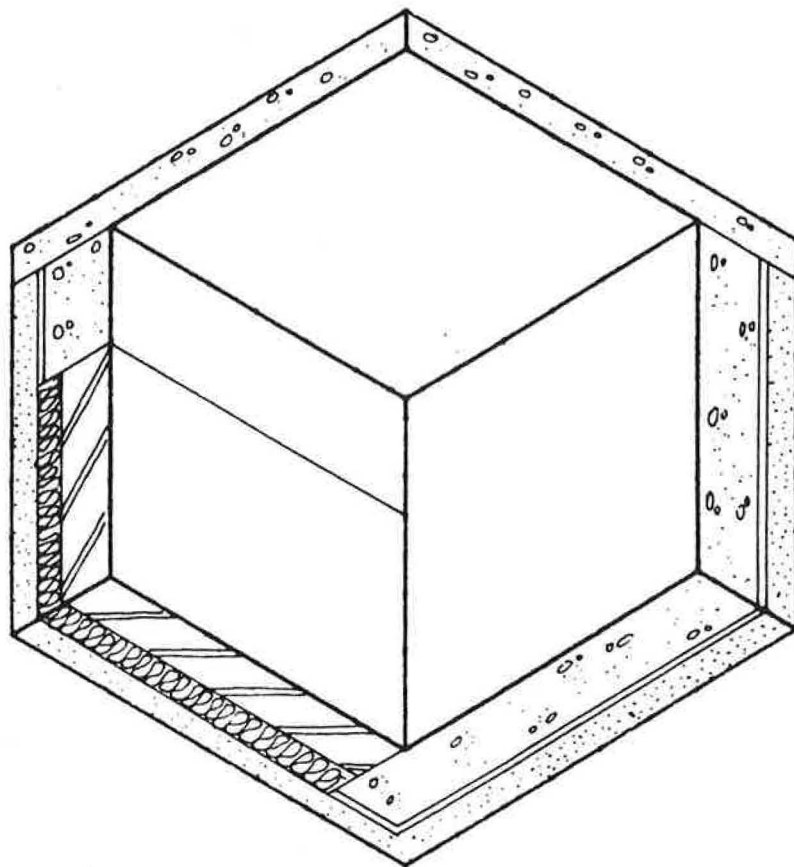
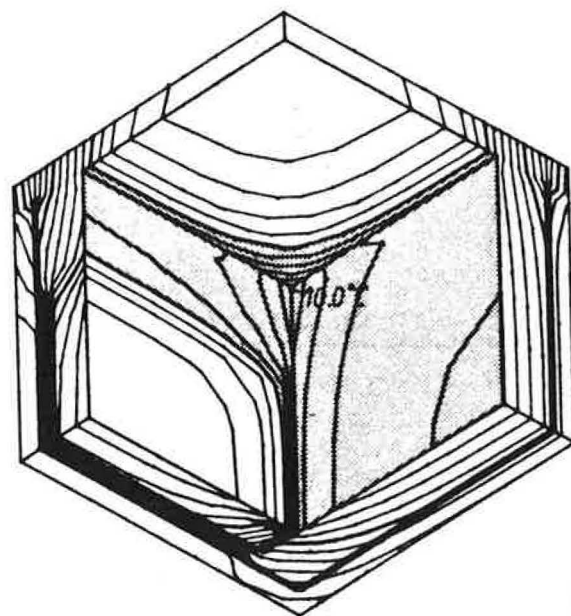


Figure 7. Axonometry of three-dimensional corner



a



b

Figure 8. (a) Mould growth in three-dimensional corner; (b) calculated isothermals

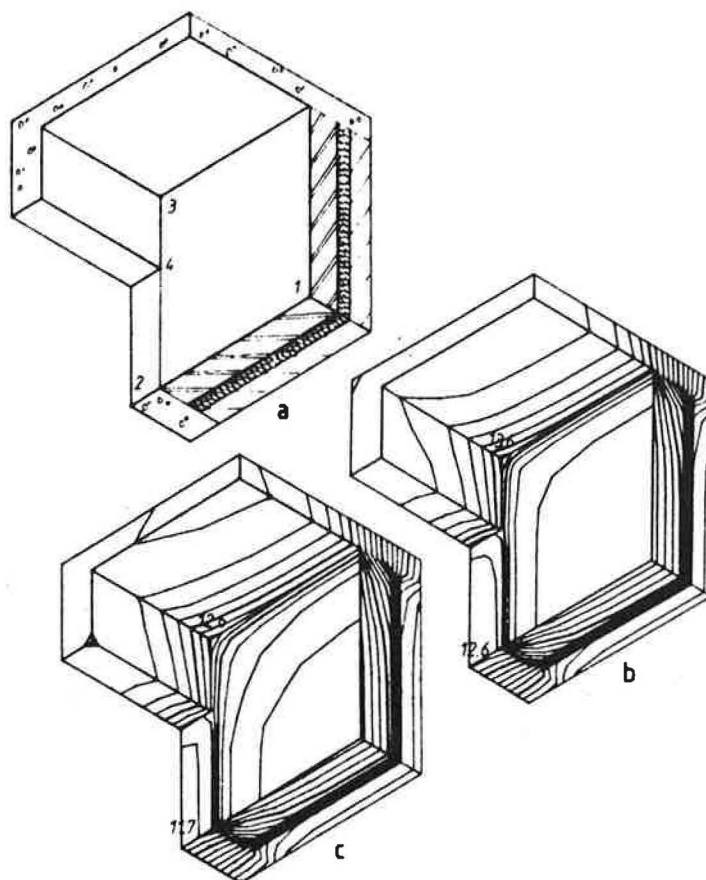


Figure 9. (a) Axonometry of three-dimensional thermal bridge; (b) calculated isothermals (assumed inner conductance); (c) calculated isothermals (calculated inner conductance)

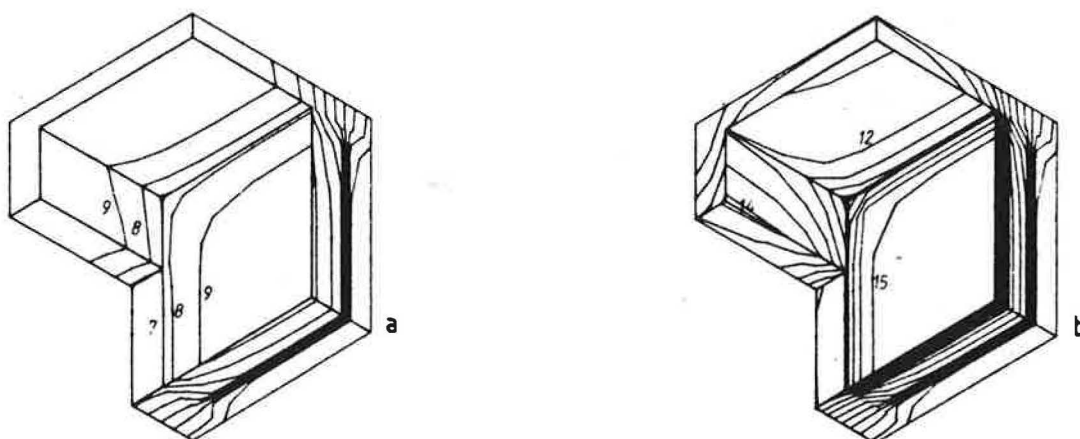


Figure 10. Calculated isothermals in transient state (a)  $t=0$  h, (b)  $t=3$  h



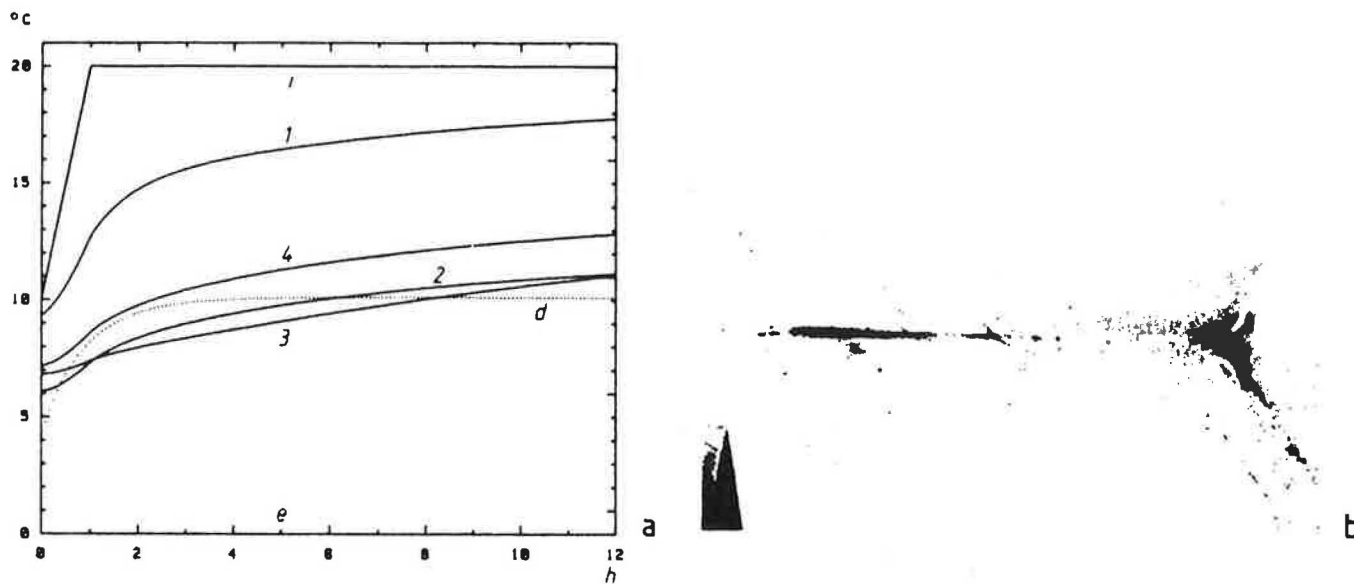


Figure 11. (a) Evolution of assumed and calculated temperatures; (b) mould growth pattern in corner

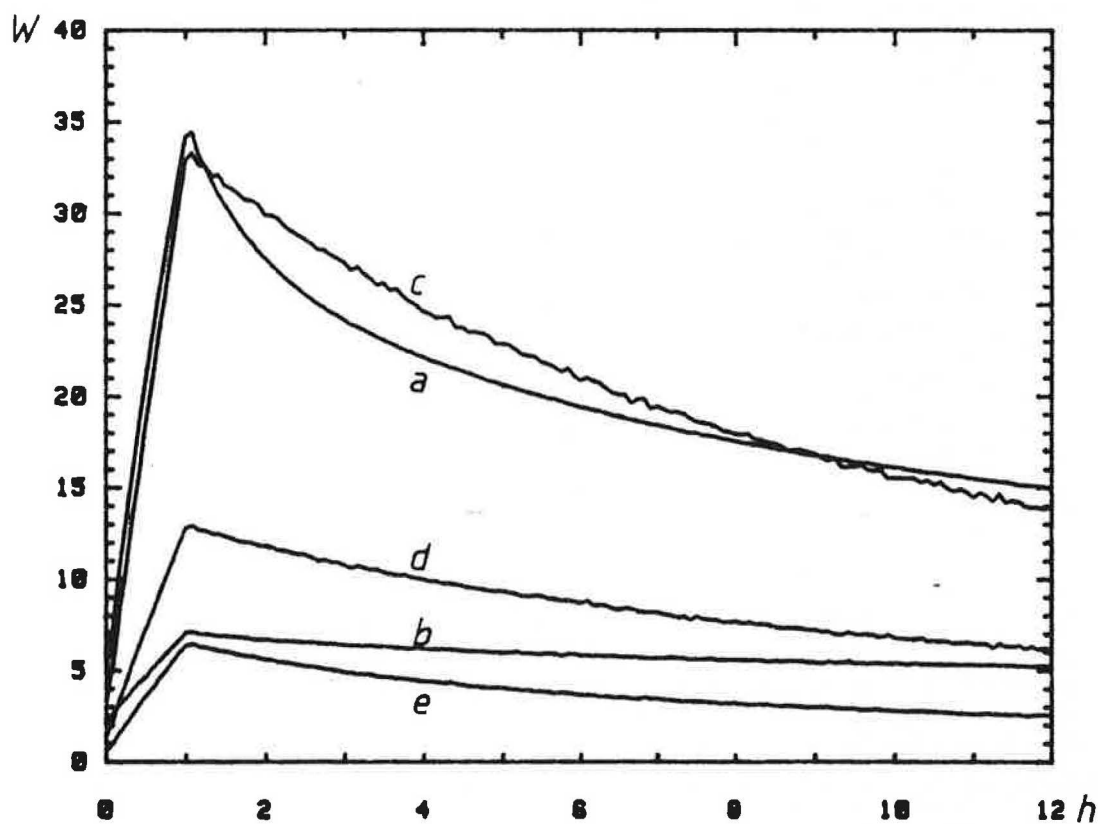


Figure 12. Evolution of heat loss through inner surfaces