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Technical Note

Summary The multiple tracer gas technique developed at UMIST has been improved so that interconnecting airflows and cell air change rates can be calculated in up to three connected cells of a building. This paper describes the mathematical analysis required to calculate intercell airflows from site measurements of tracer gas concentrations varying with time. Several results of three-cell ventilation and air movement studies in a dwelling are given, thus illustrating the usefulness of the measurement and analysis technique described.

Airflow measurement between three connected cells

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List of symbols

N_x	ventilation rate in cell x (air changes per hour (ac h^{-1}))
Q_{xo}	Amount of air flowing from cell x to outside (m^3 h^{-1})
Q_{ox}	Amount of air flow from outside to cell x (m^3 h^{-1})
Q_{xy}	Airflow from cell x to cell y (m^3 h^{-1})
C_{Ax}	Concentration of tracer gas A in cell x (ppm)
C_{0Ax}	Concentration of tracer gas A in cell x at time $t = 0$ (ppm)
t	Time (h)

1 Introduction

The measurement and analysis of ventilation rates and internal airflows between connected cells of buildings have received considerable attention in recent years$^{(1, 2)}$.

The multiple tracer gas technique developed at UMIST to measure air movements between spaces is well documented$^{(3, 4)}$. This paper is concerned with the analysis and measurement of two-directional airflows between three connected cells. The method of analysing two-directional airflows between two connected cells has been given elsewhere$^{(5)}$. Unlike existing techniques which rely on pairs of tracer gas concentration/time points, the following technique uses all data points collected during an experiment.

2 Analysis of two-directional airflows between three connected cells using a simplified analytical solution

The conservation of mass of tracer gas equations describing the variations in tracer gas concentrations between three connected cells are derived in the Appendix.

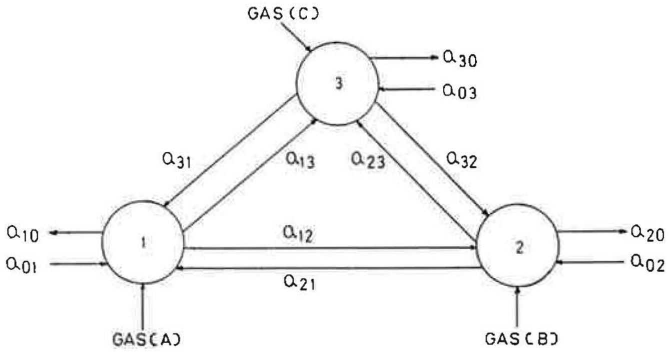

Figure 1 Airflows between three connected cells

Referring to Figure 1 using three different tracer gases, where tracer gas A is released in cell 1, tracer gas B is released in cell 2 and tracer gas C is released in cell 3, the concentration of tracer gas A in cell 1, C_{A1} at time t is given by

$$\begin{aligned}C_{A1} &= C_{0A1}\exp(-N_1t) + \frac{Q_{21}C_{0A2}}{V_1(N_1 - N_2)}\\ &\times(\exp(-N_2t) - \exp(-N_1t))\\ &+ \frac{Q_{31}C_{0A3}}{V_1(N_1 - N_3)}(\exp(-N_3t) - \exp(-N_1t))\\ &+ \frac{Q_{21}Q_{12}C_{0A1}}{V_1V_2(N_2 - N_1)}\left(\frac{\exp(-N_2t) - \exp(-N_1t)}{N_2 - N_1}\right.\\ &\left.+ t\exp(-N_1t)\right) + \frac{Q_{31}Q_{13}C_{0A1}}{V_1V_3(N_3 - N_1)}\\ &\times\left(\frac{\exp(-N_3t) - \exp(-N_1\mathrm{t})}{N_3 - N_1} + t\exp(-N_1t)\right) \qquad (1)\end{aligned}$$

The concentration of tracer gas A in cell 2 at time t, C_{A2}, is given by

$$\begin{aligned}C_{A2} &= C_{0A2}\exp(-N_2t) + \frac{Q_{21}C_{0A1}}{V_2(N_2 - N_1)}(\exp(-N_1t)\\ &- \exp(-N_2t)) + \frac{Q_{32}\mathrm{C}_{0A3}}{V_2(N_2 - N_3)}(\exp(-N_3t)\\ &- \exp(-N_2t)) + \frac{Q_{12}Q_{21}C_{0A2}}{V_1V_2(N_1 - N_2)}\\ &\times\left(t\exp(-N_2t) + \frac{\exp(-N_2t) - \exp(-N_1t)}{N_2 - N_1}\right)\\ &+ \frac{Q_{12}Q_{31}C_{0A3}}{V_2V_1(N_1 - N_3)}\left(\frac{\exp(-N_3t) - \exp(-N_2t)}{N_2 - N_3}\right.\\ &\left.+ \frac{\exp(-N_2t) - \exp(-N_1t)}{N_2 - N_1}\right) + \frac{Q_{32}Q_{13}\mathrm{C}_{0A1}}{V_2V_3(N_3 - V_1)}\end{aligned}$$

$$
\begin{aligned}
&\times \left(\frac{\exp(-N_1 t) - \exp(-N_2 t)}{N_2 - N_1}\right.\\
&\left.+ \frac{\exp(-N_2 t) - \exp(-N_3 t)}{N_2 - N_3}\right)
\end{aligned}
\tag{2}
$$

Equation (2) has been rounded to the first six terms.

The concentration of tracer gas A in cell 3 at time t, C_{A3}, is given by

$$
\begin{aligned}
C_{A3} = {}& C_{0A3}\exp(-N_3 t) + \frac{Q_{13}C_{0A1}}{V_3(N_3 - N_1)}\\
&\times (\exp(-N_1 t) - \exp(-N_3 t))\\
&+ \frac{Q_{23}C_{0A2}}{V_3(N_3 - N_2)}(\exp(-N_2 t) - \exp(-N_3 t))\\
&+ \frac{Q_{13}Q_{21}C_{0A2}}{V_1V_3(N_1 - N_2)}\left(\frac{\exp(-N_2 t) - \exp(-N_3 t)}{N_3 - N_2}\right.\\
&\left.+ \frac{\exp(-N_3 t) - \exp(-N_1 t)}{N_3 - N_2}\right) + \frac{Q_{13}Q_{31}C_{0A3}}{V_3V_1(N_1 - N_3)}\\
&\times \left(\frac{\exp(-N_3 t) - \exp(-N_1 t)}{N_3 - N_1} + t\exp(-N_3 t)\right)\\
&+ \frac{Q_{23}Q_{12}C_{0A1}}{V_3V_2(N_2 - N_1)}\left(\frac{\exp(-N_1 t) - \exp(-N_3 t)}{N_3 - N_1}\right.\\
&\left.+ \frac{\exp(-N_3 t) - \exp(-N_2 t)}{N_3 - N_2}\right) + \frac{Q_{23}Q_{32}C_{0A3}}{V_3V_2(N_2 - N_3)}\\
&\times \left(t\exp(-N_3 t) + \frac{\exp(-N_3 t) - \exp(-N_2 t)}{N_3 - N_2}\right)
\end{aligned}
\tag{3}
$$

Equation 3 has been rounded to the first seven terms. The full solutions to equations 2 and 3 are given elsewhere$^{(6)}$. The same derivation can be applied to tracer gas B, released in cell 2 and tracer gas C, released in cell 3 at the same time.

Figure 2 shows the shape of equations 1, 2 and 3. Comparison is made with the conservation of mass of tracer gas equations for known airflows and initial tracer gas concentrations (see Appendix).

The solution technique given in the Appendix relies on the assumption of good mixing between air and tracer gas in each cell. It further relies on the effects of recirculation of tracer gas between connected cells being time dependent.

For a typical three-cell case, recirculation of tracer gas between cells may cause errors in calculated airflows for time $t > 20$ min.

If recirculation of tracer gas becomes significant, then it is reflected when comparison is made with the time variations of tracer gas concentrations described by equations 1 to 3 inclusive and the conservation of mass of tracer gas equations in the Appendix.

3 Experimental

The multiple tracer gas measuring technique used to measure the concentrations of three Freon tracer gases has been fully described elsewhere$^{(5, 6)}$.

Briefly, the system consists of an AI505 portable gas chromatograph, fitted with two gas chromatographic separation columns of identical packing and length, attached in parallel to one gas chromatograph unit fitted with an electron capture detector.

Sampling valves are arranged so that air/tracer gas samples passing through each column can be passed alternately to the electron capture detector. A rapid sampling rate is achieved by virtue of the fact that the 'dead time' associated with waiting for a sample to pass through a single column is eliminated.

For three cell measurements, three Freon tracer gases are used: Freon 12, Freon 114 and BCF (Bromochlorodifluoromethane).

The sampling interval is currently 30 s for three tracer gases. The minimum number of tracer gas concentration/time points collected is 10 points per cell per test. The

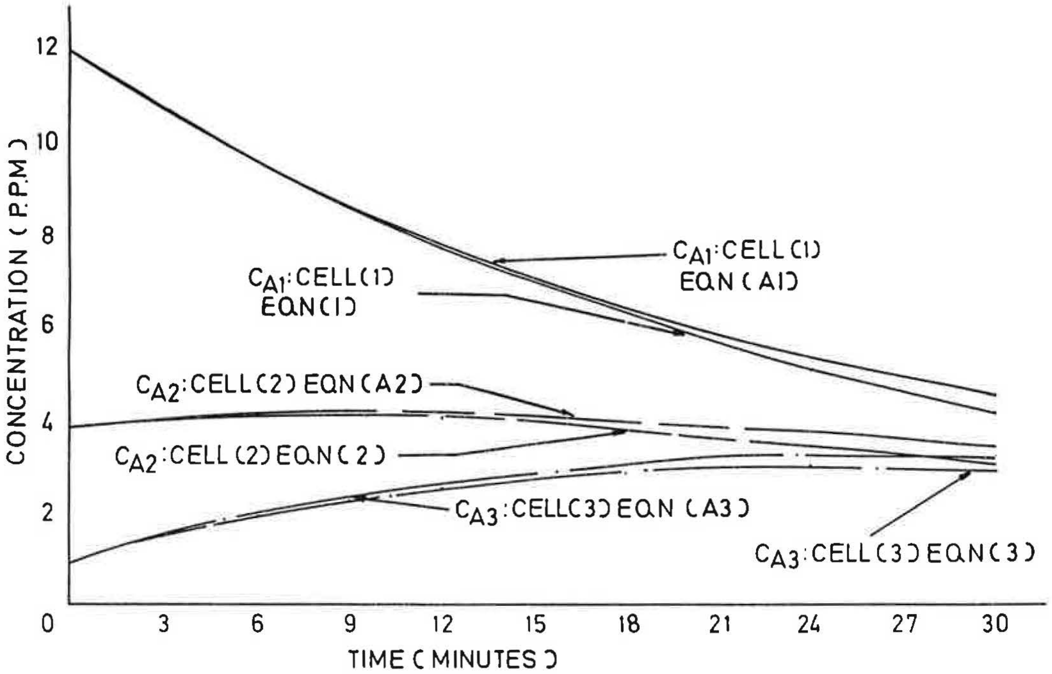

Figure 2 $C_{A(t)}$ variations in cells 1, 2 and 3

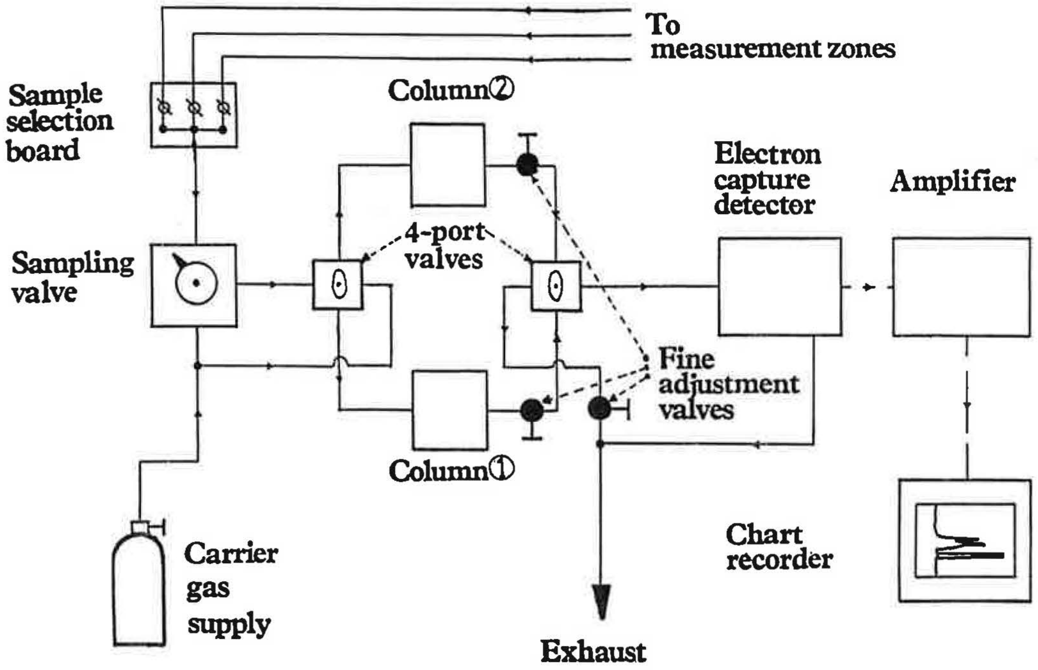

Figure 3 Measuring system

minimum time needed to obtain sufficient data points is 20 min (includes 5 min tracer gas mixing time).

A schematic diagram of the measuring system is shown in Figure 3. Validation of the technique under controlled laboratory conditions suggests errors between ± 20% on airflow rates and air change rates calculated from tracer gas concentration measurements obtained with this system(5). It should be noted that the test conditions used were made artificially severe in order to see if the mathematical analysis would fail. Under more realistic site conditions, it is anticipated that the magnitude of errors would be substantially less, probably of the order of ±10–12%.

4 Results and discussion

Table 1 shows a typical selection of results for a variety of test conditions within a single dwelling. The six results in Table 1 show the flow of air between the upstairs, downstairs and roofspace of the dwelling. Freon 114 was released downstairs, Freon 12 upstairs and BCF in the roof space. Figure 4 shows the decay of Freon 114 downstairs and its growth upstairs; Figure 5 shows the decay of Freon 12 upstairs and its growth downstairs and in the roofspace. Figure 6 shows the decay of BCF in the roofspace. The analysis of this data was made using the curve fitting technique described in the Appendix.

For the house studied downstairs to upstairs airflows were approximately 110–220 $m^3 h^{-1}$. Airflows from upstairs to downstairs were in the range 160–300 $m^3 h^{-1}$, with a one-directional airflow between house and roofspace of 28–40 $m^3 h^{-1}$. Switching a kitchen extract fan on reduces ground floor to first floor airflows by approximately 85%, the one-directional airflow between house and roofspace also being reduced by about 50%.

The measurement and analysis of airflows and ventilation rates between three connected cells where all six intercell airflows are non-zero values have been carried out using the techniques described here(6).

5 Conclusions

The simplified analytical solution of tracer gas concentration data presented here enables estimates of air change rates

Table 1 Summary of three-cell site measurements

Test no.	Cell (1) Vent. rate (ac h^{-1})	Cell (2) Vent. rate (ac h^{-1})	Cell (3) Vent. rate (ac h^{-1})	Q_{12} ($m^3 h^{-1}$)	Q_{21} ($m^3 h^{-1}$)	Q_{13} ($m^3 h^{-1}$)	Q_{31} ($m^3 h^{-1}$)	Q_{23} ($m^3 h^{-1}$)	Q_{32} ($m^3 h^{-1}$)	Comments
1	0.8 ± 0.1	0.5 ± 0.03	5 ± 0.65	120 ± 18	160 ± 21	0	0	35 ± 4	0	Windows shut; all doors open
2	1.6 ± 0.18	0.67 ± 0.01	4.8 ± 0.28	10 ± 10	170 ± 27	0	0	18 ± 6	0	Kitchen fan on
3	1.8 ± 0.09	0.85 ± 0.09	2.9 ± 0.15	15 ± 5	115 ± 12	0	0	15 ± 1	0	Kitchen fan on
4	0.9 ± 0.04	0.35 ± 0.02	3.6 ± 0.16	310 ± 42	220 ± 25	0	0	42 ± 6	0	Kitchen fan off
5	2.2 ± 0.18	0.75 ± 0.1	5.2 ± 0.45	175 ± 25	221 ± 10	0	0	30 ± 10	0	Kitchen fan off; kitchen window open
6	0.62 ± 0.07	0.86 ± 0.1	8.8 ± 1.5	220 ± 21	306 ± 40	0	0	45 ± 15	0	All windows shut; kitchen fan on

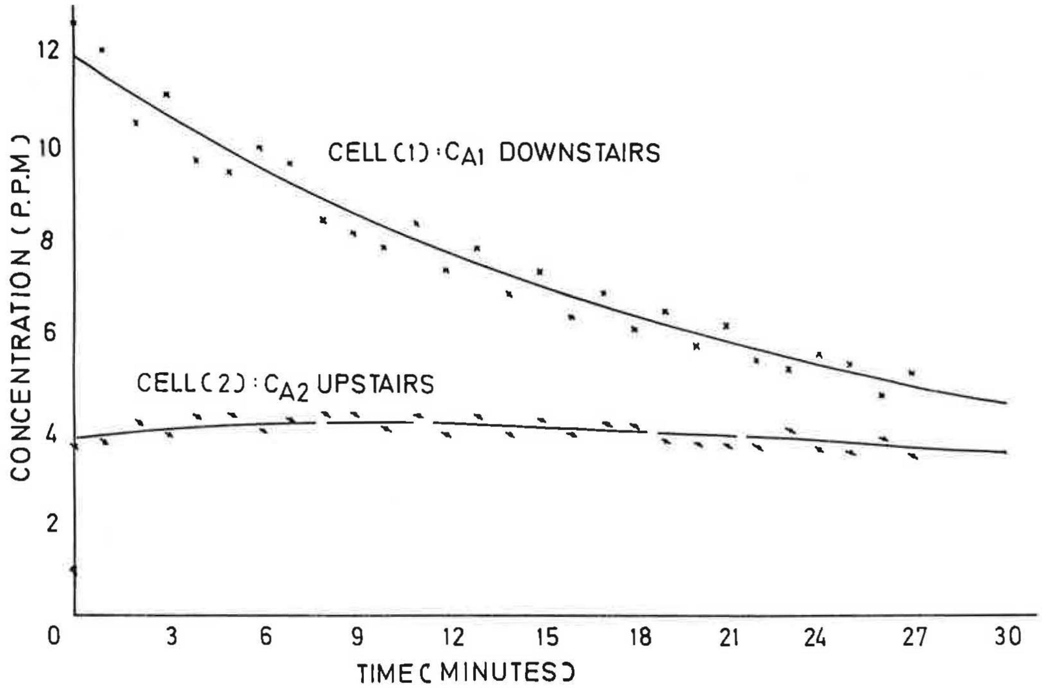

Figure 4 Freon 114 released downstairs

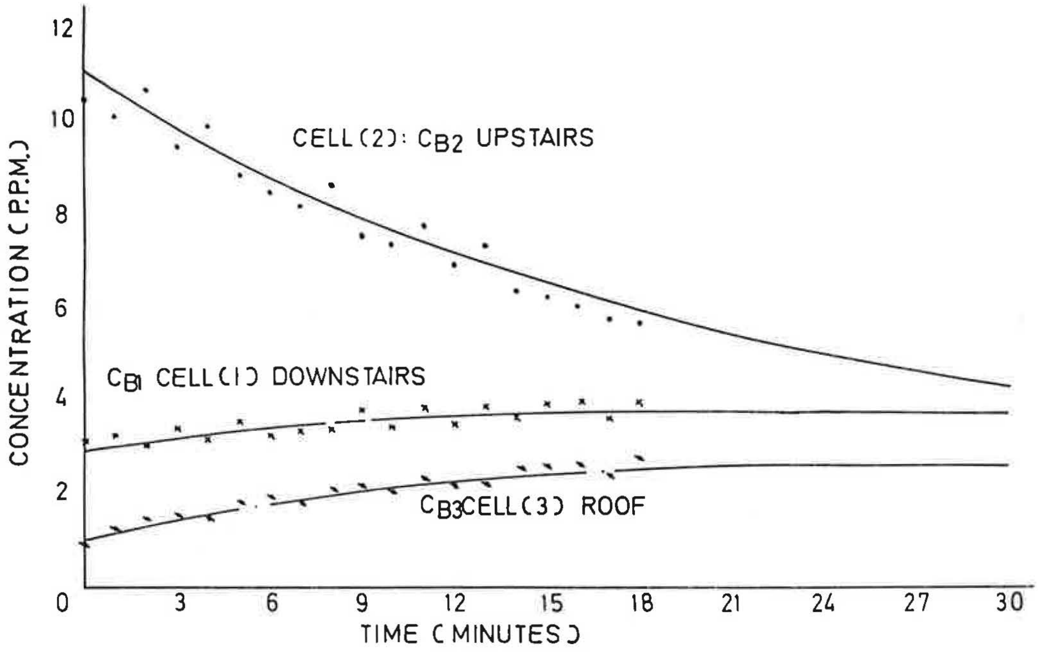

Figure 5 Freon 12 released upstairs

and intercell airflows to be calculated directly from site measurements in a short time period, typically 20 min, thus enabling errors due to variations in external wind conditions to be minimised. The usefulness of this technique for measuring air movements in buildings has been illustrated by the site measurements summarised in Table 1. This technique is currently being used to study intercell airflows and ventilation rates in a variety of domestic and industrial premises.

Appendix: Analysis of two-directional airflows between three connected cells

Considering Figure 1, if tracer gas A is released in cell 1 and allowed to mix with the air, some of it will be carried to cell 2 and cell 3, where it will also mix with the air, after which some may be returned to cell 1. Let us define $t = 0$ (seconds) as the time when initial mixing is complete: the concentrations of tracer gas A in cells 1, 2 and 3 are given by C_{0A1}, C_{0A2} and C_{0A3} respectively. Now the rate at which tracer gas A is entering cell 1 is given by $Q_{21}C_{A2} + Q_{32}C_{A3}$ and the rate at which it is leaving by S_1C_{A1} ($S_1 = Q_{1o} + Q_{13} + Q_{12}$): hence, the rate of decrease of volume of tracer gas A in cell 1 at time t is given by

$$-V_1\frac{dC_{A1}}{dt} = S_1C_{A1} - Q_{21}C_{A2} - Q_{31}C_{A3} \qquad \text{(A1)}$$

Similarly, the rate of decrease of volume of tracer A in cell 2 is given by

$$-V_2\frac{dC_{A2}}{dt} = S_2C_{A2} - Q_{12}C_{A1} - Q_{32}C_{A3} \qquad \text{(A2)}$$

where $S_2 = Q_{20} + Q_{23} + Q_{21}$

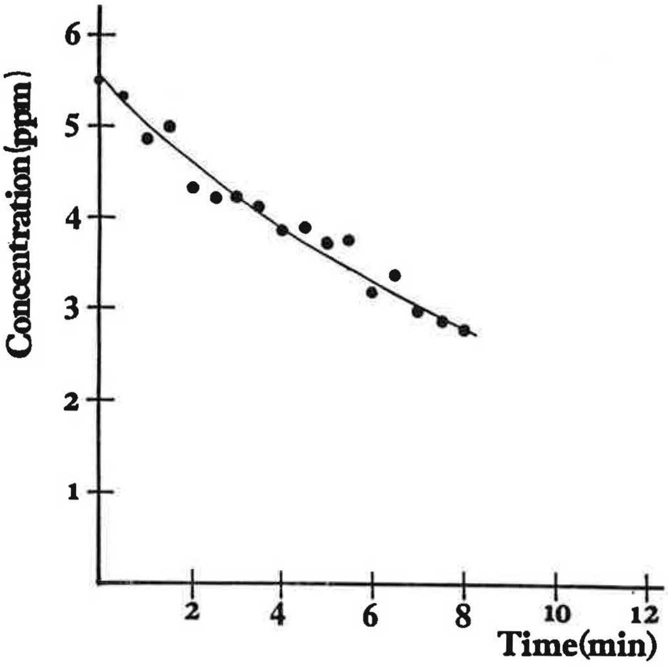

Figure 6 BCF released in roofspace

and in cell 3:

$$-V_3\frac{dC_{A3}}{dt} = S_3C_{A3} - Q_{13}C_{A1} - Q_{23}C_{A2} \qquad (A3)$$

where $S_3 = Q_{3o} + Q_{31} + Q_{32}$. Since $N_1 = S_1/V_1$, $N_2 = S_2/V_2$ and $N_3 = S_3/V_3$, substituting for N_1, N_2 and N_3 in equations A1, A2 and A3 and rearranging we obtain

$$\frac{dC_{A1}}{dt} + N_1C_{A1} = \frac{Q_{21}}{V_1}C_{A2} + \frac{Q_{31}}{V_1}C_{A3} \qquad (A4)$$

$$\frac{dC_{A2}}{dt} + N_2C_{A2} = \frac{Q_{12}}{V_2}C_{A1} + \frac{Q_{32}}{V_2}C_{A3} \qquad (A5)$$

$$\frac{dC_{A3}}{dt} + N_3C_{A3} = \frac{Q_{13}}{V_3}C_{A1} + \frac{Q_{23}}{V_3}C_{A2} \qquad (A6)$$

Equations A4, A5 and A6 are first-order differential equations which can be solved using an integrating factor. The integrating factor for equation A4 is $\exp(N_1t)$:

$$C_{A1}\exp(N_1t) = \int \frac{Q_{21}C_{A2}}{V_1}\exp(N_1t)\,dt + \int \frac{Q_{31}C_{A3}}{V_1}\exp(N_1t)\,dt + A \qquad (A7)$$

Using $\exp(N_2t)$ as the integrating factor for equation (A5):

$$C_{A2}\exp(N_2t) = \int \frac{Q_{12}C_{A1}}{V_2}\exp(N_2t)\,dt + \int \frac{Q_{23}C_{A3}}{V_2}\exp(N_2t)\,dt + B \qquad (A8)$$

and using $\exp(N_3t)$ as the integrating factor for equation (A6):

$$C_{A3}\exp(N_3t) = \int \frac{Q_{13}C_{A1}}{V_2}\exp(N_3t)\,dt + \int \frac{Q_{23}C_{A2}}{V_3}\exp(N_3t) + D \qquad (A9)$$

For equation A7 to be solved, the variation with time of C_{A2} and C_{A3} must be known or approximated. If it is assumed that, initially, no recirculation of tracer gas A occurs (i.e. $Q_{21} = 0$ and $Q_{31} = 0$) then, from Dick's equations(8), C_{A2} is given by

$$C_{A2} = C_{0A2}\exp(-N_2t) + \frac{Q_{12}C_{0A1}}{V_3(N_3 - N_1)} \times (\exp(-N_1t) - \exp(-N_2t)) \qquad (A10)$$

and C_{A3} is given by

$$C_{A3} = C_{0A3}\exp(-N_3t) + \frac{Q_{13}C_{0A1}}{V_3(N_3 - N_1)} \times (\exp(-N_1t) - \exp(-N_3t)) \qquad (A11)$$

Substituting for C_{A2} and C_{A3} in (A7) and solving:

$$\begin{aligned} C_{A1} = {} & C_{0A1}\exp(-N_1t) + \frac{Q_{21}C_{0A2}}{V_1(N_1 - N_2)} \\ & \times (\exp(-N_2t) - \exp(-N_1t)) \\ & + \frac{Q_{31}C_{0A3}}{V_1(N_1 - N_3)}(\exp(-N_3t) - \exp(-N_1t)) \\ & + \frac{Q_{21}Q_{12}C_{0A1}}{V_1V_2(N_2 - N_1)}\left(\frac{\exp(-N_2t) - \exp(-N_1t)}{N_2 - N_1}\right. \\ & \left. + t\exp(-N_1t)\right) \\ & + \frac{Q_{31}Q_{13}C_{0A1}}{V_1V_3(N_3 - N_1)}\left(\frac{\exp(-N_3t) - \exp(-N_1t)}{N_3 - N_1}\right. \\ & \left. + t\exp(-N_1t)\right) \end{aligned} \qquad (A12)$$

Similarly, equation (A8) can be solved by substituting for C_{A1} from equation A12 and C_{A3} from equation A11.

The same derivation can be applied to tracer gas B released in cell 2, and to tracer gas C released in cell 3 at the same time.

The derivation of these equations is given in full elsewhere(4).

If the concentrations of tracer gas A are monitored in cells 1, 2 and 3, then C_{A1}, C_{A2}, C_{A3}, t, V_1, V_2, V_3 are known, leaving N_1, N_2, N_3, Q_{12}, Q_{13}, Q_{32}, Q_{23}, Q_{21}, Q_{31}, C_{0A1}, C_{0A2} and C_{0A3} as unknowns. By using the following curve fitting technique on sets of experimental values of C_{A1}, C_{A2}, C_{A3} and t, these unknowns can be calculated.

The technique discussed here assumes the effects of recirculation of tracer gas between connected cells is time dependent, the contribution of recirculated tracer gas being small for time $t < 20$ min. Consequently equations 1, 2 and 3 (given in Section 2) for $C(t)$ can be simplified to:

Considering Gas A in Cell 1:

$$\bar{C}_{A1} = C_{0A1}(1 - AN_1) + \frac{Q_{21}C_{0A2}}{V_1} \times \left(\frac{\exp(-N_2't) - \exp(-N_1't)}{N_1' - N_2'}\right) + \frac{Q_{31}C_{0A3}}{V_1} \times \left(\frac{\exp(-N_3't) - \exp(-N_1't)}{N_1' - N_3'}\right) \qquad (A13)$$

Gas A in Cell 2:

$$\tilde{C}_{A2} = C_{0A2}(1 - BN_2) + \frac{Q_{12}C_{0A1}}{V_2} \times \left(\frac{\exp(-N'_1 t) - \exp(-N'_2 t)}{N'_2 - N'_1}\right) + \frac{Q_{32}C_{0A3}}{V_2} \times \left(\frac{\exp(-N'_3 t) - \exp(-N'_2 t)}{N'_2 - N'_3}\right) \qquad \text{(A14)}$$

Gas A in Cell 3:

$$C_{A3} = C_{0A3}(1 - DN_3) + \frac{Q_{13}C_{0A1}}{V_3} \times \left(\frac{\exp(-N'_1 t) - \exp(-N'_3 t)}{N'_3 - N'_1}\right) + \frac{Q_{23}C_{0A2}}{V_3} \times \left(\frac{\exp(-N'_2 t) - \exp(-N'_3 t)}{N'_3 - N'_2}\right) \qquad \text{(A15)}$$

where

$$\tilde{C}(t) = \int_{t=0}^{t=k} C(t)\,\mathrm{d}t/{}_0\Delta t_k$$

The integral is evaluated using numerical integration of site $C(t)$ data points.

$(1 - AN_1)$ is a Maclaurin series expansion of $\exp(-N_1 t)$:

$$A = -t + \frac{N'_1 t^2}{2!} - \frac{N'^2_1 t^3}{3!} + \frac{N'^3_1 t^4}{4!} - \frac{N'^4_1 t^5}{5!}$$

$(1 - BN_2)$ is a Maclaurin expansion of $\exp(-N_2 t)$:

$$B = -t + \frac{N'_2 t^2}{2!} - \frac{N'^2_2 t^3}{3!} + \frac{N'^3_2 t^4}{4!} - \frac{N'^4_2 t^5}{5!}$$

$(1 - DN_3)$ is a Maclaurin expansion of $\exp(-N_3 t)$

$$D = -t + \frac{N'_3 t^2}{2!} - \frac{N'^2_3 t^3}{3!} + \frac{N'^3_3 t^4}{4!} - \frac{N'^4_3 t^5}{5!}$$

N'_1, N'_2, N'_3 are 'first order' estimates of N_1, N_2 and N_3, found from taking $\ln C(t)$, time t for C_{A1}, C_{B2} and C_{C3} data points.

The three equations given by tracer gas A are solved together with the three equations given by tracer gas B and a further three given by tracer gas C. Using numerical iteration techniques, values for the unknown airflows and air change rates are calculated.

The calculated values of intercell flows and cell air change rates can be used in equations A1, A2 and A3 to enable comparison between theoretical curve shapes and site data points.

References

1 Perera M Review of techniques for measuring ventilation rates in multi-celled buildings, *Proc. European Community Contractors' Meeting on Natural Ventilation, Brussels* (September 1982)

2 Penman J M and Rashid A An experimental determination of ventilation rates in occupied rooms using carbon dioxide *Building Environ.* **15** (1980)

3 Irwin C, Edwards R E and Howarth A T An improved multiple tracer gas technique for calculating air movement in buildings *Air Infiltration Rev.* 5(2) (1984)

4 Edwards R E and Irwin C Multiple cell air movement measurements, ventilation strategies and measurement techniques *Proc. 6th Air Infiltration Centre Conf., The Netherlands* (September 1985)

5 Irwin C, Edwards R E and Howarth A T The measurement of airflows using a rapid response tracer gas technique *Building Serv. Eng. Res. Technol.*, **6**(4) (1985)

6 Irwin C *A method of measuring air movements in compartmentalised buildings* PhD thesis, University of Manchester (1985)

7 Irwin C and Edwards R E The validation of a multiple tracer gas technique for the determination of airflows between three interconnected cells *Air Infiltration Rev.* 7(4) (1986)

8 Dick J B Experimental studies of natural ventilation of houses *J. Inst. Heating Ventil. Eng.* **17**(X) 420 (1944)