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Heat Flow Rates and Temperature Distributions in Corners of External Walls with Non-Isothermal Surfaces

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The finite element method is employed to obtain rates of heat flow and temperature distributions in corners of insulated external walls with non-isothermal surfaces. A computer program is developed to solve steady-state heat flow problems in any two-dimensional body with various boundary conditions. The effects of positon, thickness and thermal conductivity of the insulation layer on the total rates of heat flow and temperature distributions are investigated. A quantity known as the insulation efficiency has been introduced as a measure of the percentage reduction in the total rate of heat flow due to the insulation.

NOMENCLATURE

I_R	Resultant heat flow intensity (W m^{-2})
I_{OR}	Resultant heat flow intensity at the corner (W m^{-2})
I_∞	Heat flow intensity at an infinite distance from the corner (W m^{-2})
Rsi	Inside surface resistance (m^2 K W^{-1})
Rso	Outside surface resistance (m^2 K W^{-1})
t	Thickness of insulation layer (m)
Δx	Length of interval along the surface (m)
λ	Thermal conductivity (W m^{-1} K^{-1})
λins.	Thermal conductivity of insulation (W m^{-1} K^{-1})
λcon.	Thermal conductivity of concrete (W m^{-1} K^{-1})
θ	Temperature (°C)
θai	Inside air temperature (°C)
θao	Outside air temperature (°C)
$\theta\infty$	Surface temperature at an infinite distance from the corner (°C)
η	Insulation efficiency (%)

1. INTRODUCTION

IN PREVIOUS papers [1, 2] investigations were reported of the temperatures and heat flow conditions in square corners with isothermal surfaces. The results were obtained by applying the Schwartz–Christoffel transformation technique, which can be used only if the surfaces are assumed to be isothermal. However, in practice this assumption is not valid: variations in temperature do exist along the surfaces where disturbances to the regular pattern of heat flow occur, such as at corners. In the presence of a corner the surface temperature will drop as the corner is approached, which causes the heat loss there to be greater. To limit the amount of heat being transferred from the warm inside to outside, composite or insulated corners are built.

In this work, the finite element method is employed to obtain the temperature distributions and total rates of heat flow in non-isothermal insulated corners. The objective of this paper is to demonstrate the effect of varying (a) position, (b) thickness, and (c) thermal conductivity of the insulation layer on the temperature distribution and rate of heat flow. It must be stressed that all the results for rate of heat flow are obtained for a unit length (1m) along the corner.

The finite element method allows the assumption of isothermal surface conditions to be relaxed as well as permitting the treatment of complicated geometries, composite structures and various boundary conditions, giving a closer approach to the real life situation.

2. DESCRIPTION OF THE CASES STUDIED

The study was conducted on 36 different cases. In all the cases the structure studied was a square corner between two walls, comprised of 200 mm of solid cast concrete plus a thickness of 12.5 to 50 mm of insulation. The varying parameters are:

(1) thermal conductivity of the insulation layer,
(2) thickness of the insulation layer, and
(3) position of the insulation layer.

All the thermal data were taken from the CIBSE Guide Section A3 [3]. The 200 mm corner wall was assumed to be cast concrete (dense) with a thermal conductivity of 1.4 W m^{-1} K^{-1} and density 2100 kg m^{-3}. Four different values of insulation thermal conductivity were taken, namely 0.035, 0.09, 0.28 and 0.35 W m^{-1} K^{-1} to cover a wide range of insulation materials. Three different typical thicknesses of insulant, namely 12.5, 25 and 50 mm were

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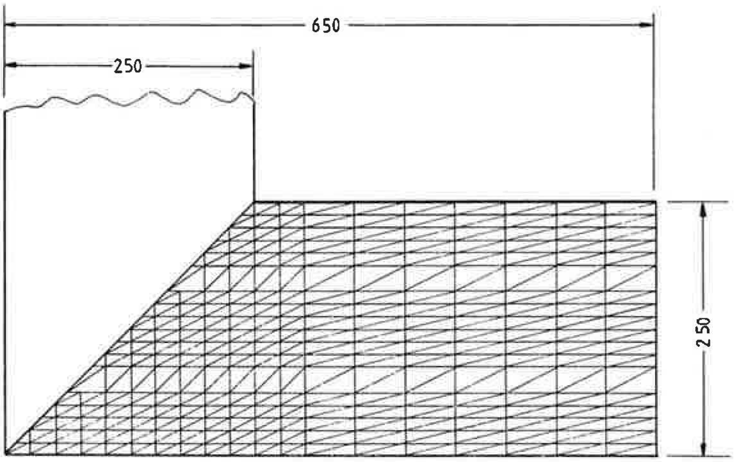

Fig. 1. The layout of the finite element model. A square corner in a 200 mm thick concrete wall is modelled, with an insulation layer up to 50 mm thick. The maximum number of elements is 512. The maximum number of nodes is 290. All dimensions are in mm.

used. The insulation positions considered were (a) inside surface, (b) middle (i.e. separating two walls each 100 mm thick) and (c) outer surface.

3. THE FINITE ELEMENT PROGRAM AND MODEL

A complete account of this method and the derivation of the finite element equations, the system equations and methods of inserting the boundary conditions is given in reference [4].

The computer program was developed to solve the Laplace equation in any two-dimensional section. The inputs to the program are (a) the geometry of the body, (b) the physical properties and (c) the boundary conditions. The outputs are (a) temperature distribution throughout the body and (b) the components of the temperature gradient of each element. The boundary conditions include:

(1) specified temperature at any selected node or nodes within the body;
(2) specified heat flux at any selected node or nodes within the body;
(3) convective heat transfer at exposed surfaces, and
(4) internal heat generation at any selected node or nodes.

The program is set up as a series of independent modules or subroutines linked by a small driving program. Each subroutine has a specific function such as data input, calculation of element equations and asssembly, solution of system equations and printing the results. The data for each case were stored in a data file, which was then attached to the master program when required. With this arrangement it was possible to compute more than one case with each program run.

A single finite element model was constructed to cater for all the cases studied. The thickness of the wall in the model varies between 200 and 250 mm, depending on the thickness of the insulation layer. The length of the uninsulated wall in the model measured from the inside corner is 2.25 times the wall thickness to ensure minimum temperature variations at the far end of the wall where the flow of heat is almost unidimensional. This was based on the results obtained by the present authors in previously reported studies [1, 2]. However, the total length of the wall measured from the inside corner decreases as the insulant thickness increases and reaches a value of 1.6 times the total wall thickness when the insulation thickness is 50 mm. Figure 1 shows the layout of the nodes for the finite element model that applies for all the cases. It can be seen that the model has different sizes of elements. A finer mesh was constructed at places where large variations in temperature were expected to occur, namely the area close to the bend (i.e. corner), insulation/concrete interfaces and air/wall surface interfaces. Larger elements are positioned in places where temperature variations are relatively small and they are always surrounded by other elements with similar thermal conductivity. The inside and outside air temperatures are assumed to be 20 and 0°C respectively. The inside and outside surface resistances are 0.12 and 0.06 $m^2\ K\ W^{-1}$ respectively.

4. RESULTS FOR THE TOTAL RATE OF HEAT FLOW

From the program the resultant heat flow intensity I_R was calculated at each element. Along the inside surface, the length of each element represents the length of an interval Δx. Therefore, the resultant intensity calculated for each surface element (the element that is in contact with the inside air) represents the mean resultant intensity for that interval. The heat flow at each interval was then obtained from

$$Q_{(\text{int})} = I_{R(\text{int})} \times \Delta x \times 1 \quad \text{W}$$

Therefore, the total rate of heat flow across the inner surface to a defined distance from the inner corner is the sum of all the rates of heat flow for all the intervals covering that distance. The extra heat flow intensity due to the presence of the corner should be zero at some defined distance along the surface where the flow of heat is completely unidimensional (i.e. $I_R = I_\infty$).

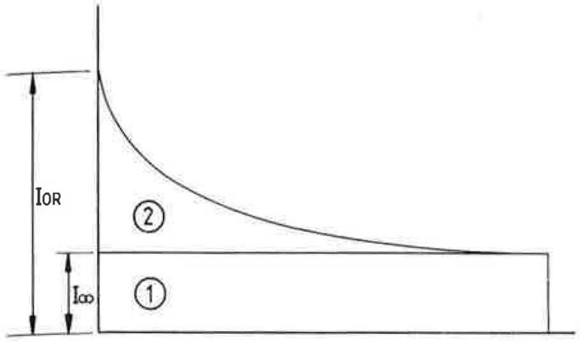

Fig. 2. The general form of the heat flow intensity distribution along the inside surface of a corner. Area (1) represents the basic heat flow through one wall. Area (2) represents the extra heat flow due to the presence of the corner.

The total rate of heat flow was used for comparison purposes instead of the total *extra* rate of heat flow, since the latter value is dependent upon I_∞ which has a different value for each case, depending on thickness and thermal conductivity of the insulation material. Figure 2 defines the quantities I_{OR} and I_∞ graphically and shows the areas of the total rate of heat flow and the total extra rate of heat flow. The origin represents the inside corner point. Area 1 plus 2 in Fig. 2 represent the total rate of heat flow across the inside surface while area 2 alone represents the extra rate of heat flow due to the presence of the corner. The total rates of heat flow for all the cases are given in Table 1. The table also gives the percentage reduction of the total rate of heat flow based on the case with no insulation, i.e.

$$\eta = 100\,(Qa - Qb)/Qa$$

where Qa and Qb are the total rates of heat flow through the uninsulated corner and the insulated corner respectively.

This will be called the insulation efficiency. It does not mean the efficiency of a particular insulation layer, but it is the efficiency of the insulation method which depends on position, thickness and thermal conductivity of the insulation used, and it also depends on the distance from the corner.

Throughout the calculations, the concrete wall was assumed to have a thermal conductivity of 1.4 W m^{-1} K^{-1}. However, for outside insulation the concrete becomes protected and therefore its thermal conductivity decreases slightly to 1.28 W m^{-1} K^{-1} [3]. The results for these additional calculations are also given in Table 1 for insulation thermal conductivities of 0.035 and 0.28 W m^{-1} K^{-1} only.

5. THE EFFECTS OF VARYING PARAMETERS ON THE TOTAL RATE OF HEAT FLOW

The total rate of heat flow across the inside surface was calculated for each case and compared with that obtained with no insulation. The results are given in Table 1 for four different insulation thermal conductivities. Figure 3 shows the variations of the total rate of heat flow with positions of insulant for different insulation thicknesses and thermal conductivities. The figure shows a steady slow increase in the total rate of heat flow as the position of insulant changes from inside to outside. The results also show that at low values of insulation thermal conductivity, (0.035 and 0.09 W m^{-1} K^{-1}) similar thicknesses of insulation of different thermal conductivities have a significant difference in the reduction of the total rate of heat flow for all the insulation positions (compare 12.5 mm; 0.035 W m^{-1} K^{-1} with 12.5 mm; 0.09 W m^{-1} K^{-1}). This difference becomes relatively small as the insulation thermal conductivity increases (i.e. compare 12.5 mm; 0.28 W m^{-1} K^{-1} with 12.5 mm; 0.35 W m^{-1} K^{-1}).

Table 1. Total rates of heat flow (Q) and insulation efficiency (η) of insulated corners with various insulation thicknesses, positions and thermal conductivities. The thermal conductivity of concrete is taken as 1.4 W m^{-1} K^{-1} throughout, except for the two cases marked *, where it is 1.28 W m^{-1} K^{-1}. The total rate of heat flow for the uninsulated case is 30.76 W.

		Insulation thickness (mm)					
		12.5		25		50	
Insulation Position	λ_{ins} (W m^{-1} K^{-1})	Q Total (W)	η (%)	Q Total (W)	η (%)	Q Total (W)	η (%)
Inside	0.035	13.46	56	8.50	72	4.72	85
Middle		15.37	50	10.16	67	5.94	81
Outside		16.41	47	11.17	64	6.71	78
Outside*		15.97	48	10.95	64	6.63	78
Inside	0.090	20.38	34	15.23	51	9.83	68
Middle		22.05	28	17.12	44	11.66	62
Outside		22.73	26	18.03	41	12.63	59
Inside	0.280	25.80	16	22.50	27	17.44	43
Middle		26.80	13	23.67	23	19.02	38
Outside		27.07	12	24.13	22	19.68	36
Outside*		26.08	15	23.30	24	19.08	38
Inside	0.350	26.50	14	23.59	23	18.90	39
Middle		27.40	11	24.68	20	20.31	34
Outside		27.61	10	25.05	19	20.87	32

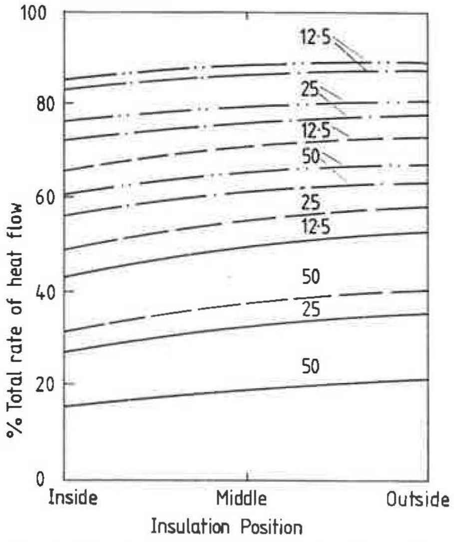

Fig. 3. The total rate of heat flow as a function of insulation position for different insulation thicknesses and thermal conductivities: ——— 0.035 W m^{-1} K^{-1}, — — — 0.09 W m^{-1} K^{-1}, —·— 0.28 W m^{-1} K^{-1}, —··— 0.35 W m^{-1} K^{-1}. The heat flow rate is expressed as a percentage of the case with no insulation.

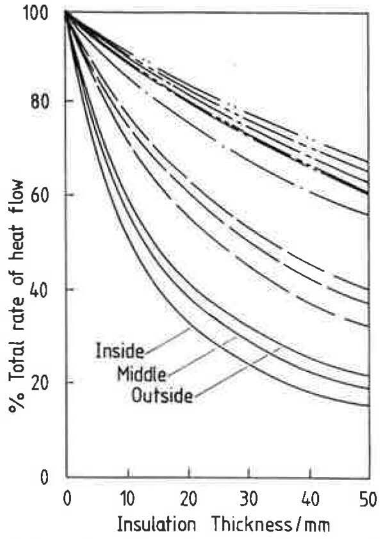

Fig. 4. The total rate of heat flow as a function of insulation thickness for different positions and thermal conductivities. Line symbols as for Fig. 3. The heat flow rate is expressed as a percentage of the case with no insulation.

The heat flow values of Table 1 are presented in an alternative way in Table 2, which gives them a more practical significance. Table 2 lists the extra length of wall which is required to carry the extra heat flow due to the presence of the corner. For the uninsulated case (200 mm solid wall) this is 96 mm, i.e. about an extra 50% of the wall thickness. For the insulated cases (212.5 to 250 mm wall) it ranges from only 13 mm up to nearly 190 mm. This can produce an increase in the surface area assumed for steady state heat loss calculation of up to 10% in a small room.

The variations of the total rate of heat flow with insulation thickness are shown in Fig. 4 for three insulation positions and four different thermal conductivities. For low values of insulation thermal conductivity (0.35 and 0.09 W m^{-1} K^{-1} Fig. 4 shows a sharp decrease in the total rate of heat flow as the insulant thickness increases to a value of around 20 mm. This decrease in the heat flow is continuous for all insulation positions, but becomes more steady for larger thicknesses of insulant. The curve for the 0.035 W m^{-1} K^{-1} and inside surface insulation position shows that increasing the insulation thickness from 0 to 20 mm caused a reduction in the total rate of heat flow of about 70%, while a further reduction of only 15% was obtained when the thickness was further increased to 50 mm. The results indicate that as the insulation thermal conductivity increases, the relationship between the rate of heat flow and thickness of insulation takes a semi-linear form. Furthermore, the position of the insulation becomes unimportant.

Table 2. Extra equivalent wall lengths for the cases of Table 1. For the uninsulated case (200 mm solid concrete) the extra equivalent wall length is 96 mm.

Insulation Position	λ ins	Insulation thickness (mm) 12.5	25	50
		Extra equivalent wall length (mm)		
Inside	0.035	58	41	13
Middle		122	127	120
Outside		158	179	188
Outside*		154	175	185
Inside	0.09	70	57	32
Middle		109	114	112
Outside		125	142	155
Inside	0.28	74	64	37
Middle		92	88	77
Outside		98	97	94
Outside*		97	96	91
Inside	0.35	75	65	40
Middle		91	86	73
Outside		95	94	86

Figures 3 and 4 and Table 1 show that for a specific insulation thermal conductivity, increasing the thickness of the insulation layer has a relatively far greater effect on reducing the total rate of heat flow than varying the insulation position. The results in Table 1 show that the increase in insulation efficiency due to the change in insulation position from outside to inside was not more than 10% for all the cases studied, while increasing the thickness of insulation from 12.5 to 50 mm for a fixed position caused an increase in insulation efficiency of between 20 and 32%.

Figure 5 shows the variations of the total rate of heat flow with thermal conductivity of insulant for the three insulation thicknesses and positions. The Figure indicates that the largest variations in insulation efficiency occur when the insulation thermal conductivity is between 0.035 and 0.23 W m^{-1} K^{-1}. The Figure also shows that for a specific insulation thickness and position, increasing the thermal conductivity to 0.35 W m^{-1} K^{-1} would not reduce the insulation efficiency by more than 10%.

5.1 *Discussion of heat flow results*

Table 1 shows that the maximum percentage reduction in the total rate of heat flow (insulation efficiency) for the

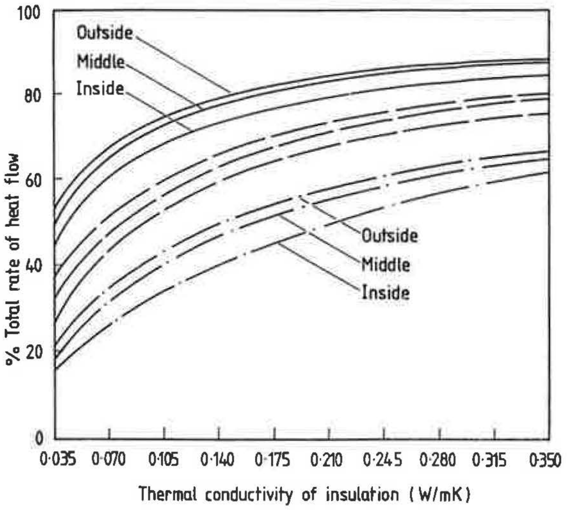

Fig. 5. The total rate of heat flow as a function of insulation thermal conductivity for different positions and thicknesses. The heat flow rate is expressed as a percentage of the case with no insulation. Insulation thickness (mm): ——— 12.5, – – – 25, —·—·— 50.

cases considered was around 85% when the wall was insulated from inside with a layer of 50 mm thick and thermal conductivity of 0.035 W m^{-1} K^{-1}. The insulation efficiency obtained by insulating the outside surface of the corner with a layer 12.5 mm thick and of thermal conductivity 0.35 W m^{-1} K^{-1} was only 10%. The insulation efficiencies for the rest of the cases lie between these two values. The results in Fig. 4 show how increasing the thickness of insulation could compensate for increasing the thermal conductivity. Increasing the thermal conductivity of insulation from a typical value of 0.035 to 0.09 W m^{-1} K^{-1} caused a reduction of around 20% in insulation efficiency for the same thickness and position of insulation. The results show that for an insulant with thermal conductivity of 0.09 W m^{-1} K^{-1} to give the same insulation efficiency as an insulant of 12.5 mm thick and 0.035 W m^{-1} K^{-1}, the thickness has to be increased by at least a factor of 2.5.

From Fig. 5 it is possible to obtain different alternatives of insulation thickness, position and thermal conductivity at a specified insulation efficiency (i.e. specified reduction in the total rate of heat flow). Therefore, 85% insulation efficiency (i.e. the rate of heat flow is 15% of that in the uninsulated case) could be achieved by having a layer 50 mm thick placed on the inside surface with thermal conductivity of 0.035 W m^{-1} K^{-1}. At an insulation efficiency of 65% (35% on the scale) there are six different insulation possibilities. These are:

1. A layer 25 mm thick placed on the outside surface with a thermal conductivity of 0.035 W m^{-1} K^{-1}.
2. A layer 25 mm thick placed at the middle with a thermal conductivity of 0.042 W m^{-1} K^{-1}.
3. A layer 25 mm placed on the inside surface with a thermal conductivity of 0.053 W m^{-1} K^{-1}.
4. A layer 50 mm thick placed on the outside surface with a thermal conductivity of 0.074 W m^{-1} K^{-1}.
5. A layer 50 mm thick placed at the middle with a thermal conductivity of 0.084 W m^{-1} K^{-1}, and finally
6. A layer 50 mm thick placed on the inside surface with a thermal conductivity of 0.116 W m^{-1} K^{-1}.

In other words, Fig. 5 defines the changes in insulation thickness and position which are needed to compensate for choosing higher insulation thermal conductivities.

When a layer of insulation is placed on the outside surface of an external wall, the wall becomes protected; hence the thermal conductivity of the concrete drops from 1.4 to 1.28 W m^{-1} K^{-1} [3]. Calculations were carried out for some cases taking into account the reduction in the thermal conductivity of the concrete wall and the results were then compared with those obtained using the original thermal conductivity. The results given in Table 1 for these cases, for insulation thermal conductivities at 0.035 and 0.28 W m^{-1} K^{-1}, show that the protected wall caused a maximum increase in the insulation efficiency of 3% at insulation thicknesses of 12.5 and 25 mm. At an insulation thickness of 50 mm the increase in insulation efficiency became negligible.

6. TEMPERATURE DISTRIBUTIONS

This section deals with the temperature distributions both along the inside warm surface and through the wall.

6.1 *Temperature distributions along the inside surface*

Figure 6 shows the temperature distribution along the inside surface of the 200 × 200 mm corner with no insulation. The overall drop in the surface temperature due to the corner is around 3 K. Furthermore, there is a significant drop in temperature of over 10 K between the air and the corner point. This shows the magnitude of error involved in the assumption of uniform surface temperatures.

A similar case was analysed by Billington and Becher [5], using the relaxation method. Their results agree well with those of the present study.

Figure 7 shows the temperature distributions along the inside surface after placing an insulation layer with thermal conductivity of 0.035 W m^{-1} K^{-1} on the inside. Figure 7 also gives the temperature distributions for the three different thicknesses. A layer of 12.5 mm thickness placed on the inside caused an increase of 4.6 K in the corner point temperature (compared with that of no insulation). At an insulation thickness of 50 mm an increase in the corner point temperature of 7.4 K was achieved. The drop in surface temperature due to the corner was around 1.6 K in the last case. The results show that,

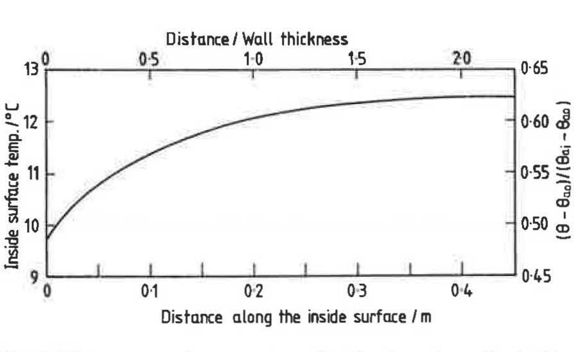

Fig. 6. The computed temperature distribution along the inside surface of the 200 mm square uninsulated corner. The temperatures at the corner and at the wall are 9.6 and 12.6°C respectively.

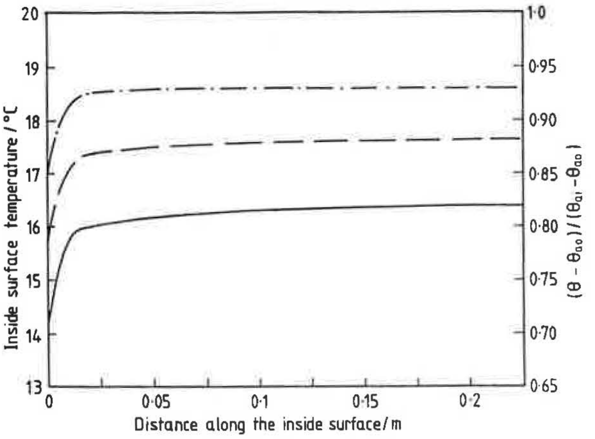

Fig. 7. The computed temperature distributions along the inside surface for various thicknesses of insulation placed on the inside. The thermal conductivity of the insulation is 0.035 W m^{-1} K^{-1}. Line symbols as for Figure 5. The temperatures at the corner and at the wall for the three thicknesses of insulation are 14.2, 15.7, 17.0°C and 16.5, 17.7, 18.6°C respectively.

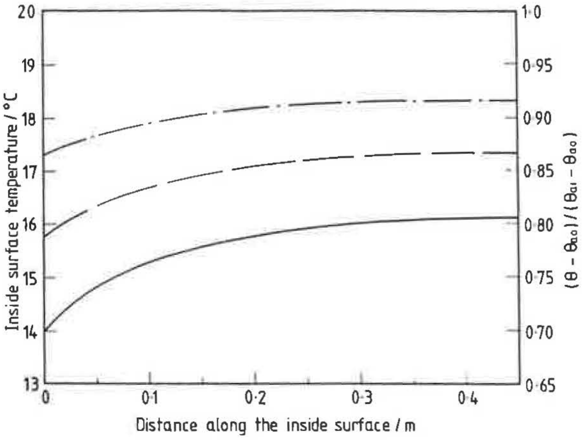

Fig. 9. The computed temperature distributions along the inside surface for various thicknesses of insulation placed on the outside. The thermal conductivity of the insulation is 0.035 W m^{-1} K^{-1}. Line symbols as for Fig. 5. The temperatures at the corner and at the wall for the three thicknesses of insulation are 14.0, 15.8, 17.3°C and 16.5, 17.7, 18.6°C respectively.

unlike the surface with no insulation, the surface temperature drops sharply just before the corner is reached and is almost steady along the entire remaining surface.

Positioning the insulant at the middle or on the outside surface (Figs 8 and 9) caused the drop in surface temperature towards the corner to be more gradual. The surface temperatures reached a steady value at relatively longer distances than those with inside insulation.

6.2 *Temperature distribution across the corners*

The isothermal lines for selected cases only are shown in Figs 10–13. Figure 10 shows the isotherms of the uninsulated corner: the maximum temperature in the wall is around 12°C. Figure 11 shows that the pattern of isothermal lines changed considerably when an insulation layer of 50 mm thickness was placed on the inside surface of the corner. As can be seen from Fig. 11, the effect of this insulation layer is quite clear in the sense that it contains all the high temperature isotherms. Consequently, the remaining part of the wall is at relatively much lower temperatures. The structural temperature towards the outside has dropped to about 0.2°C. The risk of condensation at the outer parts of the structure may be high, unless an intact vapour check layer is installed at the warm inner surface. Moving the same insulation layer to the middle position (Fig. 12) has separated the wall into two regions: the upper (towards the inside) warm part and lower (towards the outside) cold part, where again the structural temperature is at a dangerously low value. A significant change in temperature distribution was also observed as the insulation layer was moved to the outside surface as shown in Fig. 13. In this case, relatively higher temperatures were obtained throughout the wall compared with the previous cases.

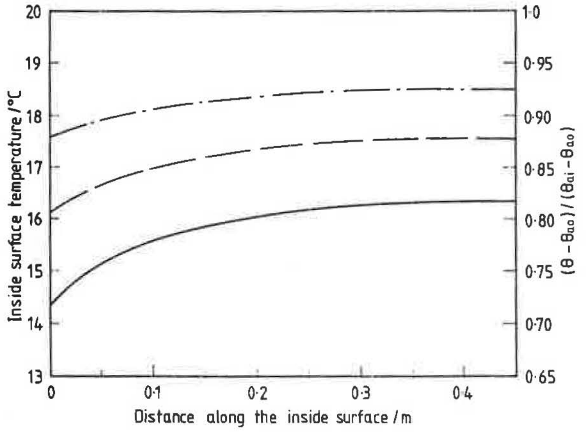

Fig. 8. The computed temperature distributions along the inside surface for various thicknesses of insulation placed in the middle. The thermal conductivity of the insulation is 0.035 W m^{-1} K^{-1}. Line symbols as for Fig. 5. The temperatures at the corner and at the wall for the three thicknesses of insulation are 14.4, 16.1, 17.6°C and 16.5, 17.7, 18.6°C respectively.

7. CONCLUSIONS

Although these conclusions are drawn from the results for the specific cases examined, they can provide a general insight into (a) the thermal analysis of an important building element, and (b) the effect of the insulation layer on the rate of heat flow and the temperature at the corner point. The conclusions are summarised as follows:

1. The finite element method has been applied to the analysis of the effects of a corner of an external wall with non-isothermal surfaces. The effects of position, thickness and thermal conductivity of the insulation layer on the temperature distributions and on the total rates of heat flow were observed.
2. As was expected, the results showed that, for a given combination of thickness and thermal conductivity of the insulation, placing the insulation layer on the inside surface of the corner gives the highest value of insulation efficiency. However, changing the position of the insulation produced relatively smaller variations in insulation efficiency compared with those obtained

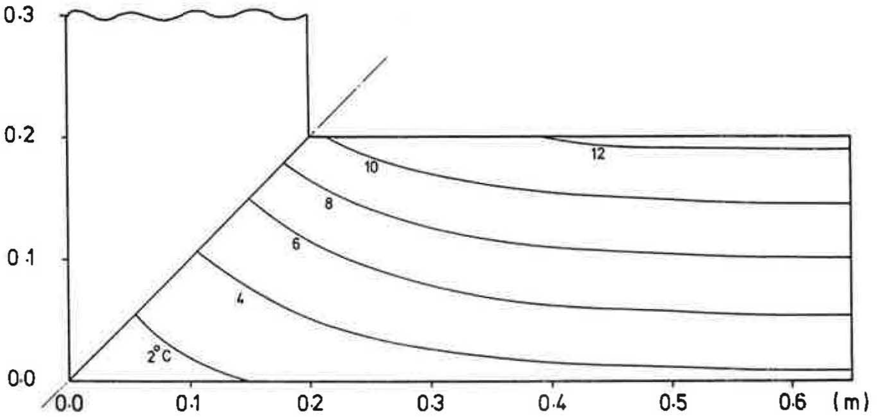

Fig. 10. Isotherms across the uninsulated 200 mm square corner. The inside and outside air temperatures are 20 and 0°C respectively. The inside and outside surface resistances are 0.12 and 0.06 m^2 K W^{-1}.

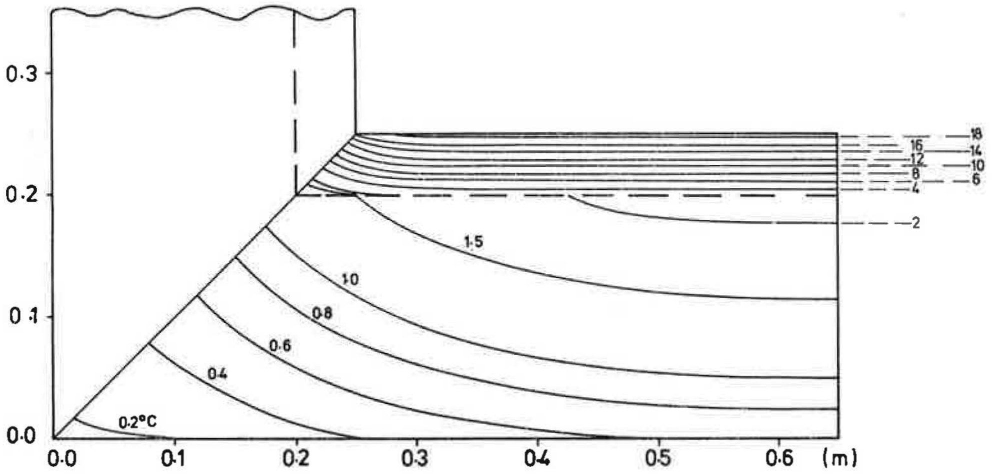

Fig. 11. Isotherms across the 200 mm square corner with 50 mm insulation ($\lambda = 0.035$ W m^{-1} K^{-1}) placed on the inside surface. All other conditions are as for Fig. 10.

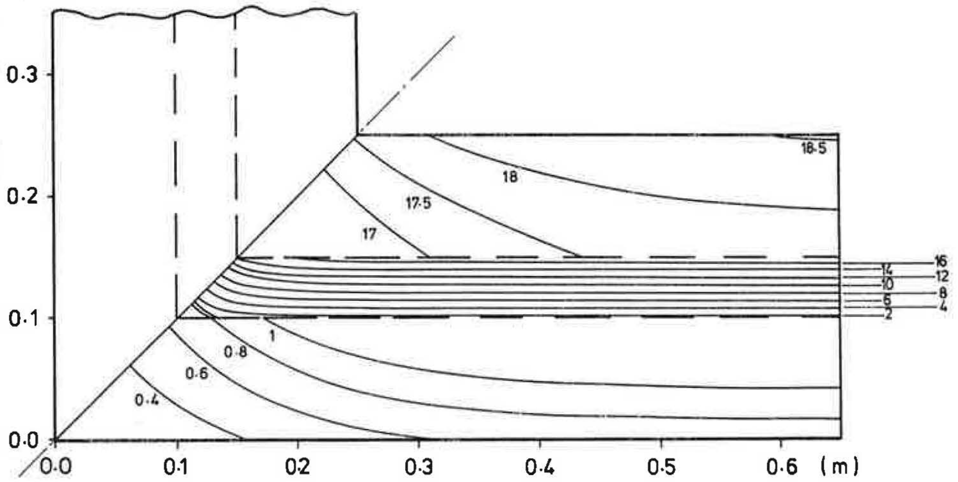

Fig. 12. Isotherms across the 200 mm square corner with 50 mm insulation placed at the middle. All conditions are as for Figs 10 and 11.

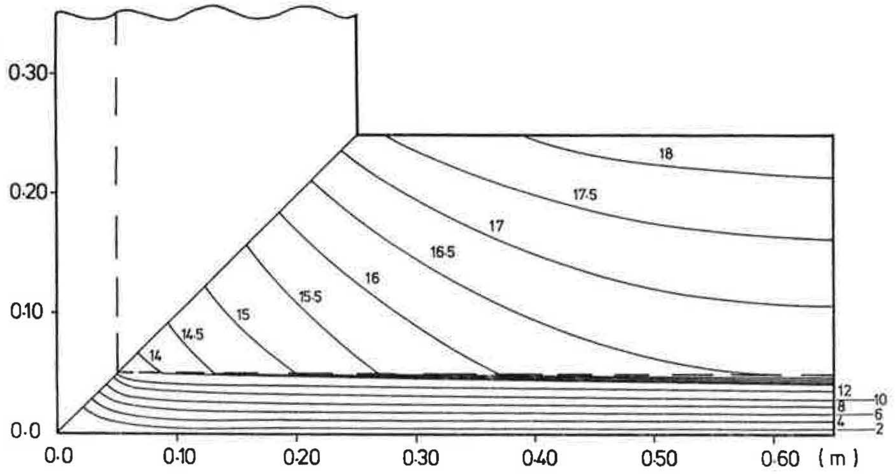

Fig. 13. Isotherms across the 200 mm square corner with 50 mm insulation placed on the outside surface. All conditions are as for Figs 10 and 11.

by varying the thickness. It was found that the increase in insulation efficiency produced by changing the position from inside to outside was not more than 10% for all the cases considered. Increasing the thickness of insulation from 12.5 to 50 mm caused an improvement of about 25% in efficiency.

3. The largest variations in insulation efficiency occur when the insulation thermal conductivity is between 0.035 and 0.28 W m^{-1} K^{-1}. Also, for a specific insulation thickness (up to 50 mm) and position, increasing the insulation thermal conductivity to 0.35 W m^{-1} K^{-1} would not reduce the insulation efficiency by more than 10%.
4. At an insulation thermal conductivity of 0.035 W m^{-1} K^{-1}, installing an insulation layer of up to 20 mm on the inside surface caused a reduction of 70% in the total rate of heat flow. A further increase in thickness produced negligible reduction in the total rate of heat flow.
5. Some to the practical consequences of the different insulation positions are summarised as follows:

Inside: Highest insulation efficiency, preferred for rapid warming up of the room, but it reduces the useful size of rooms and needs careful attention to a vapour check, as well as protection from mechanical impact. It causes a considerable drop in structural temperature with the danger of freezing damage.

Middle: Medium insulation efficiency for a given thermal resistance. It is convenient to use an existing cavity construction, giving protection from the weather and from impact. There is a considerable drop in the temperature of the external leaf, with the possibility of spalling damage due to freezing. There may also be a danger of internal condensation, which could be avoided by the installation of a vapour check on the warm surface.

Outside: This position gives the lowest insulation efficiency, but it keeps the building structure at the highest mean temperture. Variations in inside temperature due to intermittent heating are smoothed out and slowed down. The insulation layer needs to be protected from the weather and from impact damage. Condensation risk also needs to be considered.

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